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Fermion Masses and Phenomenology in SO(10) or SU(5) Superstring Compactifications

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ABSTRACT

Symmetry breaking patterns and phenomenology of the SO(10) and SU(5) compactifications of the $E_8 \times E_8'$ heterotic superstring recently proposed by Witten are examined with regard to the fermion mass matrix. Under some circumstances there exist constraints due to the presence of relatively light charged leptons.

Superstring theories may be the first consistent models which unify gravity with the other interactons. As such they must be able to predict, amongst other things, the pattern of gauge symmetry breaking and the fermion mass spectrum. Given the fact that a complete formulation of such theories is still lacking, it is not possible to attempt a detailed analysis of these questions. However, it is certainly worthwhile to pursue a a program of obtaining as much general information on these problems as is possible from the present formulation. Such a program was initiated in [1] where the $E_8 \times E_8'$ heterotic string theory [2] was compactified on a Calabi-Yau [3] manifold, K. This gave rise to an effective E_6 supersymmetric grand unified model for the zero modes in which the number of matter fields transforming as $\underset{\sim}{27}$ or $\overline{\underset{\sim}{27}}$ of E_6 was given by topological invariants of K. This program was continued in [4] where it was shown how the topology and symmetries of K could determine the possible patterns of symmetry breaking and couplings in the superpotential. Other phenomenological constraints on E_6 superstring models have been discussed in ref. [5].

In a previous publication [6] we used the topological methods of [4] and the assumption that the up and down quark mass matrices were non-zero at tree level to arrive at more constraints on the pattern of symmetry breaking and constraints due to relations between the mass matrices of the charged and neutral leptons. In this paper we will extend our previous analysis to the possible new superstring compactifications discussed by Witten [7] which give rise to SO(10) or SU(5) supersymmetric unified models as an effective low energy theory. We will see that, despite the fact that no explicit construction of these compactifications exists at present, a surprising amount of information can be derived about the pattern of SO(10) symmetry breaking (there is, of course, only one possibility for SU(5)). There is the possibility that new particles may arise in the low energy phenomenology of these theories with interesting experimental consequences.

Let us first review the mechanism of gauge symmetry breaking via Wilson loops described in [1,4,8]. We consider Calabi-Yau manifolds K of the form K_0/G where K_0 is a simply connected Calabi-Yau manifold, G is a discrete group of transformations that act holomorphically and without fixed points on K_0 and K_0/G means that we consider the points of K to be the set of equivalence classes of points of K_0 under the symmetry G. Then K is non-simply connected and, in fact, $\pi_1(K) \cong G$. Due to this fact, there may exist non-trivial gauge configurations on K that cannot be gauged away despite the fact that their field strength F_{mn} vanishes. These configurations can break the gauge group H (previously E_6, currently either SO(10) or SU(5)) to a subgroup Σ due to their contribution to the vacuum value of the Wilson loop operator given by

$$U_g = P\ \exp[\ i\int_\Gamma A_m dx^m] \tag{1}$$

Here Γ is a non contractible loop on K_0/G which is the image of a non-trivial path from x_0 to $g \cdot x_0$ on K_0. The mapping sending $g \in G$ to $U_g \in H$ is a homomorphism of G onto a discrete subgroup $\overline{G}$ of H. As demonstrated in [4], the particle fields in the spectrum on K_0/G are those fields $\Psi(x)$ on K_0 which satisfy the boundary condition

$$\Psi(x_0) = U_g^\dagger \Psi(gx_0) \tag{2}$$

Another way of saying this is that the only permissible particle fields on K_0/G are those invariant under the action $G \oplus \overline{G}$ as given in eq (2). From eq (2) and the fact that the gauge fields must be G invariant we see that Σ is the gauge subgroup that commutes with all of the U_g. In the E_6 case with only one $\overline{\underline{27}}$, corresponding to the

Kähler form*, the $G \oplus \overline{G}$ invariant components of $\overline{\underset{\sim}{27}}$ are just those that are neutral under U_g. This made the analysis very simple and resulted in the existence of Higgs doublets and singlets that were unaccompanied by any dangerous color tripplets.

We now turn to the more recent work of ref [7]. Witten has claimed that certain stable, irreducible, holomorphic vector bundles with SU(4) or SU(5) structure groups can be constructed over Calabi-Yau manifolds. These result in $E_8' \times SO(10)$ and $E_8' \times SU(5)$ $N=1$ supersymmetric gauge theories respectively in four dimensions. The $E_8' \times E_6$ model previously discussed is of this type, where the tangent bundle with SU(3) structure group** was considered. As in the E_6 case, we may now ask what the representation content of the chiral superfields appearing in the low energy theory is. In the SO(10) case we have

$$\alpha \underset{\sim}{1} + N_f \underset{\sim}{16} + \delta(\underset{\sim}{16} + \overline{\underset{\sim}{16}}) + \varepsilon \underset{\sim}{10} \tag{3}$$

The numbers α, N_f, δ and ε are given by the following topological invariants: $\alpha = \dim H^1(\mathrm{End}\,B)$, $N_f = (1/2)|C_3(B)|$, $\delta = \dim H^1(B^*)$, $\varepsilon = \dim (H^1(B \times B)_{AS})$. Here B is the SU(4) vector bundle with the fibre being the representation space of the fundamental representation of SU(4), $C_3(B)$ is its third Chern number, B^* is the dual bundle to B, $(B \times B)_{AS}$ is the antisymmetrized tensor product bundle, End B is the bundle of endomorphisms of B, and $H^1(V)$ is the first Dolbeault cohomology group with values in the vector space V [9]. For the SU(5) case, the representation content is

$$\alpha' \underset{\sim}{1} + m \overline{\underset{\sim}{5}} + n \underset{\sim}{10} + \delta(\underset{\sim}{5} + \overline{\underset{\sim}{5}}) + \varepsilon(\underset{\sim}{10} + \overline{\underset{\sim}{10}}) \tag{4}$$

*The Kähler form on K_0 is necessarily G invariant if K_0/G is to be a Calabi-Yau manifold.

**Calabi-Yau manifolds have an SU(3) holonomy and this is, by definition, the structure group of the tangent bundle.

where, if B' denotes the SU(5) vector bundle with fibre the representation space of the $\underset{\sim}{5}$ of SU(5), then $\alpha' = \dim H^1(\mathrm{End}\, B')$, $m = (1/2)|C_3(B')|$, $n = (1/2)\,|C_3((B' \times B')_{AS})|$, $\delta = \dim H^1(B')$, $\varepsilon = \dim (H^1(B' \times B')_{AS})$. One point of note is that the index theorem that gives N_f in the SO(10) case also tells us that if a state of the $\underset{\sim}{16}$ remains light after symmetry breaking by Wilson loops then it is paired up with a corresponding state from the $\overline{\underset{\sim}{16}}$. Similar statements can be made about the states within the $\underset{\sim}{10}$ of SO(10), and the $\underset{\sim}{5}$ and $\overline{\underset{\sim}{5}}$, and $\underset{\sim}{10}$ and $\overline{\underset{\sim}{10}}$ in the SU(5) case.

We are now ready to extend the analysis of ref [6] to these new manifolds. We assume that the up and down quark mass matrices do not vanish identically at tree level*. Thus, there must be sufficiently many Higgs doublets left invariant by the action of $G \oplus \overline{G}$. Before we proceed any further, we must specify the action of G on the fields. In the E_6 case there was an explicit correspondence between representations transforming as $\overline{\underset{\sim}{27}}$, say, and (1,1) forms. If there was only one of these present (i.e., the Kähler form) then it was automatically G invariant as stated earlier. Unfortunately, there is no such correspondence in the present situation. Nonetheless, we may still deduce the action of G on fields Ψ that are to be $G \oplus \overline{G}$ invariant. Since the elements of $\overline{G}$ belong to the gauge group H, their action can at most reshuffle fields <u>within</u> a fixed irreducible representation of H. Thus, if Ψ is to be $G \oplus \overline{G}$ invariant, the action of G <u>must not</u> mix it with other fields Ψ' in a different irreducible representation of H, since such a mixing cannot be undone by $\overline{G}$. In fact, the action of G on $G \oplus \overline{G}$ invariant fields can only multiply Ψ by an overall phase η_Ψ so that <u>all</u> components of an irreducible representation of H acquire the <u>same</u> phase**. Armed with this knowledge, we now proceed to derive the constraints on symmetry

*But we do not rule out either zero entries or zero eigenvalues in these matrices.
**This point seems to have been overlooked by the authors of ref [10] (which we received after this work was begun) who allowed different components of an irreducible representation fo acquire different phases under G.

breaking patterns imposed by the demand that Higgs doublets exist as zero modes on K_0/G.

Let us first deal with the SO(10) case. To do this we note [4] that G cannot be non-Abelian so the rank of the unbroken subgroup Σ must be five. We parameterize $\mathbf{U}_g$ using the $SU(4)_c \times SU(2)_L \times SU(2)_R$ basis of SO(10):

$$\mathbf{U}_g = \begin{pmatrix} \beta & & & \\ & \beta & & \\ & & \beta & \\ & & & \beta^{-3} \end{pmatrix} \times \begin{pmatrix} \gamma & \\ & \gamma^{-1} \end{pmatrix} \times \begin{pmatrix} \mu & \\ & \mu^{-1} \end{pmatrix} \tag{5}$$

where $\gamma^2 = 1$ for $SU(2)_L$ to be invariant and β, γ, μ are phases. Next we decompose the $\underset{\sim}{10}$, $\underset{\sim}{16}$ and adjoint $\underset{\sim}{45}$ representations of SO(10) under the subgroup $\Sigma_0 = SU(3)_c \times SU(2)_L \times U(1)_{15} \times U(1)_{3R}$, where $U(1)_{15}$ appears in the maximal decomposition $SU(4)_c \supset SU(3)_c \times U(1)_{15}$ and $U(1)_{3R}$ is the U(1) subgroup of $SU(2)_R$.

$$10 = (\underset{\sim}{\bar{3}}, \underset{\sim}{1}; 2, 0) + (\underset{\sim}{3}, \underset{\sim}{1}; -2, 0) + (\underset{\sim}{1}, \underset{\sim}{2}; 0, 1) + (\underset{\sim}{1}, \underset{\sim}{2}; 0, -1) \tag{6a}$$

$$16 = (\underset{\sim}{3}, \underset{\sim}{2}; 1, 0) + (\underset{\sim}{1}, \underset{\sim}{2}; -3, 0) + (\underset{\sim}{\bar{3}}, \underset{\sim}{1}; -1, 1) + (\underset{\sim}{1}, \underset{\sim}{1}; 3, 1) + (\underset{\sim}{1}, \underset{\sim}{1}; 3, -1) \tag{6b}$$

$$\begin{aligned} 45 = {} & (\underset{\sim}{8}, \underset{\sim}{1}; 0, 0) + (\underset{\sim}{1}, \underset{\sim}{3}; 0, 0) + (\underset{\sim}{1}, \underset{\sim}{1}; 0, 2) + (\underset{\sim}{1}, \underset{\sim}{1}; 0, 0) + (\underset{\sim}{1}, \underset{\sim}{1}; 0, -2) \\ & + (\underset{\sim}{3}, \underset{\sim}{2}; -2, 1) + (\underset{\sim}{3}, \underset{\sim}{2}; -2, -1) + (\underset{\sim}{\bar{3}}, \underset{\sim}{2}; 2, 1) + (\underset{\sim}{\bar{3}}, \underset{\sim}{2}; 2, -1) + (\underset{\sim}{1}, \underset{\sim}{1}; 0, 0) \\ & + (\underset{\sim}{\bar{3}}, \underset{\sim}{1}; -4, 0) + (\underset{\sim}{3}, \underset{\sim}{1}; 4, 0) \end{aligned} \tag{6c}$$

The required doublets are contained in the $\underset{\sim}{10}$ since those in the $\underset{\sim}{16}$ are useless due to the absence of a $(\underset{\sim}{16})^3$ coupling. We must demand that both types of doublets in the $\underset{\sim}{10}$, $(\underset{\sim}{1}, \underset{\sim}{2}; 0, 1)$ and $(\underset{\sim}{1}, \underset{\sim}{2}; 0, -1)$, be present in the spectrum for the following reasons: 1) both are needed in order to give the up and the down quarks a mass, 2) both are required so that both chirality states of the higgsinos be present, 3) the index theorem can only be satisfied if both doublets are present. Let us now specialize to the case where only one $\underset{\sim}{10}$ is present, i.e., $\dim H^1((B\times B)_{AS}) = 1$. Noting that the $SU(4)_c$ and $SU(2)_R$ pieces of $\mathbf{U}_g$ are proportional to the generators T_{15} and T_{3R} of $U(1)_{15}$ and $U(1)_{3R}$ respectively, and denoting the phase acquired by the $\underset{\sim}{10}$ under G by η_{10}, we have:

$$\mu\,\eta_{10} = \mu^{-1}\eta_{10} = 1\ ,\ \text{or}\ \ \mu^2 = \eta_{10}{}^2 = 1 \tag{7}$$

From eqs (5) and (7) we immediatly deduce that $SU(2)_R$ is always unbroken by the $\mathbf{U}_g$ (as in the E_6 case [6]). Furthermore, if $\mu \neq 1$, G must contain a Z_2 subgroup which then implies that the order of G is even. The smallest unbroken subgroup in this case is $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{15}$. If $\beta^4 = 1$ we have $SU(4)_c\times SU(2)_L\times SU(2)_R$ as the unbroken subgroup, while if $\mu = 1$ and $\beta^2 = 1$, then SO(10) remains unbroken. This exhausts the possibilities for Σ if only one $\underset{\sim}{10}$ exists. If more $\underset{\sim}{10}$'s appear as zero modes then groups such as $SU(3)_c\times SU(2)_L\times U(1)_{15}\times U(1)_{3R}$ may appear [10]. It is very interesting to see the tight correlations between the topology of B (i.e., the index theorem), phenomenology (quark masses), the allowed groups of transformations G on the Calabi-Yau manifold K_0 , and the allowed non-trivial vacuum configurations of SO(10) gauge fields on K_0/G. In particular it appears that under certain circumstances some vacuum configurations of gauge fields (i.e., values of $\mathbf{U}_g$) are not permitted since they would lead to a particle spectrum inconsistent with the index theorem.

Since Σ always contains $SU(2)_R$ in the case considered, we must have a mechanism to break Σ to the standard model. Witten [7] showed that it is plausible that the SU(5) singlet in the $\underset{\sim}{16}$ could acquire an intermediate scale vacuum value

($\gtrsim 10^9$ GeV), thus breaking $SU(2)_R$ at a scale very large compared to M_W. If the singlet is to come from an "incomplete" multiplet (i.e., from one of the δ ($\underset{\sim}{16} + \overline{\underset{\sim}{16}}$)), we arrive at the condition that $\eta_{16}\, \beta^3\, \mu^{-1} = 1$. $SU(2)_R$ invariance then implies that the "positron" state of this $\underset{\sim}{16}$ is $G\oplus\overline{G}$ invariant and the index theorem implies that these states have counterparts coming from the $\overline{\underset{\sim}{16}}$. We may now list the $G\oplus\overline{G}$ transformation laws of the (single) $\underset{\sim}{10}$ and of the $\underset{\sim}{16}$ whose phase η_{16} satisfies the above condition. This is done in Table (1). If the number of $\underset{\sim}{10}$'s is larger than one and $\Sigma = SU(3)_c \times SU(2)_L \times U(1)_{3R} \times U(1)_{15}$, then the singlet may be taken from one of the N_f $\underset{\sim}{16}$'s, since in this case one may break Σ at a scale O(TeV) without incuring any problems with phenomenology [5]. Under this circumstance η_{16} is unrelated to β and μ.

The SU(5) case is much simpler since there is only one pattern of symmetry breaking. The U_g's take the form $\exp(3i\alpha Y)$, where Y is is the weak hypercharge and is proportional to $(1/3)\, T_{15} + T_{3R}$. The Higgs doublets are contained in the ($\underset{\sim}{5} + \overline{\underset{\sim}{5}}$)'s. If η_5 is the phase acquired by one of these $\underset{\sim}{5}$'s, then the condition that its doublet remains in the spectrum is $\eta_5 \exp(-3i\alpha) = 1$. The index theorem requires a doublet from a $\overline{\underset{\sim}{5}}$, so one of the $\overline{\underset{\sim}{5}}$'s must have a G phase opposite to η_5. The requirement that no color triplets appear from the $\underset{\sim}{5}$'s is $\eta_5 \exp(2i\alpha) \neq 1$ which implies that G cannot be Z_5. One must also ensure the lack of color triplets coming from the $\underset{\sim}{10}$'s, which imposes conditions on η_{10}' (the G phase of the $\underset{\sim}{10}$ of SU(5)) relative to α.

We now turn to some phenomenological considerations that were absent in the E_6 analysis. The question is: what happens to the superpartners of the Higgs scalars? In E_6 they were all able to acquire a large mass and thus leave present phenomenology untouched [6]. It is not at all clear that this happens in the SO(10) and SU(5) models. We will first consider the SO(10) case. There are two possibilities to consider, depending on whether or not SO(10) singlet superfields are present (i.e., on whether or not dim H^1(End B) is nonzero). If such singlets exist <u>and</u> can acquire sufficiently large vacuum expectation values, couplings such as $\underset{\sim}{10}\cdot\underset{\sim}{10}\cdot\underset{\sim}{1}$ and $\underset{\sim}{16}\cdot\underset{\sim}{\overline{16}}\cdot\underset{\sim}{1}$ will allow these states to decouple from the low energy spectrum. However, it is certainly possible that dim H^1(End B) is zero, in which case the situation must be examined more carefully. For simplicity let us consider the case where there is a single $\underset{\sim}{10}$ and where the intermediate scale breaking is triggered by the SU(5) singlets coming from a single "incomplete" ($\underset{\sim}{16} + \underset{\sim}{\overline{16}}$) multiplet (i.e., we take $\delta = 1$). Then we have the following new (left handed, by convention) fermions:

--two $SU(2)_L$ doublets from the $\underset{\sim}{10}_H$ (which also form two $SU(2)_R$ doublets)

--two neutral states together with an $SU(2)_L$ singlet "positron" state and its antiparticle from the $\underset{\sim}{16}_H + \underset{\sim}{\overline{16}}_H$ respectively

In order to consider the charged lepton mass matrix we first list the relevant charged particles. They are:

-- the standard charged leptons e_i ($i = e, \mu, \tau, \ldots$) and their right handed conjugates e_i^c

--the charged components of the two $SU(2)_L$ doublets from the $\underset{\sim}{10}_H$, E and E^c

--the $SU(2)_L$ singlets, a^- and $\hat{e}^c$, from the $\underset{\sim}{\overline{16}}_H$ and $\underset{\sim}{16}_H$ respectively

--and the charged $SU(2)_L \times SU(2)_R$ gauginos, $\lambda_\pm^{L,R}$.

The gauginos can only appear in mass terms with fermions whose scalar partners have obtained a vacuum value. The mass terms are given by (schematically):

$$y^{ij}\, \underset{\sim}{16}_i \underset{\sim}{16}_j < \underset{\sim}{10}_H > \;\rightarrow\; y^{ij}\, e_i^c\, e_j <\phi> + \text{h.c.} \tag{8a}$$

$$y^{i}\, \underset{\sim}{16}_i \underset{\sim}{16}_H < \underset{\sim}{10}_H > \;\rightarrow\; y^{i}\, \hat{e}^c\, e_i <\phi> + \text{h.c.} \tag{8b}$$

$$\tilde{y}^{i}\, \underset{\sim}{16}_i \underset{\sim}{10}_H < \underset{\sim}{16}_H > \;\rightarrow \tilde{y}^{i}\, E^c\, e_i <V> + \text{h.c.} \tag{8c}$$

where the subscript H denotes an incomplete multiplet that contributes a Higgs scalar. The gaugino mass terms are (where $g_{L,R}$ is the $SU(2)_{L,R}$ gauge coupling):

$$g_L (\lambda_+^L E <\phi> + \lambda_-^L E^c <\phi>) + h.c. \tag{9a}$$

$$g_R (\lambda_+^R E <\phi> + \lambda_-^R E^c <\phi> + \lambda_+^R a^- <V> + \lambda_-^R \hat{e}^c <V>) + h.c. \tag{9b}$$

Here $<\phi>$ denotes an $SU(2)_L$ doublet vacuum value from the $\underset{\sim}{10}_H$ (we use the same symbol, $<\phi>$, to denote both of the distinct $SU(2)_L$ doublet vacuum values from the $\underset{\sim}{10}_H$, since we are just performing an order of magnitude analysis at the present), while $<V>$ denotes an $SU(2)_L$ singlet vacuum value from the $\underset{\sim}{16}_H$. The mass matrix takes the schematic form

	e_j^c	E^c	$\hat{e}^c$	λ_+^L	λ_+^R
e_i	$y^{ij} <\phi>$	$\tilde{y}^i <V>$	$y^i <\phi>$	0	0
E	0	0	0	$g_L <\phi>$	$g_R <\phi>$
a^-	0	0	0	0	$g_R <V>$
λ_-^L	0	$g_L <\phi>$	0	m_1	0
λ_-^R	0	$g_R <\phi>$	$g_R <V>$	0	m_2

(10)

where we have added diagonal gaugino terms arising from supersymmetry breaking with $m_{1,2} \approx O(m_{3/2})$ [11].

In the E_6 model the incomplete multiplet mechanism entailed the existence of quantum numbers that distinguished between $\underset{\sim}{27}$ and $\underset{\sim}{27}_H$. If this also occurs in the SO(10) model, then $\underset{\sim}{16}\cdot\underset{\sim}{16}_H\cdot\underset{\sim}{10}_H$ cannot exist if $\underset{\sim}{16}\cdot\underset{\sim}{16}\cdot\underset{\sim}{10}_H$ does (which must since it is this coupling that gives the standard fermions their masses). This sets y^i and $\tilde{y}^i$ to zero and prevents mixing between the standard leptons and any of the new particles. Let us assume that this is what happens in what follows. If Σ contains $SU(2)_R$, then $g_R\langle V\rangle \gg m_{1,2} \approx O(\text{Tev})$ [5] and the mass matrix may be approximately diagonalized. The eigenvalues (not including those of the standard leptons) are of the order of: M_w^2/m_1, m_1, $g_R\langle V\rangle$, $g_R\langle V\rangle$. If m_1 were 1 Tev we would predict a charged lepton of mass $\approx O(10\ \text{Gev})$ which could have easily been seen in e^+e^- experiments. If $m_1 \approx M_w$, then a similar analysis shows that this mass gets pushed up to $O(M_w)$ so we would predict two new charged leptons with such mass. Note that for standard Yukawa couplings ($\approx 10^{-2} \to 10^{-3}$), terms that would arise from $\underset{\sim}{16}\cdot\underset{\sim}{16}_H\cdot\underset{\sim}{10}_H$ would not affect this result. Thus, under the assumptions that $g_R\langle V\rangle \gg m_{1,2}$, $m_1 < 1$ Tev, $\varepsilon = \delta = 1$ and $\dim H^1(\text{End } B) = 0$, we would predict new light charged leptons that may be seen in the next generation of experiments.

What happens if some of the parameters are changed? If $\varepsilon > 1$, so that μ^2 need not equal one and $SU(2)_R$ need not be present in Σ, $\hat{e}^c$ and a^- are absent from the spectrum and the new leptons have masses $\approx m_1$, and $g_R\langle V\rangle$. If $\varepsilon=1$ and $\delta > 1$, we may have (depending upon whether or not the additonal $(\underset{\sim}{16} + \overline{\underset{\sim}{16}})$'s have the same G transformation properties as the original one) at most δ copies of $\hat{e}^c$ and a^- which do <u>not</u> mix with one another. We would expect that "low" mass leptons would exist in this case also. If $\mu^2 \neq 1$ and $\Sigma = SU(3)_c \times SU(2)_L \times U(1)_{15} \times U(1)_{3R}$, the intermediate scale vacuum value may arise from the SU(5) singlet component of one of the standard $\underset{\sim}{16}$'s (so that $\hat{e}^c$ and a^- may be absent). This mixes $\hat{e}^c$ with λ_-^R but with small mixing angle. Finally, if we assume that $m_{1,2} \gg g_R\langle V\rangle$ (the case of a supermassive gravitino), $m_1 \approx m_2$, and $(g_L/g_R) > (g_R<V>/m_2)$ (which is consistent with $g_L \approx g_R$ and $m_{1,2} \gg g_R<V>$), our results above still obtain.

Let us turn next to the neutral leptons of the SO(10) model. Again, we will assume that $\varepsilon = \delta = 1$ and dim H^1(End B) = 0 (the case where gauge singlets exist has been partially treated in [7, 12]). The relevant particles in this case are: ν_i, ν_i^c (i = e, μ, τ,...), the left and right handed (conjugate) components of the standard neutrinos, the $SU(2)_L$ partners of E and E^c, N and N^c respectively, the $SU(2)_R$ partners of $\hat{e}^c$, a^-, ν, and ν^c respectively and the neutral gauginos λ_{3L}, λ_{3R}, and λ_{15} associated with T_{3L}, T_{3R}, and T_{15} respectively. This leads to a $(2N_f + 7) \times (2N_f + 7)$ mass matrix about which we say little except that the various mechanisms that were proposed to give rise to a sensible neutrino mass spectrum in the E_6 case might work similarly in this case as well [6, 12].

Finally we turn to the phenomenology of the SU(5) models. We have nothing new to say about the neutral sector and concentrate instead on the charged lepton sector. Let us assume that we have only one pair of doublets coming from $(\underset{\sim}{5} + \bar{\underset{\sim}{5}})$ and nothing from $(\underset{\sim}{10} + \overline{\underset{\sim}{10}})$. Then the charged leptons at our disposal are the standard ones e_i and e_i^c, E and E^c from the $\underset{\sim}{5}$ and $\bar{\underset{\sim}{5}}$ respectively, and the charged $SU(2)_L$ gauginos λ_+^L, λ_-^L. The only Yukawa couplings available lead to the mass terms $\bar{\underset{\sim}{5}}\,\underset{\sim}{10} <\bar{\underset{\sim}{5}}_H>$, $\bar{5}_H\,\underset{\sim}{10} <\bar{\underset{\sim}{5}}_H>$ and the gaugino coupling $g_L\,\lambda_+^L\,E <\phi>$. These give rise to the following mass matrix:

$$
\begin{array}{c|ccc}
 & e_j^c & E^c & \lambda_+^L \\
\hline
e_i & y^{ij}<\phi> & 0 & 0 \\
E & y^j<\phi> & 0 & g_L<\phi> \\
\lambda_-^L & 0 & g_L<\phi> & m_{3/2}
\end{array}
\qquad (11)
$$

In the limit that $m_{3/2} \gg g_L <\phi> = M_W \gg y <\phi>$, the approximate eigenvalues are $m_{3/2}$, $y <\phi>$ and $M_W^2/m_{3/2}$. For $m_{3/2} \approx 1$ Tev we again find a charged lepton of mass $\approx O(10$ Gev$)$. If $m_{3/2} \approx M_W$, we have two charged leptons of mass $\approx O(M_W)$. If we allow the "positron" states of $(\underset{\sim}{10}_H + \overline{\underset{\sim}{10}}_H)$ to remain in the spectrum then the analysis is similar to that of SO(10) with states $\hat{e}^c$ and a^- (but no $\lambda_\pm^R$).

We have seen that phenomenological considerations arising from the fermion mass matrices can in fact constrain the possible bundles used in the new compactifications used in ref [7]. In particular bundles B for which dim H^1(End B)=0 have a potential problem with the appearance of light charged leptons.

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$\underline{SO(10)}$	$\underline{\Sigma_0}$	$\underline{G\oplus\overline{G}}$
$\underset{\sim}{10}$	$(\underset{\sim}{\bar{3}}, \underset{\sim}{1}; 2, 0)$	$\beta^2 \mu^{-1}$
	$(\underset{\sim}{3}, \underset{\sim}{1}; -2, 0)$	$\beta^2 \mu$
	$(\underset{\sim}{1}, \underset{\sim}{2}; 0, 1)$	1
	$(\underset{\sim}{1}, \underset{\sim}{2}; 0, -1)$	1
$\underset{\sim}{16}$	$(\underset{\sim}{3}, \underset{\sim}{2}; 1, 0)$	$\beta^{-2} \mu$
	$(\underset{\sim}{1}, \underset{\sim}{2}; -3, 0)$	$\beta^{-6} \mu$
	$(\underset{\sim}{\bar{3}}, \underset{\sim}{1}; -1, 1)$	β^{-4}
	$(\underset{\sim}{\bar{3}}, \underset{\sim}{1}; -1, -1)$	β^{-4}
	$(\underset{\sim}{1}, \underset{\sim}{1}; 3, 1)$	1
	$(\underset{\sim}{1}, \underset{\sim}{1}; 3, -1)$	1

Table 1: $G\oplus\overline{G}$ transformations of the components fo the "incomplete" $\underset{\sim}{10}$ and $(\underset{\sim}{16} + \underset{\sim}{\overline{16}})$ multiplets for the case of only one $\underset{\sim}{10}$ irrep.
$\Sigma_0 = SU(3)_c \times SU(2)_L \times U(1)_{15} \times U(1)_{3R}$.