



Fermi National Accelerator Laboratory

FERMILAB-Pub-86/42-T

March, 1986

AN AMPLITUDE FOR n GLUON SCATTERING

STEPHEN J. PARKE and T. R. TAYLOR

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510.

Abstract

A non-trivial, squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.



For the n gluon scattering amplitude, there are $\lfloor (n+2)/2 \rfloor$ independent helicity amplitudes. At the tree level, the two helicity amplitudes which most violate the conservation of helicity are zero. This is easily seen by embedding the Yang-Mills theory in a supersymmetric theory^{1,2}. Here we give an expression for the next helicity amplitude, also at tree level, to leading order in the number of colors in $SU(N)$ Yang-Mills theory.

If the helicity amplitude for gluons $1 \dots n$, of momenta $p_1 \dots p_n$ and helicities $\lambda_1 \dots \lambda_n$ is $\mathcal{M}_n(\lambda_1, \dots, \lambda_n)$, where the momenta and helicities are labelled as though all particles are outgoing, then the three helicity amplitudes squared of interest are

$$|\mathcal{M}_n(+++\dots)|^2 = c_n(g, N) [0 + \mathcal{O}(g^4)] \quad (1)$$

$$|\mathcal{M}_n(-++++\dots)|^2 = c_n(g, N) [0 + \mathcal{O}(g^4)] \quad (2)$$

$$|\mathcal{M}_n(- - + + + \dots)|^2 = c_n(g, N) [(1 \cdot 2)^4 \sum_P \frac{1}{(1 \cdot 2)(2 \cdot 3)(3 \cdot 4) \dots (n \cdot 1)} + \mathcal{O}(N^{-2}) + \mathcal{O}(g^2)] \quad (3)$$

where $c_n(g, N) = g^{2n-4} N^{n-2} (N^2 - 1) / 2^{n-4} n$ and $(i \cdot j) = p_i \cdot p_j$. The sum is over all permutations, P , of $1 \dots n$. Eqn(3) has the correct dimensions for a n particle scattering amplitude squared and also agrees with the known results^{3,4} for $n=4, 5$ and 6 . The agreement for $n=6$ is numerical.⁴ More importantly, this set of amplitudes is consistent with the Altarelli and Parisi⁵ relationship for all n , when two of the gluons are made parallel. This is trivial for the first two helicity amplitudes but is a highly non-trivial statement for the last amplitude, as shown below,

$$|\mathcal{M}_n(- - + + + \dots)|^2 \xrightarrow{1 \parallel 2} 0 \quad (4)$$

$$|\mathcal{M}_n(- - + + + \dots)|^2 \xrightarrow{2 \parallel 3} 2g^2 N \frac{z^4}{z(1-z)} \frac{1}{s} |\mathcal{M}_{n-1}(- - + + \dots)|^2 \quad (5)$$

$$|\mathcal{M}_n(- - + + + \dots)|^2 \xrightarrow{3 \parallel 4} 2g^2 N \frac{1}{z(1-z)} \frac{1}{s} |\mathcal{M}_{n-1}(- - + + \dots)|^2 \quad (6)$$

where s is the corresponding pole and z is the momentum fraction. The result for

particles 2 and 3 near parallel, eqn(5), is only simple because $\mathcal{M}_{n-1}(-+++...)$ is zero to this order in g so that there is no interference term and therefore azimuthal averaging is not required.

The surprise about this result is that all denominators are simple dot products of two momenta. The Feynman diagrams for $n (> 5)$ gluon scattering contain propagators $(p_i + p_j + p_k)^2$, $(p_i + p_j + p_k + p_m)^2$, These propagators must cancel for eqn(3) to be correct; this occurs for $n=6$. Of course, Altarelli and Parisi have taught us that many cancellations are expected.

Another numerical fact worth mentioning is that to leading order in g but to all orders in N , the amplitude $|\mathcal{M}_{n=6}(- - + + +)|^2$ is permutation symmetric apart from the factor $(1 \cdot 2)^4$. This allows all permutations of this amplitude to be trivially calculated from one such permutation.

We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that eqn(3) is correct.

References

- [1] M. T. Grisaru, H. N. Pendleton and P. van Nieuwenhuizen, Phys. Rev. **D15**, 997 (1977);
M. T. Grisaru and H. N. Pendleton, Nucl. Phys. **B124**, 81 (1977).
- [2] S. J. Parke and T. R. Taylor, Phys. Lett. **157B**, 81 (1985).
- [3] T. Gottschalk and D. Sivers, Phys. Rev. **D21**, 102 (1980);
F. A. Berends, R. Kleiss, P. de Causmacker, R. Gastmans and T. T. Wu,
Phys. Lett. **103B**, 124 (1981).
- [4] S. J. Parke and T. R. Taylor, Fermilab Preprint Pub-85/118-T;
Z. Kunszt, CERN Preprint TH-4319 (1985).
- [5] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).