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NONPERTURBATIVE QCD CORRECTIONS TO $R = \sigma_L/\sigma_T$

S. Fajfer

Institut za fiziku, University of Sarajevo
Sarajevo, Yugoslavia
and
Fermi National Accelerator Laboratory
Batavia, Illinois, 60510, U.S.A.

and

R.J. Oakes

Department of Physics and Astronomy, Northwestern University
Evanston, Illinois, 60201, U.S.A.
and
Fermi National Accelerator Laboratory
Batavia, Illinois, 60510, U.S.A.

ABSTRACT

We have calculated the twist-four, spin-two Quantum Chromodynamic (QCD) corrections to the ratio $R = \sigma_L/\sigma_T$ and the Callan-Gross relation for electroproduction on proton and deuteron targets. Using the Wilson operator product expansion (OPE) of the bilocal product of the electromagnetic currents, we determined the coefficient functions using perturbative techniques, neglecting their anomalous dimensions. The MIT bag model was used to evaluate the proton and deuteron matrix elements of the (local) quark operators appearing in the expansion. The corrections to R were found to be small, increasing R by at most a few percent, contrary to a previously reported result which was substantially larger and was based on a quite different approach to estimating higher twist effects. For completeness we present the higher twist corrections to the structure functions W_1, and νW_2, as well as R, in the kinematical region of interest and note that these are in agreement with the results implicitly contained in previous independent calculations using similar techniques.

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Recently, Cortes, et al.[1] have estimated the higher-twist corrections to the ratio

$$R = \sigma_L/\sigma_T = \frac{W_2(Q^2,\nu)}{W_1(Q^2,\nu)} \left[1+\nu^2/Q^2\right] - 1 \qquad (1)$$

in electroproduction, finding unexpectedly large effects, with the implication that there are also substantial higher twist corrections to the Callan-Gross relation among the electroproduction structure functions: $W_1(Q^2,\nu) = (1+\nu^2/Q^2)W_2(Q^2,\nu)$ or $F_1(x) = 2\ x\ F_2\ (x)$ in the scaling limit. We have calculated the twist-four, spin-two corrections to the structure functions $W_1(Q^2,\nu)$ and $W_2(Q^2,\nu)$ (and therefore R) using the more conventional techniques for estimating higher twist effects. We find the corrections to R to be considerably smaller than Cortes, et al.[1] and, consequently, no evidence for important higher twist corrections to the Callan-Gross relation. Our independent calculations are in agreement with the results that are implicitly contained in the work of Castorina and Mulders[2] on higher twist effects.

After briefly describing our calculations we give our results for the twist-four, spin-two QCD corrections to the structure functions in terms of two model-dependent matrix elements of quark operators for protons and deuterons in the kinematical regime where R has been measured at SLAC[3]. Finally, we emphasize our main conclusion: the nonperturbative higher twist corrections to R (and to the Callan-Gross relation) that we have estimated are negligible compared to the usual perturbative corrections calculated in leading log approximation[4]. In fact, the higher twist corrections are small compared to the present experimental uncertainties[3] in R.

The hadronic tensor entering in deep inelastic electron scattering is

$$W_{\mu\nu} = \frac{1}{4\pi M} \int dz e^{iqz} \langle p \mid \left[J_\mu(z),\ J_\nu(0) \right] \mid p \rangle \tag{2}$$

and can be written in terms of the familiar nucleon structure functions $\nu W_{L,2}\ (\nu,Q^2) = F_{L,2}\ (\nu,Q^2)$:

$$W_{\mu\nu} = (-g_{\mu\nu} + q_\mu q_\nu/q^2)\ W_L\ (\nu,Q^2)$$
$$+ \left[g_{\mu\nu} + p_\mu p_\nu q^2/(p\cdot q)^2 - (p_\mu q_\nu + p_\nu q_\mu)/(p\cdot q)\right](\nu^2/q^2) W_2(\nu,Q^2) \tag{3}$$

It is convenient to define

$$W_1\ (\nu,Q^2) = W_L(\nu,Q^2)\ -(q^2/\nu^2)\ W_2\ (\nu,Q^2) \tag{4}$$

where the notation is standard: $\nu = p.q/M$, $p^2 = M^2$, $Q^2 = -q^2$ and the average over target spins in Eq. (2) is understood.

Following the systematic procedure[4] for calculating the nonperturbative higher-twist effects, we first expand the bilocal product of currents in Eq. (2) in a basis of local operators, using the Wilson[5] operator product expansion (OPE). The coefficients in this OPE obey[6] renormalization group equations and can be evaluated using perturbative techniques[7-9] by choosing the renormalization point $\mu^2 = Q^2$, which is tantamount to neglecting the anomalous dimensions of the local operators in the OPE. A more detailed description of the technical aspects of this procedure can be found in previous calculations of other higher-twist effects[2,7,10,11]. Finally, to evaluate the nucleon matrix elements of the quark operators appearing in the OPE we have used the MIT bag model[12].

The twist-four, spin-two corrections to the proton structure functions are

$$\delta W_1^p = (\nu^2/Q^2)\delta W_2^p + 2\alpha_s(Q^2)/Q^4(\nu^2/Q^2-1)\left[-(4/3)I_1+(64/81)I_2\right] \tag{5}$$

and

$$\delta W_2^P = 4\alpha_s(Q^2)/Q^4(\nu^2/Q^2-1)\left[(77/27)I_1 + (112/27)I_2\right] \quad (6)$$

where $\alpha_s(Q^2) = g^2/4\pi$ is the QCD strong coupling constant. The integrals, I_1 and I_2, contain the part of the matrix elements which involve the spatial dependence of the particular quark model assumed. For the MIT bag model[12] $I_1 = 20.36 \times 10^4$ GeV^3 and $I_2 = 3.21 \times 10^4$ GeV^3. (For several other plausible confinement models these integrals have also been evaluated and shown to be quite model dependent.[11])

For deuterium (isosinglet) targets one similarly finds

$$\delta W_1^D = (\nu^2/Q^2)\delta W_2^D + 2\alpha_s(Q^2)/Q^4(\nu^2/Q^2-1)\left[-(10/9)I_1 + (80/81)I_2\right] \quad (7)$$

and

$$\delta W_2^D = 4\alpha_s(Q^2)/Q^4\left[(131/27)I_1 - 8I_2\right] \quad (8)$$

The resulting twist-four, spin-two correction to R is

$$\delta R = \left[(1 + \nu^2/Q^2)\ \delta W_2 - (1+R)\ \delta W_1\right]/W_1 \quad (9)$$

assuming $W_{1,2}$ are small, as is evident below.

The ratio $R = \sigma_L/\sigma_T$ has been measured at SLAC[3] for values of Q^2 and ν in the respective regions of about 3 - 12 GeV^2 and 3 - 14 GeV. We have numerically evaluated the twist-four, spin-two corrections to W_1, W_2 and R for both proton and deuteron targets and have compared the results with these data. For the QCD strong coupling constant we have used the one-loop result $\alpha_s(Q^2) = (4\pi/9)/\ln(Q^2/\Lambda^2)$ with $\Lambda = 100$ MeV. (Note that we have assumed effectively three active flavors in this kinematical regime.)

In Table I the numerical results are presented for protons while Table II refers to deuterons. In both cases, it is evident that the twist-four, spin-two functions W_1 and W_2 are quite small ($\leq 1\%$). It is also clear from Tables I and II that the corrections to R are very small, too; in fact, δR is negligible compared to the experimental uncertainty in R.

Our results are in agreement with those implicitly contained in the calculation of Castorina and Mulders[2]. (Indeed, the fact that we find R to be numerically negligible provides support, ex post facto, for our neglecting corrections to the Callan-Gross relation in our previous work[10,11]. However, our calculations do not support the much larger estimates of the higher twist effects on R given by Cortes et al.[1], nor their conclusion that for present energies R is essentially given entirely by the twist-four correction. We suspect that their analysis, which is quite different from ours, is highly sensitive to the rather large experimental errors in R.

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Q^2 (GeV^2)	ν(GeV)	W_1^{exp}(GeV^{-1})	δW_1 (GeV^{-1})	W_2^{exp}(GeV^{-1})	δW_2(GeV^{-1})	R^{exp}	δR
3	3.26	0.2157	1.62×10^{-3}	0.0564	5.54×10^{-4}	0.186 ± 0.152	2.81×10^{-3}
	4.86	0.4439	3.44×10^{-3}	0.0598		0.195 ± 0.110	1.81×10^{-3}
	6.46	0.5970	5.97×10^{-3}	0.0528		0.319 ± 0.226	6.3×10^{-4}
	8.06	0.8459	9.23×10^{-3}	0.0438		0.174 ± 0.217	2.02×10^{-3}
6	4.86	0.0621	3.38×10^{-4}	0.0014	1.08×10^{-4}	0.120 ± 0.186	2.5×10^{-3}
	6.46	0.0170	5.74×10^{-4}	0.0247		0.157 ± 0.095	1.15×10^{-3}
	8.06	0.2880	8.77×10^{-4}	0.0293		0.203 ± 0.116	7.7×10^{-4}
9	6.46	0.0227	1.64×10^{-4}	0.0049	4.52×10^{-5}	0.209 ± 0.119	2.47×10^{-3}
	8.06	0.0756	2.49×10^{-4}	0.0112		0.220 ± 0.103	9.0×10^{-4}
	9.66	0.1476	3.52×10^{-4}	0.0157		0.208 ± 0.131	6.0×10^{-4}
	11.26	0.2048	4.74×10^{-4}	0.0198		0.458 ± 0.188	-4×10^{-5}
12	8.06	0.0102	1.03×10^{-4}	0.0020	2.44×10^{-5}	0.260 ± 0.145	2.69×10^{-3}
	9.66	0.0397	1.44×10^{-4}	0.0005		0.202 ± 0.116	1.03×10^{-3}
	11.26	0.0856	1.94×10^{-4}	0.0087		0.167 ± 0.149	6.6×10^{-4}
	12.86	0.1256	2.51×10^{-4}	0.0121		0.422 ± 0.300	3.7×10^{-5}
	14.46	0.1627	1.94×10^{-4}	0.0169		0.908 ± 0.169	-9.4×10^{-4}

Table I. Higher twist corrections to the structure functions W_1 and W_2 and the ratio R for proton targets. All data are taken from reference 3.

Q^2 (Gev^2)	ν(GeV)	W_1^{exp}(GeV^{-1})	δW_1 (GeV^{-1})	W_2^{exp}(GeV^{-1})	δW_2(GeV^{-1})	R^{exp}	δR
3	3.26	0.3178	1.14×10^{-3}	0.0893	3.98×10^{-4}	0.274±0.16	1.12×10^{-3}
	4.86	0.6901	2.40×10^{-3}	0.1002		0.287±0.11	6.3×10^{-4}
	6.46	1.1225	4.17×10^{-3}	0.0893		0.185±0.22	9.4×10^{-4}
	8.06	1.5335	6.43×10^{-3}	0.0761		0.122±0.19	1.17×10^{-3}
6	4.86	0.0899	2.79×10^{-4}	0.0211	8.84×10^{-5}	0.158±0.18	1.26×10^{-3}
	6.46	0.2442	4.75×10^{-4}	0.0385		0.254±0.12	4.4×10^{-4}
	8.06	0.4223	7.26×10^{-4}	0.0485		0.354±0.13	1.5×10^{-5}
9	6.46	0.0326	1.36×10^{-4}	0.0069	3.70×10^{-5}	0.217±0.01	1.33×10^{-3}
	8.06	0.1078	2.06×10^{-4}	0.0167		0.268±0.13	4.0×10^{-4}
	9.66	0.2249	2.92×10^{-4}	0.0236		0.194±0.14	5.5×10^{-5}
	11.26	0.3347	3.93×10^{-4}	0.0305		0.374±0.18	5.7×10^{-5}
12	8.06	0.0147	8.49×10^{-5}	0.0029	2.00×10^{-5}	0.271±0.13	1.40×10^{-3}
	9.66	0.0520	1.20×10^{-4}	0.0081		0.358±0.13	2.6×10^{-4}
	11.26	0.1271	2.07×10^{-4}	0.0151		0.179±0.15	-9×10^{-6}
	12.86	0.2112	2.66×10^{-4}	0.0170		0.185±0.25	-1.1×10^{-4}

Table II. Higher twist corrections to the structure functions W_1 and W_2 the ration R for deuterium targets. All data are taken from reference 3.

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