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Heavy and smooth strings in QCD[1]

Robert D. Pisarski
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

Abstract

Several applications of string models to QCD are studied. This is motivated by a new string model of Polyakov, where a term involving the extrinsic curvature of the surface is added to the action. This acts to smooth out a surface and so produce a "smooth" string; it represents a consistent theory of strings in four dimensions. Also considered are "heavy" strings — strings with heavy fermions tied to their ends.

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I. Introduction

The original motivation of string theory was as an attempt to describe the strong interactions. While in their present incarnation strings have gone far beyond such matters, they might still be of value in describing low energy physics.

In this note I summarize some recent work done in applying string models to QCD. At the outset, I confess that there is no effort made to derive a string model from first principles, in the large N limit or otherwise. For the most part my interest is merely in applying strings to problems where they obviously apply as an effective theory of QCD. The classic example is the flux sheet formed by infinitely massive quarks — the Wilson loop. Neglecting its thickness, over large distances it should be reasonable to describe the flux sheet by the purely geometrical variables of the string. I do not apply strings to systems like light quarks, where it is not apparent how a string picture might arise.

This revival of string theory in the strong interactions was motivated by a new string model proposed last year by Polyakov;[1] essentially the same theory had been proposed over a decade ago by Helfrich,[2] as a model of interfaces. In this string theory the action contains not only the usual Nambu term but a novel term that involves the extrinsic curvature of the surface.

Viewing this theory of surfaces as just another Euclidean field theory shows why it is essential to include the curvature term. The coupling of the curvature term is dimensionless, so the renormalization group instructs us that it is a (marginal) operator whose effect cannot be neglected. The coupling of the curvature term is asymptotically free,[1,3–5] so its effects are dominant in the ultraviolet limit.

There is a simpler and more physical reason, though, why such a term should be included. If the dynamics of a flux tube is controlled only by the Nambu term, the flux tube has an extraordinary property: as long as its length is the same, we can twist it any which way we can without changing the action. This contradicts the intuition that it should cost effort not only to stretch the flux tube lengthwise but to bend it sideways. The resistance to bending is controlled by the extrinsic curvature, for bending necessitates describing how the surface is embedded in space-time. The simplest example of this is a cylinder, which has no intrinsic curvature and yet is bent in one direction. Because the curvature term acts to smooth out the world sheet, I refer to it as a smooth string.

The curvature term dramatically changes things. For the Nambu action, with the proper choice of gauge the theory is free on the world sheet. Smooth strings have complicated interactions in any gauge which can be strongly coupled in the infrared. On the other hand, the extrinsic curvature term does not possess the local conformal symmetry of the Nambu action. Without conformal symmetry there is no reason for there to be a critical dimension, and so the quantum theory of smooth strings should be consistent in a wide range of dimensions, including four.

I begin my discussion not with smooth strings, but with what John Stack and I call "heavy" strings, where very massive fermions are attached to the ends of a rotating string.[6] The long-distance part of the spin-dependent potential is obtained by solving for the classical motion of a rotating string. This quantity is of experimental significance, and cannot be calculated directly in any model except that of heavy strings.

For heavy, open strings, it turns out that even if a curvature term is added to the action, it does not affect the classical solutions we use. To obtain a consistent quantum theory in four dimensions, however, a curvature term must be included.

Turning to smooth strings, I first describe the calculation of the static potential by Eric Braaten, S.-M. Tse, and I.[7] Rather easily, the leading behavior of the static potential at both large and small distances can be computed. If R is the distance, for both large and small R there are terms $\sim 1/R$ in the static potential whose coefficients depend only upon the dimension of space-time, but not on the coupling of the curvature term. While the behavior of the static potential at small distances is certainly not of much relevance to QCD, this is important in establishing that the static potential for smooth strings avoids a pathology that occurs when just the Nambu action is used.

The next topic is an exercise: in the same way that Helfrich and Polyakov introduced theories of smooth surfaces, I investigate a model of smooth paths.[8] The action for a path in d space-time dimensions involves not only its length but its curvature. The theory can be solved in the limit of infinite d, with an unusual solution — an action that starts out with a non-polynomial form ends up looking like a theory with long-range interactions.

I conclude by addressing some general questions concerning smooth strings.[9] While there is no reason to expect a critical dimension, it is still reasonable to ask if the model is well-behaved in all dimensions. In particular, somewhere in smooth strings should be buried the usual Liouville action that appears in the quantization of the Nambu string.[10] But the Liouville action only makes sense if the dimension of space-time, d, is less than 26 — so are smooth strings sensible if $d \geq 26$? Answering this question involves understanding the role of ghosts, the conformal anomaly, and the like.

II. Heavy strings

QCD strings represent an effective theory which remains after the gluonic degrees of freedom have been integrated out. How do matter fields appear in such a string theory?

There is an essential difference between matter fields in the fundamental representation and those in the adjoint. Here I rely upon intuition gained from the large N expansion of an $SU(N)$ gauge theory. If the fermions lie in the fundamental representation, as N becomes large the N fermions are sparse relative to the $N^2 - 1$ gluons. Mesons can always be characterized by a definite number of fermions, which act as sources or sinks for $Z(N)$ flux. Thus fundamental fermions are attached to the string only at isolated points along it. Shortly I show that to consistently attach matter fields to a string at isolated points requires that they live exclusively on the ends of the string.

The string theory is very different for matter fields in the adjoint representation. At large N any adjoint fields are as plentiful as the gluons themselves, so it only seems natural that the string should be composed of the matter fields as much as they are of gluons. This is done by letting the matter fields reside along the entire length of the string.

Strings with matter fields along their length are most familiar in string theory, but strings with matter on the ends have been studied.[11] Putting fermions on the ends leads to a theory originally proposed by Bars.[12]

I note that unlike other possible string theories there is no color label attached to the matter fields. After all, the string itself is formed from the gluons and yet carries no explicit color index, so why should the matter fields? Attaching explicit color factors to the matter fields would seem to be overcounting for QCD strings. All that is required of the matter fields is that they have the same Lorentz transformation properties as the original fields.

I take the Nambu action for the string,

$$S_{string} = \mu \int \sqrt{(\dot{x}\cdot x')^2 - \dot{x}^2(x')^2}\, d\sigma\, d\tau + \ldots \tag{2.1}$$

To obtain a consistent string theory in four dimensions a term for the extrinsic curvature must be added to the string action. For the classical solutions of interest here, however, the effects of the curvature term are negligible (following Curtright *et al.*,[13] as discussed in the next section).

I assume that space-time has a Minkowski signature. This is done only as a matter of expediency, to make it easier keeping track of the signs. The coordinate $x = x^\alpha(\sigma,\tau)$ represents the embedding of the world-sheet in four Minkowski dimensions, with signature (+ - - -). The Lorentz index α on x^α is often dropped: $\dot{x} = \partial x^\alpha/\partial\tau$, $x' = \partial x^\alpha/\partial\sigma$. The world-sheet is parametrized by σ and τ, with τ a time-like variable of infinite extent, while σ describes the spatial extent of the string, $\sigma : 0 \to \pi$. The string tension equals μ. The Nambu action is invariant under arbitrary reparametrizations, $\sigma \to \tilde{\sigma}(\sigma,\tau)$ and $\tau \to \tilde{\tau}(\sigma,\tau)$, subject to the condition that the length of the string is always equal to π,

$$\left(\frac{\partial\tilde{\sigma}}{\partial\tau}\right)_{\sigma=0,\pi} = 0. \tag{2.2}$$

I start by adding scalar particles to the string. By the stated philosophy they are attached only at isolated points along the string. Suppose the action for the scalar of mass m has the usual form, proportional to the length of its world-line:

$$S_{scalar} = m \int_{\sigma=?} \sqrt{\dot{x}^2}\, d\tau, \tag{2.3}$$

In general, this action breaks the reparametrization invariance of the original string action, for it picks out a preferred point in σ along the string. Mathematically, $\dot{x}$ transforms inhomogeneously under an arbitrary reparametrization:

$$\dot{x} \to \frac{\partial\tilde{\sigma}}{\partial\tau}\frac{\partial x}{\partial\tilde{\sigma}} + \frac{\partial\tilde{\tau}}{\partial\tau}\frac{\partial x}{\partial\tilde{\tau}}, \tag{2.4}$$

It is easy to preserve reparametrization invariance simply by restricting the scalar to live exclusively on the ends of the string, $\sigma = 0$ or π. Because of the restriction that the length of the string is fixed, eq. (2.2), at the ends $\dot{x}$ transforms homogeneously under reparametrizations, which are just reparametrizations of τ.

In other words, the boundary conditions of an open string automatically treat the ends as special points, so matter fields can be added there without upsetting the reparametrization symmetry. Having constructed a meson in this way, baryons are for by tying open strings together. These simple arguments also lead to conclusions about glueballs, which are represented by closed strings. As closed strings have no preferred point along their length, in a string model it is not possible to form "mixed" states composed of quarks and a glueball.

Eq. (2.3) is the invariant action for a scalar field propogating in a curved, one dimensional manifold with a metric tensor $g_{00} = \dot{x}^2$. To introduce fermions on the ends the *ein-bein* $e_0^\alpha = \dot{x}^\alpha$ is used. The invariant action for a fermion of mass m at $\sigma = \pi$ is[12,14]

$$S_{fermion} = \int_{\sigma=\pi} \left(-\frac{i}{2} \frac{\dot{x}_\alpha}{\sqrt{\dot{x}^2}} \left(\bar{\psi} \gamma^\alpha \overleftrightarrow{\partial}_\tau \psi \right) + m \sqrt{\dot{x}^2}\, \bar{\psi}\psi \right) d\tau, \tag{2.5}$$

with a similar term for the fermion at the other end of the string; $\overleftrightarrow{\partial}_\tau = \overrightarrow{\partial}_\tau - \overleftarrow{\partial}_\tau$. The fermion field ψ is a Dirac spinor under Lorentz transformations but a scalar under reparametrizations of τ.

The action of eq. (2.5) was first written down by Bars[12] and studied by him, Kikkawa, Sato, Burden, Tassie, and others.[14] While we build on these results, in a fundamental way we differ from ref. (14) in what should be calculated to determine the spectrum of bound states in this theory.

The first thing to do is to determine the equations of motion. I just write that for ψ,

$$D_+ \psi = \left(-i \frac{\dot{\not{x}}}{\dot{x}^2} \partial_\tau - \frac{i}{2\sqrt{\dot{x}^2}} \partial_\tau \left(\frac{\dot{\not{x}}}{\sqrt{\dot{x}^2}} \right) + m \right) \psi = 0. \tag{2.6}$$

While eq. (2.5) is the most natural action for a fermion at the end of a string, it is but the simplest of an infinite class. For example, if a term such as

$$\int_{\sigma=\pi} \left(\frac{\kappa}{2} \frac{\dot{x}_\alpha}{\sqrt{\dot{x}^2}} \partial_\tau (\bar{\psi} \gamma^\alpha \psi) \right) d\tau \tag{2.7}$$

is added to the action, for large fermion masses the spin density if multiplied by $1+\kappa^2$. Thus eq. (2.7) shows how to write an anomalous magnetic moment for the fermion in terms of string variables.

As I said before, it is not clear to me that string models should be used for light quarks. Though for no good reason, it is still interesting to consider how to introduce massless fermions. I argue in analogy to scalar fields in two dimensions, where the scalar field is dimensionless, and the critical behavior is governed by a non-linear model. Fermion fields in one dimension (as on the ends of a string) are dimensionless, so perhaps their critical limit is also controlled by a non-linear model. Thus for massless fermions we propose that the action is the same as in eq. (2.5), but subject to the constraint

$$(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 = \frac{1}{e^2}. \tag{2.8}$$

As appropriate to massless fermions, the constraint and the original action are both chirally symmetric. In the constraint e^2 is a dimensionless coupling constant which we speculate is asymptotically free.

In applying a string model to a meson composed of very heavy quarks, the fermion action of eq. (2.5) should be used. We computed the potential between heavy fermions on the end of a string in this case. The end at $\sigma = 0$ is nailed down by making the fermion at that end infinitely massive; the fermion at $\sigma = \pi$ has mass m. To work in a limit where a string picture certainly applies, we take the fermion mass to be much larger than the mass scale set by the string tension, $m \gg \sqrt{\mu}$. In this limit the simplest possible motion is one fermion encircling the other at constant angular velocity. The total energy of the fermion plus the string is

$$E = \frac{1}{2}mv^2 + m + \mu r + \frac{\vec{l}\cdot\vec{s}}{2m^2}\frac{\mu}{r}, \tag{2.9}$$

where v is the magnitude of the velocity for the rotating fermion, $\vec{l} = mrv\,\hat{z}$ its angular momentum, and $\vec{s}$ its spin.

Eq. (2.9) reproduces the results of Kikkawa, Sato, Burden, Tassie, and others,[14] extrapolated to the non-relativistic limit. They argued that the spectrum of bound states can be extracted from the energy: we disagree. Following a similar treatment of spin-dependent effects in QCD by Eichten and Feinberg,[15] to determine the spectrum it is necessary to evaluate the propagator for the fermion at the end of a string. It turns out that for large fermion mass this doesn't make a difference until one gets to the spin-orbit terms, which as in are eq. (2.9) are small, $\sim 1/m^2$.

Evaluating the fermion propagator gives a rather remarkable result. The fermion propagator is given by $1/D_+$, eq. (2.6). Define D_- from eq. (2.6) by taking the opposite sign for the fermion mass, and form the product of D_+ and D_-:

$$-D_+D_- = \left(\frac{1}{\sqrt{\dot{x}^2}}\left(\partial_\tau + \frac{[\dot{\not{x}}, \ddot{\not{x}}]}{4\dot{x}^2}\right)\right)^2 + m^2. \tag{2.10}$$

There is a very easy way of proving this relation. Begin by assuming that $\dot{x}^2$ is a constant; then it is only necessary to keep track of the terms like $[\dot{\not{x}}, \ddot{\not{x}}]$. To show that it is true for arbitrary $\dot{x}^2$, observe that since D_+ and D_- are each reparametrization invariant, their product must be as well. But it is not to difficult to show that eq. (2.10) is already in a form that is manifestly reparametrization invariant. The essential point is that while $\ddot{\not{x}}$ transforms inhomogenously under reparametrizations of τ, the combination $[\dot{\not{x}}, \ddot{\not{x}}]$ transforms homogenously. Thus eq. (2.10) is correct for arbitary $\dot{x}^2$.

Eq. (2.10) shows that squaring the inverse fermion propagator produces an inverse propagator that looks like that for a scalar field coupled to a background gauge field A_τ,

$$A_\tau = \frac{-i}{4\dot{x}^2}[\dot{\not{x}}, \ddot{\not{x}}]. \tag{2.11}$$

Except for the appearance of A_τ, D_+D_- is the right covariant laplacian for a scalar field propagating in a curved, one-dimensional manifold with metric $g_{00} = \dot{x}^2$. Of course, in this string model A_τ is not really a gauge field — it just looks like one.

As shown by Peskin,[16] since $1/D_+ = (1/D_+D_-)D_-$, it is only necessary to evaluate the scalar type propagator $1/D_+D_-$. Following Feynman,[16] the scalar propagator is written as a path integral:

$$G = \frac{1}{D_+D_-} = \int_0^\infty d\xi_t \int [d\tau]\, exp(-iS_\tau) \tag{2.12}$$

where S_τ is the action

$$S_\tau = \int_0^{\xi_t} \left(\frac{\dot{x}^2}{2}\left(\frac{d\tau}{d\xi}\right)^2 - \frac{i}{4\dot{x}^2}[\dot{\not{x}}, \ddot{\not{x}}]\frac{d\tau}{d\xi} + \frac{1}{2}m^2\right) d\xi. \tag{2.13}$$

This sum over paths looks familiar, but in fact the variables are rather different from what one is used to. In the path integral, the sum over paths is over those in τ space, not in x space; the x coordinates are just other variables which are implicitly determined by the string equations of motion. Similarly, ξ and ξ_t are, respectively, the proper time and total proper time for the paths in τ space.

From eq. (2.13), the only place where the spin of the fermion enters explicitly is in the term which looks like a "Wilson loop" for A_τ, $exp(-i\int A_\tau d\tau)$. Otherwise, the path integral is that for a scalar particle in a background x field.

Using this representation for the fermion propagator, it is easy to determine the potential energy:

$$V_{string}(r) = m + \mu r - \frac{\vec{l}\cdot\vec{s}}{2m^2}\frac{\mu}{r}. \tag{2.14}$$

Compare eqs. (2.9) and (2.14): the rest mass of the fermion and the linear potential of the string are the same, but the sign of the spin-orbit term is opposite to that found from the energy. This difference can be understood from the discussion of spin dependent effects in QCD by Eichten and Feinberg.[15] In a basis in which the upper part of the fermion wave-function is large, the propagation of the fermion includes virtual transitions from the upper to the lower part, and back again, that are missed by the energy.

The sign of the spin-orbit term is a quantity of experimental significance. The sign in eq. (2.14) was first argued to hold in QCD by Buchmüller,[17] on the basis of an intuitive physical picture. This result was also argued by Gromes,[18] who used a general formalism of Eichten and Feinberg.[15]

Further, data from heavy quark spectroscopy and from Monte Carlo simulations support the sign found in eq. (2.14). Thus we cannot claim that our derivation of the spin-orbit term in the string model gives anything other than the expected result. Even so, given the importance of this term we think it worthwhile to have a model in which it can be calculated *analytically.*

The origin of the sign flip for the spin-orbit term between eqs. (2.9) and (2.14) is not transparent when the fermion propagator is evaluated by the path integral method. A more direct method has been worked out by John Stack,[19] in analogy to the approach of Eichten and Feinberg.[15] This involves identifying what the field stength is in terms of the string variables and being very careful in evaluating quantities on the ends of the string.

In all of these considerations only classical solutions of the string enter. To be consistent, one should consider expanding in quantum fluctuations about these classical fields; this would generate corrections $\sim 1/(m^2r^3)$ to the spin-orbit term of eq. (2.14). Since the quantum string with either massless or infinitely massive ends[20] is only consistent in twenty-six dimensions, it should be true as well for ends that have a finite but non-zero mass. Showing this is completely opaque in a canonical formalism but trivial in the functional approach.[6] Hence to consider a consistent string theory in four dimensions I turn to the extrinsic curvature term and smooth strings.

III. Smooth strings

The action for smooth strings is

$$S_{smooth} = \frac{1}{2\alpha}\int d^2z \left(\sqrt{g}\,(\Delta x)^2 + i\lambda^{ab}\left(\partial_a x\cdot\partial_b x - g_{ab}\right)\right) + \mu\int d^2z\,\sqrt{g}, \qquad (3.1)$$

where Δ is the covariant laplacian for the metric g_{ab},

$$\Delta x = \frac{1}{\sqrt{g}}\partial_a\left(\sqrt{g}\,g^{ab}\partial_b\right)x. \qquad (3.2)$$

The vector $x = x^{\alpha}(z^a)$ describes the embedding of the surface in d Euclidean dimensions. The space-time metric is positive definite; the corresponding index α is often suppressed. Instead of parametrizing the world-sheet by the Minkowski variables σ and τ, henceforth I use the Euclidean variables z^a, $a = 1,2$.

The first term in eq. (3.1) is the curvature term. This can be reexpressed as an integral over the square of the second fundamental form for the surface.[1] While this makes the underlying geometry transparent, the above form is more convienient for calculation. In the second term λ^{ab} is a constraint field which fixes the metric to be that intrinsic to the surface, $g_{ab} = \partial_a x\cdot\partial_b x$. The last term is the usual Nambu action.

Both x and z are lengths and so have dimensions of inverse mass. Counting dimensions shows that the coupling of the curvature term, α, is dimensionless, while the string tension μ has dimensions of $(mass)^2$. As a theory with a dimensionless coupling constant it should be renormalizable. Explicit calculations to one loop order show that it is also asymptotically free.[1,3-5]

Using the value for g_{ab}, the Nambu term is equal to

$$\frac{\mu}{2}\int d^2z\,\sqrt{g}\,g^{ab}\,\partial_a x\cdot\partial_b x\,. \tag{3.3}$$

Polyakov[10] first noticed that for the Nambu model it is possible to start just with the action of eq. (3.3); g_{ab} is fixed to be proportional to the intrinsic metric by the equations of motion. This is not possible for the action of eq. (3.1) — for smooth strings it is essential to introduce an explicit constraint to enforce this relation. As will be seen in sec. VI, the presence of the constraint field λ^{ab} complicates the model enormously.

For the Nambu action of eq. (3.3) the metric g_{ab} only has to be proportional to the intrinsic metric, not equal to it. This is because there is a local conformal symmetry of $g_{ab}\rightarrow exp(\phi)\,g_{ab}$. In two dimensions the combination $\sqrt{g}\,g^{ab}$ is conformally invariant, so the curvature term transforms simply,

$$\sqrt{g}\,(\Delta x)^2\rightarrow e^{-\phi}\sqrt{g}\,(\Delta x)^2\,, \tag{3.4}$$

but it is not conformally invariant. In the Nambu model explicit conformal symmetry in the quantum theory requires that the dimension $d=26$. For smooth strings, there is no conformal symmetry to begin with and so they should be consistent in a wide range of dimensions.

At the outset one aspect of smooth strings should be emphasized. Unlike Nambu strings, smooth strings involve higher derivatives on the world-sheet. This means that in Minkowski space-time they will inevitably be plagued with the usual diseases of higher-derivative theories, such as ghosts, acausality, and a lack of unitarity.[21]

Smooth strings are still a perfectly good field theory in Euclidean space-time. The class of sensible Euclidean field theories is far larger than their Minkowski cousins, for matters such as unitarity and the like do not constrain the behavior of correlation functions in Euclidean space-time.

For this reason smooth strings cannot play a role in unified theories, but it is still perfectly good as an effective theory of the strong interactions. As with any effective theory it should only be applicable over distances much larger than the scale set by the fundamental dynamics, which is determined here by the underlying theory of QCD.

The equations of motion for smooth strings have been studied by Curtright, Ghandour, Thorn, and Zachos.[13] Without writing them out in detail it is possible to make a simple but useful observation. Under an aribitrary variation δ the curvature term becomes

$$\delta\left(\sqrt{g}\,(\Delta x)^2\right) = \Delta x\left(\Delta x\,\delta\left(\sqrt{g}\right) + 2\sqrt{g}\,\delta\left(\Delta x\right)\right). \tag{3.5}$$

The equations of motion for the Nambu model are simply $\Delta x = 0$, so by eq. (3.5) the variation of the curvature term automatically vanishes when this is true. That is, on the world-sheet any solution of the Nambu string is a solution for smooth strings as well. The converse does not hold.

There is a caveat attached to this: the string had better not have any singularities which would produce a divergent curvature term. This is essential for understanding closed strings,[13] but is of no concern for open strings.

This observation can be used to conclude that the results of the previous section apply to strings that are smooth as well as heavy. As appropriate to my caution above the calculations should really have been done in Euclidean space-time, but this requires no effort.

There are terms not written in eq. (3.1) which are only sensitive to the global topology of the surface. In any dimension d it is always possible to add $\int \sqrt{g}\,R$, R the Ricci scalar, to the action; this quantity is proportional to the Euler characteristic of the surface. There is another topological invariant which is unique to four dimensions:[1]

$$\nu = \frac{1}{4\pi}\int d^2z\,\sqrt{g}\,g^{ab}\,\epsilon^{\alpha\beta\delta\gamma}\,\partial_a t_{\alpha\beta}\,\partial_b t_{\delta\gamma}, \tag{3.6}$$

with

$$t_{\alpha\beta} = \frac{\epsilon^{ab}}{\sqrt{g}}\,\partial_a x_\alpha\,\partial_b x_\beta. \tag{3.7}$$

The quantity ν is an integer, measuring how many times a surface embedded in four dimensions intersects itself.[1,22] Thus a θ term, $i\,\theta\,\nu$, can be added to the action.

Polyakov[1] suggested that like other theories with a θ term, it is possible that when $\theta = \pi$ smooth strings have a non-trivial infrared stable fixed point in α. If so, about this fixed point the correlation length of smooth strings could always be tuned to be much larger than the characteristic scales of QCD: hence the effective theory of strings in QCD might be determined by this fixed point.

My concerns here are more modest, in trying to calculate certain basic quantities from a theory of smooth strings. It is not that I would not like to use Polyakov's θ term — rather, if this fixed point exists it is surely in strong coupling, inaccesible by pedestrian techniques.

The ν term can be generalized to other dimensions. For dimensions greater than four its coefficient would have dimensions of inverse mass and violate renormalizability. In contrast, in three dimensions the term

$$\frac{\varsigma}{2} \int d^2z \sqrt{g}\, \epsilon^{\alpha\beta\delta}\, \Delta x_\alpha\, t_{\beta\delta} \tag{3.8}$$

can be added to the action. As the coefficient ς has dimensions of mass renormalizability is not disturbed.

Eq. (3.8) can be rewritten in another way. In any number of dimensions the identity

$$\Delta x^\alpha\, n^i_\alpha = K^i_{ab}\, g^{ab} \tag{3.9}$$

holds;[13] n^i_α is a normal vector to the surface and K^i_{ab} the second fundamental form, with the index i running over the $d-2$ normal directions. What is special to three dimensions is that the normal at each point is uniquely defined, $n^\alpha = \epsilon^{\alpha\beta\delta}\, t_{\beta\delta}/2$, so eq. (3.8) becomes

$$\varsigma \int d^2z \sqrt{g}\, K_{ab}\, g^{ab}. \tag{3.10}$$

Eq. (3.10) is only invariant in three dimensions, where the index on the normal direction can be dropped from K^i_{ab}.

In this form the ς term is familiar from the study of interfaces.[2,3] Remember that the curvature term can be written as the square of the second fundamental form, $\sim \int d^2z \sqrt{g}\, (K^i_{ab})^2$. The quantity ς can have either sign, and acts like a mass term for a scalar field. The curvature K^i_{ab} plays the role of the scalar: when ς is negative the surface spontaneously curves up, with K^i_{ab} developing a vacuum expectation value, while for positive ς the surface tries to remain flat. The critical points where ς and/or μ vanish are of interest for the study of interfaces.

These and other parity odd terms are also induced by massless fermions propagating on the world sheet. For examples, see the works of Mazur and Nair[22] and Kavalov, Kostov, and Sedrakyan[23].

IV. The (smooth) static potential

One of the first things required of a theory of flux sheets is the behavior of the static potential. Eric Braaten, S.-M. Tse and I have shown that it is simple to extract certain universal terms for the potential computed from smooth strings.[7] This problem has also been considered by Olesen and Yang.[24]

Choose a physical gauge in which the coordinates of the flux sheet are equal to the physical distances in space and time, $z^1 = r$ and $z^2 = t$; the length of the flux sheet is much greater than its width, $T \gg R$. In this physical gauge it is only necessary to integrate over the $d-2$ transverse degrees of freedom, x_{tr}. To one loop order

$$S_{smooth} = \mu R T + \int d^2 z \left(\frac{1}{2\alpha} \left(-\partial^2 x_{tr} \right)^2 + \mu \left(\partial_a x_{tr} \right)^2 \right), \tag{4.1}$$

so integration over x_{tr} gives

$$S_{smooth} = \mu R T + \frac{d-2}{2} \, tr \, log \left(\frac{-\partial^2}{\alpha} \left(-\partial^2 + m^2 \right) \right), \tag{4.2}$$

with $m^2 = \alpha \mu$.

The transverse fluctuations must vanish at the sides of the flux sheet, $x_{tr}(R) = x_{tr}(0) = 0$, which defines the trace. To discuss eq. (4.2) it helps to think of the system as if it were at a finite temperature $\tilde{T} \sim 1/R$. This analogy is not precise, for at finite temperature the fluctuations only need be periodic in $1/\tilde{T}$, but it is good enough for my purposes.

For any distance R the massless modes in eq. (4.2) give

$$tr \, log \left(-\partial^2 \right) = -\frac{\pi}{12} \frac{T}{R}, \tag{4.3}$$

which is like the free energy of a massless gas at a temperature $\tilde{T}$. The massive modes give a contribution that depends on R in an involved way. At high temperatures the free energy of a massive gas approaches that of a massless gas, so for small R

$$tr\ log\left(-\partial^2+m^2\right)\sim\ tr\ log\left(-\partial^2\right) \tag{4.4}$$

For large distances the massive modes act like a massive gas at low temperature. Thus there is a contribution at zero temeprature $= c\,m^2\,R\,T$ (where c is some constant) and a zero-point (free) energy $\sim m\,T$. That's it, for at low temperature the free energy of a massive gas has the usual Boltzman form, $exp(-\,m/\tilde{T}) = exp(-\,m\,R)$, which is completely negligible at large R.

The static potential is $V(R) = S_{smooth}/T$, so at large R

$$V(R)\simeq\mu_{ren}\,R+V_0-\frac{\pi(d-2)}{24\,R}+\ldots. \tag{4.5}$$

The free energy at zero "temperature" produces a renormalized string tension $\mu_{ren} = \mu + c\,m^2 = \mu(1 + c\,\alpha)$, while the zero point energy $V_0 \sim m \sim \sqrt{\alpha\,\mu}$. At small R both the massless and the massive modes contribute the same amount to give a $1/R$ term twice that at large R:

$$V(R)\simeq-\frac{\pi(d-2)}{12\,R}+\ldots. \tag{4.6}$$

So what? At large R the $1/R$ term is just that familiar from Lüscher, Symansik, and Weisz,[25] while that at small R is obvious after a little thought. But why should there be much significance to this trivial calculation?

Because the $1/R$ terms at both large and small R have coefficients that are exact: although there is a dimensionless coupling constant floating about, these terms are not renormalized by α. Moreover, eq. (4.6) is the dominant term at small R.

The exactness of the $1/R$ term at large R follows from the old analysis of Lüscher,[26] who argued that this term should be universal in any string model. In the context of the present model his conclusion is all the more remarkable, for while both the string tension and the zero point energy are renormalized order by order in perturbation theory, the $1/R$ term (at large R) is not.

The argument at small R is very different. Here I return to the analogy at finite temperature and ask for how the free energy behaves at high temperature. The system acts as an essentially massless gas with non-ideal corrections in α. Since the coupling is asymptotically free, however, the gas approaches the ideal limit as $\tilde{T} \to \infty$, with corrections $\sim 1/log(\tilde{T})$:

$$V(R) \simeq -\frac{\pi(d-2)}{12\,R}\left(1+\frac{a}{log(R)}\right)+\dots \tag{4.7}$$

at small R where a is some constant that depends only upon the dimension d and not upon the coupling α.

As with any general argument it should only be believed after it is checked explicitly. Eric Braaten and S.-M. Tse[27] have carried out involved calculations to two loop order at large d to verify these arguments. This is especially important for the $1/R$ term at large R, for it is only at two loop order that non-universal corrections would first show up; Braaten and Tse find they do not.

This result is in contrast to that obtained from the Nambu model. A calculation by Alvarez[28] at large d and by Arvis[20] in $d = 26$ give the same result:

$$V_{Nambu}(R) = \mu\,R\sqrt{1-\frac{R_c^2}{R^2}}, \tag{4.8}$$

$R_c^2 = \pi(d-2)/(12\,\mu)$. Expanding about large R gives $\mu\,R$ plus the Lüscher term, but for $R < R_c$ the potential is imaginary. Arvis[20] showed how the imaginary part of V_{Nambu} is related to the tachyon of the 26-dimensional bosonic string.

We have not been able to compute the static potential for smooth strings over all distances (even in the limit $d \to \infty$), but there is no reason to suspect any pathology like that of Nambu strings. Over intermediate distances the static potential will certainly depend upon α in a complicated way, but given the behavior it must obey at small and large R, V_{smooth} can only grow monotonically in a rather uninteresting way.

To be consistent in its interpretation as an effective theory, V_{string} should only be taken seriously over large distances. At short distances the static potential in QCD is controlled not by strings but by perturbation theory. Indeed, at short distances strings give a potential $1/R$ for any d, while single gluon exchange gives $1/R^{d-3}$; it is only an accident that even the powers agree in four dimensions.

Pushing coincidence beyond the limits of reason, I observe that the Cornell model,[29] which fit charmonium spectroscopy with a potential $\mu\,R + \kappa/R$, found a best fit for a value $\kappa = .52$. Making the unjustified assumption that κ should be taken from V_{smooth} at small R gives $\kappa = \pi(d-2)/12$ or $k \simeq .524\ldots$ in four dimensions.

Of course it would be preposterous to claim this numerical agreement as a phenomenological success of string theory.

But it is.

V. Smooth paths

As the addition of a curvature term tends to smooth out surfaces, so it is possible to produce "smooth paths".[8] I find this exercise of interest because of the unusual solution it has in the limit of high dimension. This model has also been studied by Alonso and Espriu[5].

Let $\overline{x}$ represent a path in d flat, Euclidean dimensions. As a path, $\overline{x}$ is a function of a single parameter t, $\overline{x} = \overline{x}(t)$. I wish to construct a type of one-dimensional gravity, where the action is invariant under arbitrary reparametrizations of t, $t \to \tilde{t}(t)$. This can be done by dealing only in quantities that are manifestly reparametrization invariant.

The fundamental invariant is the arc length s, which is defined by the relation that $d\overline{x}/ds$ be the tangent vector of unit norm, $(d\overline{x}/ds)^2 = 1$. The arc length is clearly invariant, for with a given parametrization t, $ds = \sqrt{(d\overline{x}/dt)^2}\, dt$.

Any action constructed only from x and s will automatically be invariant as long as the parameter t doesn't appear explicitly. For the action I take

$$S = \frac{1}{\alpha}\int ds \sqrt{\left(\frac{d^2\overline{x}}{ds^2}\right)^2} + m\int ds\ , \tag{5.1}$$

Both x and s have dimensions of length, so α is a dimensionless coupling constant; m is a mass, $m \geq 0$.

The last term in the action typical for a relativistic particle of mass m. The first has a geometric interpretation: at a given point the curvature of a path is given by $k = \sqrt{(d^2\overline{x}/ds^2)^2}$, so the first term in eq. (4.1) is the total curvature of the path. This action is unusual in that it is non-polynomial, involving k and not k^2. I choose eq. (4.1) so that the coupling of the k term is dimensionless, like that of smooth strings.

Theories of paths naturally have relevance for polymers in dilute solution.[30] Because this action is non-polynomial in x it probably is of limited significance for polymers; at best it applies only to an isolated point in the phase diagram. As a field theory, however, it does exhibit some novel features.

There is one aspect special to paths in three dimensions. As defined above k is the absolute curvature, and is positive semi-definite, $k \geq 0$. In three dimensions it is possible to define a signed curvature, k_s, whose magnitude is equal to k but which can have either sign. The integral of this signed curvature (over ds) is a topological invariant, proportional to the number of self-intersections of a curve with itself. Thus the curvature term of eq. (4.1) is an integral over the absolute value of k_s. There is no signed curvature is more than three dimensions, with the absolute curvature the only invariant.

To calculate it is convienient to use reparametrization invariance to choose a gauge in which the curve is parametrized directly by the arc length. In this arc length gauge, local properties of the theory are determined by the Lagrangian

$$\tilde{\mathcal{L}} = \frac{1}{\alpha}\sqrt{\ddot{\overline{x}}^2} + \frac{\omega}{2\alpha}\left(\dot{\overline{x}}^2 - 1\right) , \tag{5.2}$$

with $\omega = \omega(s)$ a constraint field to enforce the gauge condition; $\dot{\overline{x}} = d\overline{x}/ds$ and $\ddot{\overline{x}} = d^2\overline{x}/ds^2$. Unlike the original action there is no trace left of the mass m: the term $m \int ds$ only contributes to the total free energy and does not affect the local dynamics. This is in contrast to theories of surfaces, where in conformal gauge the Nambu term remains to affect the local equations of motion.

Dropping the mass as in eq. (5.2) shows that the theory has not one dimensionless coupling constant and one mass parameter, as first appeared from eq. (5.1), but just one dimensionless coupling constant. Eq. (5.2) has the form of a non-linear sigma model, with $\dot{\overline{x}}$ acting as the d-component sigma field. The kinetic energy for the sigma field $\dot{\overline{x}}$ is unusual in that it is the square root of what one would expect, $\ddot{\overline{x}}^2$. The language of the sigma model is helpful in discussing smooth paths.

The theory is soluble for paths in a space-time of infinitely high dimension:

$d \to \infty$ with $\alpha\, d$ held fixed. To do this I use the following identity:

$$\int_{-\infty}^{+\infty} d\lambda\, \exp\left(-a^2\,\lambda^2 - \frac{b^2}{\lambda^2}\right) = \frac{\sqrt{\pi}}{a} \exp\left(-2\,a\,b\right), \tag{5.3}$$

for $a > 0$, $b \geq 0$. This identity appears unremarkable. To prove it, the obvious thing is to expand about the stationary point in λ, $\lambda^2 = b/a$. This gets the coefficient of the exponential right, but misses the important feature of eq. (5.3) — that the prefactor is independent of b. When corrections to the leading stationary point approximation are considered, it appears certain that they will introduce a dependence upon b. Nevertheless they do not; I leave the proof of eq. (5.3) as an exercise to amuse the reader.

What's so important about the prefactor? I use this identity to rewrite the action of eq. (5.2) as

$$\mathcal{L}' = \frac{1}{2\alpha}\,\lambda^2 + \frac{1}{2\alpha}\,\frac{\ddot{\vec{x}}^{\,2}}{\lambda^2} + \frac{1}{2\alpha}\,\omega\,\left(\dot{\vec{x}}^{\,2} - 1\right). \tag{5.4}$$

In this Lagrangian λ is a function of s, $\lambda = \lambda(s)$, which acts as another constraint field. From eq. (5.3), integration over λ should reproduce the original Largrangian. In doing this, I am sloppy about various constants that might appear in the measure of the integration, but because b does not appear in the prefactor of eq. (5.3), I am certain that curvature dependent terms will not. This is of crucial importance — if factors of $\ddot{\vec{x}}^{\,2}$ did appear in the measure, they would surely alter the dynamics unless I was very careful to treat them properly. With the above identity, I can happily ignore this.

The vector $\vec{x}$ only appears quadratically in eq. (5.4), so immediately it can be integrated out to give an effective Lagrangian,

$$\mathcal{L}_{eff}(\lambda,\omega) = \frac{1}{2\alpha}\,\lambda^2 - \frac{1}{2\alpha}\,\omega + \frac{d}{2}\; tr_p\; log\; G^{-1}(\lambda,\omega)\,. \tag{5.5}$$

The inverse propagator for the x field is G^{-1},

$$G^{-1}(\lambda,\omega) = D^2 \left(\frac{1}{\lambda^2}\right) D^2 - D\omega D\,, \tag{5.6}$$

$D = d/ds$. The trace in eq. (5.5), tr_p, is only over the momentum p, $D^2 = -p^2$.

Having obtained this effective Lagrangian, it is amenable to the usual sort of approximation at large d. I expand $\lambda(s)$ and $\omega(s)$ about fields λ_c and ω_c: $\lambda = \lambda_c + \lambda_{qu}$, $\omega = \omega_c + \omega_{qu}$. For λ_c and ω_c to give the true ground state at large d, the terms linear in λ_{qu} and ω_{qu} must vanish. This gives the equations

$$\lambda_c d\left[\frac{1}{\alpha d} - \operatorname*{tr}_p\left(\frac{p^2}{\lambda_c^4}\,\frac{1}{p^2/\lambda_c^2+\omega_c}\right)\right] = 0\,, \tag{5.7a}$$

$$-\frac{d}{2}\left[\frac{1}{\alpha d} - \operatorname*{tr}_p\left(\frac{1}{p^2/\lambda_c^2+\omega_c}\right)\right] = 0\,, \tag{5.7b}$$

where I assume that λ_c and ω_c commute with p.

The type of solution typical at large d — λ_c and ω_c both constant — can be shown not to work. I have written eq. (5.7) so that its peculiar solution is apparent. Let ω_c be a constant, but take λ_c to be an operator in *momentum* space: $\lambda_c = (-D^2)^{1/4}$. Within the trace, $p^2/\lambda_c^2 = |p|$, and both equations give

$$\frac{1}{\alpha d} - \operatorname*{tr}_p \frac{1}{|p|+\omega_c} = 0, \tag{5.8}$$

which always has a solution for constant ω_c.

To interpret this solution, I use the value for λ_c and ω_c to write what the Lagrangian for the x's is like:

$$\tilde{L} \sim \int ds\left(\frac{1}{2}\dot{x}\,(-D^2)^{1/2}\,\dot{x} + \frac{1}{2}\,\omega_c\left(\dot{x}^2 - 1\right)\right) \tag{5.9}$$

To leading order in $1/d$, what started out looking out like a non-linear sigma model with a square root kinetic energy ends up looking like a sigma model with long-range interactions. As shown by Brézin, Zinn-Justin, and Le Guillou,[31] sigma models with a kinetic term $(-D^2)^{1/2} = |p|$ have a critical dimension equal to one.

The solution of eq. (5.8) is standard for a sigma model in its critical dimension. The coupling α is asymptotically free (this is true for all $d > 1$). Constant ω_c corresponds to the dynamical generation of mass in the disordered phase, and depends upon the renormalized coupling constant according to dimensional transmutation. The value of ω_c does depend upon the boundary conditions applicable, such as the total arc length and whether the ends are held fixed or allowed to wriggle freely.

The equivalence between smooth paths and a non-linear sigma model with long-range interactions breaks down beyond leading order in $1/d$. Both theories involve the constraint field ω in the same way, but the constraint field λ is special to smooth paths. Thus the theories differ at finite d when fluctuations in λ_{qu} are included.

The integration over fluctuations in λ introduces features that are not typical of sigma models in their critical dimension. The correlation length is always finite in a sigma model. For smooth paths, at finite d there appears to be a logarithmic dependence on the total arc length $\sim 1/d$. Consider the expansion of $D^2(1/\lambda^2)D^2$, which enters into the inverse x propagator and so the effective Largrangian, eqs. (5.5) and (5.6). Writing just the terms quadratic in λ_{qu} in momentum space,

$$D^2 \frac{1}{\lambda^2} D^2 = \ \ldots + |p''| \, \lambda_{qu} \frac{1}{\sqrt{|p|}} \lambda_{qu} \sqrt{|p'|} + \sqrt{|p''|} \, \lambda_{qu} \frac{1}{\sqrt{|p|}} \lambda_{qu} \, |p'|$$
$$+ \sqrt[3/2]{|p''|} \, \lambda_{qu} \frac{1}{|p|} \lambda_{qu} \, \sqrt[3/2]{|p'|} + \ldots . \tag{5.10}$$

The problem is the last term in eq. (5.10): it involves $1/|p|$ in a virtual state, which in one dimension produces a logarithm of the total arc length. These infrared divergences do not worsen to higher order in λ_{qu}, so at finite d these logarithms could be tamed by working at finite total arc length with fixed ends.

VI. Smooth strings at large d

Smooth strings can be solved when the number of dimensions, d, is very large.[4,5,7,24] I use this to investigate whether smooth strings are well defined in more than twenty-six dimensions. For now I merely outline the calculation and present some partial results.[9]

For metrics in two dimensions, by a general coordinate transformation it is always possible to specialize to a conformal gauge, $g_{ab} = \rho\, \delta_{ab}$. In this gauge the action of eq. (3.1) becomes

$$S_{smooth} = \frac{1}{2\alpha} \int d^2z \, \rho \left(\left(\frac{-1}{\rho} \partial^2 x \right)^2 + i \, \frac{\lambda^{ab}}{\rho} \left(\partial_a x \cdot \partial_b x - \rho\, \delta_{ab} \right) \right)$$
$$+ \int d^2z \, \mu \, \rho \; + \; S_{ghost}(\rho). \tag{6.1}$$

To be complete, the contribution of the Fadeev-Popov ghosts for general coordinate invariance is included in S_{ghost}. The ghosts only depend upon properties of the metric (and its measure in the functional integral), so S_{ghost} is a function of ρ alone.

Compare this action with that for a higher derivative sigma model (HDSM) which I proposed and studied a few years ago:[21]

$$S_{HDSM} = \frac{1}{2\tilde{\alpha}} \int d^2z \left(\left(-\partial^2\sigma\right)^2 + i\,\lambda^{ab}\left(\partial_a\sigma\cdot\partial_b\sigma - \delta_{ab}\right)\right), \tag{6.2}$$

where σ is a d dimensional vector and $\tilde{\alpha}$ a dimensionless coupling constant. The similarity between smooth strings (in conformal gauge) and this HDSM is obvious — the HDSM corresponds to a sum over flat surfaces, containing the dynamics of the constraint field λ^{ab} but not that of the metric field ρ. Although related only in part to smooth strings, the calculations done in the HDSM are helpful nevertheless.

For smooth strings the x's appear quadratically in the action, so they can be integrated out to give the effective action

$$S_{eff}(\rho,\lambda) = \frac{d}{2}\, tr\; log\left(\frac{-1}{\rho}\partial^2\left(\frac{-1}{\rho}\partial^2\right) - \frac{i}{\rho}\,\partial_a\lambda^{ab}\partial_b\right)$$
$$+\int d^2z\left(\mu\,\rho - \frac{i}{2\alpha}\,\rho\,\lambda^{ab}\delta_{ab}\right) + S_{ghost}(\rho). \tag{6.3}$$

In integrating over the x's the only subtlety is that it must be done over the invariant measure on the world-sheet, $\int d^2z\,\rho$.

For simplicity I assume that the world-sheet is an infinite disc, and expand about the obvious stationary point in ρ and λ^{ab}:

$$\rho = \rho_0\left(1+\sqrt{\frac{8\pi}{d}}\,\frac{\tilde{\rho}}{m}\right),\quad \lambda^{ab} = \frac{m}{\rho_0}\left(-im\,\delta^{ab} + \sqrt{\frac{8\pi}{d}}\,\tilde{\lambda}^{ab}\right). \tag{6.4}$$

At large d it is consistent to expand the action in powers of $\tilde{\rho}$ and $\tilde{\lambda}^{ab}$. Expansion to linear order fixes ρ_0 and m^2 in terms of the string tension μ and the coupling α, as has been done by others.[4,5,7,24] I describe here what happens in expanding to quadratic order, where questions of stability can be addressed.

To quadratic order in $\tilde{\rho}$ and $\tilde{\lambda}$,

$$S_{eff}(\rho,\lambda) = S_{eff}(\rho_0, -im^2/\rho_0)$$

$$+\frac{1}{2}\int d^2z\left(\tilde{\rho}\,\Delta^{-1}(\tilde{\rho},\tilde{\rho})\,\tilde{\rho}+2\tilde{\rho}\,\Delta^{-1}(\tilde{\rho},\tilde{\lambda})\,\tilde{\lambda}+\tilde{\lambda}\,\Delta^{-1}(\tilde{\lambda},\tilde{\lambda})\,\tilde{\lambda}\right)+\dots\,, \tag{6.5}$$

$\Delta^{-1}(\tilde{\rho},\tilde{\lambda})=\Delta^{-1}(\tilde{\lambda},\tilde{\rho})$. Wherever possible the indices on the constraint field $\tilde{\lambda}$ are dropped.

I begin with the diagonal term for the metric field, $\Delta^{-1}(\tilde{\rho},\tilde{\rho})$. To determine this it is possible to ignore $\tilde{\lambda}$ and write

$$S_{ghost}(\rho)+\frac{d}{2}\,tr\;log\left(\frac{-1}{\rho}\partial^2\right)+\frac{d}{2}\,tr\;log\left(\frac{-1}{\rho}\partial^2+\frac{m^2}{\rho_0}\right)$$

$$=\frac{1}{2}\int d^2z\;\tilde{\rho}\,\Delta^{-1}(\tilde{\rho},\tilde{\rho})\,\tilde{\rho}\;+\;\dots\,, \tag{6.6}$$

The first two terms on the left hand side are identical to that in Polyakov's formulation of the Nambu model and give the usual factor of the Liouville action.[10] The last term on the left hand side is new, but for this term it is possible to expand perturbatively in $\tilde{\rho}$. To express the result I introduce the dimensionsionless variable P, which is related to the magnitude of the momentum by $P=p^2/m^2$. Then

$$\Delta^{-1}(\tilde{\rho},\tilde{\rho})=-\left(\frac{d-26}{6d}\,P+2\,L_4(P)\right) \tag{6.7}$$

where $L_4(P)$ equals

$$L_4(P)=\frac{1}{\sqrt{P(P+4)}}\,log\left(\frac{\sqrt{P+4}+\sqrt{P}}{\sqrt{P+4}-\sqrt{P}}\right). \tag{6.8}$$

The first term in eq. (6.7) is the Liouville action, written in an unfamiliar way after rescaling ρ and pulling out various constants. The second term is due to the integration over the massive mode.

At high momentum, $P\gg 1$, all that matters is the part due to the Liouville action,

$$\Delta^{-1}(\tilde{\rho},\tilde{\rho})\approx-\frac{d-26}{6d}P+O\left(\frac{log(P)}{P}\right). \tag{6.9}$$

The cause of concern is that in more than twenty-six dimensions $\Delta^{-1}(\tilde{\rho},\tilde{\rho})$ is negative, so apparently the theory is unstable.[4]

This conclusion might be premature. While $\Delta^{-1}(\tilde{\rho},\tilde{\rho})$ is negative, what is of significance are physical correlation functions. To determine these, it is not sufficient to look just at the inverse propagators — the full propagators must be calculated. For example, in calculating the two-point function for $\tilde{\rho}$, both $\Delta^{-1}(\tilde{\rho},\tilde{\rho})$ and the off-diagonal components contained in $\Delta^{-1}(\tilde{\rho},\tilde{\lambda})$ enter. These can and do change things.

Getting the full inverse propagator is easy. The worst part of this is $\Delta^{-1}(\tilde{\lambda},\tilde{\lambda})$, but this can be read off directly from the calculations done in the HDSM. The remaining term, $\Delta^{-1}(\tilde{\rho},\tilde{\lambda})$, is simple to determine perturbatively.

It is more difficult to determine the propagator from the inverse propagator, for in all there are eight scalar functions in the propagator — five for $\Delta(\tilde{\lambda},\tilde{\lambda})$, two for $\Delta(\tilde{\rho},\tilde{\lambda})$, and one for $\Delta(\tilde{\rho},\tilde{\rho})$. These can be disentangled using the results from the HDSM. I quote just the result for the propagator in the $\tilde{\rho}-\tilde{\rho}$ channel at high momemtum ($P \gg 1$):

$$\Delta(\tilde{\rho},\tilde{\rho}) \approx \frac{+3}{P\,log^2(P)} + \ldots \tag{6.10}$$

At high momenta the off-diagonal components in the inverse propagator overwhelm the negative part of eq. (6.9) to give a propagator that is positive in this channel. This suggests that smooth strings are at least perturbatively stable in all (positive) dimensions.

At infinite d the results from the HDSM can be used to investigate the stability of smooth strings not just at high momenta, but over all momenta. The solution shows that this can only be true if the renormalized value of $\alpha\, d$ is less than some critical value ($\alpha\, d$ is held fixed at large d). These calculations are left for another day.

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