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Spin dependent potential for heavy fermions on the ends of a string

Robert D. Pisarski
Fermi National Accelerator Laboratory[1]
P.O. Box 500, Batavia, Illinois 60510

John D. Stack[2]
Department of Physics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

Abstract

Mesons with heavy quarks are modeled by putting fermions on the ends of a string. The spin dependence of the fermion potential over large distances is found by solving for the classical motion of a rotating string. Some general remarks about such theories of "heavy" strings are also made.

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The potential between heavy quarks is a problem of fundamental as well as practical significance. For infinitely massive quarks the potential is of necessity a scalar quantity, but for quarks of finite mass more detailed questions arise, such as its spin dependence [1-7]. Eichten and Feinberg [1] developed a general formalism to calculate the potential's spin dependence in QCD. They find

$$V_{spin-orbit} = \left(\frac{\vec{l}_1 \cdot \vec{s}_1}{2m_1^2} - \frac{\vec{l}_2 \cdot \vec{s}_2}{2m_2^2} \right) \frac{1}{r} \frac{d}{dr} \left(V(r) + 2V_1(r) \right)$$

$$+ \left(\frac{\vec{l}_1 \cdot \vec{s}_2}{m_1 m_2} - \frac{\vec{l}_2 \cdot \vec{s}_1}{m_1 m_2} \right) \frac{1}{r} \frac{d}{dr} \left(V_2(r) \right), \tag{1}$$

for quarks of mass m_1 and m_2, angular momentum $\vec{l}_1$ and $\vec{l}_2$, and spin $\vec{s}_1$ and $\vec{s}_2$, respectively, separated by a distance r. $V(r)$ is the static potential, while $V_1(r)$ and $V_2(r)$ are determined by an insertion of a chromo-electric and a chromo-magnetic field into a Wilson loop. Only the spin-orbit terms are written in eq. (1). The spin-spin terms are related to functions $V_3(r)$ and $V_4(r)$, which are given by two insertions of a chromo-magnetic field into a Wilson loop. The functions V and $V_1 \ldots V_4$ determine the complete spin dependence of the quark potential to leading order in $1/m^2$.

In a confining theory the scalar potential $V(r)$ depends linearly on r over large distances. The question of interest is whether any of the functions $V_1 \ldots V_4$ have terms which are significant over large distances.

Based upon a picture of "electric" confinement, a first guess might be that all of the functions $V_1 \ldots V_4$ are short ranged. Buchmüller was the first to suggest that this might not be true [2]. He argued that for an electric flux tube which rotates with the heavy quark, the field in the rest frame of the heavy quark is purely electric, so the only spin-orbit term in the potential comes from Thomas precession. The resulting spin-orbit term is opposite in sign to that found in an atom, where the electron moves through a static electric field which does not rotate. Buchmüller's conclusion was reached in another way by Gromes [3], who showed that Lorentz invariance implied a relation between V, V_1, and V_2: $V_2 - V_1 = V$. Under the plausible assumption that V_2 is short-ranged compared to V and V_1, one obtains $V_1 = -V$ over large distances. From eq. (1), this long-range part of V_1 flips the sign of the spin-orbit term from that which would result if both V_1 and V_2 were short-ranged. These arguments also suggest that only the spin-orbit terms have long-ranged pieces: the spin-spin terms V_3 and V_4 are expected to be short-ranged.

This sign of the spin-orbit term, which is sometimes described as scalar-like, is in good agreement with charmonium spectroscopy [2-6] and with Monte Carlo simulations of QCD [7]. Nevertheless, there has been no simple model which both exhibits confinement and allows for the long-distance behavior of the spin-dependent potential to be computed directly.

In this paper we present such a model. Confinement is built into theories of strings, since even at the classical level the potential is linear. Based upon the analogy with lattice gauge theories [8], we model heavy quark systems by putting massive fermions on the ends of a string [9-12]. The model which results was originally proposed by Bars [10], and studied by him, Kikkawa and Sato [11], and others [10-12]; it is closely related to flux-tube models of confinement [13].

We compute the spin-dependent part of the potential for this model of "heavy" strings by solving for the classical motion of a rotating string. The results we find are very similar to those expected in the electric flux-tube picture: the sign of the spin-orbit term is scalar-like, with no spin-spin interactions over large distances. (Spin-spin interactions in string theories have been calculated by Kogut and Parisi [14], but the effects they find are of higher order in $1/m^2$.)

The spectrum of bound states for heavy strings has been studied previously [11,12], but in these works the potential was determined from the energy of the system. In contrast, following Eichten and Feinberg [1] we determine the potential by evaluating the propagator for heavy fermions coupled to a string. It is only in this way that the (correct) spin-orbit term emerges.

This model of heavy strings might be of interest beyond the admittedly technical question which we have set out to answer here. For this reason, along the way in our discussion we pause to discuss such matters as other models of heavy strings and the quantization of these theories.

Theories of strings in QCD presumably arise when the original gauge degrees of freedom are integrated out to yield an effective theory of flux sheets and the like that only involves purely geometrical variables. To describe a flux sheet we take the usual Nambu action,

$$S_{string} = \mu \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 (x')^2}\, d\sigma\, d\tau. \tag{2}$$

The coordinate $x = x^\alpha(\sigma, \tau)$ represents the embedding of the world-sheet in four (Minkowski) dimensions, with signature (+ - - -). The Lorentz index α on x^α will often be dropped: $\dot{x} = \partial_\tau x = \partial x^\alpha / \partial \tau$, $x' = \partial_\sigma x = \partial x^\alpha / \partial \sigma$. The world-sheet is parametrized by σ and τ, with τ a time-like variable of infinite extent, while σ describes the spatial extent of the string, which is always chosen to run from 0 to π. The string tension equals μ. The Nambu action is invariant under arbitrary reparametrizations,

$$\sigma \rightarrow \tilde{\sigma}(\sigma, \tau), \quad \tau \rightarrow \tilde{\tau}(\sigma, \tau), \tag{3}$$

subject to

$$\left(\frac{\partial \tilde{\sigma}}{\partial \tau} \right)_{\sigma = 0, \pi} = 0. \tag{4}$$

Consider an idealized limit of QCD in which the effects of light quarks can be neglected. For a bound state of heavy quarks, the quarks act as the sources of color flux, so to introduce quarks into the string model, it is most natural to attach them only to the *ends* of the string. (It is possible to put quarks on the entire world-sheet [15], but this does not seem relevant for heavy quarks; it might be so for light quarks in the adjoint representation.) Following the example of the world-sheet, we use purely geometric variables to describe the matter fields on the ends of the string. We assume that the matter fields are invariant under reparametrizations of the world-sheet, but transform as usual under Lorentz rotations.

This is trivial for scalar quarks, as the action is simply proportional to the length of the world-line for the ends [9]:

$$S_{scalar} = m_1 \int_{\sigma=0} \sqrt{\dot{x}^2}\, d\tau + m_2 \int_{\sigma=\pi} \sqrt{\dot{x}^2}\, d\tau, \tag{5}$$

for a quark of mass m_1 at one end and mass m_2 at the other. It is important to stress that the reparametrization invariance of the string theory is the same with eq. (5) as without. Having fixed the length of the string to be π, terms on the ends need only be invariant under reparametrizations of τ, which eq. (5) clearly is. (Note that while in the interior of the world-sheet $\dot{x}$ transforms inhomogeneously under reparametrizations, because of eq. (4) it transforms homogeneously on the boundary.)

Eq. (5) is the invariant action for a scalar field propogating in a curved, one dimensional manifold with a metric tensor $g_{00} = \dot{x}^2$. To introduce fermions on the ends, we use the *ein-bein* $e_0^\alpha = \dot{x}^\alpha$. The spin connection automatically vanishes in one dimension, so the invariant action for a fermion of mass m at $\sigma = \pi$ is [10]

$$S_{fermion} = \int_{\sigma=\pi} \left(-\frac{i}{2} \frac{\dot{x}_\alpha}{\sqrt{\dot{x}^2}} \left(\bar{\psi} \gamma^\alpha \overleftrightarrow{\partial_\tau} \psi \right) + m \sqrt{\dot{x}^2}\, \bar{\psi}\psi \right) d\tau, \tag{6}$$

with a similar term for the fermion at the other end of the string; $\overleftrightarrow{\partial_\tau} = \overrightarrow{\partial_\tau} - \overleftarrow{\partial_\tau}$. The fermion field ψ is a Dirac spinor under Lorentz transformations, but is unaltered by reparametrizations of τ; then the fermion action is obviously reparametrization invariant. The total action is the sum of the string and fermion actions.

The equation of motion for ψ is

$$D_+\psi = \left(-i \frac{\not{\dot{x}}}{\dot{x}^2} \partial_\tau - \frac{i}{2\sqrt{\dot{x}^2}} \partial_\tau \left(\frac{\not{\dot{x}}}{\sqrt{\dot{x}^2}} \right) + m \right) \psi = 0. \tag{7}$$

Away from the ends the equations of motion for x are unaffected by the boundary terms, so in orthonormal gauge,

$$\dot{x} \cdot x' = \dot{x}^2 + (x')^2 = 0, \tag{8}$$

x satisfies the wave equation for a free massless field in two dimensions,

$$(\partial_\tau^2 - \partial_\sigma^2) x^\alpha = 0. \tag{9}$$

To compute the boundary conditions that relate ψ and x at the ends of the string requires the fermion contribution to the canonical momentum, Π^α_{ferm}:

$$\Pi^\alpha_{ferm} = \frac{\delta S_{ferm}}{\delta \dot{x}_\alpha} = m \frac{\dot{x}^\alpha}{\sqrt{\dot{x}^2}} \bar{\psi}\psi - \frac{i}{2(\dot{x}^2)^{\frac{3}{2}}} \left(\dot{x}^2 \delta^\alpha_\beta - \dot{x}^\alpha \dot{x}_\beta \right) \left(\bar{\psi} \gamma^\beta \overleftrightarrow{\partial_\tau} \psi \right). \tag{10}$$

Using the fermion's equation of motion, eq. (7),

$$\Pi^\alpha_{ferm} = m \frac{\dot{x}^\alpha}{\sqrt{\dot{x}^2}} \bar{\psi}\psi + \frac{i}{8(\dot{x}^2)^{\frac{3}{2}}} \bar{\psi} \{ \gamma^\alpha, [\not{\dot{x}}, \not{\ddot{x}}] \} \psi. \tag{11}$$

For massive fermions ψ can be normalized so that $\bar{\psi}\psi = 1$. The first term in eq. (11) is just that which would be obtained for a scalar of mass m. The second term is special to fermions and is proportional to their spin density.

In orthonormal gauge the boundary conditions become

$$\partial_\tau \Pi^\alpha_{ferm} = \pm\, \mu\, (x')^\alpha, \tag{12}$$

where the sign is $(+)$ at $\sigma = 0$ and $(-)$ at $\sigma = \pi$.

Classical solutions for heavy strings are given by solving eq.'s (7), (9), and (12). Before going to the special case of a rotating string, we remark that while eq. (6) is the simplest and most natural action for a massive fermion, it is by no means unique. For example, sticking to terms that are bilinear in the fermion fields, it is possible to add

$$\int_{\sigma=\pi} \left(\tilde{m}\, \bar{\psi}\dot{\not{x}}\psi + \frac{\kappa}{2} \frac{\dot{x}_\alpha}{\sqrt{\dot{x}^2}}\, \partial_\tau (\bar{\psi}\gamma^\alpha\psi) \right) d\tau \tag{13}$$

to the action of eq. (6). The canonical momentum for this generalized theory is, after using the equations of motion,

$$\Pi^\alpha_{ferm} = m\, \frac{\dot{x}^\alpha}{\sqrt{\dot{x}^2}}\, \bar{\psi}\psi + \tilde{m}\, \frac{\dot{x}^\alpha}{\dot{x}^2}\, \bar{\psi}\dot{\not{x}}\psi + \frac{i}{8(\dot{x}^2)^{\frac{3}{2}}}\, (1+\kappa^2)\, \bar{\psi}\{\gamma^\alpha, [\dot{\not{x}}, \ddot{\not{x}}]\}\psi$$

$$-\frac{im\kappa}{2\sqrt{\dot{x}^2}}\, \bar{\psi}\, [\gamma^\alpha, \ddot{\not{x}}]\, \psi. \tag{14}$$

In the non-relativistic limit, this can be shown to be equal to

$$\Pi^\alpha_{ferm} = (m + \tilde{m})\, \frac{\dot{x}^\alpha}{\sqrt{\dot{x}^2}}\, \bar{\psi}\psi + \frac{i}{8(\dot{x}^2)^{\frac{3}{2}}}\, (1+\kappa^2)\, \bar{\psi}\{\gamma^\alpha, [\dot{\not{x}}, \ddot{\not{x}}]\}\psi. \tag{15}$$

By comparison with eq. (11), we see that for the action of eq. (13), the effective mass of the fermion is given by $m + \tilde{m}$, while the spin density is multiplied by $1+\kappa^2$. That is, the κ dependent term in eq. (13) acts like an anomalous magnetic moment for the fermion. For the action to be hermitian κ must be real, so the anomalous magnetic moment is always greater than the bare one. In what follows we treat the simplest case, the action of eq. (6).

Eq. (6) is a reasonable action for very massive quarks, but it is not well defined in the chiral limit, $m = 0$. Massless ends move at the speed of light, so $\dot{x}^2 = 0$

and the fermion action blows up. It might be possible to approach the chiral limit by setting $m \neq 0$ and then carefully sending $m \to 0$. (For an example of this, see Bardeen *et al.* [9]). We propose an alternate approach. Unlike most field theories of fermions, in the action of eq. (6) the fermion field is dimensionless. Thus it is possible to take as an action

$$S_{fermion} = \int_{\sigma=\pi} \left(-\frac{i}{2e^2} \frac{\dot{x}_\alpha}{\sqrt{\dot{x}^2}} \left(\bar{\psi} \gamma^\alpha \overleftrightarrow{\partial_\tau} \psi \right) \right) d\tau, \tag{16}$$

subject to the nonlinear constraint

$$(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 = 1. \tag{17}$$

This model is chirally symmetric, but because of the constraint the ends do not necessarily move at the speed of light, and so the presence of $\dot{x}^2$ in the action need not cause a problem. The constraint fixes the scale of ψ, with e^2 in eq. (16) a dimensionless coupling constant; as a type of nonlinear model we suspect that it is asymptotically free. In any case, the constraint generates a series of four-, six- and higher point interactions between the fermion on the end of the string with itself.

We now return to the fermion action of eq. (6), to determine the classical solution which represents a rotating string. We work in the extreme non-relativistic limit, where the fermion masses are much larger than the scale set by the string tension, $m \gg \sqrt{\mu}$. Further, we nail down the end of the string at $\sigma = 0$ by taking the fermion at that end to be infinitely massive. This last assumption is not essential and is only done to simplify the algebra.

In the non-relativistic limit, the time-like component of the canonical momentum for the fermions, eq. (11), is

$$\Pi^0_{ferm} = \frac{m}{\sqrt{\dot{x}^2}} \dot{x}^0 + \frac{1}{(\dot{x}^2)^{\frac{3}{2}}} \epsilon^{ijk} \dot{x}^i \ddot{x}^j s^k, \tag{18}$$

with the space-like components equal to

$$\Pi^i_{ferm} = \frac{m}{\sqrt{\dot{x}^2}} \dot{x}^i + \frac{\dot{x}^0}{(\dot{x}^2)^{\frac{3}{2}}} \epsilon^{ijk} \ddot{x}^j s^k, \tag{19}$$

where s^i is the fermion's spin density,

$$s^i = \frac{i}{4}\,\epsilon^{ijk}\,\psi^\dagger\gamma^j\gamma^k\psi. \tag{20}$$

If the masses of the fermions are large they move very slowly, with the string responding essentially instantaneously to the fermions' motion. Because of this, to first approximation we can assume that the fermion at $\sigma = \pi$ simply moves in a circle about that at $\sigma = 0$: it moves with frequency ω in the x-y plane, at a distance r from the origin.

The solution for $x^\alpha(\sigma,\tau)$ is especially simple:

$$x^0 = p\,\tau, \qquad x^i = r\,\frac{sin(\omega\sigma)}{sin(\omega\pi)}\,(cos(\omega\tau), sin(\omega\tau))\,. \tag{21}$$

The constant p is set by the gauge condition of eq. (8), $p = \omega\, r/sin(\omega\pi)$. For small ω, $p \sim r/\pi$ and the string is straight.

The boundary condition on the time-like component x^0, eqs. (12) and (19), is satisfied trivially. The boundary condition on the spatial components x^i, eqs. (12) and (20), determines the frequency ω :

$$\omega^2 = \omega_o^2 - \frac{\pi s^z \omega_o^3}{mr} + \ldots, \tag{22}$$

$m\,(\pi\,\omega_o)^2 = \mu\,r$. This equation for ω expresses the condition that the string tension must balance against the centrifugal force felt by the fermion. The frequency ω_o is the same as for a scalar particle, with the second term in eq. (22) the (leading) correction to ω from the fermion's spin.

The total energy of the system, E, is equal to the sum of the energies for the fermion and the string. The latter is just μr, while the former can be found from eq. (18) to give

$$E = \frac{1}{2}mv^2 + m + \mu r + \frac{\vec{l}\cdot\vec{s}}{2m^2}\,\frac{\mu}{r}, \tag{23}$$

where $v = \pi\omega_o$ is the magnitude of the velocity for the rotating fermion, $\vec{l} = mrv\,\hat{z}$ its angular momentum, and $\vec{s}$ its spin. To obtain the $\vec{l}\cdot\vec{s}$ term in eq. (23), it is necessary to recognize that $m\dot{x}^0/\sqrt{\dot{x}^2} = m + m(\omega\pi)^2/2 + \ldots$ and keep track of all terms $\sim O(\omega_o^3)$. The sign of the spin-orbit term in eq. (23) is vector-like, and is opposite to the scalar-like sign given by Buchmüller's electric flux tube argument [2].

At this point we have only duplicated the results of Kikkawa, Sato, and others [11,12], extrapolated to the non-relativistic limit. This is not the whole story, however — we must go on to evaluate the propagator for the fermion on the end of the string, as Eichten and Feinberg [1] did for the quark propagator in QCD. In this we follow the treatment of Peskin [4].

The fermion propagator is given by $1/D_+$, eq. (7). Define D_- from eq. (7) by taking the opposite sign for the fermion mass, and form the product of D_+ and D_-:

$$-D_+D_- = \left(\frac{1}{\sqrt{\dot{x}^2}}\left(\partial_\tau + \frac{[\not{\dot{x}}, \not{\ddot{x}}]}{4\dot{x}^2}\right)\right)^2 + m^2. \tag{24}$$

To verify this relation it is simplest to first assume that $\dot{x}^2$ is a constant. Because D_+D_- is already in a form that is manifestly reparametrization invariant, it must then be correct for arbitrary $\dot{x}^2$.

Eq. (24) is rather surprising: by squaring the inverse fermion propagator we obtain an inverse propagator that looks like that for a *scalar* field coupled to a background gauge field A_τ,

$$A_\tau = \frac{-i}{4\dot{x}^2}[\not{\dot{x}}, \not{\ddot{x}}]. \tag{25}$$

Except for the appearance of A_τ, D_+D_- is the right covariant laplacian for a scalar field propagating in a curved, one-dimensional manifold with metric $g_{00} = \dot{x}^2$. We emphasize that our calling A_τ a gauge field is meant as nothing more than a helpful analogy which is useful in evaluating the propagator. A_τ is completely determined by the string fields x, and there is no (explicit) gauge field nor any gauge invariance associated with it.

Since $1/D_+ = (1/D_+D_-)D_-$, it is only necessary to evaluate the scalar type propagator $1/D_+D_-$ [4]. For this we can use an old trick of Feynman's [16] to write the scalar propagator as a path integral:

$$G = \frac{1}{D_+D_-} = \int_0^\infty d\xi_t \int [d\tau]\, exp(-iS_\tau) \tag{26}$$

where S_τ is the action

$$S_\tau = \int_0^{\xi_t} \left(\frac{\dot{x}^2}{2} \left(\frac{d\tau}{d\xi}\right)^2 - \frac{i}{4\dot{x}^2}\,[\not{\dot{x}}, \not{\ddot{x}}]\,\frac{d\tau}{d\xi} + \frac{1}{2}m^2 \right)\, d\xi. \tag{27}$$

Eqs. (26) and (27) should be familiar, at least after learning how to translate from the usual variables to those relevant for heavy strings. The sum over paths is over those in τ space (not in x space), with ξ the proper time for these paths. Integrating over the total proper time ξ_t produces the propagator G.

In this representation it is easy to evaluate the propagator. The term which is like a Wilson loop for A_τ, $exp(-i\int A_\tau\, d\tau)$, is independent of ξ, while the remaining terms are identical to those for a scalar particle. Doing the integrals by stationary phase gives [4,16]

$$G \sim exp\left(-i\int (m\sqrt{\dot{x}^2} + A_\tau)\, d\tau\right), \tag{28}$$

where x represents a given solution to the equations of motion. (In the exponential we have written $m\sqrt{\dot{x}^2}$ times the total length in τ space as an integral over τ.) The vector potential A_τ can be decomposed into two pieces, $A_\tau = A_\tau^{el} + A_\tau^{mag}$,

$$A_\tau^{el} = \frac{1}{2\dot{x}^2}\,\dot{x}^0\ddot{x}^i\,\sigma^{0i}, \qquad A_\tau^{mag} = -\frac{1}{2\dot{x}^2}\,\dot{x}^i\ddot{x}^j\,\sigma^{ij}. \tag{29}$$

While A_τ^{el} is larger than A_τ^{mag} in the non-relativistic limit, it can be shown that when $exp(-i\int A_\tau^{el}\, d\tau)$ is sandwiched between the non-relativistic wave-functions found from eq. (7), for large τ it gives a term that is purely real. Such a real term does not contribute to the action for the system, and only serves to renormalize the wave-functions [4]. Evaluating $exp(-i\int A_\tau^{mag}\, d\tau)$ is direct for a rotating string. It is convienient to write the result in terms of the physical time $t = x^0 = r\,\tau/\pi$. Including the action for the string gives

$$\bar{\psi}\, G\, \psi\; exp(-iS_{string}) \sim exp\left(i\int (\frac{1}{2}mv^2 - V_{string}(r))dt\right), \tag{30}$$

where $V_{string}(r)$ is the potential

$$V_{string}(r) = m + \mu r - \frac{\vec{l}\cdot\vec{s}}{2m^2}\,\frac{\mu}{r}. \tag{31}$$

The coefficient of the $\vec{l}\cdot\vec{s}$ term in eq. (31), $= -\frac{1}{2}$, is a sum of two terms. A_τ^{mag} contributes -1, while the expansion of $\sqrt{\dot{x}^2}$ contributes $+\frac{1}{2}$ (see the discussion following eq. (23)). The overall factor of $\frac{1}{2}$ is the correct value for Thomas precession.

Eq. (31) is our principal result: over large distances the sign of the spin-orbit term is scalar-like, and there is no spin-spin interaction.

From the form of eq. (1), the reader might wonder whether our results are affected by the fact that we have assumed one fermion is infinitely massive. We have solved for the case of two fermions with equal mass, and verified that the solution (up to factors of two and the like) is of the same form, with a scalar-like spin-orbit term and no spin-spin interaction. Relative to the QCD potential of eq. (1), this corresponds to $V_1(r) = -V(r) = -\mu r$ and $V_2(r) = V_3(r) = V_4(r) = 0$.

The sign of the spin-orbit term found from the fermion propagator in eq. (31) is *opposite* that found from the energy in eq. (23). The reason for this difference is not apparent from our treatment, but would be so if we had followed the approach of Eichten and Feinberg directly, as opposed to Peskin's elegant variation. Calculating the spin-orbit term requires keeping track of quantities that are small in the non-relativistic limit. Choose a basis in which the upper components of the fermion wave-function are large in the non-relativistic limit. One source of small terms arises from an upper component that turns into a lower component, propagates for a short period of time, and then flips back [1]. Evaluating the energy as in eq. (23) neglects the virtual mixing of upper and lower components in the fermion's propagation. In contrast, an especially painless way of keeping track of all of this is to evaluate the fermion propagator by the path integral representation used above [4].

There are many other problems that could be addressed using heavy strings. We conclude by discussing their quantization. It is known that if the ends of the string are either massless [17] or infinitely massive [18], that the theory is only consistent in 26 dimensions. By following the usual approach [17,18] it is not clear whether this carries over to strings with ends that have a finite but non-zero mass. Consider a string with scalars on both ends, eq. (5), taking one end to be infinitely massive and the other of mass m: $m_1 = \infty$, $m_2 = m$. It is possible to use the gauge invariance left over in orthonormal gauge to require that $\dot{x}^2$ be a constant, independent of τ, for $\sigma = \pi$; *i.e.*, that along that end of the string τ is proportional to the proper time for the scalar particle. In this instance the boundary condition like that of eq. (12) is

$$\ddot{x}^{\alpha} = -c\,(x')^{\alpha}, \tag{32}$$

where $c = \mu\sqrt{\dot{x}^2}/m$, evaluated at $\sigma = \pi$. The solution to eqs. (9) and (32) for the spatial components of x is

$$x^i = \sum_n a_n^i\, sin(p_n\sigma)\, exp(ip_n\tau) + c.c. \tag{33}$$

with a similar solution for x^0 [18]. The sum in eq. (33) is over all p_n that satisfy the transcendental equation

$$p_n\ tan(\pi p_n) = c. \tag{34}$$

If the mass is neither zero or infinite, the sum of any two solutions to eq. (34) is not itself a solution. This means that when the orthonormal gauge condition of eq. (8) is imposed upon eq. (33), the usual Virasoro conditions are not obtained if $\infty > m > 0$, and it is obscure how to implement the gauge condition.

An argument for a critical dimension can be given, however, by using the functional approach of Polyakov [19]. Introduce a metric tensor g_{ab} on the world sheet (a,b=σ,τ) and a scalar function λ on the boundary, with an action

$$\frac{\mu}{2}\int\left(\sqrt{g}\,g^{ab}\,(\partial_a x)\cdot(\partial_b x)\right)\,d\sigma\,d\tau + \frac{m}{2}\int_{\sigma=\pi}\left(\lambda^2 + \frac{\dot{x}^2}{\lambda^2}\right)\,d\tau. \tag{35}$$

Integration over the fields g_{ab} and λ reproduces the original action. As usual for the Nambu string, eq. (35) is invariant under local conformal transformations of g_{ab}. The x fields appear quadratically in eq. (35), so they can be integrated out to give [19]

$$\frac{26-d}{48\pi}\int\left(\frac{1}{2}\,(\partial_a log(\rho))^2 + \mu_{ren}\,\rho\right)\,d\sigma\,d\tau + boundary\ terms, \tag{36}$$

in conformal gauge, $g_{ab} = \rho\,\eta_{ab}$, for an arbitrary number of dimensions d. The boundary terms in eq. (36) will be complicated, but on the world sheet there is just the standard Liouville action. Requiring that the local conformal symmetry be manifest implies that $d = 26$.

Does this mean that our results are only valid for the quarks in 26 dimensions? Not at all. Helfrich [20] and Polyakov [21] have proposed models of "smooth" strings, where a term involving the extrinsic curvature of the world sheet is added to the Nambu action. Smooth strings do not have a local conformal symmetry and so are surely a consistent string theory in four space-time dimensions. Moreover, for the simple kind of open string solutions which we have used here, on the world sheet these solutions are unaffected by the presence of a curvature term [22]. The curvature term does alter the boundary condition of eq. (12), but over large distances ($rm \gg 1$) this can be ignored. Thus our entire analysis should be viewed as an exercise in strings that are smooth as well as heavy.

For any kind of string, over large distances fluctuations to one loop order in the string variables generate a correction $\sim 1/r$ to the linear term in the static potential [23]. Such fluctuations will also produce spin dependent terms $\sim 1/r^3$; it would be interesting to see what these look like.

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