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A PRECISE FORMULATION OF THE EFFECTIVE-VECTOR-BOSON METHOD FOR HIGH ENERGY COLLISIONS*

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Abstract

The effective vector boson method for high energy collisions is put on a sound basis. Simple exact expressions of the left-handed, right-handed, and longitudinal distribution functions are derived for vector bosons with arbitrary (V, A) couplings. Using group-theoretical arguments, non-diagonal terms in the vector boson polarization index are shown to vanish identically. No approximation is introduced other than the on-mass-shell continuation of a regularized vector boson hard scattering amplitude. The new improved effective vector boson formula extends the applicability of this method to a wide kinematic region beyond that of the usual formulation, and it obviates the need to do case-by-case numerical tests.

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INTRODUCTION

The conventional derivation of the effective vector boson method,[1,2] fashioned after the classical Weizsacker-Williams equivalent photon approach, invokes a number of approximations of uncertain accuracy. The list of (inter-related) approximations include: small-angle (or 'collinear') approximation, 'leading log' expansion, arbitrariness in the definition of vector-boson polarization vectors (as the result of the above approximations), the (unjustified) neglect of non-diagonal terms in the polarization index, and the extrapolation to on-shell vector-boson momentum. Previous studies of the reliability of this method depend almost exclusively on numerical comparisons of the effective-vector-boson calculations with 'exact' ones for test examples on a case-by-case basis.[3] Although such studies have yielded encouraging results so far, they do not provide an understanding of the reliability of the method in general. Neither do they suggest a systematic way of including corrections to the above-mentioned approximations.

Applying a recently proposed factorization technique for analyzing Feynman diagrams to this problem,[4] we develop a precise formulation of the effective vector-boson method. This implementation of the effective vector boson idea does not require any approximation other than the on-mass-shell continuation of a regularized hard-scattering amplitude. We also show that, due to group-theoretic considerations, off-diagonal terms in the vector-boson polarization sum are identically zero[5] for scattering off massless partons. The use of precisely-defined vector-boson distribution functions provides an 'improved effective vector-boson formula' which extends the region of applicability of this method, and renders the case-by-case numerical verification unnecessary. Due to spatial limitation, only salient features of the new results can be reported here.[6] In its present form, the formalism does not address problems associated with contributions from diagrams other than the vector-boson-exchange type.[7]

FACTORIZATION OF THE SCATTERING AMPLITUDE

Consider the vector-boson exchange contribution to the generic process:

$$f + A \longrightarrow f' + X \tag{1}$$

where f, f' are light fermions (leptons or quarks), A is any light particle, and X represents an arbitrary final state consisting of at least one heavy particle. (The heavy particle might represent 'new physics'.) The momenta associated with these particles are defined in Fig.1. We use k^2 =

$k'^2 = p^2 = 0$, $-p_x^2 = M_x^2$, $q = k - k' = p_x - p$, and $s = -(k + p)^2$. We shall use dimensionless invariant parameters and variables

$$x = \frac{M_x^2}{s}\ ,\quad \Delta = \frac{M_V^2}{s-M_x^2}\ ,\quad \zeta = \frac{q^2}{s-M_x^2} = \frac{1}{2}\,(1-\cos\theta_C) \tag{2}$$

where θ_C is the CM scattering angle of f'.

The method of Ref.4 involves applying group-theoretical analysis to the numerator of the vector-boson propagator factor $g^{\mu}{}_{\nu} + q^{\mu}q_{\nu}/\,M_V^2$ (the polarization matrix) to write the scattering amplitude for this process in the explicitly factorized form:

$$T = J_m(q^2,M_X^2,\ldots)\ \frac{D(\xi_B,\phi_B)^m{}_n}{q^2 + M_V^2}\ j^n(q^2) \quad . \tag{3}$$

Here fermion helicity indices have been suppressed, $j^n(q^2) = \langle k'|J\cdot\epsilon_n{}^*|k\rangle$ is the (exactly calculable) helicity vertex function for the upper vertex[4] and $J_m(q^2,M_X^2,\ldots) = \langle p_x|J_\mu{}^+\cdot\epsilon_m|p\rangle$ is the (not necessarily calculable) helicity amplitude for the 'hard process'

$$V^* + A \longrightarrow X \quad . \tag{4}$$

The superscript * indicates a (space-like) virtual V-boson. $D(\xi,\phi)^m{}_n$ is the 'spin 1' SO(2,1) transformation, an element of the little group of q^μ, which relates the lower vertex to the upper vertex configurations in the Brick-Wall frame for which $q^\mu:(0,0,0,\sqrt{q^2})$. The D-function consists of a Lorentz boost along the x-axis (by the hyperbolic 'angle' ξ) and a rotation around the z-axis (by the angle ϕ),

$$D(\xi,\phi)^m{}_n = e^{-im\phi}\ d(\xi)^m{}_n \ . \tag{5}$$

It is the analog of the familiar spin 1 rotation matrix associated with a time-like vector meson exchanged in the s-channel. See Ref.4 for details and the explicit expression for $d(\xi)$.

Cross-Section Formula

The relevant cross-section is obtained by taking the square of the scattering amplitude (3) and integrating over the phase-space volume of the scattered fermion f'. A careful study of the kinematics[6] reveals that the only ϕ dependence in the integrand occurs in the exponential factor of the D-function, Eq.(5), provided A is massless. Hence, the integration over the (Brick-wall frame) azimuthal angle forces the sum over vector boson polarization indices to be diagonal. We choose the remaining integration variable to be ς, Eq.(2), and obtain an exact formula for the cross-section

$$\frac{d\sigma}{dx\ d\Gamma'} = \frac{x}{16\pi^2} \sum_n \int \frac{\varsigma\ d\varsigma}{(\varsigma+\Delta)^2}\ h^n(\varsigma)\ \frac{d\hat{\sigma}_n^*}{d\Gamma'}\ (q^2, M_x, \ldots) \tag{6}$$

where $d\Gamma'$ is the phase-space element for the state X after factoring out $(2\pi)^4 d^4p_x$,

$$h^n = g_R^2 \left[d(\xi)^n_{\ 1}\right]^2 + g_L^2 \left[d(\xi)^n_{\ -1}\right]^2 \tag{7}$$

is the square of $d^n{}_\lambda\ j^\lambda$ averaged over the fermion helicities, and

$$\frac{d\hat{\sigma}_n^*}{d\Gamma} = \frac{1}{2M_x{}^2}\ J_n^*(q^2, M_X{}^2, \ldots)\ J_n(q^2, M_X{}^2, \ldots) \tag{8}$$

(no sum over n) is the 'hard scattering' cross-section (HSXS) for a virtual vector boson of helicity n, Eq(4). Eq.(6) is the <u>exact</u> cross-section formula for process (1) due to the exchange of a vector boson (Fig.1).

THE EFFECTIVE VECTOR-BOSON APPROXIMATION AND THE EXACT VECTOR-BOSON DISTRIBUTION FUNCTIONS

In order to derive the 'effective vector-boson' formula, we need to replace the HSXS ($d\sigma_n{}^*$) for a virtual V^* by that for an on-shell V ($d\sigma_n$), and take $d\sigma_n$ out of the integration sign in Eq.(6). This is a reliable procedure provided the q^2-dependence of $d\sigma_n{}^*$ is 'smooth' compared to that of the other factors in the integrand of Eq.(6). We note, of course, that the propagator factor is peaked in the small ς region with typical width Δ. Hence the reference scale for measuring 'smoothness' is Δ.

There is no distinction between the transverse polarization vectors of a virtual vector boson and those of an on-shell boson; they are both independent of q^2. Well-established experience in dispersion relations leads one to believe that $d\sigma_n{}^*$ is indeed smooth, hence it can be safely replaced by the on-shell $d\sigma_n$. In contrast, the longitudinal polarization vector for the virtual particle,

$$\epsilon_0(q^2)\ :\ \left(\frac{|\vec{q}|}{\sqrt{q^2}},\ 0,\ 0,\ \frac{q_0}{\sqrt{q^2}}\right)\quad , \tag{9}$$

has a 'kinematic singularity' at $q^2 = 0$ which is absent in its on-shell counter-part. Therefore, we must explicitly take this factor into account before replacing $d\sigma_0^*$ by $d\sigma_0$ to avoid incurring unacceptable errors in the approximation.

We obtain, therefore,

$$\frac{d\sigma_n}{dx\, d\Gamma'} = f_n(x) \frac{d\hat{\sigma}_n}{d\hat{\Gamma}'}(M_x^2,\ldots) \quad , \tag{10}$$

where $n = -1, 0, 1$; $d\hat{\sigma}_n/d\hat{\Gamma}'$ is the HSXS for an on-shell vector boson of helicity n, and

$$\begin{aligned} f_n(x) &= \frac{x}{16\pi^2}\int_0^1 \frac{\varsigma\, d\varsigma}{(\varsigma+\Delta)^2}\, h_n(\xi) \quad , \qquad n = \pm 1 \\ f_0(x) &= \frac{x}{16\pi^2}\int_0^1 \frac{\Delta\, d\varsigma}{(\varsigma+\Delta)^2}\, h_0(\xi) \quad . \end{aligned} \tag{11}$$

These are the <u>exact</u> vector boson distribution functions (VBDF) for helicity n. The SO(2,1) boost parameter ξ which enters the d-functions in h^n, Eq.(7), is specified by

$$\cosh\xi + 1 = 2 \,/\, [x + (1-x)\varsigma] \quad . \tag{12}$$

It is convenient to introduce $F_\pm = f_1 \pm f_{-1}$. (F_+ is just the 'transverse' vector boson distribution function of the original literature.[1,2]) <u>The explicit expressions for the exact VBDFs are:</u>

$$\begin{aligned} F_+(x) &= \alpha^+ x \int_0^1 \frac{\varsigma\, d\varsigma}{(\varsigma+\Delta)^2}\,(\cosh^2\xi + 1) \quad , \\ f_0(x) &= \alpha^+ x \int_0^1 \frac{\Delta\, d\varsigma}{(\varsigma+\Delta)^2}\,(\cosh^2\xi - 1) \quad , \end{aligned} \tag{13}$$

and

$$F_-(x) = \alpha^- x \int_0^1 \frac{\varsigma\, d\varsigma}{(\varsigma+\Delta)^2}\,\cosh\xi \quad ,$$

where $\alpha^\pm = (g_R^2 \pm g_L^2)/32\pi^2$. These integrals only involve simple rational fractions, and can easily be done in closed form. The results will be given elsewhere.[6]

APPROXIMATE DISTRIBUTION FUNCTIONS

In order to compare the exact distributions with the 'leading log' approximations (in the expansion of the small parameter Δ) in the literature,[1,2] we note that the leading term in each integral arises from the lower end of the integration range, which can be extracted by replacing $\cosh\zeta$ with its value at $\zeta = 0$, i.e., $(2-x)/x$. The results obtained for F_+ and f_0 agree with those given in refs. 1 and 2. We have, in addition[6,8],

$$F_-(x) \simeq \alpha^- (2-x) \log(1/\Delta) \quad . \qquad (14)$$

As is well known[1,2], the leading term in f_0 is independent of Δ, and sub-leading terms vanish in the limit $\Delta \to 0$; thus the asymptotic formula should be quite accurate. Conversely, the leading terms in $F_\pm$ are proportional to $\log \Delta$, as shown above, and the sub-leading terms do not vanish as $\Delta \to 0$. From the exact expressions for $F_\pm$, we can easily derive the terms independent of Δ.[6] These terms turn out to be numerically significant compared to the leading-log terms and are opposite in sign for most relevant kinematic regions. It is, therefore, necessary to use the exact formulas whenever transverse polarization cross-sections are non-negligible. The asymptotic formulas for the transverse distributions, including the constant terms, are

$$\begin{aligned} F_+ &= 2\alpha^+ \{ [1 + (1-x)^2] [\log(1/\Delta) - 1] \\ &\qquad - 2(1-x) [\log(1/x) + 1] \} / x \\ F_- &= \alpha^- \{ (2-x) [\log(1/\Delta) - 1] - 2 \log(1/x) \} \quad . \end{aligned} \qquad (15)$$

These formulas are, in principle, accurate to the same degree as that for f_0. Terms neglected are of order $\Delta \log \Delta$. (These terms can be significant when the two leading terms cancel each other, as happens for small Δ and $x \simeq 10\Delta$.)

In Fig. 2 we compare the exact distribution functions as given by Eq.(13) to the 'leading-log' distributions given by refs. 1,2 for a sub-process energy of 1 TeV. For $x > 10^{-2}$, the 'leading-log' approximation for the longitudinal distribution matches the exact distribution quite well (never worse than a factor of 2). However, the transverse distributions differ greatly from the exact functions for reasons described above. This difference will not be noticed as long as we deal with processes where the longitudinal HSXS totally dominates the transverse one (such as heavy quark and Higgs boson production). Fig. 2 does show, however, that calculations based on leading-log VBDF can give quite erroneous results if the transverse HSXS happens to be comparable or larger than the longitudinal one. When we are looking for 'new physics', this possibility should not be ignored a priori.

AN EXAMPLE: HEAVY QUARK PRODUCTION.

To illustrate the above ideas, we consider the specific example of heavy quark production as a special case of the situation depicted in Fig.1. In this simple case, the final state phase space factor $d\Gamma'$ is trivial, and we make the replacement $dx\, d\Gamma'=2\pi/s$ in Eq.(10). Using techniques of ref.4 for evaluating $J(M_X)$, we easily find the exact (off-shell) HSXS is:

$$\frac{d\hat{\sigma}_+^*}{d\Gamma'} = \frac{g^2}{4}\frac{\varsigma+x}{x} \quad , \quad \frac{d\hat{\sigma}_0^*}{d\Gamma'} = \frac{g^2}{8}\frac{\varsigma+x}{\varsigma} \quad , \tag{16}$$

and the minus helicity cross-section vanishes for a W-boson. Note the kinematic singularity at $\varsigma = 0$ in the longitudinal HSXS. We now compute the helicity cross-sections in three ways, and compare the results.

The exact cross-sections can be obtained by substituting the precise HSXS of Eq.(16) into Eq.(6), and then performing the complete integration. The results are plotted in Fig.3 for parton-parton sub-energy $\sqrt{s}=1$ TeV as solid lines.

Next we make the 'on-shell' approximation by evaluating the regularized HSXS at $\varsigma=\Delta$ and pulling it out of the ς-integration. Regularization involves incorporating the singular factor $1/\varsigma$ of $d\sigma_0^*$ in Eq.(16) into f_0, and replacing it by $1/\Delta$, appropriate for an on-shell amplitude. We then use the exact expression for the VBDF's $f_n(x)$, Eq.(13), to obtain an 'improved effective-vector-boson approximation' to the overall cross-section. The results, shown as dashed lines in Fig.3, agree well with the exact results. This is expected, as the regularized HSXS's are indeed smooth on the scale Δ.

Finally, we compute the cross-sections in the same way, but use the conventional 'leading-log' formulas for the VBDF's. The corresponding curves in Fig.3 show that the 'leading-log' results for the longitudinal cross-section is satisfactory except for small-x; but, those for the transverse cross-section yield gross over-estimates of the real values.

By performing similar comparisons for a wide range of $\sqrt{s}$ and M_X values, we conclude that the effective vector boson method is very accurate if the exact distribution functions are used. In contrast, the 'leading-log' approximation is reliable only if the longitudinal cross-section overwhelmingly dominates and, in addition, if $\sqrt{s}$ (parton-parton total energy) is above 1 TeV. We find that the leading-log transverse cross-section is too big by a significant factor (2 to 10) for most relevant values of $\sqrt{s}$ due to an interesting cancellation of the leading term with the remainder.

CONCLUSIONS

We have given a precise formulation of the effective vector boson method which eliminates most of the approximations of the conventional approach. Exact vector boson distributions are given. Reliability of the method is tied uniquely to the smoothness of the hard-scattering cross-section in the variable q^2 over the range $q^2 \simeq M_V{}^2$, and is totally freed from restrictions on the range of kinematic variables. Due to the generality of the formulation, numerical results presented in the last section should be quite representative of many applications.

FIGURE CAPTIONS

FIGURE 1: A general Feynman diagram for the process $f+A \to V \to f'+X$.

FIGURE 2: The VBDF'S at $\sqrt{s}=1$ TeV.

FIGURE 3: The helicity cross-sections (in GeV^{-2}) for heavy quark production at $\sqrt{s}=1$ TeV.

REFERENCES

[1]G. L. Kane, W. W. Repko, and W. B. Rolnick, Phys. Lett. **148B**, 367 (1984).

[2]S. Dawson, Nucl. Phys **B249**, 42 (1985); R. N. Cahn and S. Dawson, Phys. Lett. **136B**, 196 (1984).

[3]S. Dawson and S. Willenbrock, LBL-21924, July 1986; J. F. Gunion, J. Kalinowski, and A. Tofighi-Niaki, UCD-86-19, August 1986; Also see refs. 1 and 2.

[4]F. Olness and W.-K. Tung, IIT-TH-86-17; to appear in Phys. Rev. **D1**.

[5]This was noticed by Lindfors for the heavy quark production process in the rest frame of the heavy quark. J. Lindfors, Z. Phys. **C 28**, 427 (1985).

[6]P. Johnson, F. Olness, W.-K. Tung, Illinois Inst. of Tech. preprint in preparation.

[7]See Gunion et al., ref. 3. Also see the Effective-W Group summary by W. Repko and W.-K. Tung in these proceedings.

[8]J. P. Ralston and F. Olness, Intrinsic Polarization, in these proceedings.

Corrected Figures

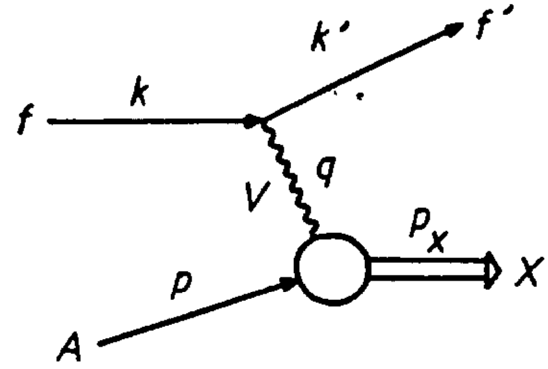


FIG. 1

VECTOR BOSON DISTRIBUTION FUNCTIONS

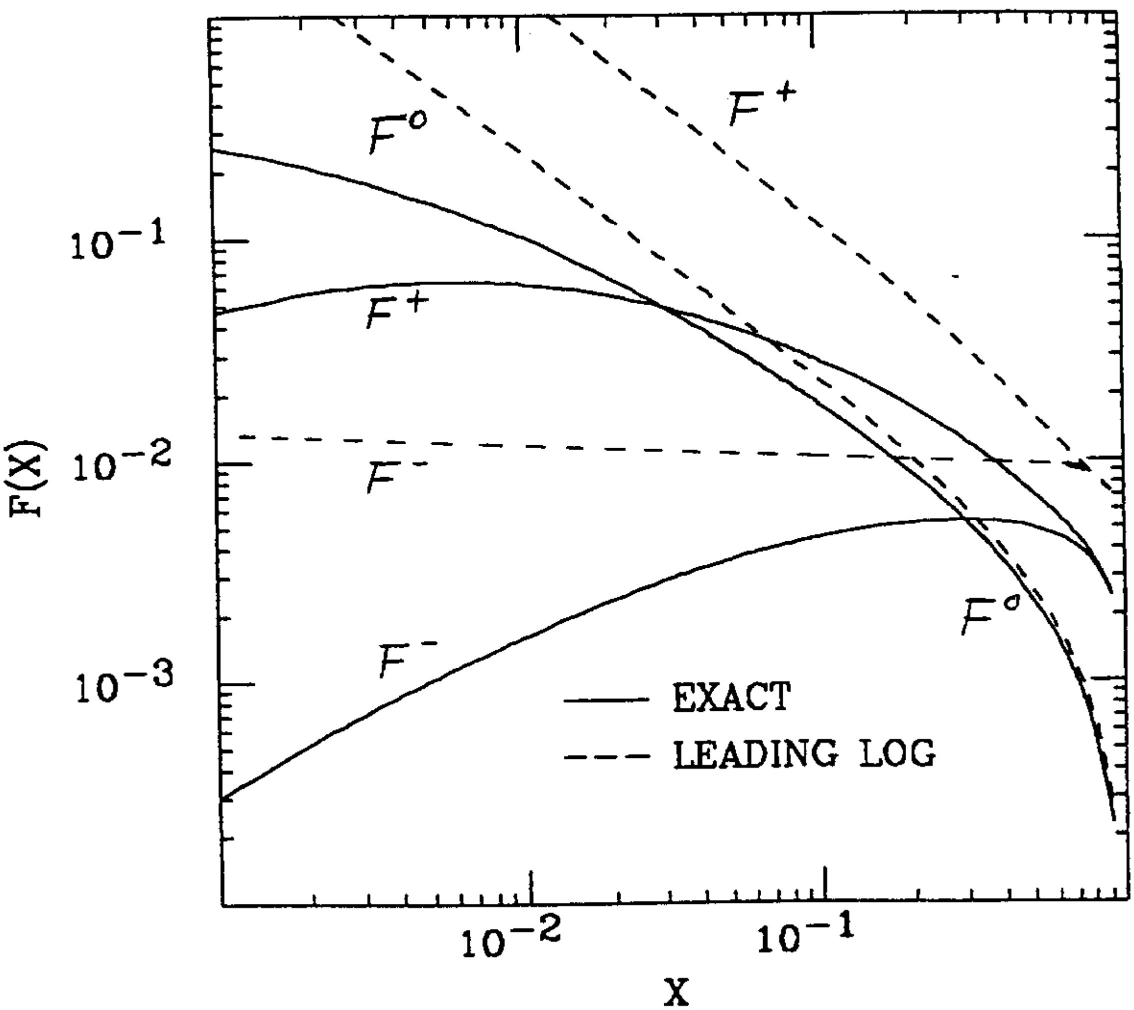
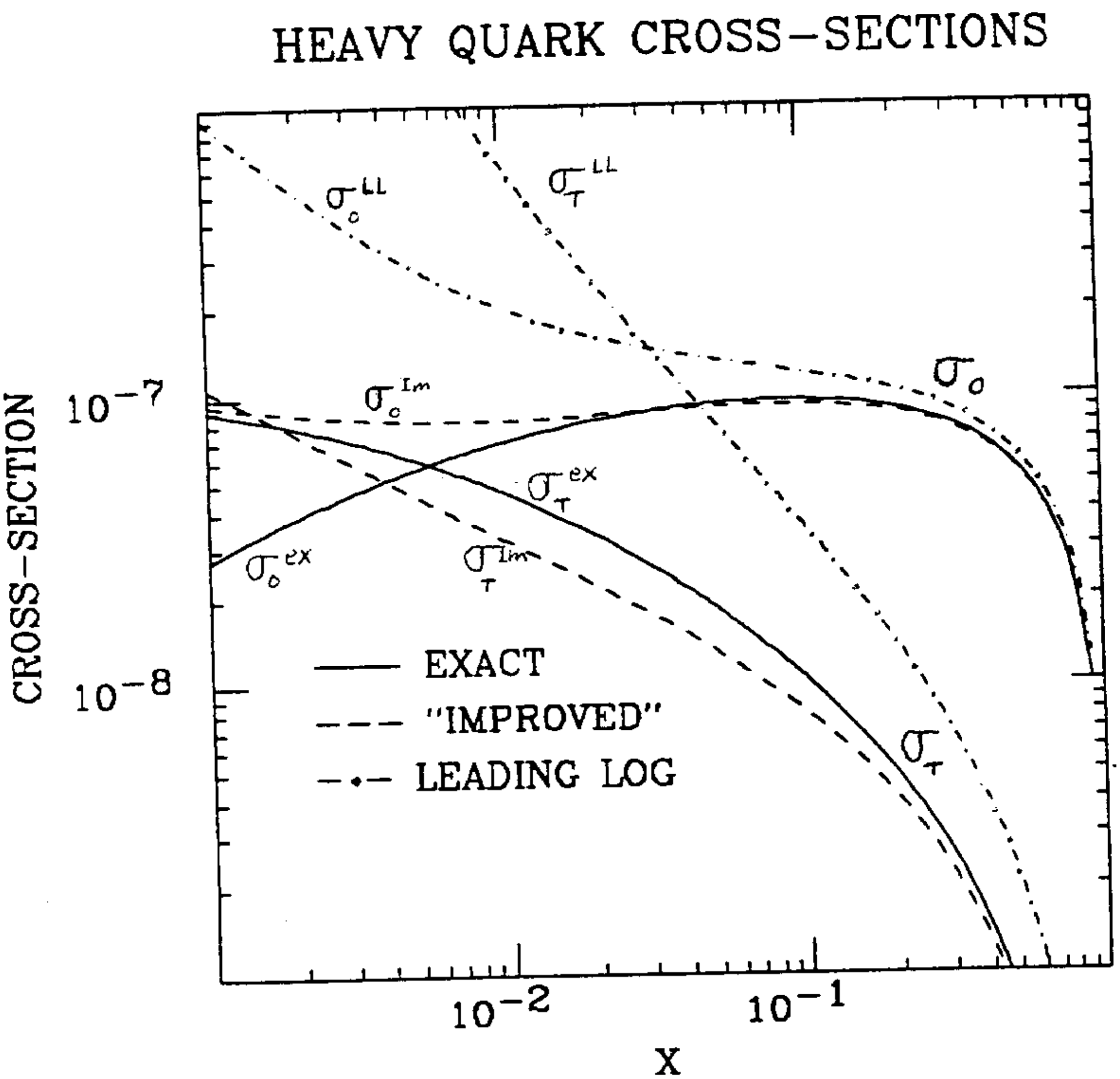


FIG. 2



HEAVY QUARK CROSS-SECTIONS
CROSS-SECTION
10^{-7}
10^{-8}
10^{-2}
10^{-1}
X
σ_0^{LL}
σ_T^{LL}
σ_0^{Im}
σ_0
σ_T^{ex}
σ_0^{ex}
σ_T^{Im}
σ_T
EXACT
"IMPROVED"
LEADING LOG

FIG. 3