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N=2 NO-SCALE SUPERGRAVITY

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Abstract

N=2 extended supergravity is discussed and an assessment is made of the problems encountered in applying it to the construction of phenomenological models of particle physics. A specific class of so-called no-scale models is discussed, in which the two supersymmetries are spontaneously broken in flat space-time, with naturally vanishing cosmological constant and the scale of supersymmetry breaking undetermined at the classical level. Supergravity induced supersymmetry breaking generates effective mass terms for spin $\frac{1}{2}$ components of the vector gauge multiplets and spin 0 components of the scalar matter multiplets. For finite globally supersymmetric models, this supersymmetry breaking preserves the finiteness. Possible connections of N=2 no-scale supergravity with superstrings and finite range antigravity are mentioned.

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I. Introduction

Supersymmetric field theories are attractive because they are less divergent than ordinary field theories. Due to so-called non-renormalization theorems, supersymmetric matter-gauge systems are free of quadratic divergences. This is in contrast with non-supersymmetric theories, where for example, the Higgs particles receive quadratically divergent radiative corrections. These corrections give rise to the naturalness problem for scalar fields; even if one starts with a small bare mass for scalars, they generate huge masses of the order of the cutoff of the theory, which is presumably the Planck mass, $M_{Planck} \approx 1.2 \cdot 10^{19} GeV$.

Since no mass-degenerate fermion-boson pairs have been observed so far, nature is not exactly supersymmetric. Hence construction of phenomenologically viable models must always involve introduction of supersymmetry breaking terms in the Lagrangian, preferably in a way so that quadratic divergences remain absent. The coupling of matter to supergravity provides a fundamental mechanism for generating such soft terms (for a review with an extensive list of references, see [1]). In order to induce spontaneous supersymmetry breaking in such models, one introduces a so-called hidden sector, which couples gravitationally to the known low-energy sector. This coupling gives rise to effective supersymmetry breaking terms for the low-energy sector. Using this mechanism many phenomenologically viable models have been constructed. A class of these models, the so-called no-scale models [2], has the interesting property that the scalar potential is identically equal to zero. In this way, the cosmological constant vanishes naturally, that is, without fine tuning of any parameters in the Lagrangian. The electro-weak scale is generated radiatively in this class of models.

The scenario presented above has been extensively studied within the framework of N=1 supergravity. A natural question is whether it is possible to extend it to models with two supersymmetric charges [3] (see also [4] for some recent phenomenological applications of N=2 supersymmetry). For N=2 supersymmetric matter-gauge systems the non-renormalization theorems are much stronger. Except for a one loop beta function renormalization, the theory is finite. A careful choice of representations for the matter fields allows one to cancel even this divergence, and a completely finite theory can be obtained [5]. Moreover it is possible to introduce explicit supersymmetry breaking terms that preserve the finiteness to all orders [6].

The problem of supersymmetry breaking in N=2 supergravity theories has been extensively studied before. The most important conclusion of these studies is that partial breaking from N=2 to N=1 supersymmetry is impossible in Minkowski space-time [7] (for a possible exception, see [8]). Therefore the two supersymmetries must be broken at one scale. An interesting class of models with such a property has been described by Cremmer et al. [9]. One couples a set of vector multiplets containing a vector field, two spin $\frac{1}{2}$ fields and a complex scalar, to supergravity. The couplings of the fields are chosen so that the kinetic terms are not minimal and the potential for the scalar fields is completely flat, generalizing the no-scale N=1 supergravity models.

In this paper, we study models containing besides vector multiplets, also scalar multiplets consisting of spin $\frac{1}{2}$ fields and complex scalar fields. We make extensive use of the formalism due to de Wit et al. [10]. A subset of vector multiplets is introduced to generate supersymmetry breaking via the mechanism of a flat potential. Supersymmetry breaking in this sector feeds down to the rest of the theory to generate effective soft breaking terms.

Our work has been stimulated in part by the recent revival of interest in antigravity [11], a gravitational strength repulsive force. Extended supergravities have the unique property of naturally incorporating the graviphoton, the spin 1 component of the supergravity multiplet, whose exchange may give rise to extremely feeble repulsive interactions. We address the question whether it is possible to generate a small mass for the graviphoton, necessary to give a finite range to its interactions. Another motivation for studying N=2 no-scale supergravity stems from the recent result in superstring theory, that N=1 no-scale models naturally arise [12] from superstring compactification on Calabi-Yau manifolds [13]. It is likely that N=2 no-scale models emerge from alternative superstring compactifications.

This paper is organized as follows. In section II, we give a presentation of the formalism of N=2 supergravity, in the formulation due to Cremmer et al. [9] and de Wit et al. [10]. We derive formulas which are particularly important for phenomenological applications of N=2 supergravity. In section III, we study N=2 no-scale models. We discuss the effects of supergravity induced supersymmetry breaking, in particular mass spectrum, low-energy effective Lagrangian and finiteness. Finally, we conclude in section IV by discussing phenomenological prospects of N=2 extended supergravity. Notation and conventions are summarized in the Appendix.

II. Physical content of gauged N=2 supergravity

In this section we give a brief account of the structure of N=2 supergravity coupled to scalar and vector multiplets. The study of extended supergravity over several years has made it clear that the construction of the theory is most succinctly derived, both in a conceptual and pragmatic sense, from the off-shell formulation of conformal supergravity. We will not dwell on the procedure of obtaining Poincaré supergravity from conformal supergravity [14]. It is, however, worth mentioning, that a set of gauge conditions, which result in the standard kinetic terms of Poincaré supergravity, combined with the elimination of auxiliary fields, yields a set of sigma-model type subsidiary conditions. The physical fields are defined as their solutions.

The content of this section is essentially devoted to the presentation of the formalism of N=2 supergravity [9,10], hopefully in a way accessible to non-experts. We derive formulas which are particularly important for phenomenological applications. Unless explicitly stated, we work in units, in which the gravitational constant is equal to one, that is, $M_{Planck}/\sqrt{8\pi} = 1$.

II-A. Introductory remarks

The physical degrees of freedom of N=2 supergravity coupled to vector and matter multiplets are the following. The only physical fields arising from the Weyl multiplet of conformal supergravity are the vierbein and two gravitinos ψ^i_μ, $i = 1, 2$. One vector multiplet consists of one complex scalar X, two fermions Ω^i and one vector field W_μ. One scalar multiplet consists of two complex scalars represented by the matrix $Q^\alpha{}_i$, $\alpha = 1, 2,\ i = 1, 2$, and two fermions ς^α. Let us recall [3] that under global N=1 supersymmetry, one N=2 vector multiplet decomposes into a vector multiplet and a chiral multiplet, whereas one N=2 scalar multiplet decomposes into two chiral multiplets. We refer the reader to the Appendix for a full explanation of the notation and conventions.

The theory can accommodate an arbitrary number of scalar and vector multiplets. One (or one linear combination) of vector multiplets always acts as a compensating multiplet, and its vector field is supplied to the supergravity sector, transmuting itself into the graviphoton of the N=2 supergravity multiplet $(2,\frac{3}{2},\frac{3}{2},1)$. After solving the aforementioned subsidiary conditions, the remaining components of the compensating multiplet become functions of the other fields, and effectively are eliminated from the physical spectrum. Also one scalar multiplet is eliminated in a similar way. The remaining scalar and vector multiplets represent the physical degrees of freedom. In particular, the vector components of physical vector multiplets can gauge nonabelian as well as abelian groups.

II-B. Constraints and kinetic energy terms

Given the field content and the representation property under the gauge group, the theory is completely specified by the chiral density $F(X^I)$ - a holomorphic function of second degree, that is, $F(\lambda X^I) = \lambda^2 F(X^I)$. The choice of the chiral density determines, among other things, the form of the kinetic terms for the scalar fields X^I. Some choices which lead to non-minimal kinetic terms are of particular interest, since they give rise to flat potentials. This fact was found by Cremmer et al. [9] for gauged N=2 supergravity coupled to vector multiplets only. We will later find a generalization to the case which also includes scalar multiplets.

Let us consider the case of n physical vector multiplets $(X^I, \Omega^{iI}, W_\mu^I)$, $I = 1,\ldots,n$, and r physical scalar multiplets $(Q^\alpha{}_i, \varsigma^\alpha)$, $\alpha = 3,4,\ldots,2r+2$. We label the compensator and the unphysical scalar multiplet by $I = 0$ and $\alpha = 1,2$, respectively. As mentioned before, the requirement that conformal supergravity gives rise to the Einstein action yields a set of subsidiary conditions [9,10] on the scalar fields:

$$\begin{cases} N_{IJ}X^I\bar{X}^J = 1 \ , & I,J = 0,\ldots,n \\ Q^\alpha{}_i d_\alpha{}^\beta Q_\beta{}^i = -2 \ , & \alpha,\beta = 1,2,\ldots,2r+2 \ , \end{cases} \tag{2.1}$$

where $Q_\alpha{}^i \equiv \bar{Q}^\alpha{}_i$. The matrices N and d are given by:

$$N_{IJ} = \tfrac{1}{4}\left(F_{IJ} + \bar{F}_{IJ}\right) \ , \tag{2.2}$$

$$d = diag\,(-\mathbf{1},\mathbf{1},\ldots,\mathbf{1}) \ , \tag{2.3}$$

with $\mathbf{1}$ a unit 2×2 matrix and F_{IJ} defined as the second derivative of $F(X^I)$.

The scalar fields $Q^\alpha{}_i$, in addition, must satisfy a reality constraint:

$$Q_\alpha{}^i = \epsilon^{ij}\rho_{\alpha\beta}Q^\beta{}_j \ , \tag{2.4}$$

where $\rho_{\alpha\beta} = \rho = diag(\epsilon,\ldots,\epsilon)$, and ϵ is a 2×2 completely antisymmetric matrix, $\epsilon^{12} = 1$. Let us write the $(2r+2)\times 2$ matrix $Q^\alpha{}_i$ as:

$$Q = (\,q^0,\ldots,q^r\,)^\sim \ , \tag{2.5}$$

with each q^p, $p = 0,\ldots,r$, a 2×2 matrix. The transposition symbol $^\sim$ means here that Q should be considered as a column in the p-indices, rather than a row. It is not difficult to see, that eq.(2.4) is implemented if one takes each element q^p to be a quaternion:

$$q^p = q_0^p\,\mathbf{1} + i\,\vec{q}^{\,p}\cdot\vec{\sigma} \ , \tag{2.6}$$

where $\vec{\sigma}$ are the usual Pauli matrices. Here q_0 and $\vec{q}$ are real coefficients. Another parametrization for the quaternions, more suitable for the discussion of the mirror symmetry of N=2 supergravity, will be given in the next subsection.

The reality constraint, eq.(2.4), must also be maintained under the gauge transformations:

$$\delta q^p \simeq T^p{}_{p'}\, q^{p'} \ . \tag{2.7}$$

The matrix T must then be a quaternion valued $(r+1) \times (r+1)$ matrix. Given an antihermitean gauge group generator t acting on complex fields, one can explicitly construct T as:

$$T = Re\, t \otimes \mathbf{1} + Im\, t \otimes \epsilon \ , \tag{2.8}$$

where $\mathbf{1}$ and ϵ act on the 2×2 internal quaternionic degrees of freedom. Other forms of generators T, which act in a more complicated way on the internal quaternionic degrees of freedom, are also possible, nevertheless we restrict our attention to the simplest form of eq.(2.8).

Matrices T can generate nonabelian as well as abelian gauge groups. When the generator T of an abelian group acts nontrivially on the first quaternion q^0, eq.(2.5), it also gauges the $SO(2)$ subgroup of the $SU(2)$ automorphism of the supersymmetry algebra; we will return to this point later on. We always denote this group, associated with the graviphoton, by $U(1)_0$, and its generator by T_0.

Eqs.(2.1) are solved explicitly by:

$$X^I \ = \ X^0 z^I \qquad (\, z^0 \equiv 1\,) \ , \tag{2.9}$$

$$|X^0|^{-2} \ \equiv \ Y \ = \ z^I N_{IJ} \bar{z}^J \ , \tag{2.10}$$

$$Q \ = \ c\, (\, 1, q^1, \ldots, q^r\,)^{\sim} \ , \tag{2.11}$$

$$c^{-2} \ \equiv \ 1 - Tr\{q^\dagger q\}/2 \ . \tag{2.12}$$

There is a similar set of conditions for fermions:

$$\begin{cases} X^I N_{IJ} \Omega^{iJ} = 0 & I,J = 0,\ldots,n \\ Q^i{}_\alpha d^\alpha{}_\beta \varsigma^\beta = 0\ , & \alpha,\beta = 1,2,\ldots,2r+2\ , \end{cases} \tag{2.13}$$

which reduces the number of fermionic degrees of freedom. Eqs.(2.13) are solved by:

$$\Omega^{iI} \;=\; \Lambda^{iI} - N_{JK}\bar{X}^I X^J \Lambda^{iK} \qquad (\,\Lambda^{i0} \equiv 0\,), \tag{2.14}$$

$$\varsigma^\alpha \;=\; \xi^\alpha + \{\,QQ^\dagger \xi\,\}^\alpha \qquad (\,\xi^{1,2} \equiv 0\,). \tag{2.15}$$

The fermions Λ^{iI} and ξ^α are now independent fields representing the gauginos and the physical matter fermions, respectively.

The subsidiary conditions discussed so far do not exhaust the list of constraints one has to impose on the fields of N=2 supergravity. The Lagrangian, expressed in terms of the physical fields z^I, Λ^{iI} $(I = 1,\ldots,n)$, and q^α_i, ξ^α $(\alpha = 3,4,\ldots,2r+2)$, defines a nonlinear sigma model [9,10], with the following non-canonical kinetic energy terms:

$$\begin{aligned} \mathcal{L}_{kin} \;=\; & -\mathcal{M}_{IJ} Y^{-1} \partial_\mu z^I \partial^\mu \bar{z}^J - \tfrac{1}{4}\mathcal{M}_{IJ} \bar{\Lambda}^{iJ} \overleftrightarrow{\not\partial} \Lambda^I_i \\ & -c^2\, \Delta^\alpha{}_\beta\, \partial_\mu q^i{}_\alpha \partial^\mu q^\beta_i - 2\,\Delta^\alpha{}_\beta \bar{\xi}_\alpha \overleftrightarrow{\not\partial}\, \xi^\beta\ , \end{aligned} \tag{2.16}$$

where

$$\mathcal{M}_{IJ} \;=\; -N_{IJ} + (N_{IK}\bar{z}^K)(N_{JL} z^L) Y^{-1}\ , \tag{2.17}$$

$$\Delta^\alpha{}_\beta \;=\; \delta^\alpha_\beta + c^2\, q^\alpha_i q^i{}_\beta\ . \tag{2.18}$$

The physical field domains are restricted by the positivity requirement for $\mathcal{L}_{kin}$, that is:

$$Y > 0\ , \qquad c^2 > 0\ ,$$

$$\mathcal{M}_{IJ}\ , \qquad \Delta^{\alpha}{}_{\beta} \qquad \text{positive definite.} \tag{2.19}$$

II-C. Scalar potential

Let us consider the case of a semi-simple gauge group which contains one non-abelian, and an arbitrary number of abelian factors. In this case, the scalar potential [10] is given by:

$$V = V_1 + V_2 + V_3\ , \tag{2.20}$$

$$V_1 \;=\; 2\,(N^{-1})^{IJ} Tr\{\, Q^\dagger d\,(g_I T_I)\, Q\, Q^\dagger d\,(g_J T_J)\, Q\,\} \tag{2.21}$$

$$V_2 \;=\; -4\,Tr\{\, Q^\dagger d\,(g_I T_I)(g_J T_J)\, Q\,\}\, \bar{X}^I X^J \tag{2.22}$$

$$V_3 \;=\; g^2\, N_{IJ}\, f_{KL}^{\ \ I}\, \bar{X}^K X^L\, f_{MN}^{\ \ J}\, \bar{X}^M X^N\ , \tag{2.23}$$

where g and f are the coupling constant and the structure constants, respectively, associated with the nonabelian group. For the abelian factors, the coupling constants g_I correspond to the generators T_I. We normalize the generators by:

$$Tr\{T_I T_J\} = 2\,Tr\{t_I t_J\} = -\delta_{IJ}\,. \tag{2.24}$$

For physical applications, it is useful to decompose a quaternion into two complex scalars. In general, quaternion valued fields which transform as the representation R under the gauge group, are decomposed into the representation $R \oplus \overline{R}$ for complex fields. This decomposition can be readily done by the following projection:

$$Q^{\pm} = \frac{\mathbf{1} \mp i\rho}{2} Q \ . \tag{2.25}$$

The action of the generator $T = Re\, t \otimes \mathbf{1} + Im\, t \otimes \epsilon$ on the projected components $Q^{\pm}$ is reduced to the action of t or its complex conjugate. More explicitly, by writing each quaternion q^p in eq.(2.5) as:

$$q^p = \tfrac{1}{2} \begin{pmatrix} x^p + y_p & i\bar{x}_p - i\bar{y}^p \\ ix^p - iy_p & \bar{x}_p + \bar{y}^p \end{pmatrix} , \tag{2.26}$$

one observes that x and y transform as R and $\overline{R}$, respectively. The particle y is usually called a mirror partner of the particle x. In this way, the quaternionic matrix Q, eq.(2.11), which satisfies all previously mentioned constraints, is reexpressed in terms of the complex fields

$$x \equiv (1, \vec{x})^{\sim} \quad ; \quad y \equiv (1, \vec{y})^{\sim} \ . \tag{2.27}$$

The normalization factor c of eq.(2.11) is given by:

$$c^{-2} = 1 - |\vec{x}|^2/2 - |\vec{y}|^2/2 \ . \tag{2.28}$$

Straightforward calculation allows us to reexpress the scalar potential V, eqs.(2.20)-(2.23), in terms of the physical complex fields. We obtain:

$$V_1 = -(N^{-1})^{IJ} c^4 \left[(A_I - C_I)(\bar{A}_J - \bar{C}_J) + 4\, B_I \bar{B}_J \right] \tag{2.29}$$

$$V_2 = -2\, c^2 Y^{-1} \left(A_{IJ} + C_{IJ} \right) \bar{z}^I z^J \tag{2.30}$$

$$V_3 = g^2 Y^{-2} N_{IJ}\, f_{KL}^{\ \ I}\, \bar{z}^K z^L\, f_{MN}^{\ \ J}\, \bar{z}^M z^N \ , \tag{2.31}$$

where

$$A_I = \bar{x}\, g_I t_I d\, x \tag{2.32}$$

$$B_I = y\, g_I t_I d\, x \tag{2.33}$$

$$C_I = y\, g_I t_I d\, \bar{y} \tag{2.34}$$

$$A_{IJ} = g_I g_J\, \bar{x}\, (t_I t_J + t_J t_I)\, d\, x \tag{2.35}$$

$$C_{IJ} = g_I g_J\, y\, (t_I t_J + t_J t_I)\, d\, \bar{y}\ . \tag{2.36}$$

The $(r+1) \times (r+1)$ dimensional matrix d is defined here as $d = diag(-1, 1, \ldots, 1)$.

Eqs.(2.29)-(2.31) describe the potential in various models which are special cases of the models considered here. For example, if we keep two quaternions ($r = 1$), turn off the vector multiplets except for the compensator ($n = 0$), and set $F(X^0) = (X^0)^2$, $g_0 t_0 \simeq i\, diag(e, \kappa m)$, we recover the potential of the model considered by Zachos [15]. On the other hand, if we turn off the scalar multiplets except for the unphysical one ($r = 0$), and set $g_I t_I = i g_I/\sqrt{2}$, we obtain the potential for the models studied by Cremmer et al. in section 3 of [9].

II-D. Yukawa couplings and supersymmetry breaking

As discussed in the previous subsection, the spectrum of matter scalars exhibits a mirror symmetry, that is, for every scalar in a given representation of the gauge group there exists a mirror partner in the conjugate representation. The spectrum of matter fermions is also mirror-symmetric. By using CPT, this property of the fermion spectrum is often expressed as a left-right symmetry; namely, for every left-handed fermion there exists a right-handed partner in the same representation of the gauge group.

In order to display the mirror symmetry explicitly, we follow a procedure similar to the treatment of quaternions in the previous subsection. We write the fermion ς^α, in the representation R of the gauge group, as:

$$\varsigma = (\varsigma^0, \ldots, \varsigma^r)^\sim \ , \tag{2.37}$$

with each ς^p, $p = 0, \ldots, r$, a two-component column:

$$\varsigma^p = \begin{pmatrix} \chi^p + \eta_p \\ i\chi^p - i\eta_p \end{pmatrix} \ . \tag{2.38}$$

The action of the gauge group generator $T = Re\, t \otimes \mathbf{1} + Im\, t \otimes \epsilon$ on ς is reduced to the action of t and its complex conjugate on χ and η, respectively; χ transforms as R, and its mirror partner η as $\overline{R}$.

The matter fermions ς are subject to the subsidiary conditions of eqs.(2.13), therefore they should be expressed in terms of the physical fields

$$\xi^p = \begin{pmatrix} \varrho^p + \vartheta_p \\ i\varrho^p - i\vartheta_p \end{pmatrix} \qquad (\varrho^0 = \vartheta_0 = 0) \ . \tag{2.39}$$

By using eqs.(2.15) and (2.26), we obtain:

$$\chi^p = \varrho^p + \tfrac{1}{2}c^2\, [\, x^p\, (\bar{x}\varrho + \vartheta\bar{y}) + \bar{y}^p\, (y\varrho - \vartheta x)\,] \tag{2.40}$$

$$\eta_p = \vartheta_p + \tfrac{1}{2}c^2\, [\, y_p\, (\bar{x}\varrho + \vartheta\bar{y}) - \bar{x}_p\, (y\varrho - \vartheta x)\,] \ . \tag{2.41}$$

We are interested in the part of the action $\mathcal{L}_Y$, which contributes to the fermion mass matrices [10]. It is given by:

$$\mathcal{L}_Y = \mathcal{Y}^I_{ij}\, \bar{X}^I\, \bar{\psi}^i_\mu \sigma^{\mu\nu} \psi^j_\nu + \tfrac{1}{2}\mathcal{Y}^I_{ij}\, \bar{\psi}^i \cdot \gamma \Omega^{jI} - 4\bar{X}^I\, \bar{\psi}^i \cdot \gamma \Sigma^{jI} \epsilon_{ij}$$

$$+\tfrac{1}{16}\mathcal{Y}^I_{ij}(N^{-1})^{IJ}\bar{F}_{JKL}\,\bar{\Omega}^{iK}\Omega^{jL}+\tfrac{1}{2}g\,N_{IJ}\,f_{KL}{}^{J}\,X^K\,\bar{\Omega}^{iI}\Omega^{jL}\epsilon_{ij}$$

$$+16i\,\bar{X}^I\,\eta^{\sim}g_It_Id\,C\chi+4\,\bar{\Omega}^{iI}\Sigma^{jI}\epsilon_{ij}+c.c.\;, \tag{2.42}$$

where the charge conjugation matrix C has been defined in the Appendix, eq.(A.7), and F_{IJK} denotes the third derivative of the chiral density $F(X^I)$. The matrices $\mathcal{Y}^I_{ij}$ and the fermion fields Σ^I_i are defined below:

$$\mathcal{Y}^I \;=\; c^2\begin{pmatrix} -2iB_I & A_I-C_I \\ A_I-C_I & 2i\bar{B}_I \end{pmatrix}\;, \tag{2.43}$$

$$\Sigma^I \;=\; c\;(\;\bar{x}\,g_It_Id\,\chi-\eta\,g_It_Id\,\bar{y}\;,\quad i\,y\,g_It_Id\,\chi+i\,\eta\,g_It_Id\,x\;)\;. \tag{2.44}$$

The first term in $\mathcal{L}_Y$, eq.(2.42), is the usual gravitino mass term, the next two are the gravitino-goldstino couplings, and the remaining ones are conventional Yukawa couplings of gauginos and matter fermions. Since the theory is uniquely specified by the gauge group and the chiral density, the list of Yukawa couplings is rather small.

In the next section, we will be considering models with supersymmetry breaking in flat space-time, with zero cosmological constant. In order to determine whether supersymmetry is broken or not, one usually examines the action of supersymmetry transformations on the fermion fields. However, if one is interested in supersymmetry breaking with vanishing cosmological constant only, the part of the action contained in $\mathcal{L}_Y$ provides all information necessary to determine the gravitino masses and eventually, to identify the goldstino fields, whose degrees of freedom are absorbed by gravitinos through the super-Higgs mechanism. Gravitino mass generation is a necessary and sufficient condition for supersymmetry breaking in

Minkowski space-time.

II-E. Graviphoton

We will discuss here some properties of the graviphoton, the gauge boson of the $U(1)_0$ gauge symmetry, already mentioned in the previous subsection. Exchanges of graviphotons may give rise to antigravity forces, which are recently receiving some theoretical attention [11].

We assume here that, of all the gauge group generators, only the generator T_0 of $U(1)_0$ acts nontrivially on the compensating quaternion q^0. We will consider a simple case given by:

$$g_0 T_0 = diag(e_0, e_1, \ldots, e_r) \otimes \epsilon \ . \tag{2.45}$$

Generalization to other forms of the generator T_0 is straightforward.

The compensating quaternion $(q^0)^{\alpha}{}_i = c\delta^{\alpha}_i$ remains invariant under the following symmetry transformation:

$$(\delta q^0)^{\alpha}{}_i = e_0 \epsilon_{\alpha\beta} (q^0)^{\beta}{}_i - e_0 (q^0)^{\alpha}{}_j \epsilon^{ji} = 0 \ . \tag{2.46}$$

Eq.(2.46) implies that the graviphoton effectively gauges the diagonal combination of $U(1)_0$ (acting on the indices α, β) and the $SO(2)$ subgroup of the $SU(2)$ automorphism of the supersymmetry charges (acting on the indices i, j). The relevant couplings of the graviphoton W_0^{μ} [10] are contained in:

$$\mathcal{L}_0 = -Tr(D^{\mu} Q^{\dagger}) d\, (D_{\mu} Q) + \tfrac{1}{4} Tr(Q^{\dagger} d \overleftrightarrow{D}^{\mu} Q)(Q^{\dagger} d \overleftrightarrow{D}_{\mu} Q) \ + \ \text{fermion terms} \ , \tag{2.47}$$

where

$$(D^{\mu} Q)^{\alpha}{}_i = \partial^{\mu} Q^{\alpha}{}_i + g_0 W_0^{\mu} (T_0)^{\alpha}{}_{\beta}\, Q^{\beta}{}_i \tag{2.48}$$

and the fermion terms include the matter fermions as well as the gauge fermions and the gravitinos, since the latter rotate under the $SO(2)$ symmetry transformations.

Suppose now that some scalar fields q^p, $p = 1, 2, \ldots, s$, acquire non-zero vacuum expectation values. Under the symmetry $[U(1)_0 \otimes SO(2)]_{diag}$ these vacuum expectation values transform as:

$$\delta q^p = e_p \, \epsilon \, q^p - e_0 \, q^p \epsilon = e_p \, \epsilon \, q^p - e_0 \, \epsilon \, \bar{q}^p \ , \tag{2.49}$$

where we used the reality constraint, eq.(2.4). We are interested in the criterion for zero graviphoton mass, that is vanishing of the right-hand side of eq.(2.49). Clearly, the right-hand side of eq.(2.49) is non-zero if $|e_p| \neq |e_0|$ for some p. The graviphoton may be massless only for the form of the generator T_0 corresponding to:

$$|e_1| = |e_2| = \ldots = |e_s| = |e_0| \ . \tag{2.50}$$

The criterion for zero graviphoton mass corresponds then to the following condition on the scalar vacuum expectation values:

$$(q^p)^\alpha{}_i = \mathrm{sgn}(e_p/e_0) \, (\bar{q}^p)^\alpha{}_i \tag{2.51}$$

Indeed, by using eq.(2.47) one can easily check that the graviphoton mass is zero if and only if eqs.(2.50) and (2.51) are satisfied.

It is important to note that the symmetry gauged by the graviphoton enters in the supersymmetry algebra in the form of a central charge [16]. The supersymmetry algebra of conformal supergravity [10,14] contains some field-dependent gauge transformations:

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_G(-4\bar{X}^I \bar{\epsilon}_1^i \epsilon_2^j \, \epsilon_{ij} + h.c.) \ + \ \ldots \tag{2.52}$$

As already explained, in the procedure of deriving Poincaré supergravity, X^0 receives a non-zero value, which generates a central charge. Therefore, the supersymmetry algebra acting on the gauge field W_0^μ forces this central charge to be gauged. In this way, the graviphoton becomes the spin 1 component of the $(2, \frac{3}{2}, \frac{3}{2}, 1)$ supergravity multiplet.

III. No-scale models

In this section, we first construct a simple model with a physical vector multiplet and a physical scalar multiplet, where the two supersymmetries are spontaneously broken in flat space-time and the scale of supersymmetry breaking remains undetermined at the classical level. This model enjoys the essential features of N=2 no-scale models with scalar multiplets in a simple way, and can be used as a so-called hidden sector in extended models with a more realistic particle content. Later, we pursue this idea to construct a class of models in which an extra gauge group is built into the theory, with physical fields assigned to some nontrivial representations.

III-A. Flat potential

Let us now come back to the expression for the potential, eqs.(2.29)-(2.31), and consider the case of $n = r = 1$, that is, one physical vector multiplet (z, Λ^i), and one physical scalar multiplet $(x, y, \varrho, \vartheta)$. We further demand that the potential possesses a minimum with a vanishing cosmological constant.

An interesting case, which satisfies the above requirements, arises if we set:

$$F(X^0, X^1) = \frac{-i}{X^0}(X^1)^3, \qquad g_0 t_0 = \frac{ie}{2} diag(1,1) \ , \qquad g_1 t_1 = 0 \ . \tag{3.1}$$

This choice of the chiral density is motivated by [9], where a family of models which exhibit a vanishing potential has been constructed for the theory with vector multiplets only; the above choice corresponds to the simplest possibility. The resultant potential is given by:

$$V = 2e^2c^4Y^{-1}\,|\,x - \bar{y}\,|^2 \tag{3.2}$$

$$c^{-2} = 1 - |x|^2/2 - |y|^2/2 \;, \qquad Y = 2(Im\, z)^3. \tag{3.3}$$

This potential is positive semi-definite and flat along the zero-value direction of $x = \bar{y}$.

In the zero-energy vacuum corresponding to the scalar field configuration of $x = \bar{y}$, the part of the action $\mathcal{L}_Y$, eq.(2.42), responsible for the gravitino masses and the gravitino-goldstino couplings, can be written as:

$$\mathcal{L}_Y^g = -e\,Y^{-1/2}\left[\bar{\psi}_\mu^i + \tfrac{1}{2}i(Im\,z)^2Y^{-1/2}\bar{\Lambda}^i\gamma_\mu\right]\sigma^{\mu\nu}\left[\psi_\nu^i - \tfrac{1}{2}i(Im\,z)^2Y^{-1/2}\gamma_\nu\Lambda^i\right] \;. \tag{3.4}$$

From eq.(3.4) it follows that the two supersymmetries are spontaneously broken. By the super-Higgs mechanism, the two gravitinos absorb the physical degrees of freedom of the gauginos Λ^i, acquiring equal masses of:

$$m_g = e(Im\,z)^{-3/2}/\sqrt{2}\;. \tag{3.5}$$

Since the vacuum expectation value of the scalar field z is arbitrary at the classical level, the gravitino masses, and hence the scale of supersymmetry breaking, remain undetermined in the classical approximation. Together with the natural vanishing of the cosmological constant, these are the characteristic features of the so-called no-scale models, which have been previously studied within the framework of N=1 supergravity [2].

III-B. Model for N=2 no-scale supergravity

We proceed now to the construction of a model which is general enough to allow incorporation of the physical fields in a nontrivial representation of a non-abelian gauge group. Eventually, after including radiative corrections, models of this type might serve as a starting point towards phenomenological applications of N=2 supergravity. The main purpose of this section is to analyse the effects of supersymmetry breaking on the spectrum of physical particles.

The model under consideration is constructed by adding an extra $SU(2)$ gauge group with its vector multiplet, and a scalar matter multiplet in the fundamental representation, to the $U(1)_0 \otimes U(1)_1$ model of the previous subsection. The $SU(2)$ group is being used here as a prototype of any physically interesting symmetries, like the grand-unified or the electro-weak ones. Generalization to other groups and matter representations is straightforward. The model is completely specified by the chiral density F. Motivated by the results of [9], we choose:

$$F(X^0, X^1, \vec{X}) = \frac{-i}{X^0}\left[(X^1)^3 + \alpha X^1 \vec{X}^2\right] \ , \tag{3.6}$$

where $\vec{X} = X^a$ are the gauge scalars in the adjoint representation of $SU(2)$ and α is an arbitrary real negetive parameter, $\alpha < 0$.

The generators of the $U(1)_0 \otimes U(1)_1 \otimes SU(2)$ gauge transformations are defined below:

$$g_0 t_0 = \frac{ie}{2} diag(1,1,0) \ ; \quad g_1 t_1 = 0 \ ; \quad g\vec{t} = \frac{ig}{2} diag(0,0,\vec{\sigma}) \ . \tag{3.7}$$

The generators t_0, t_1 and $\vec{t}$ are block-diagonal, with the upper two blocks acting on the compensators and the hidden sector, respectively. The hidden sector contains the scalars x and y, and chiral fermions ϱ and ϑ, all $SU(2)$ singlets. The lower blocks of the gauge group generators, eq.(3.7), act on the observable sector of matter multiplets. This sector contains scalars $\vec{u}$ and chiral fermions $\vec{\omega}$, in the fundamental representation of $SU(2)$, and their mirror partners $\vec{v}$ and $\vec{\tau}$, in the conjugate representation.

In the following calculations, we will need the matrix N_{IJ} and its inverse, $(N^{-1})^{IJ}$. The matrix N is easily calculated from the chiral density by using eq.(2.2). Inverting this matrix is somewhat tedious, therefore for completeness we list the result below. Let us first introduce the following notation for the real and imaginary parts of the physical scalar components z and z^a of the vector multiplets:
$a = Re\, z$, $b = Im\, z$, $A^a = Re\, z^a$, $B^a = Im\, z^a$.
The matrix $(N^{-1})^{IJ}$ is given by:

$$N^{-1} = \begin{pmatrix} S^{-1} & aS^{-1} & \vec{A}^{\sim} S^{-1} \\ aS^{-1} & a^2S^{-1} - b^2T^{-1} & a\vec{A}^{\sim}S^{-1} + b\vec{B}^{\sim}T^{-1} \\ \vec{A}S^{-1} & a\vec{A}S^{-1} + b\vec{B}T^{-1} & \delta^{ab}(\alpha b)^{-1} + A^aA^bS^{-1} - B^aB^bT^{-1} \end{pmatrix} , \tag{3.8}$$

with

$$S = -\tfrac{1}{2}Y = -b^3 - \alpha b\vec{B}^2 \tag{3.9}$$

$$T = -3b^3 + \alpha b\vec{B}^2 \ . \tag{3.10}$$

Before calculating the potential, let us note that the domains of physical fields are restricted by the positivity requirement for the kinetic energy terms, eqs.(2.19). For the model under consideration, they are:

$$c^{-2} = 1 - |x|^2/2 - |y|^2/2 - |\vec{u}|^2/2 - |\vec{v}|^2/2 > 0 , \tag{3.11}$$

$$b > 0 \qquad b^2 + \alpha\vec{B}^2 > 0 \ . \tag{3.12}$$

The scalar potential is calculated by using eqs.(2.29)-(2.31). We obtain:

$$V = e^2V' + egV'' + g^2V''' \quad , \tag{3.13}$$

$$V' = 2\,c^4Y^{-1}|\,x - \bar{y}\,|^2 + (\,1 - |x|^2/2 - |y|^2/2\,)\,c^4Y^{-1}\,(\,|\vec{u}|^2 + |\vec{v}|^2\,) \tag{3.14}$$

$$\begin{aligned} V'' &= -2c^4Y^{-1}[\,\vec{u}^*A\vec{v}^*(1 - xy) + \vec{v}A\vec{u}(1 - \bar{x}\bar{y})\,] \\ &\quad + c^4Y^{-1}(\,|x|^2 - |y|^2\,)(\,\vec{u}^*A\vec{u} - \vec{v}A\vec{v}^*) \end{aligned} \tag{3.15}$$

$$\begin{aligned} V''' &= -c^4(\alpha b)^{-1}[\,\tfrac{1}{4}(\,\vec{u}^*\sigma^a\vec{u} - \vec{v}\sigma^a\vec{v}^*\,)^2 + (\,\vec{v}\sigma^a\vec{u}\,)(\,\vec{u}^*\sigma^a\vec{v}^*\,)\,] \\ &\quad - c^4S^{-1}[\,\tfrac{1}{4}(\,\vec{u}^*A\vec{u} - \vec{v}A\vec{v}^*\,)^2 + (\,\vec{v}A\vec{u}\,)(\,\vec{u}^*A\vec{v}^*\,)\,] \\ &\quad + c^4T^{-1}[\,\tfrac{1}{4}(\,\vec{u}^*B\vec{u} - \vec{v}B\vec{v}^*\,)^2 + (\,\vec{v}B\vec{u}\,)(\,\vec{u}^*B\vec{v}^*\,)\,] \\ &\quad + c^2Y^{-1}(\vec{A}^2 + \vec{B}^2)(\,|\vec{u}|^2 + |\vec{v}|^2) - \alpha bY^{-2}(\,f_{abc}A^bB^c\,)^2 \,, \end{aligned} \tag{3.16}$$

where:

$$A = \vec{A}\cdot\vec{\sigma}\;, \qquad B = \vec{B}\cdot\vec{\sigma} \tag{3.17}$$

The most interesting property of the potential V, eqs.(3.13)-(3.16), is that it exhibits a zero-value flat direction along the configuration of arbitrary fields $x = \bar{y}$ and an arbitrary field b, with $\vec{u} = \vec{v} = 0$ and $[A, B] = 0$. From now on we restrict our attention to the $SU(2)$ symmetric configuration of $A = B = 0$. It is easy to check that this flat direction corresponds to a local minimum of the potential, ensuring the existence of a zero-energy vacuum. Since the potential is unbounded from below, the vacuum may be unstable for large fluctuations of the scalar fields.

The study of this problem is beyond the scope of this investigation, therefore we assume here, that the tunnelling amplitude is small enough to justify perturbation theory around this zero-energy ground state.

Let us discuss now the Yukawa part of the action, $\mathcal{L}_Y$, which gives rise to the fermion masses in the presence of non-zero vacuum expectation values of the scalar fields. By using eq.(2.42), we obtain:

$$\mathcal{L}_Y = \mathcal{L}_Y^g - \tfrac{1}{8}e\alpha b^{-2}Y^{1/2}\bar{\Lambda}^{ia}\Lambda^{ia} - 8ec^2Y^{-1/2}\varrho^{\sim}C\vartheta \ + \ c.c., \tag{3.18}$$

where $\mathcal{L}_Y^g$ is given in eq.(3.4).

As in the simple model discussed before, the gravitinos acquire equal masses:

$$m_g = eb^{-3/2}/\sqrt{2} \tag{3.19}$$

and the scale of supersymmetry breaking is undetermined at the classical level.

We are now in a position to determine the full mass spectrum of the model. The only ingredient, which remains to be calculated in order to obtain the masses, is the kinetic energy part of the Lagrangian, $\mathcal{L}_{kin}$. From eq.(2.16), we obtain:

$$\begin{aligned}\mathcal{L}_{kin} \ &= \ -\tfrac{1}{4}\alpha b\,\bar{\Lambda}^{ia}\overleftrightarrow{\not\partial}\Lambda_i^a - 4c^2\bar{\varrho}\overleftrightarrow{\not\partial}\varrho - c^4\partial_\mu x\,\partial^\mu\bar{x} - c^2\partial_\mu\vec{u}\,\partial^\mu\vec{u}^* \\ &\quad + (\,x\to y,\ \vec{u}\to\vec{v},\ \varrho\to\vartheta\,)\ .\end{aligned} \tag{3.20}$$

In eq.(3.20) we omitted the kinetic energy terms for the particles, which are explicitly massless due to the form of the scalar potential, eqs.(3.13)-(3.16), and the Yukawa terms $\mathcal{L}_Y$, eq.(3.18), that is, all scalar components of the vector multiplets, and the matter fermions $\vec{\omega}$ and $\vec{\tau}$. Also, the kinetic terms for the goldstinos Λ^i have been suppressed, since by the standard lore of the super-Higgs mechanism, they contribute to the gravitino mass term.

By using eqs.(3.13)-(3.16), (3.18) and (3.20), one easily computes the mass spectrum of the model under consideration. The masses of all particles turn out to be integer multiples of the gravitino mass. The hidden sector contains one Dirac fermion and one scalar, with masses equal to two gravitino masses, and due to the trough nature of the scalar potential, one massless dilaton.

In the observable sector, the gauginos Λ^{ia} of the $SU(2)$ gauge group acquire masses equal to the gravitino mass. However, the most interesting effect of supersymmetry breaking is that it generates a mass splitting inside the scalar matter multiplets. The scalars $\vec{u}$ and $\vec{v}$ receive masses equal to the gravitino mass whereas their fermionic partners, $\vec{\omega}$ and $\vec{\tau}$, remain massless. Such a mass generation for scalars is well known in N=1 supergravity, where it provides a starting point for the construction of some realistic models [1]. Our simple model provides the first example of such a mechanism in the framework of N=2 supergravity. A systematic study of mass matrices would be helpful to understand, to what extent such a mass pattern is general. We summarize the mass spectrum of particles with spin 0 and $\frac{1}{2}$ in the table below:

Spin 0, $\frac{1}{2}$ Mass Spectrum			
Spin 0	Mass	Spin $\frac{1}{2}$	Mass
$\frac{1}{\sqrt{2}}(x-\bar{y})$	$2m_g$	$\varrho\ \vartheta$	$2m_g$
$\frac{1}{\sqrt{2}}(x+\bar{y})$	0		
$a\ b$	0		
$A^a\ B^a$	0	Λ^{ia}	m_g
$\vec{u}\ \vec{v}$	m_g	$\vec{\omega}\ \vec{\tau}$	0

We close this subsection with a couple of remarks on the $U(1)_0 \otimes U(1)_1$ gauge interactions. By construction, $t_1 = 0$, which means that $U(1)_1$ is essentially decoupled, playing a sort of auxiliary role. From eqs.(2.50) and (2.51) it follows that the $U(1)_0$ group remains unbroken for the vacuum expectation values of the scalar fields corresponding to the minimum of the potential V, eqs.(3.13)-(3.16). The massless graviphoton couples to matter with the coupling constant

$$e = \sqrt{2b^3}\, m_g \ , \tag{3.21}$$

which is proportional to the gravitino mass.

III-C. Effective Lagrangian and finiteness

In this subsection, we study in more detail the structure of the Lagrangian for the 'observable' $SU(2)$ sector in the limit, where it decouples from the 'hidden' $U(1)_0 \otimes U(1)_1$ sector. As in the N=1 supergravity case, the effective Lagrangian is expected to describe a globally supersymmetric Yang-Mills model, with possible soft supersymmetry breaking terms. Interaction between the hidden and observable sectors should be suppressed by factors of $O(m_g/M_{Planck})$.

In order to derive such an effective Lagrangian for the model under consideration, we make the following rescalings:

$$\begin{aligned}
g \ &\rightarrow \ (-\alpha b)^{1/2} g \ ; & \vec{W}_\mu \ &\rightarrow \ (-\alpha b)^{-1/2} \vec{W}_\mu \\
\{\vec{A},\, \vec{B}\} \ &\rightarrow \ (-\alpha b/Y)^{-1/2} \{\vec{A},\, \vec{B}\} \ ; & \vec{\Lambda} \ &\rightarrow \ (-\alpha b)^{-1/2} \vec{\Lambda} \\
\{\vec{u}\,, \vec{v}\} \ &\rightarrow \ c^{-1} \{\vec{u}\,, \vec{v}\} \ ; & \{\vec{\omega}\,, \vec{r}\} \ &\rightarrow \ c^{-1} \{\vec{\omega}\,, \vec{r}\} \ ,
\end{aligned} \tag{3.22}$$

where the scalar fields in the scaling factors should be understood as their vacuum expectation values.

After these rescalings, it is straightforward to take the limit $M_{Planck} \to \infty$, with m_g kept constant. In this limit, there is indeed no interaction between the $U(1)_0 \otimes U(1)_1$ and $SU(2)$ sectors of the model. The effective Lagrangian for the observable $SU(2)$ sector is precisely given by the standard N=2 globally supersymmetric Yang-Mills Lagrangian. There are however some explicit supersymmetry breaking terms present. They are given by:

$$\mathcal{L}_{SB} = -m_g \bar{\Lambda}^{ia}\Lambda^{ia} - m_g^2(|\vec{u}|^2 + |\vec{v}|^2) - 2gm_g(\vec{u}^* A \vec{v}^* + \vec{v} A \vec{u}) \ . \tag{3.23}$$

Similarly to the case of spontaneously broken N=1 supergravity, these terms are soft, in the sense that they do not generate quadratic divergences [6,17].

In the context of N=2 supergravity, it is very interesting to consider a theory that would be finite without the soft breaking terms. For an $SU(2)$ theory this would be a model analogous to the model considered in the previous subsection, but with four pairs of doublets $\vec{u}_n, \vec{v}_n$, $n = 1,2,3,4$, since then the beta function vanishes [5,6]. The generated soft terms are of the same form for each doublet.

For further discussion, it is convenient to decompose the multiplets of N=2 supersymmetry into the multiplets of N=1 supersymmetry. The vector multiplet $(A^a, B^a, \Lambda^{ia}, W_\mu^a)$ decomposes into the chiral multiplet (A^a, B^a, Λ^{1a}) in the adjoint representation of the gauge group, and the vector multiplet (Λ^{2a}, W_μ^a). The scalar multiplet $(\vec{u}, \vec{v}, \vec{\omega}, \vec{r})$ decomposes into the chiral multiplets $(\vec{u}, \vec{\omega})$ and $(\vec{v}, \vec{r})$, in the fundamental representation and its conjugate, respectively.

The results of [6] indicate that there are basically four different types of soft terms that preserve the finiteness of the theory:

(i) N=1 supersymmetric mass terms, for example a supersymmetric mass term for the chiral multiplet in the adjoint representation:

$$-m\bar{\Lambda}^{1a}\Lambda^{1a} - gmA^a(\vec{u}^*\sigma^a\vec{v}^* + \vec{v}\sigma^a\vec{u}) - igmB^a(\vec{u}^*\sigma^a\vec{v}^* - \vec{v}\sigma^a\vec{u}) - m^2(\vec{A}^2 + \vec{B}^2) \tag{3.24}$$

(ii) A mass term for the gaugino, obtained by an $SO(2)$ rotation of (3.24):

$$-m\bar{\Lambda}^{2a}\Lambda^{2a} - gmA^a(\vec{v}\sigma^a\vec{u} + \vec{u}^*\sigma^a\vec{v}^*) - igmB^a(\vec{v}\sigma^a\vec{u} - \vec{u}^*\sigma^a\vec{v}^*) - m^2(\vec{A}^2 + \vec{B}^2) \tag{3.25}$$

If the $SO(2)$ subgroup of the $SU(2)$ automorphism of the supersymmetry algebra is unbroken, as in the models studied in this section (more precisely, the unbroken group gauged by the graviphoton corresponds to $[SO(2) \otimes U(1)_0]_{diag}$), this term must combine with the mass term (3.24) for the chiral multiplet in the adjoint representation, to form the combination:

$$\mathcal{L}_{F_1} = -m\bar{\Lambda}^{ia}\Lambda^{ia} - 2gm(\vec{u}^*A\vec{v}^* + \vec{v}A\vec{u}) - 2m^2(\vec{A}^2 + \vec{B}^2) \tag{3.26}$$

(iii) Terms of the form $(\vec{A}^2 - \vec{B}^2)$, which break parity and are therefore absent in our model.

(iv) Parity-conserving mass terms of the form:

$$-\sum_R[\mu_R^2(C_R^2 + D_R^2) \quad + \quad \mu_{\bar{R}}^2(C_{\bar{R}}^2 + D_{\bar{R}}^2)] \ + \ \mu^2(\vec{A}^2 + \vec{B}^2) \tag{3.27}$$

$$\mu_R^2 + \mu_{\bar{R}}^2 \quad = \quad \mu^2 \quad \text{for each representation } R, \tag{3.28}$$

where $C_{R,\bar{R}}$ and $D_{R,\bar{R}}$ are the scalar components of chiral multiplets in the representations R and $\overline{R}$, respectively. Due to the $SO(2)$ symmetry, in the model under consideration, (3.27) corresponds to:

$$\mathcal{L}_{F_2} = -\mu^2[\,|\vec{u}_n|^2 + |\vec{v}_n|^2 - 2(\vec{A}^2 + \vec{B}^2)] \tag{3.29}$$

We notice that the supersymmetry breaking terms $\mathcal{L}_{SB}$, eq.(3.23), generated by N=2 supergravity, are equal to the sum $\mathcal{L}_{F_1} + \mathcal{L}_{F_2}$, eqs.(3.26) and (3.29), with $m = \mu = m_g$. From the construction it is obvious, that this result holds not only for an $SU(2)$ gauge group, but also for arbitrary gauge groups. We conclude therefore, that the soft breaking terms, induced by supergravity in a finite N=2 supersymmetric Yang-Mills theory, preserve the finiteness of that theory. This shows that it is a meaningful procedure to demand a N=2 supersymmetric Yang-Mills theory to be finite, since at least its coupling to supergravity does not spoil the finiteness. Another way to generate some finiteness preserving terms is through a dimensional reduction from a higher dimensional theory [18]. This suggests that the no-scale models considered here correspond to some self-consistent superstring compactifications.

IV. Conclusions and outlook

In this paper, we discussed the N=2 extended supergravity theory, as formulated by Cremmer et al. [9] and de Wit et al. [10]. We derived formulas which are particularly useful for phenomenological applications. We presented a specific class of the so-called no-scale models, which exhibit spontaneous supersymmetry breaking in flat space-time. In these models, the two supersymmetries are broken at one scale, undetermined at the classical level. The cosmological constant vanishes naturally in the classical Lagrangian, without fine tuning of any parameters.

We analysed the effective low-energy Lagrangian, in the limit $M_{Planck} \to \infty$, with m_g kept constant. In the presence of supergravity induced supersymmetry breaking terms, the theory remains free of troublesome quadratic divergences. Moreover, it is possible to choose a representation content in such a way, that the model is finite in the presence of these soft breaking terms.

Supergravity induced supersymmetry breaking generates effective mass terms for spin $\frac{1}{2}$ components of the vector gauge multiplets and spin 0 components of the scalar matter multiplets. In this way supergravity offers a mechanism for generating phenomenologically desirable non-zero masses of squarks, sleptons and gauginos. On the other hand, phenomenologically viable models always involve spontaneous breaking of local gauge symmetries, like the electro-weak symmetry, by the vacuum expectation values of some scalar fields. In the presence of positive mass terms, it is very difficult to induce such vacuum expectation values, at least in the classical approximation. This problem is common to all supergravity theories, including the N=1 models. Due to N=2 supersymmetry, the models under consideration exhibit also a mirror symmetry, which is unwanted at low energies. The spectrum of fermions is left-right symmetric and Yukawa couplings are extremely restrictive. The problems of gauge symmetry breaking, left-right symmetry breaking and fermion mass generation present therefore a serious challenge for model builders.

The mechanism commonly employed to resolve the problem of gauge symmetry breaking in the framework of N=1 supergravity is based on the observation that in some situations, after supersymmetry has been broken spontaneosly, the radiative corrections may be large enough to generate non-zero vacuum expectation values of some scalar fields [1,2]. Hence it is very important to analyse the radiative corrections to the N=2 supergravity action, in order to resolve the question whether it is possible to radiatively induce spontaneous breaking of the gauge and mirror symmetries. It is a logical possibility, that radiative corrections also determine the gravitino masses of order of the electro-weak scale, therefore solving the naturalness problem.

In spite of supersymmetry breaking, at the classical level the graviphoton remains massless. For the gravitino mass of order of the electro-weak scale, the graviphoton coupling constant $e \sim m_g/M_{Planck}$ is very small, of order 10^{-17}. The assumption, that radiative corrections break the $U(1)_0$ symmetry at a scale comparable to the electro-weak scale, leads to an interesting conjecture, that the graviphoton is very light, with a mass of order $e\, m_g \sim 10^{-6} eV$. In this way, N=2 supergravity would give rise to extremely feeble antigravity forces, with a characteristic length scale of order 1 meter.

Finally, we would like to comment on the possible connection of N=2 no-scale models with superstrings. If it were possible to obtain the N=2 no-scale models by superstring compactification from ten dimensions, this would provide a natural mechanism for supersymmetry breaking, which is absent in the N=1 case. Moreover, at least at the classical level, baryon number would be conserved for the simple reason that no superpotential is allowed, therefore the baryon number violating Yukawa couplings are absent.

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Appendix: Conventions and spinor algebra

The conventions used in this paper follow [9,10]. We list here the features most relevant to practical calculations.

A convenient representation for the Dirac matrices is given by:

$$\vec{\gamma} = \begin{pmatrix} 0 & i\vec{\sigma} \\ -i\vec{\sigma} & 0 \end{pmatrix} \qquad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{A.1}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.2}$$

The matrices $\sigma^{\mu\nu}$ are defined as:

$$\sigma^{\mu\nu} = \tfrac{1}{4}\,[\,\gamma^\mu, \gamma^\nu\,] \tag{A.3}$$

Gravitinos, gauge fermions and matter fermions are represented by Weyl spinors, with respective chiralities determined by:

$$\gamma_5\,\psi_\mu^i = \psi_\mu^i\,, \qquad \gamma_5\,\Omega_i^I = \Omega_i^I\,, \qquad \gamma_5\,\zeta_\alpha = \zeta_\alpha\,. \tag{A.4}$$

Spinorial Lorentz indices are always suppressed. The indices $i, j, \ldots = 1, 2$ label representations of the $SU(2)$ automorphism of the supersymmetry algebra. The indices $I, J, \ldots = 0, 1, \ldots, n$ label the vector multiplets and the generators which they gauge. The indices $\alpha, \beta, \ldots = 1, 2, \ldots, 2r+2$ label scalar multiplets; the gauge group may act on these indices.

The indices of type $m = \alpha, \beta, \ldots;\ i, j, \ldots$ are raised and lowered by complex conjugation, up to multiplication by γ_2; namely, for a generic fermion λ_m ,

$$\lambda^m \equiv \gamma_2(\lambda_m)^* \qquad (\, m = \alpha, \beta, \ldots ;\ i, j, \ldots\,). \tag{A.5}$$

Hence lowering or raising the indices of type m flips the chirality of a fermion. Eq.(A.5) implies the following property of the Dirac conjugation:

$$\bar{\lambda}^m \equiv (\lambda_m)^\dagger \gamma_4 = (\lambda^m)^\sim C \ , \tag{A.6}$$

where the charge conjugation matrix C is given by:

$$C = \gamma_2 \gamma_4 = \begin{pmatrix} i\sigma_2 & \mathbf{0} \\ \mathbf{0} & -i\sigma_2 \end{pmatrix} . \tag{A.7}$$

Under complex conjugation:

$$(\bar{\lambda}^m \lambda^n)^* = \bar{\lambda}_m \lambda_n \ . \tag{A.8}$$

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