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# Possible Significance of a New Dimensionles Ratio in Cosmology

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**Observations suggest that the mass density of the Universe is dominated, not by ordinary matter, but by exotic particles which are a relic of the Big Bang. In this case, a new dimensionless cosmological ratio arises, the ratio of the mass density in ordinary matter to that in exotic matter, whose value is about 0.1. *A priori* it might seem remarkable that this ratio should be so close to unity. However, we point out that, for many exotic dark matter candidates, the ratio is related to the fundamental scales of particle physics. A value of order unity arises naturally providing rather simple relationships exist between these scales.**

Dimensionless numbers play a crucial role in cosmology[1] and attempts to explain them often lead to important insights. For example, the fractional primordial abundances of the light elements D, $^3$He, $^4$He and $^7$Li relative to hydrogen ($\sim 10^{-5}$, $\sim 10^{-5}$, $10^{-1}$ and $\sim 10^{-10}$, respectively) are remarkably concordant with the abundances predicted by Big Bang nucleosynthesis.[2] The baryon to entropy ratio ($n_B/s \sim 10^{-10}$), and possibly the lepton to entropy ratio (whose value $n_L/s$ is unknown but certainly constrained by primordial nucleosynthesis[2] to be less than 1), can be understood as deriving from processes occuring at the grand unification epoch.[3] The ratio of the present neutrino temperature to the present photon temperature (expected to be $(4/11)^{1/3}$, though not yet observable) is related to the number of particle species less massive than a few MeV, since such species transfer their entropy to the photons (but not the neutrinos) when they annihilate.[4] The total entropy within the present horizon ($\sim 10^{88}$), as well as the dimensionless amplitude associated with adiabatic perturbations upon entering the horizon ($\lesssim 10^{-4}$), can be explained by the inflationary theory[5,6] for the early Universe.

Another set of dimensionless numbers is associated with the various contributions to the density of the Universe. Nowadays cosmologists differentiate between four contributions to the cosmological density: (1) the density of baryonic material (i.e. ordinary matter, be it visible or dark), (2) the density of visible material (stars and galaxies), (3) the density of the dark material which clusters on scales less than 10-30 Mpc, and (4) the density of any background dark matter which remains unclustered. None of these contributions is known precisely, but they are all constrained to lie in the range 0.01-1, in units of the critical density. For example, the baryonic density is constrained by primordial nucleosynthesis[2] to be $0.014h^{-2} \leq \Omega_b \leq 0.035h^{-2}$ where $h$ is the Hubble parameter in units of $100kms^{-1}Mpc^{-1}$; observational data indicate that the visible density is $\Omega_{vis} \simeq 0.01$ or perhaps slightly less;[7] dynamical studies of galactic halos and clusters of galaxies[8] suggest that the clustered dark matter density is $\Omega_{dark} \simeq 0.1 - 0.3$; theoretical prejudice (the naturalness of the flat Einstein-de Sitter Universe and the ideas of inflation) suggests that the total cosmological density is $\Omega = 1$, in which case the unclustered dark matter must have a density $\Omega_X$ close to 1.

A particularly crucial question is whether the baryonic density should be identified with $\Omega_{vis}$ or $\Omega_{dark}$. Since h lies between 0.5 and 1, $\Omega_b$ must be in the range 0.014-0.15. If

$h = 1$, one could just about have $\Omega_b = \Omega_{vis}$; in this case, all the dark material would be non-baryonic. On the other hand, if $h = 0.5$, one could just about have $\Omega_b = \Omega_{dark}$; in this case, a large fraction of the baryonic material must itself be dark, presumably having been converted into the black hole or Jupiter remnants of a first generation of "Population III" stars.[9] In either case, the discrepancy between the baryonic density and the total density (at least if $\Omega = 1$) implies there must be a substantial non-baryonic component to the mass of the Universe: $\Omega_X = \Omega - \Omega_b$. We thus have a new dimensionless ratio[10] $r \equiv \Omega_b/\Omega_X$ whose value seems to be of order unity. If we accept the inflationary theory and adopt $\Omega = 1$, $\Omega_b \simeq 0.1$ and $\Omega_X \simeq 0.9$, we have $r \simeq 0.1$. If we reject inflation and adopt $\Omega = 0.1$, $\Omega_b \simeq 0.01$ and $\Omega_X \simeq 0.09$, we still have $r \simeq 0.1$.

We now address the question of why r is so close to 1, and not say, $10^{-20}$ or $10^{20}$. Of all the various candidates suggested for the non-baryonic dark matter,[11] only one naturally predicts $r \sim 1$. This is the quark nugget picture suggested by Witten.[12] He pointed out that, if the stable form of matter for very large baryon number density is quark matter rather than nuclear matter, then it might be possible for all but a fraction of the quarks in the Universe to become concentrated into quark nuggets during the quark/hadron transition at $10^{-5}s$ after the bang. Although Witten and others[13] have concluded that such a scenario is rather unlikely, the fraction of free nucleons surviving would naturally be of order 0.1 if it did occur.

In the context of all the other candidates for the dark matter (e.g., axions, light or heavy neutrinos, superheavy magnetic monopoles, photinos, sneutrinos, Higgsinos, axinos, primordial black holes), it would seem rather miraculous that r should be of order 1. Indeed, in the case of primordial black holes or magnetic monopoles, it would require incredibly fine tuning of either the amplitude of the initial density fluctuations[14] or the amount of inflation.[15] However we will argue that this miracle can be understood for the other candidates providing there exist certain well-defined relationships between the various energy scales which arise in particle physics.

First consider the number of baryons in the Universe. It is convenient to express the net baryon number density as a fraction of the entropy density: $n_B/s \simeq 10^{-10}$. In the absence of large entropy production, this ratio is expected to be conserved at temperatures ($T < 10^{14} GeV$) for which baryon conservation pertains. Essentially all the entropy density today is contained in the cosmic photon and neutrino backgrounds, in which case it is about 7 times the number density of microwave photons, $n_\gamma \simeq 400 cm^{-3}$. Assuming there are very few antiprotons in the Universe today, as all observations suggest, the net baryon per entropy ratio is also the baryon per entropy ratio, so the mass density of baryons is conveniently expressed in terms of $n_B/s$ as

$$\rho_b \simeq m_b(\frac{n_B}{s})(7n_\gamma). \tag{1}$$

where $m_b$ is the mass of a nucleon ($\simeq 1GeV$). In the modern view, the ratio $n_B/s$ evolves at around $10^{-34}s$ due to non-equilibrium processes which do not conserve B,C and CP. The resultant baryon number to entropy ratio can be expressed as[3]

$$\frac{n_B}{s} = \frac{\epsilon}{g_*(10^{14} GeV)} \tag{2}$$

where $g_*(10^{14} GeV)$ counts the number of very relativistic species in the Universe at the time of baryogenesis ($T \simeq 10^{14} GeV$) and $\epsilon$ parameterizes the amount of C and CP violation. The value of $\epsilon$ (which evidently must be $\sim 10^{-7} - 10^{-8}$) depends upon the particular grand unification model but it must be less than about $\alpha/\pi$ where $\alpha = g^2/4\pi$ and g is a coupling constant. The appropriate coupling is probably that associated with the Higgs field, in which case $\alpha/\pi$ is likely to be $<< 10^{-4}$. The ratio r can then be expressed in terms of $n_X/s$, the ratio of the relic density of exotic particles to the entropy density:

$$r = (\frac{m_b}{m_X})(\frac{n_B}{s})(\frac{n_X}{s})^{-1} = \epsilon(\frac{m_b}{m_X})(\frac{n_X}{s})^{-1} g_*(10^{14}GeV)^{-1}. \tag{3}$$

We now show how one can calculate the ratio $n_X/s$ for various dark matter candidates.

Currently, axions are a popular solution to the dark matter problem. These are the hypothetical particles associated with a spontaneously broken symmetry (Peccei-Quinn, or PQ symmetry) introduced to solve the strong CP problem.[16] Their mass is related to the PQ symmetry breaking scale $f_{PQ}$ by

$$m_a = \frac{N m_\pi^2}{f_{PQ}} \simeq 10^{-5} eV (\frac{f_{PQ}}{10^{12}GeV})^{-1}, \tag{4}$$

where $m_\pi$ is the pion mass ($\simeq 140MeV$). (We have normalized $f_{PQ}$ to the value $\sim 10^{12}GeV$ required to give $\Omega_a \sim 1$). Axions come into existence as coherent oscillations of the axion field ($a = \theta f_{PQ}$) because this field is initially misaligned with the minimum of its potential at $\theta = 0$. From the particle point of view, they correspond to a highly non-relativistic condensate. Their relic abundance can be calculated[17] in terms of the symmetry-breaking scale $f_{PQ}$ and the initial misalignment angle $\theta_1$. The value of $\theta_1$ must be in the interval $[-\pi/N, \pi/N]$ where N is an integer which depends upon the details of how the PQ symmetry is implemented. (N=6 in the simplest models.) In general, we have

$$\frac{n_a}{s} \simeq \frac{f_{PQ}^2 \theta_1^2}{g_*(1GeV)^{1/2} m_b m_{pl}}, \tag{5}$$

where $m_{pl} = 1.2 \times 10^{19} GeV$ is the Planck mass. [In eqn. (5) the factor $m_b$ should actually be the temperature $T_1$ at which the coherent oscillations commence which is roughly equal to the nucleon mass; more precisely, $T_1 \simeq 1GeV$ $(2\text{N} \times 10^{11} GeV/f_{PQ})^{0.18}$.] Eqns. (3) and (4) then imply

$$r \simeq \epsilon g_*(10^{14}GeV)^{-1} g_*(1GeV)^{1/2}(N\theta_1)^{-2}(\frac{m_b}{m_\pi})^2(\frac{Nm_{pl}}{f_{PQ}}). \tag{6}$$

If $N\theta_1 = 0(1)$, $g_*(10^{14}GeV) = 0(10^2)$, $g_*(1GeV) = 0(10)$ and $N = 0(10)$, the condition that r be of order unity just reduces to

$$\frac{f_{PQ}}{m_{pl}} \simeq 10^{-1}\epsilon(\frac{m_b}{m_\pi})^2 \simeq 10\epsilon. \tag{7}$$

Thus $f_{PQ}$ should differ from the Planck scale only by the C and CP violating factor $\epsilon$. If we assume that $\epsilon \simeq \alpha/\pi$, condition (7) just becomes $f_{PQ} \simeq \alpha m_{pl}$.

Let us now consider those hypothetical particles whose relic abundances are determined by when their annihilations become ineffective. These particles include the heavy neutrino[18] and the lightest of the supersymmetric partners of some of the more familiar particles;[19] e.g., depending upon the model, the photino, sneutrino, or higgsino. They all have masses in the range around 1 GeV (i.e. close to $m_b$) and interactions which are roughly characterized by the weak interaction (Fermi) scale. Their relic abundances are determined from their annihilation cross-section $(\sigma v)_{ann}$ and mass $m_X$:

$$\frac{n_X}{s} \simeq \frac{10\xi g_*(GeV)^{1/2}}{m_X m_{pl}(\sigma v)_{ann}}. \tag{8}$$

Here $\xi \simeq ln[m_X m_{pl}(\sigma v)_{ann}]$ is a logarithmic factor which is about 20 for all these particles.[20] For an order-of-magnitude calculation, we can take $(\sigma v)_{ann} \simeq m_X^2 G_F^2$ where $G_F \simeq 10^{-5} GeV^{-2}$ is the Fermi coupling constant. From eqn. (3), the ratio r is then

$$r \simeq 10^{-1}\xi^{-1}\epsilon g_*(10^{14}GeV)^{-1} g_*(GeV)^{-1/2} m_{pl} m_b m_X^2 G_F^2. \tag{9}$$

If we put $\xi = 20$ and $g_*(10^{14}GeV)g_*(GeV)^{1/2} \simeq 10^4$, the condition for r to be of order unity can be expressed as

$$\frac{m_F}{m_{pl}} \simeq (10^{-6}\epsilon)^{1/4}(\frac{m_b m_X^2}{m_{pl}^3})^{1/4} \simeq (10^{-6}\epsilon)^{1/4}(\frac{m_b}{m_{pl}})^{3/4} \tag{10}$$

where $m_F = G_F^{-1/2} \simeq 300GeV$ is the Fermi scale and we have put $m_X \simeq m_b$. This tells us that the Fermi scale must be very much less than the Planck scale. Rather remarkably, eqn. (10) is almost exactly equivalent to the "anthropic condition" that the weak fine structure constant ($\alpha_W$) be the quarter power of the gravitational fine structure constant ($\alpha_G$). This is so because the condition $\alpha_W \simeq \alpha_G^{1/4}$ can be written as

$$\frac{m_F}{m_{pl}} \simeq 10^2 \alpha_e^{5/2}(\frac{m_b}{m_{pl}})^{3/4}. \tag{11}$$

where $\alpha_e \equiv e^2/\hbar c = 1/137$ is the electric fine structure constant. This agrees with eqn. (10), not only as regards the $(m_b/m_{pl})$ dependence, but also as regards the value of the coefficient. The condition $\alpha_W \simeq \alpha_G^{1/4}$ is "anthropic" in the sense that it is required for cosmological nucleosynthesis and also perhaps for supernovae.

Finally, we consider those "light" exotic particles which do not undergo appreciable annihilation because their interactions freeze out at a temperature exceeding their rest mass. Such particles should have a number density close to $n_\gamma$ (i.e. $n_X/s \simeq 10^{-1}$). In this case, (3) implies that r is of order unity providing

$$m_X \simeq \frac{10\epsilon m_b}{g_*(10^{14}GeV)} \simeq 10^{-1}\epsilon m_b. \tag{12}$$

In particular, this condition pertains for massive neutrinos. There are now independent reasons[21] for believing that neutrinos themselves are not good candidates for providing $\Omega = 1$ but eqn. (12) is still of interest in a more general context. We note that many grand unified theories predict some sort of relation[22] between $m_\nu$ and the other fermion masses, of the form $m_X \sim m_f^2/M$ where $M$ is some very large mass-scale. In this case, taking $m_f \sim m_b$, eqn. (12) just reduces to a condition of the form $m_b \simeq \epsilon M$.

To summarize, it seems that three kinds of exotic relics will have a value for r of order unity providing one imposes special (but rather simple) relationships between the fundamental energy scales of particle physics. This obviously begs the question of why these relationships should pertain. It is conceivable that one of conditions (7), (10) and (12) will turn out to be predicted by particle physics itself. In this case, it would be natural to conclude that the particle which actually provides the dark matter is the particle associated with that condition. It is even conceivable that more than one of the conditions will be predicted, in which case there might be two non-baryonic contributors to the dark matter. At present, however, none of the relevant conditions is unambiguously predicted by particle physics. An alternative (albeit rather exotic) possibility is to appeal to the anthropic principle, since this is often invoked to explain otherwise inexplicable relationships between the constants of physics. This possibility is accentuated by the fact that condition (10) already has a well-known anthropic significance. Indeed, on this basis, one might argue that the heavy neutrino or one of the supersymmetric particles is the most plausible dark matter candidate. On the other hand, it is hard to see any anthropic reason for the condition $r \sim 1$ itself. There are possible anthropic reasons for why r should not be too small (e.g. if there were too much dark matter, it would be hard to form structures out of ordinary visible matter), but there are no obvious anthropic arguments which preclude it being too large.

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