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LIMITS ON PREDICTABILITY OF SUPERCOLLIDER PHYSICS*

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ABSTRACT

We discuss present limitations on the predictability of ultra-high energy cross sections in QCD. Assuming coupled Altarelli-Parisi evolution, we exhibit kinematic boundaries in the x, Q^2 plane beyond which no reliable predictions can now be made. The boundaries occur for any large Q^2 for small enough x, and follow from the effects of boundary conditions on small-x asymptotic estimates. A typical boundary occurs at $1/x \gtrsim [\ln(Q^2/\Lambda^2_{QCD})/\ln(Q_o^2/\Lambda^2_{QCD})]^{mb/8N}$ where m depends on theoretical models. Such boundaries occur in phenomenologically important regions, and are distinct from the unitarity boundaries discussed by Gribov, Levin, and Ryskin: the unpredictable regions could be predicted with new data at medium Q^2 and small x. We also discuss the possibility of significant effects throughout the SSC kinematic regions from unexpected consequences of unitarity boundary conditions.

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SMALL-X

One of the prerequisites for considering hadron physics in the energy regime of supercolliders[1] is an accurate description of parton distributions. The typical values of momentum scales Q^2 involved are large, say $10^2 < Q^2(\mathrm{GeV}^2) < 10^6$, but this does not cause much uncertainty since scaling violations are a slowly varying effect. However, as the c.m. energy $\sqrt{s}$ increases, the typical momentum fraction x of partons decreases, and this is an important effect for very large s. For reference, the annihilation of partons carrying fractions x_1, x_2 to produce an object of mass M is constrained by $x_1x_2 = M^2/s$; if M = 20 GeV and $\sqrt{s}$ = 20 TeV, we have $x_1x_2 = 10^{-6}$. There are few direct measurements of the parton distributions for $10^{-2} < x < 10^{-1}$, so present supercollider predictions are heavily dependent on theoretical assumptions.

We present results here on the dependence of parton distributions as various levels of theoretical uncertainty are exposed. One of the striking effects we will be able to explain is the observation of EHLQ[2] that the differences between different inputs is ironed out by the QCD evolution as one goes to larger Q^2 for fixed and small x. This "empirical" result, the outcome of numerical experiments, follows from our analysis. At the same time we will be able to predict regions in the Q^2 and x plane which remain overly sensitive to unreliable input. These regions seem to be phenomenologically important. The general description of our results is that the scale Q^2 must be increased in a specific manner as $\sqrt{s}$ is increased if one wants a fixed level of theoretical predictability.

The evolution of parton distributions in the Q^2 and x plane can be determined with the Altarelli-Parisi (AP) equations.[3] It is important to recognize that the history of the subject has emphasized the renormalization-group Q^2 dependence of moments, but that this in turn implies an interplay of the evolution in the x-Q^2 plane. In fact, the small-x limit of the equations exhibits an important symmetry in certain variables representing the x and Q^2 dependence, as we will show below. However, the region of experimental interest is one where the x-dependence is much more dramatic and interesting than the Q^2 dependence - the reverse of the traditional study of scaling violations.

The kernels of the AP equations suggest a self-consistent ansatz in which the gluon distribution $G(x,Q^2)$ is much larger than the sea-quark distributions $q_S(x,Q^2)$. A preliminary approach can be made by dropping quarks in the gluon evolution, except for running coupling effects:

$$\frac{dG(x,Q^2)}{dt} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dx'}{x'} P_{GG}(\frac{x'}{x}) G(x',Q^2) \qquad (1)$$

A good approximation to the asymptotically small-x dependence of (1) is given by replacing $P_{GG}(z)$ by its small-z limit. It is convenient to study $xG(x,Q^2)$ in order to smooth the singularity, and to use new variables y, ξ:

$$y = \frac{8N}{b} \ln 1/x \quad ; \quad b = 11 - 2/3\, n_f,\ N = 3$$

$$\xi = \ln(\ln Q^2/\Lambda^2_{QCD})/\ln(Q_o^2/\Lambda^2_{QCD})),$$

$$\bar{G}(y,\xi) = xG(x,Q^2)\,.$$

It is straightforward to show[4,10] that (1) leads to the differential equation

$$\frac{\partial^2 \bar{G}(y,\xi)}{\partial\xi\partial y} - \frac{1}{2}\bar{G}(y,\xi) = 0 \qquad (2)$$

with corrections of inverse powers of $y\xi$. A key point we wish to emphasize is the local character and symmetric evolution in the y and ξ variables given by Eq. 2. We will see that this leads to a fast and powerful method of making quantitative statements on the small-x region. Another key point is that y, because of $8N/b \cong 3\,1/4$ in its definition, is truly a large logarithm: $y(10^{-4}) \sim 29$. Thus (2) is a good starting point in the SSC region of interest.

The discussion of some consequences of Eq. 2 by Gribov, Levin and Ryskin[4] (GLR) has drawn much attention. Those authors choose not to discuss the coupled quark and gluon AP equations, but bring up important issues of unitarity in the context of a solution to (2). The conclusion of GLR and other workers,[5] however, has been that for the perturbatively large Q^2 of interest the unitarity problem has little practical effect. Although we will question that conclusion below, our main purpose is to discuss gluon and quark distributions in regions of interest for the SSC, and to separate those regions which can be predicted with present data from those which cannot.

As shown in Ref. (6), the quark evolution can be self-consistently incorporated, to leading power of y, by similar leading approximations. We find that $\bar{G}(y,\xi)$ continues to satisfy (2), while the leading order sea-quark distributions $u_s \cong d_s \cong s_s \cong 2C_s \cong 2b_s$ can be shown to obey

$$xu_s(y,\xi) = \frac{2}{b}\frac{\partial}{\partial y}\, xG(x,Q^2). \qquad (3)$$

The valence quarks are much smaller at large y and will be dropped. The differential relations[6] (2,3) are superior to more traditional quantities such as moments for the purposes of imposing boundary conditions (b.c.'s), as we now discuss.

The b.c.'s on the AP equations are data, but this is known only in a limited region of x and Q^2. Numerical methods and some modeling are needed to convert data to distributions at large y and ξ where (2) and (3) are applicable. That work has been done by EHLQ[2] and Johnson and Tung,[7] among others. Our approach here is to write a general solution to (2) for $\bar{G}(y,\xi)$, and impose b.c.'s given by evolved data, rather than low-energy data. Solutions to (2) are not difficult to obtain; the solutions that grow in $y\xi$ are given by

$$\bar{G}(y,\xi) = \sum_{\nu=0}^{\infty} [A_\nu (\frac{2\xi}{y})^{\nu/2} + B_\nu (\frac{y}{2\xi})^{\nu/2}]\ I_\nu(\sqrt{2\xi y}) \qquad (4)$$

The coefficients A_ν and B_ν are determined by the arbitrary b.c.'s. To emphasize the importance of these, a physical analogy is useful.

Consider (2) as representing a wave propagating in the y, ξ plane, with y and ξ playing the role of light-cone variables. One can verify that a "particle" (wave-packet) of mass-squared -1, i.e. a free tachyon, is associated with $\bar{G}(y,\xi)$. Such an object has exponentially growing amplitudes rather than the oscillatory dependence of familiar wave equations, but otherwise acts like an ordinary wave-packet.

The b.c. coefficients A_ν and B_ν parameterize initial conditions on the wave packet as a function of y/ξ, a function of the "velocity," while the Bessel functions $I_\nu(\sqrt{2\xi y})$ parameterize the unfolding of plane "waves." We are interested in a region far from the origin, $2\xi y >> 1$, where the exponential growth is evident:

$$I_\nu(\sqrt{2\xi y}) \sim I(\xi y)\ [1 - \frac{4\nu^2 - 1}{8\sqrt{2\xi y}} + \ldots .]$$
$$I(\xi y) = \exp(\sqrt{2\xi y})/\sqrt{2\pi}\ (2\xi y)^{1/4} \qquad (5)$$

The $\exp(\sqrt{\ln 1/x})$ behavior, discussed in GLR[4] and elsewhere[3,5,6], comes out this way. Note that the first term in (5) is independent of ν for asymptotically large $y\xi$. Combining this[11] with (4) gives

$$\bar{G}(y,\xi) = K(y/\xi) I(\xi y) \qquad (6)$$

in which the function $K(y/\xi)$ carries all the information on b.c.'s.

The large $y\xi$ problem is thus determined by the amplitude at a given y/ξ, a "ray," which then propagates out. One way a prediction can be made without regard to boundary conditions is by relating points on the same ray $y = m\xi$, for $y\xi >> 1$:

$$\frac{\bar{G}(y',y'/m)}{\bar{G}(y,y/m)} \sim \exp((y'-y)\sqrt{2/m})(y/y')^{1/2} .$$

On the other hand, the boundary conditions are essential to make contact with data. The assumptions of GLR seem to be that all coefficients A_ν and B_ν in (4) are zero except for $\nu = 0$. That does not agree well with numerical work, however.

From the ray tracing analogy, the region $y/\xi >> 1$ actually probes many b.c. coefficients, but might look simpler in terms of ξ at fixed $y = y_o$. In fact, one can show[6] by direct differentiation that

$$\bar{G}(y,\xi) = K_A(\xi)e^{\sqrt{2\xi y}} \tag{7}$$

is another form of solution to (2) for $y >> \xi$. This ansatz is more convenient than (4) because $K_A(\xi)$ can be fit more directly. The functional form (7) is consistent with conventional b.c.'s of setting $\bar{G}(x,Q_o^2) \sim$ const as $x \to 0$, so that $B_\nu \equiv 0$, $\nu > 0$. We incorporate data by fitting the boundary value $K_A(\xi)$ at $y = y_o$ ($x = 10^{-4}$). A suitable source is the Set 1 distributions for $xG(x,Q^2)$ of EHLQ, which give for $\Lambda_{QCD} = 200$ MeV, $Q_o^2 = 5$ GeV^2:

$$K_A(\xi) = 50.4(e^{\xi} - 0.957)e^{-7.957\sqrt{\xi}} \tag{8}$$

A test of the approach is given in Fig. 1, which shows $xG(x,Q^2)$ at different x values as a function of Q^2. The prediction is excellent for all $Q^2 \geq 10^2$ GeV^2 and $x < 10^{-2}$, and acceptable even for $x \lesssim 10^{-1}$. This demonstrates that the numerical size of corrections to the $y\xi >> 1$, $y >> \xi$ formalism are indeed small. In fact using (4), (7) and (8) we have a convenient analytic description[6] of the EHLQ distributions. The sea quarks also compare well[6] using (3).

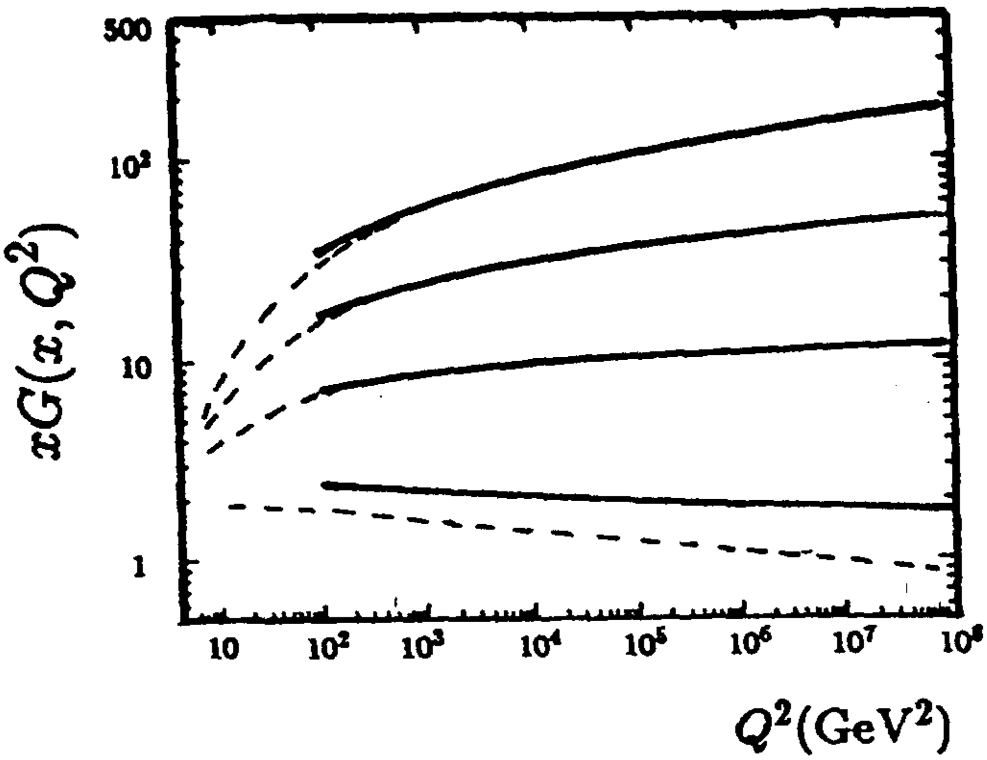

Fig. 1. Comparison of asymptotic estimates (7,8) for $xG(x,Q^2)$ (solid lines) with the numerically integrated distributions of Ref. 2 (dashed lines), at $x = 10^{-4}$, 10^{-3}, 10^{-2}, 10^{-1} (top to bottom).

PREDICTABLE REGIONS

Now we turn to the sensitivity of the output to the input. Recall that (6) showed us that b.c.'s can be set at fixed ξy by a function of y/ξ. Data does not exist to predict $K(y/\xi)$ for all y/ξ, however; we would need the distributions for $x \to 0$ at fixed large Q_o^2. Nevertheless, a measure of the theoretical uncertainty can be gained by modeling data in the small x, Q^2 region. If we are willing to accept a fixed fractional error in $\bar{G}$, say $(\Delta\bar{G}/\bar{G}) \leq 2$ as physically reasonable boundary conditions are varied, we can determine the maximum y/ξ for which $K(y/\xi)$ is reliably known.

Thus we write for $y\xi >> 1$,

$$\bar{G}_\ell = K_\ell(y/\xi) I(\xi y),$$

where ℓ is an index representing different input b.c.'s. Let the extremes in input be denoted by $\ell = 1$ and 2. The fractional uncertainty,

$$\Delta\bar{G}/\bar{G} = (\bar{G}_2 - \bar{G}_1)/\bar{G}_1$$

$$= K_2(y/\xi)/K_1(y/\xi) - 1, \tag{9}$$

is a function of $y/\xi = m$. We associate with each $\Delta\bar{G}/\bar{G}$ a value of m, $m(\Delta\bar{G}/\bar{G})$, in this way. For definiteness the two extremes in the boundary conditions are taken to be:

1) $xG_1(x,Q_o^2) \sim O(1)$, $x < 10^{-2}$.
2) $xG_2(x,Q_o^2) \sim O(1/\sqrt{x})$, $x < 10^{-2}$.

Although the ξy region covered in the available numerical work is only marginally appropriate for the above discussion, let us illustrate the idea by finding $\Delta\bar{G}/\bar{G}$ for cases 1) and 2) by using evolved data as presented by EHLQ[2] and as discussed further by Collins.[9] The $1/\sqrt{x}$ behavior ($G(x,Q^2) \sim x^{-3/2}$) has been advocated by Regge theorists.[9] From the different cases we get curves of bounded fractional uncertainty, defined by $y/\xi \lesssim m(\Delta\bar{G}/\bar{G})$, or the bound

$$1/x \lesssim [\ln(Q^2/\Lambda^2)/\ln(Q_o^2/\Lambda^2)]^E , \tag{10}$$

$$E(\Delta\bar{G}/\bar{G}) = bm(\Delta\bar{G}/\bar{G})/8N.$$

In Fig. (2) the results are illustrated. Rather than fitting K_2 and K_1, as implied by (9), $\Delta\bar{G}/\bar{G}$ was fixed at large $y_o \cong 29$ ($x = 10^{-4}$) and then a value of $\xi = \xi_o$ and $y_o/\xi_o = m_o$ was determined. Thus at $x = 10^{-4}$, $Q^2 = 10^3\,\mathrm{GeV}^2$, $\Delta\bar{G}/\bar{G} \cong 220\%$ and $y/\xi = 38.9$. For an allowable error of a factor of 2, the exponent $E(2) = 12.4$ is the value to be used in (10). Similarly, at $x = 10^{-4}$, $Q^2 = 10^2\,\mathrm{GeV}^2$, $\Delta\bar{G}/\bar{G} \cong 450\%$, and $m(4.5) = 59.7$, giving $E(4.5) = 19.1$.

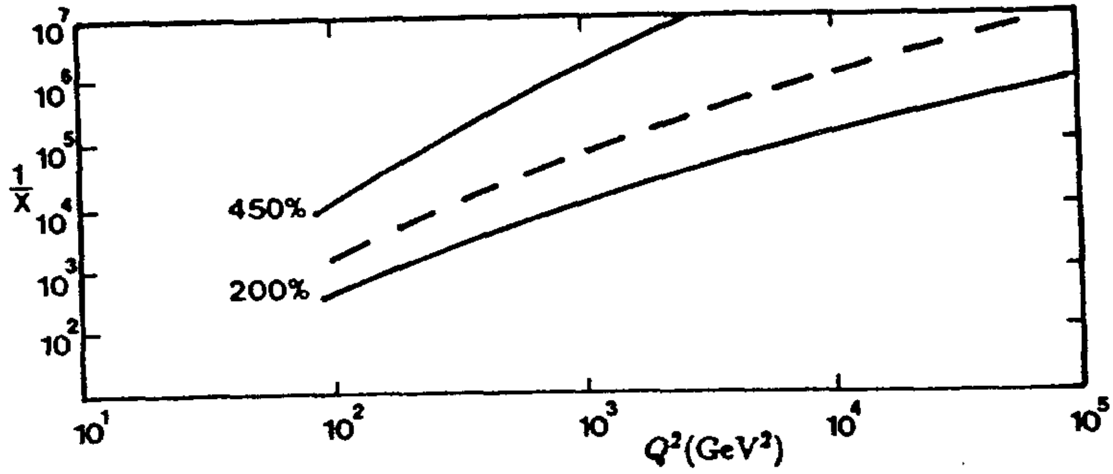

Fig. 2. Solid lines: Curves of fixed uncertainty $\Delta\bar{G}/\bar{G}$ in the gluon distribution. Dashed line: an estimate of the line of $\Delta\sigma/\sigma \cong 10$ for σ evaluated at typical Q^2 and $1/x$.

Another consequence of these numbers is an estimate of the error of a cross section $\sigma(x,Q^2)$ going like the product of two parton distributions. (An integral over the distributions can also be estimated[10] but is too detailed for our purposes here.) For a rough approximation to the point at which $\Delta\sigma/\sigma \cong 10$, a fractional uncertainty of a factor of about $3\frac{1}{2}$ in the distributions, we can interpolate between E = 12 and E = 19 in (10). Given the other uncertainties, this defines a line

$$1/x \Big|_{\Delta\sigma/\sigma \cong 10} \leq [\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]^{15}, \tag{11}$$

which is intermediate between the two lines in Fig. (2).

UNITARITY BOUNDARY CONDITIONS

Another interesting issue to consider with our methods is the new physics introduced by unitarity considerations. As emphasized in Refs. (4,5), the AP evolution is not consistent with more general bounds on the total cross section, even when probed at large enough Q^2 for the perturbative QCD to be naively applicable.

We will only mention here a preliminary consequence of the unitarity issue; a more thorough discussion will be presented elsewhere.[10] Our approach will be based on the observation that the new physics from a region where the unitarity issue is important can be represented by non-trivial boundary conditions on $\bar{G}(y,\xi)$. Note the b.c.'s are imposed in a region where the AP evolution, represented by (2), is applicable. This is because we are not specifically interested in the details of the breakdown of the AP equations, say, at some Q^2 and x point. Such breakdown is physically inevitable, but only constitutes a new b.c. in a nearby region where the AP evolution is applicable. (Note that the local character of (2,3) is important here.)

New boundary conditions on partial differential equations lead to global effects, however. One would not expect the new effects to be isolated into the unitarity region, in fact, because disturbances propagate with exponential growth.

To illustrate this, we write a more general superposition which is a solution to (2) in the large $y\xi >> 1$ region:

$$\bar{G}(y,\xi) = \int dy'd\xi'\delta(y'-f(\xi'))K(y'/\xi')\ I[(\xi-\xi')(y-y')] \tag{12}$$

In this expression, $y' = f(\xi)$ defines a curve along which specific b.c.'s are imposed. An example is the choice of GLR[4] of the line $y = e^{2\xi}/\xi$ where the quantity $xG(x,Q^2)/Q^2$ is held fixed. There is nothing sacred about that choice.

Now we interpret (12) in the light of our previous discussion. The point (y',ξ') in the integrand in (12) has been given an initial amplitude $K(y'/\xi')$ which propagates with exponential growth according to $I[(\xi-\xi')(y-y')]$ to a distant point (y,ξ). In other words, there is every reason to suspect that imposing certain new unitarity b.c.'s at small Q^2 and moderate $1/x$ will significantly change the predictions in the distant regions of large $1/x$ and large Q^2 needed for SSC physics.

Physically, the abrupt collisions and coalescence that occurs among partons near the unitarity boundaries changes the parton distributions. In the linearized description of (2), the partons rescatter (reflect) and act as new sources for the next generations of partons at larger y and ξ. The explosive growth of the distributions in $y\xi$ may amplify what appears to be a small effect.

This point requires further investigation and detailed numerical work to see if it can become an important issue. We emphasize that this is a point where the different physics choices must be examined systematically to get an estimate of the order of magnitude. Within the context of the previous discussion, there is some reason to anticipate that the new effects of unitarity boundary conditions may lead to large effects.

ACKNOWLEDGMENT

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