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YET ANOTHER POSSIBLE EXPLANATION OF THE SOLAR-NEUTRINO PUZZLE

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Abstract

Mikheyev and Smirnov have shown that the interactions of neutrinos with matter can result in the conversion of electron neutrinos produced in the center of the sun to muon neutrinos. Bethe has exploited this and has pointed out that the solar-neutrino puzzle can be resolved if the mass difference squared of the two neutrinos is $m_2^2 - m_1^2 \simeq 6 \times 10^{-5}$ eV^2, and the mixing angle satisfies $\sin\theta_V > 0.0065$. We discuss a qualitatively different solution to the solar-neutrino puzzle which requires $1.0 \times 10^{-8} < (m_2^2 - m_1^2)(\sin^2 2\theta_V / \cos 2\theta_V) < 6.1 \times 10^{-8}$ eV^2. Our solutions result in a much smaller flux of neutrinos from the $p - p$ process than predicted by standard solar models, while Bethe's solution results in a flux of neutrinos from the $p - p$ process that is about the same as standard solar models.

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If neutrinos are massive, there is no reason to believe the weak interaction eigenstates and the mass eigenstates are identical. We denote the weak interaction eigenstates as $|\nu_e\rangle$ and $|\nu_\mu\rangle$, and the mass eigenstates as $|\nu_1\rangle$ and $|\nu_2\rangle$. [1] The two basis are related by a rotation of angle θ_V

$$|\nu_e\rangle = \cos\theta_V|\nu_1\rangle + \sin\theta_V|\nu_2\rangle, \qquad |\nu_\mu\rangle = -\sin\theta_V|\nu_1\rangle + \cos\theta_V|\nu_2\rangle \tag{1}$$

where θ_V is the vacuum mixing angle (assumed to be less than 45°).

If we write the neutrino wavefunction $|\nu(t)\rangle$ in the mass eigenstate basis, $|\nu(t)\rangle = \nu^i(t)|\nu_i\rangle$ $(i = 1,2)$, the time evolution of $\nu^i(t)$ is determined by the Wigner - Weisskopf equation

$$i\frac{d}{dt}\begin{bmatrix}\nu^1\\ \nu^2\end{bmatrix} = \mathbf{H}\begin{bmatrix}\nu^1\\ \nu^2\end{bmatrix}, \tag{2}$$

with $\mathbf{H}$ for vacuum oscillations given by

$$\mathbf{H} = \begin{pmatrix} E_1 & 0\\ 0 & E_2\end{pmatrix}, \tag{3}$$

where $E_i = (k^2 + m_i^2)^{1/2}$. The solution for $\nu^i(t)$ is $\nu^i(t) = \exp(-iE_it)\nu^i(0)$ (no sum on i).

As a result of the difference in the time evolution of ν^1 and ν^2, a state $|\nu(t)\rangle$ that at time $t = 0$ is pure $|\nu_e\rangle$ (i.e, $\nu^1(0) = \cos\theta_V$, $\nu^2(0) = -\sin\theta_V$), becomes a mixture of $|\nu_1\rangle$ and $|\nu_2\rangle$ after propagating in vacuum a distance $x = t$.[2]

$$\begin{aligned} |\nu(t)\rangle &= \cos\theta_V e^{-iE_1t}|\nu_1\rangle - \sin\theta_V e^{-iE_2t}|\nu_2\rangle \\ &= \cos\theta_V|\nu_1\rangle - \sin\theta_V \exp(-i2\pi x/l_V)|\nu_2\rangle, \end{aligned} \tag{4}$$

where l_V is the vacuum oscillation length given by

$$l_V \equiv 2\pi/(E_2 - E_1) \simeq 4\pi k/\Delta. \tag{5}$$

In the definition of l_V we have made the assumption that the neutrino momentum k is much greater than m_i, and that $\Delta \equiv m_2^2 - m_1^2$ is positive. The probability that a state originally $|\nu_e\rangle$ has oscillated into a state $|\nu_\mu\rangle$ is given by

[1] For simplicity we only consider two neutrino flavors. Extention of our results to more than two flavors is straightforward. The flavor state labeled ν_μ could just as well be ν_τ.

[2] The *overall* phase of $|\nu(t)\rangle$ is irrelevant, and throughout we will take advantage of the freedom to remove any *overall* phase factor. Only the *relative* phase between $|\nu_1\rangle$ and $|\nu_2\rangle$ is important for flavor oscillations.

$$P(\nu_\mu;t) = |\langle\nu_\mu|\nu(t)\rangle|^2 = \frac{1}{2}\sin^2 2\theta_V \left(1 - \cos\frac{2\pi x}{l_V}\right). \tag{6}$$

Because $\nu_e - e$ scattering proceeds via both charged and neutral current processes, while $\nu_\mu - e$ scattering proceeds only via neutral currents, the matter interactions of ν_e and ν_μ are different. For this reason, the vacuum energy eigenstates are not the same as the matter energy eigenstates, and $\mathbf{H}$ in matter must be replaced by $\mathbf{H}_M$, given by [1, 2]

$$\mathbf{H}_M = \begin{pmatrix} E_1 + \sqrt{2}G_F n_e \cos^2\theta_V & \sqrt{2}G_F n_e \cos\theta_V \sin\theta_V \\ \sqrt{2}G_F n_e \cos\theta_V \sin\theta_V & E_2 + \sqrt{2}G_F n_e \sin^2\theta_V \end{pmatrix}, \tag{7}$$

where G_F is Fermi's constant and n_e is the electron density. [3]

The energy eigenvalues in matter, found by diagonalizing $\mathbf{H}_M$, are not the same as the energy eigenvalues in vacuum, and the fact that $\mathbf{H}_M$ is not diagonal implies the energy eigenstates in matter are not the same as the energy eigenstates in vacuum. The energy eigenvalues are given by

$$\lambda_\pm = \frac{1}{2}(E_2 + E_1 + A) \pm \frac{1}{2}\left[\left(\frac{\Delta}{2k}\cos 2\theta_V - A\right)^2 + \left(\frac{\Delta}{2k}\right)^2 \sin^2 2\theta_V\right]^{1/2}, \tag{8}$$

where $A \equiv \sqrt{2}G_F n_e = 3.67 \times 10^{-14} 2\rho Y_e$ eV (ρ is in g cm^{-3} and Y_e is the number of electrons per nucleon).

The oscillation length in matter l_M is determined by the difference of the energy eigenvalues:

$$2\pi l_M^{-1} = \lambda_+ - \lambda_- = \left[\left(\frac{\Delta}{2k}\cos 2\theta_V - A\right)^2 + \left(\frac{\Delta}{2k}\right)^2 \sin^2 2\theta_V\right]^{1/2}. \tag{9}$$

The difference between the energy eigenvalues is density dependent since $A = A(\rho)$, and has the form of a resonance, with width $\delta A = 2(\Delta/2k)\sin 2\theta_V$. The resonance occurs at a value of A which minimizes $\lambda_+ - \lambda_-$, $A_{res} = (\Delta/2k)\cos 2\theta_V$. Qualitatively, at high density the mass eigenstates are almost pure ν_e (λ_+) and pure ν_μ (λ_-), while at low density the mass eigenstates are the vacuum mass eigenstates, which for $\theta_V \ll 1$ and $\Delta > 0$ are ν_μ (λ_+) and ν_e (λ_-). At resonance, the mass eigenstates are fully mixed, $\nu^{1,2} = (\nu_e \pm \nu_\mu)/\sqrt{2}$.

[3] We have ignored the neutral-current contribution to $\mathbf{H}_M$ since it is the same for ν_e and ν_μ and can give only an overall phase.

Due to the resonance, it is possible to fully rotate an electron neutrino to a muon neutrino [3]. For this to occur two conditions are necessary. First, the neutrino must be created at a density large enough that $A > A_{res}$, or $2\rho Y_e > 1.36 \times 10^7 (\Delta_{eV}/k_{MeV}) \cos 2\theta_V$, where Δ_{eV} is Δ in units of eV^2, and k_{MeV} is k in units of MeV. The second condition for near complete extinction of ν_e is that the neutrino must pass through the resonance in an adiabatic manner. Adiabatic here means that a neutrino that begins in the λ_+ (λ_-) eigenstate traverses the resonant region and emerges in the λ_+ (λ_-) eigenstate without level mixing. For $\Delta > 0$ this means a ν_e created at high density emerges as a ν_μ at low density. The condition for adiabaticity is that the matter oscillation length in resonance,

$$l_M(res) = l_M(A = A_{res}) = \frac{4\pi k}{\Delta \sin 2\theta_V}, \tag{10}$$

be smaller than the characteristic length δr associated with the width of the resonance region. Physically, this means $|\nu(t)\rangle$ should have several oscillations in the resonance region. Over the width of the resonance, the density changes by $\delta n_e = \delta A/(\sqrt{2}G_F) = 2(\Delta/2k)\sin 2\theta_V/(\sqrt{2}G_F)$, or $\delta\rho/\rho_{res} = \delta A/A_{res}$. The corresponding distance δr is

$$\delta r = \left[-\frac{1}{\rho}\frac{d\rho}{dr}\right]^{-1} (\delta\rho/\rho_{res}) = \left[-\frac{1}{\rho}\frac{d\rho}{dr}\right]^{-1} 2\tan 2\theta_V. \tag{11}$$

For regions in the sun where it is possible to describe the density profile as an exponential atmosphere, i.e., $\rho(r) = \rho_0 \exp(-r/R_S)$ (R_S is the scale height), $-\rho^{-1}d\rho/dr = R_S^{-1}$ is a constant, so $\delta r \simeq 2R_S \tan 2\theta_V$. This is a good approximation for 44 g cm$^{-3} \geq 2Y_e\rho \geq 10^{-3}$ g cm^{-3}, which corresponds to $0.23R_\odot \lesssim r \lesssim R_\odot$, $R_\odot = 6.96 \times 10^{10}$ cm is the radius of the sun. For the remainder of the paper we will assume resonance occurs in such a region, and we will take $R_S = 0.092R_\odot$.

The degree of adiabaticity at resonance depends upon the parameter [4, 5]

$$\xi = \delta r/l_M(res) = \frac{R_S}{2\pi}\frac{\Delta}{k}\frac{\sin^2 2\theta_V}{\cos 2\theta_V} \simeq 2.2 \times 10^8 \frac{\Delta_{eV}}{k_{MeV}}\theta_V^2. \tag{12}$$

In the first three Figures, we show results of a numerical evaluation of Eq. 2 with $\mathbf{H}_M$ given by Eq. 3. We have assumed that the neutrino is pure ν_e at high density, and we integrate the equations to find $\nu^1(t)$ and $\nu^2(t)$. We then calculate the probability that the neutrino is a ν_e by evaluating $P(\nu_e; t) = |\langle\nu_e|\nu(t)\rangle|^2$. In Figure 1 the resonance is adiabatic, while in Figure 2 the resonance is non-adiabatic. In Figure 3 the resonance is in the transition region between adiabatic and non-adiabatic. To a very good approximation for small θ_V, the transmission probablity depends upon Δ, k, and θ_V only through the scaling variable ξ. The transmission probability for a ν_e generated at $\rho \gg \rho_{res}$ as a function of ξ is shown in Figure 4. As expected, for $\xi \ll 1$, the resonance is non-adiabatic and the transmission factor $P(\nu_e) \equiv P(\nu_e; t = \infty) \simeq 1$, while for $\xi \gtrsim 1$, the resonance is adiabatic and $P(\nu_e) \simeq 0$.

The resonant oscillation of neutrinos in the sun has been proposed as a possible solution to the solar neutrino problem [3, 5]. Bethe [5] has pointed out that if $\Delta \cos 2\theta_V = 4.5 \times 10^{-5}$ eV^2, and passage through the resonance occurs adiabatically, then electron neutrinos with $k \gtrsim 5$ MeV will be created in the center of the sun above resonance density and emerge as muon neutrinos. Neutrinos with $k \lesssim 5$ MeV will be created in the sun below resonance density and will emerge unscathed. According to standard solar models [6], electron neutrinos from decay of ^{8}B with $k \gtrsim 5$ MeV contribute about 5 SNU to the ^{37}Cl solar-neutrino experiment, while neutrinos from all the other processes have $k \lesssim 5$ MeV and contribute about 2 SNU. In order for the high-energy neutrinos to go through resonance adiabatically, Bethe estimates that $\tan^2 2\theta_V \gtrsim 4 \times 10^{-4}$. In Bethe's solution the low-energy neutrinos are unaffected by the resonance. The proposed ^{71}Ga solar-neutrino experiment is sensitive to these low-energy neutrinos, and the ^{71}Ga SNU rate from the low energy neutrinos should be close to the prediction of the standard solar model.

We will now point out another solution that is qualitatively and observationally different, in that all the low-energy neutrinos are also rotated to muon neutrinos. For our solution, we arrange that the ^{8}B neutrinos are created well above ρ_{res}, and then traverse the resonance region quasi-adiabatically (i.e., $\xi \lesssim 1$), so that $P(\nu_e) \simeq \frac{1}{3}$, thereby cutting out all but 2 SNU from the ^{8}B process. The lower-energy neutrinos from all other processes have lower energy (larger ξ) and go through resonance adiabatically, leading to their near-complete rotation to ν_μ.

In the region $-4 < \ln(\xi) < 0$, the transmission factor can be fit as

$$P(\nu_e) \simeq -\frac{1}{4} \ln \xi \simeq \frac{1}{4} \ln(ax), \tag{13}$$

where $a^{-1} = 1.57 \times 10^7 \Delta_{eV} \theta_V^2$, and $x = k/Q$ (Q is the Q-value of ^{8}B decay, $Q = 14$

MeV). The flux of ^{8}B neutrinos given by the standard solar model is (with no oscillation) [6]

$$\frac{dN_\nu}{dx} \propto x^2(1-x)^2 \tag{14}$$

and the cross-section for neutrino capture by ^{37}Cl is [7]

$$\sigma \propto x^n, \tag{15}$$

where $n = 2.85$ for $1 < E_\nu < 5$ MeV, and $n = 3.7$ for $8 < E_\nu < 14$ MeV.

If the neutrinos created in the center of the sun go through resonance in the region of the sun where $\rho(r) = \rho_0 \exp(-r/R_S)$, and $-4 < \ln\xi < 0$, then the differential flux of ν_e at earth is

$$\frac{dN_\nu}{dx} \propto x^2(1-x)^2 \frac{1}{4}\ln(ax). \tag{16}$$

The ^{8}B neutrinos contribute to the ^{37}Cl experiment a number of SNUs proportional to

$$SNU \propto \int_{x_1}^{1} x^{n+2}(1-x)^2 P(\nu_e)\ dx \tag{17}$$

where $x_1 = E_{th}/Q$ ($E_{th} = 0.82$ MeV is the detector threshold). With oscillation, the SNUs from ^{8}B is reduced by the factor

$$F = \frac{\int_{x_1}^{1} x^{n+2}(1-x)^2 \frac{1}{4}\ln(ax)\ \ dx}{\int_{x_1}^{1} x^{n+2}(1-x)^2\ \ dx} \tag{18}$$

Requiring that the neutrinos from ^{8}B give 2.1 ± 0.3 SNU [8] rather than the 4.3 ± 0.7 SNU from the standard solar model [6, 8] (all uncertainties are 1σ) implies

$$1.0 \times 10^{-8} < \Delta_{eV} \frac{\sin^2 2\theta_V}{\cos 2\theta_V} < 6.1 \times 10^{-8} \tag{19}$$

We note that with the value of ξ necessary to give 2.1 ± 0.3 SNU from the ^{8}B neutrinos, the lower energy neutrinos from all other processes will be almost completely rotated to ν_μ. For instance, there are two monoenergetic neutrinos from ^{7}Be decay with energies 1.44 and .861 MeV, which contribute 0.23 and 1.02 SNUs to the ^{37}Cl experiment [6]. For the value of $\Delta \sin^2 2\theta_V / \cos 2\theta_V \simeq 1.0 \times 10^{-8}$, $\ln\xi \simeq -0.96$ for the 1.44 MeV neutrinos, which results in $P(\nu_e) \leq 0.1$. Lower energy neutrinos have a larger value of ξ, and hence are even more suppressed.

Let us review the conditions necessary for our scenario to lead to $\simeq 2.1$ SNU from ^{8}B with a drastically reduced flux of low-energy neutrinos. (1) In the adiabatic regime ($\xi \gtrsim 1$) $P(\nu_e) \propto \sin^2\theta_V \cos^2\theta_V$, and θ_V must be small in order for ν_e to be rotated completely into ν_μ. We estimate that $\sin\theta_V \lesssim 0.3$ will result in a sufficient rotation of ν_e to drastically reduce the low-energy neutrinos. We note that since Bethe requires near complete extinction of the high-energy neutrinos from ^{8}B, his solution also requires $\sin\theta_V \lesssim 0.3$. (2) Neutrinos above the ^{71}Ga threshold of 0.236 MeV must be created at a density ρ_c larger than the resonance density. This corresponds to $\Delta_{eV}\cos 2\theta_V < 1.73 \times 10^{-8}\rho_c Y_e$. For $\cos 2\theta_V \approx 1$, and $Y_e = \frac{1}{2}$, this corresponds to $1.4 \times 10^{-6} > \Delta_{eV}$. If neutrinos with $E = 0.236$ MeV are created above resonance, then all neutrinos of greater energy will be also. (3) Both the low-energy ($E = 0.236$ MeV) and high-energy ($E = 14$ MeV) neutrinos must go through resonance in the region of the sun where the exponential atmosphere is a good approximation. For the low-energy neutrinos this corresponds to $1.6 \times 10^{-6} > \Delta_{eV}$. For the high-energy neutrinos, the limit is $\Delta_{eV} > 2.2 \times 10^{-9}$. For the limits on Δ, we have assumed $\cos\theta_V \approx 1$. The comparison of our solution with Bethe's solution is shown in $(\Delta, \sin^2 2\theta_V/\cos 2\theta_V)$ plane in Figure 5.

In the standard solar model, ^{8}B neutrinos account for 15-17 SNUs in the ^{71}Ga experiment [9, 10]. The capture cross section for the ^{8}B neutrinos in ^{71}Ga can be fit by Eq. 15 with $n \approx 5$. For values of ξ that predict 2 SNUs for the ^{37}Cl experiment, 4-13 SNUs are predicted for the ^{71}Ga experiment. This number is far less than the 108 SNUs predicted by Bethe, or the 120 SNUs predicted by the standard solar models.

If we suppose that $m_1 \ll m_2$, then Bethe's solution requires $m_2 \simeq 8 \times 10^{-3}$ eV and $0.3 \gtrsim \sin\theta_V \gtrsim 0.01$. With the same assumption that $m_1 \ll m_2$, our solution requires $m_2 \simeq (0.5 - 1)\theta_V^{-1} \times 10^{-4}$ eV, with $\sin\theta_V \lesssim 0.3$.

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Note added: After completion of this work we learned of a paper by Rosen and Gelb [11] who also found solutions to the solar-neutrino puzzle which predict a reduced flux of low-energy neutrinos.

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Figure Captions:

Figures 1-3. $P(\nu_e)$ as a function of ρ.

Figure 4. The final value of $P(\nu_e)$ as a function of the adiabaticity parameter ξ.

Figure 5. A comparison of our solution and Bethe's solution of the solar-neutrino problem.

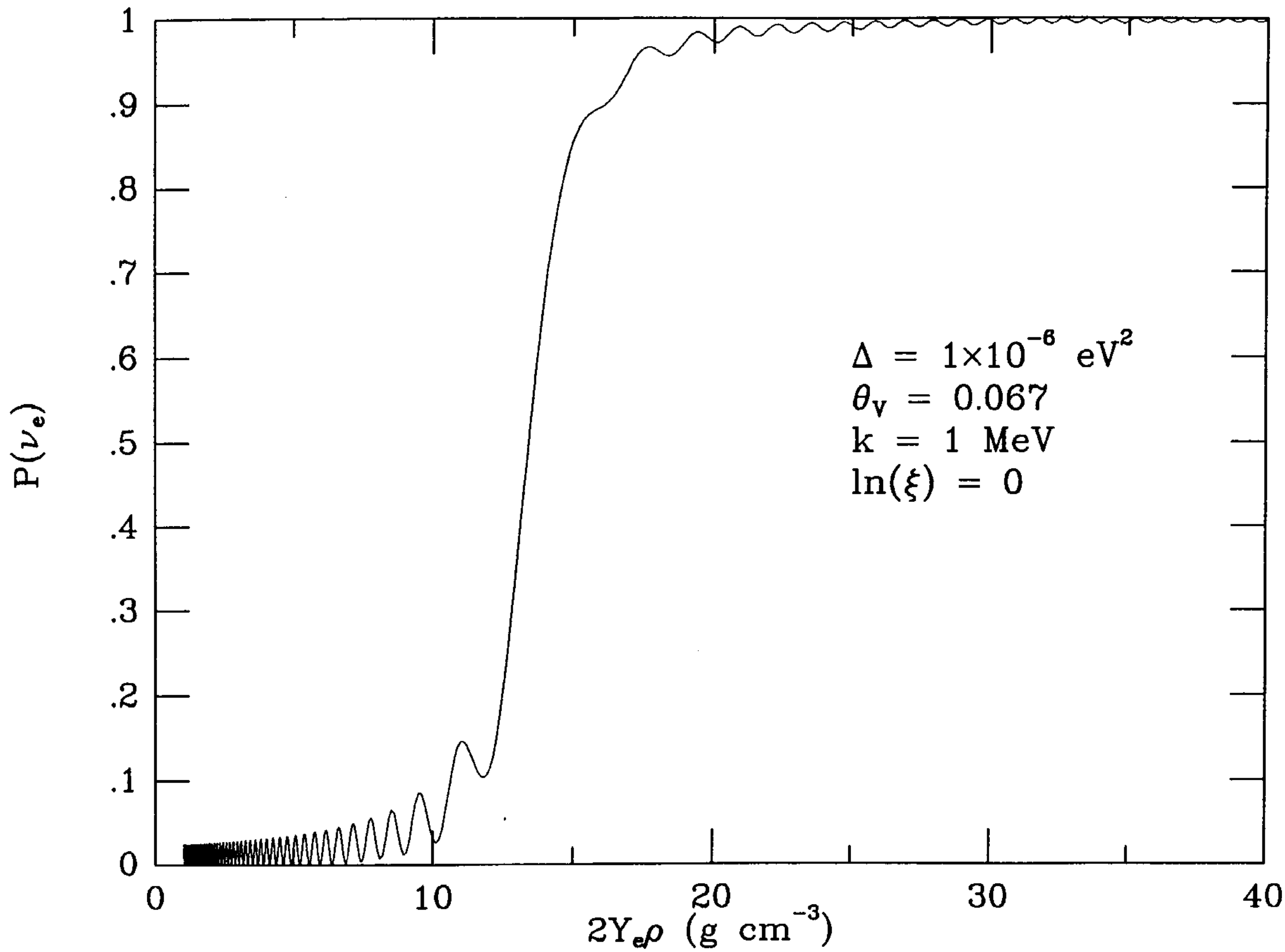
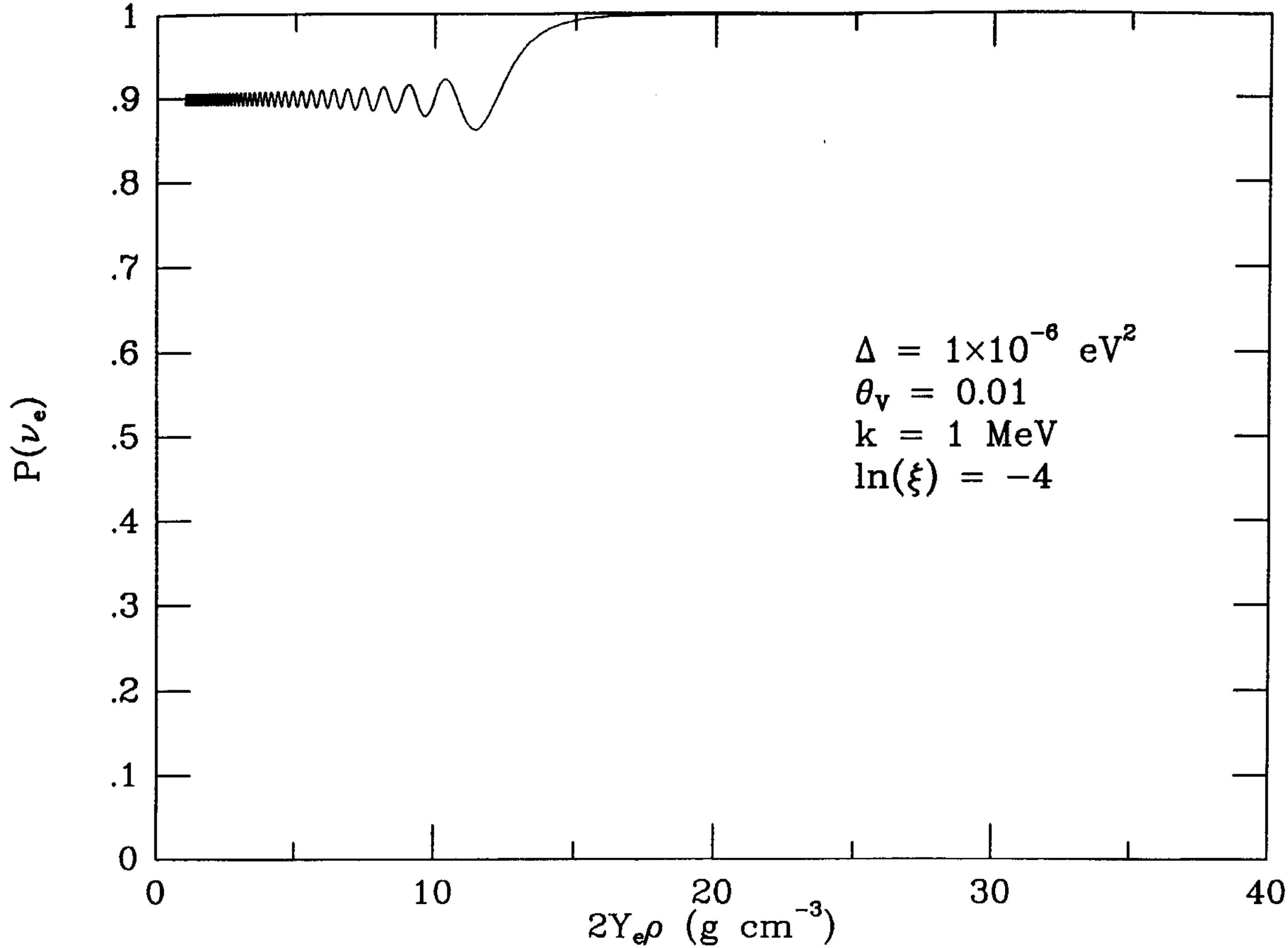

P(νe)
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
10
20
30
40
2Yeρ (g cm-3)
Δ = 1×10-6 eV2
θv = 0.067
k = 1 MeV
ln(ξ) = 0

FIGURE 1



1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
P(νe)
Δ = 1×10^-6 eV^2
θv = 0.01
k = 1 MeV
ln(ξ) = −4
0
10
20
30
40
2Yeρ (g cm^-3)

FIGURE 2

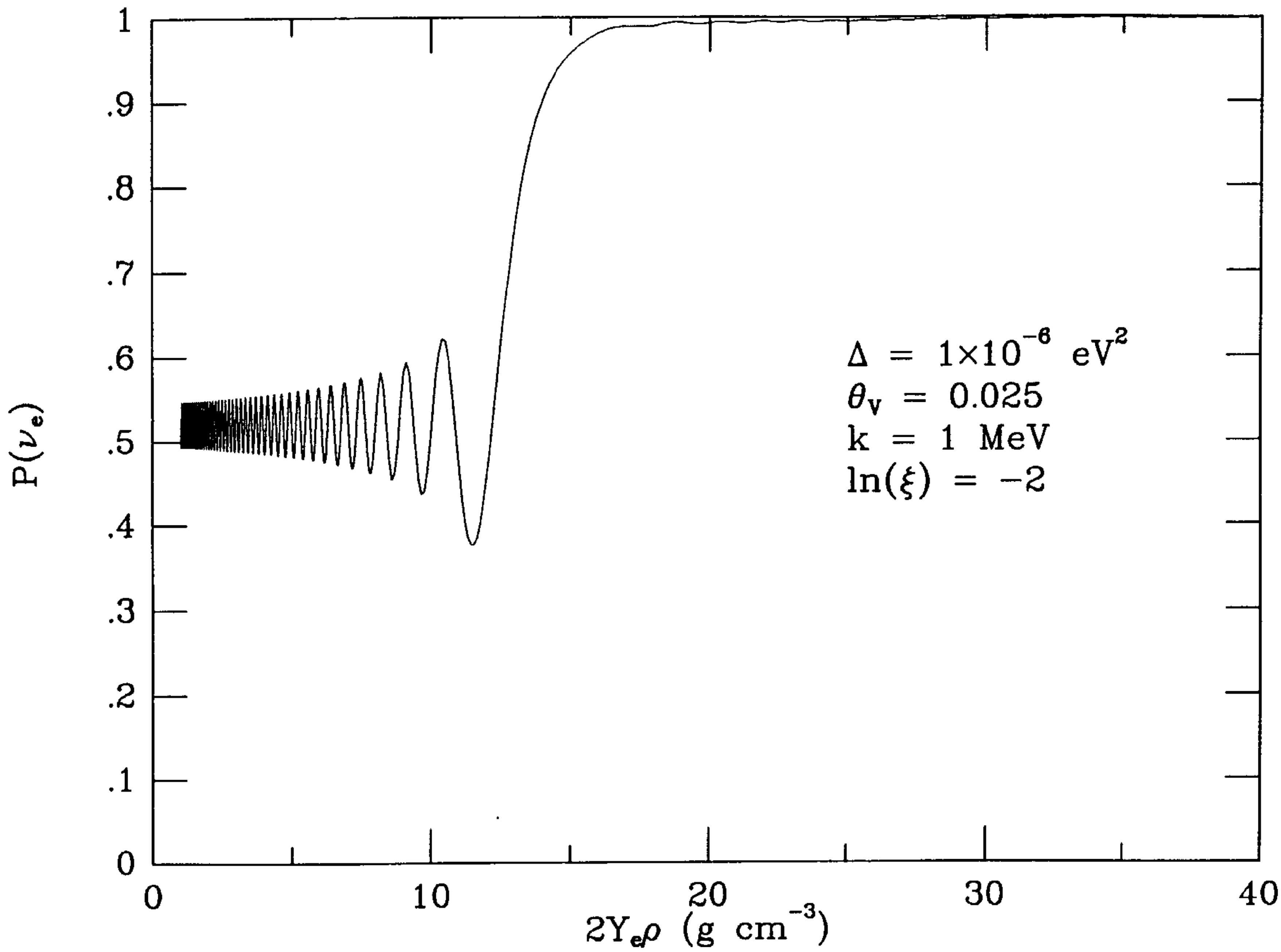

1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
10
20
30
40
P(νe)
2Yeρ (g cm^-3)
Δ = 1×10^-6 eV^2
θv = 0.025
k = 1 MeV
ln(ξ) = −2

FIGURE 3

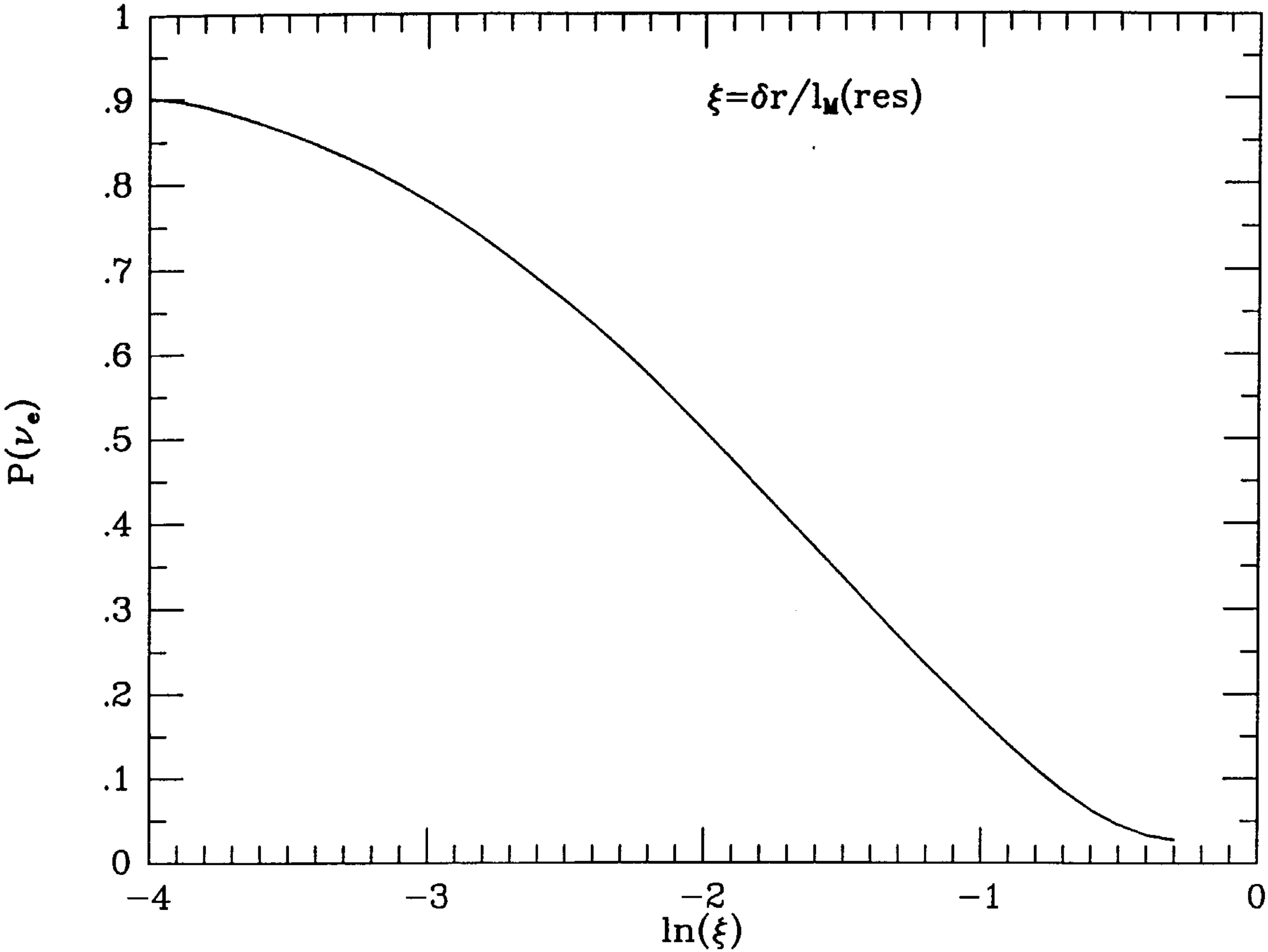


FIGURE 4

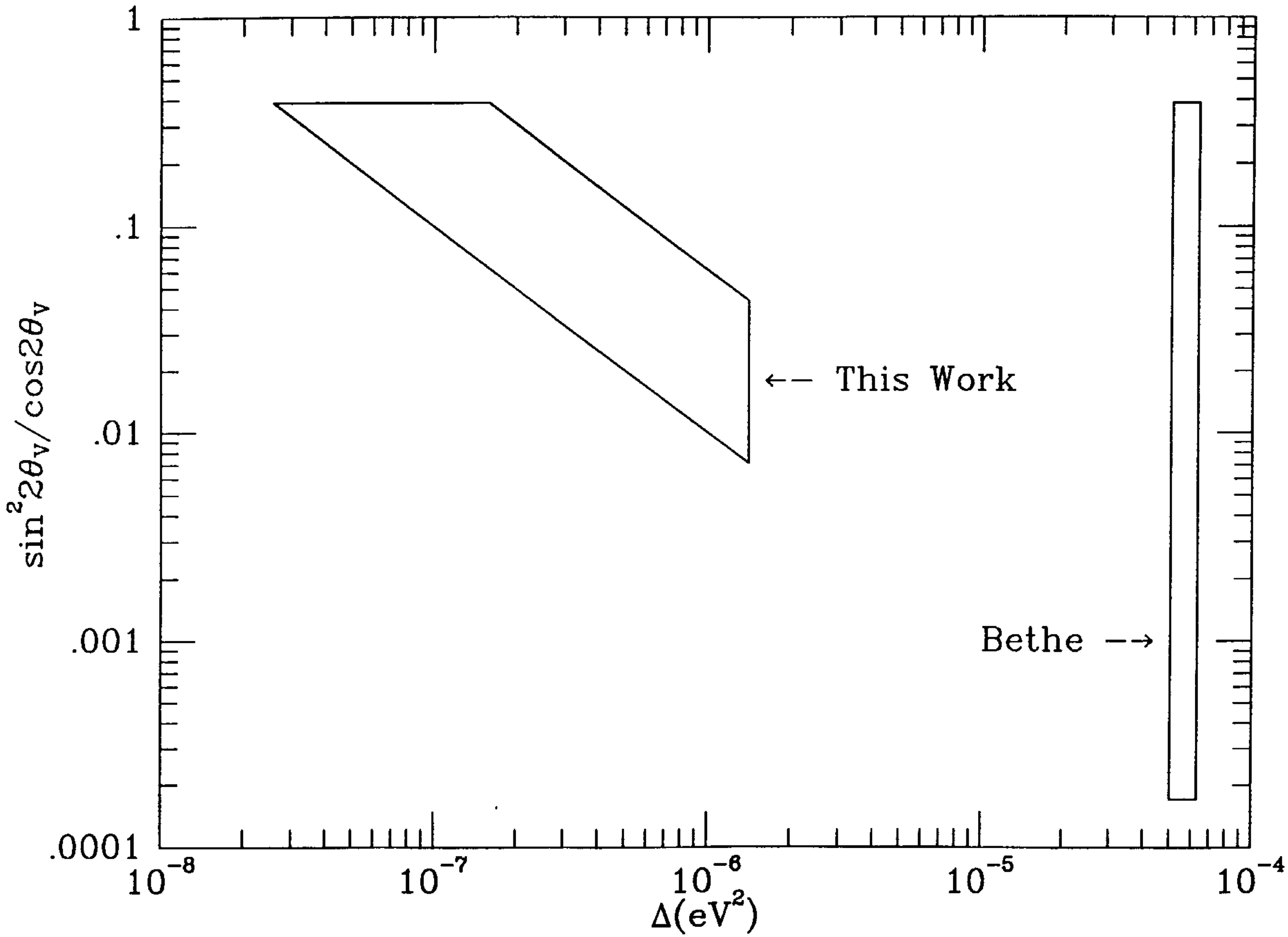

1
.1
.01
.001
.0001
$\sin^2 2\theta_V/\cos 2\theta_V$
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
$\Delta(eV^2)$
←– This Work
Bethe –→

FIGURE 5