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Revised

Stringy Corrections To Calabi-Yau Compactification

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Abstract

The counter terms constructed by Green and Schwarz in the field theory limit of superstrings are put into a form of Calabi-Yau compactification. Dimension six operators are explicitly extracted. Some modifications of the gauge kinetic terms and of the Kähler potential are suggested. Axion couplings and some CP violating interactions are given. With suitable manifolds ($b_{11} > 1$), some axion-like symmetries remain unbroken, promising $\Lambda = 0$.

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There has been much interest in the unified theories of H superstrings[1] since Green and Schwarz sparked the field by the discovery of anomaly free string theories.[2] The subsequent developments of Heterotic construction[3] of superstrings and the compactification on certain Ricci flat Kähler manifolds[4,5] seem to have demonstrated that one can argue a realistic unification and its phenomenological properties, based on $E_8 \times E_8$ superstring theory, within certain ansatze and approximations. Certainly, one of the central issues is to extract dynamics involving massless particles in some convincing manner.

Much of the recent phenomenological developments along these lines are, in practise, based on the field theory (or zero slope) limit of superstrings and a few results,[5,6] on complete tree string amplitudes: One may naturally wonder what we obtain from quantum corrections of superstrings. At the present, a workable approach to Calabi-Yau compactification in full-fledged string theory does not appear to exist. There is, however, a relatively simple way to give some of the characteristic terms among those corrections, coming back to the original work by Green and Schwarz[2]: the "counterterms" S_1', S_2 and S_3' necessary to have an anomaly free $d = 10, N = 1$ supergravity do come from string quantum corrections.

Superstring theories are one-loop finite and believed to be finite to all orders. The anomaly free properties are closely tied down to this fact.[2] One can even view the former as a consequence of the latter. Upon truncation to the massless sector (field theory limit), finiteness is lost, but the anomaly free property is maintained with the addition of those counter terms. The point of view[2] pursued here is that they represent some of the contributions coming from massive modes, thereby implementing constraints coming from anomaly free theory.

The aim of this letter is to investigate their significance in the context of Calabi-Yau compactification. Below, we explicitly perform the decomposition of ten dimensional fields, with its basis on Calabi-Yau space. We mostly use the language of differential forms.

The starting point is a truncation of the E_8 vector potential one form in $M_4 \times$ (Calabi-Yau) space following the recipe of Ref. 7;

$$A(x, y) = 1 \otimes \hat{\alpha}_6(y) + A_4(x) \otimes 1 + 1 \otimes A_6(x, y) \ , \tag{1}$$

where $A_4 \equiv A_4^{(1,78)}$ is the E_6 gauge field in M_4, $A_6 \equiv A^{(3,27)+(\overline{3},\overline{27})}$ is a massless scalar field in M_4 arising from the zero modes in Calabi-Yau space[8] and is expanded in terms of the available harmonic forms[7] (see below). $\hat{\alpha}_6 \equiv \alpha^{(8.1)}$ is a $c-number$ background configuration; it must be a holomorphic, stable $1-form$. The corresponding field strength two form is

$$F = 1 \otimes \hat{F}_6 + F_4 \otimes 1 + 1 \otimes \left\{\hat{D}_6, A_6\right\} + \{D_4, A_6\} + 1 \otimes A_6^3 \,, \tag{2}$$

where $D_4 \equiv d_4 + A_4, \hat{D}_6 \equiv d_6 + \hat{\alpha}_6$ and $\hat{F}_6 \equiv d_6\hat{\alpha}_6 + \hat{\alpha}_6^2$. The last quantity must satisfy $(\hat{F}_6)_{mn} = 0$ and $g^{m\bar{n}}(\hat{F}_6)_{m\bar{n}} = 0$ due to the properties of $\hat{\alpha}_6$.[4]

The gauge field for the second $E_8(E_8')$ undergoes a trivial truncation $A' \equiv A_4' \otimes 1$ according to the above recipe. The antisymmetric tensor field decomposes like

$$B = B_4 \otimes 1 + 1 \otimes B_6 \,. \tag{3}$$

The background spin connection $\hat{\omega}_6$, obtained from truncation $\omega = \hat{\omega}_6 + \omega_4$ (and $R = \hat{R}_6 + R_4$), is related to the background gauge field through $tr_1\hat{F}_6^2 = 30\mathrm{tr}_1\hat{R}_6^2$.[4] The truncated expressions we obtain for S_1', S_2 and S_3' are respectively

$$\begin{aligned}
S_1' \;=\; & \frac{c}{108000}\Bigg[30\int B_4\Bigg(\left[4\mathrm{tr}_1F_4A_6^2 + 2\mathrm{tr}_1\{D_4,A_6\}^2\right]G \\
& +4\left[\mathrm{tr}_1\{D_4,A_6\}\{\hat{D}_6,A_6\} + \frac{1}{3}d_4\mathrm{tr}_1A_6^3\right]^2\Bigg) \\
& +30\int B_6\left(2(\mathrm{tr}_1F_4^2)G + \left[\mathrm{tr}_1\{D_4,A_6\}^2 + 2\mathrm{tr}_1F_4A_6^2\right]^2 - (\mathrm{tr}_2F'^2)G\right) \\
& -\int \mathrm{tr}_1\left[\{D_4,A_6\}A_6\right]G\omega_{3Y_2}^{(d=4)} \\
& -2\int\left(\hat{\omega}_{3Y_1}^{(d=6)} + \mathrm{tr}_1\left[A_6\{\hat{D}_6,A_6\}\right] + \frac{2}{3}\mathrm{tr}_1A_6^3\right) \\
& \quad\cdot\left[\mathrm{tr}_1\{D_4,A_6\}\{\hat{D}_6,A_6\} + \frac{1}{3}d_4\mathrm{tr}_1A_6^3\right]\cdot\omega_{3Y_2}^{(d=4)}\Bigg] \,,
\end{aligned} \tag{4}$$

$$S_2 \;=\; -\frac{c}{16}\int(\mathrm{tr}R_4^2)(\mathrm{tr}\hat{R}_6^2)B_6 \,, \tag{5}$$

and

$$\begin{aligned}
S_3' \;=\; & \frac{c}{7200}\Bigg[30\int B_6(\mathrm{tr}R_4^2\cdot G + \mathrm{tr}\hat{R}_6^2\mathrm{tr}F_4^2) + 30\int B_4\mathrm{tr}\hat{R}_6^2\mathrm{tr}_1\left[\{D_4,A_6\}^2 + 2F_4A_6^2\right] \\
& +5\int\hat{\omega}_{3L}^{(d=6)}\left[\mathrm{tr}_1A_6\{\hat{D}_6,A_6\} + \frac{2}{3}\mathrm{tr}_1A_6^3\right]\mathrm{tr}R_4^2 + 5\int\omega_{3L}^{(d=4)}\mathrm{tr}_1\left[\{D_4,A_6\}A_6\right]\mathrm{tr}\hat{R}_6^2
\end{aligned}$$

$$
\begin{aligned}
&+\int \mathrm{tr}_1\left[\{D_4, A_6\}A_6\right]\omega_{3L}^{(d=4)}G \\
&+2\int\left[\hat{\omega}_{3Y_1}^{(d=6)}+\mathrm{tr}_1 A_6\{\hat{D}_6, A_6\}+\frac{2}{3}\mathrm{tr}_1 A_6^3\right]\omega_{3L}^{(d=4)}\left[\mathrm{tr}_1\{D_4, A_6\}\{\hat{D}_6, A_6\}+\frac{1}{3}d_4\mathrm{tr}_1 A_6^3\right] \\
&+2\int \omega_{3Y_1}^{(d=4)}\hat{\omega}_{3L}^{(d=6)}\left[\mathrm{tr}_1\{D_4, A_6\}\{\hat{D}_6, A_6\}+\frac{1}{3}d_4\mathrm{tr}_1 A_6^3\right] \\
&+\int\left[\mathrm{tr}_1 A_6\{\hat{D}_6, A_6\}+\frac{2}{3}\mathrm{tr}_1 A_6^3\right]\hat{\omega}_{3L}^{(d=6)}(\mathrm{tr}_1 F^2-\mathrm{tr}_2 F'^2) \\
&+30\int B_6 \mathrm{tr}\hat{R}_6^2 \mathrm{tr}_2 F'^2 \\
&+2\int \hat{\omega}_{3L}^{(d=6)}\omega_{3Y_2}^{(d=4)}\left[\mathrm{tr}_1\{D_4, A_6\}\{\hat{D}_6, A_6\}+\frac{1}{3}d_4\mathrm{tr}_1 A_6^3\right]\Big] \; .
\end{aligned}
\tag{6}
$$

where $G = \mathrm{tr}_1\hat{F}_6^2 + \mathrm{tr}_1\hat{D}_6,\ A_6\}^2 + \mathrm{tr}_1 A_6^4 + 2\mathrm{tr}_1\ \hat{F}_6 A_6^2 + 2\mathrm{tr}_1\hat{D}_6,\ A_6\}A_6^2$ and tr_1 and tr_2 are the trace of 248 dimensional representations of E_8 and E_8' respectively. We generically denote Chern-Simon forms by ω. For instance, $\hat{\omega}_{3Y_1}^{(d=6)}$ is the Chern-Simon three forms for the $\hat{\alpha}_6$ of E_8 where all differentials are in six dimentional Calabi-Yau space. A hat denotes that the object is defined in terms of background fields alone. Finally $\{D_4, A_6\} \equiv d_4 A_6 + \{A_4, A_6\}$. The rest of the notation is self-explanatory.

To exhibit the four-dimensional couplings which arise from the above structure, it is necessary to express A_6 and B_6 more explicitly as follows;

$$B_6 = \sum_{i=1}^{b_{11}} a^{(i)}(x)\Omega_{m\overline{n}}^{(i)}(y)dz^m \wedge dz^{\overline{n}} \tag{7}$$

and

$$A_6 = A_m^{(3,27)}dz^m + A_m^{(\overline{3},\overline{27})}dz^m + \overline{A_{\overline{m}}^{(3,27)}}dz^{\overline{m}} + \overline{A_{\overline{m}}^{(\overline{3},\overline{27})}}dz^{\overline{m}}\ , \tag{8}$$

where $a^{(i)}(x)$'s are model-dependent axion fields, due to the gauge invariance $B_6 \rightarrow B_6 + d\Lambda$ in the ten-dimensional supergravity Lagrangian. $A_m^{(3,27)}$ and $A_m^{(\overline{3},\overline{27})}$ also arise from the zero modes in Calabi-Yau space, and can be written as

$$A_m^{(3,27)} = \sum_{i=1}^{b_{12}}\sum_{\substack{a,X\\ \overline{n},\overline{l}}} T^{Xa}C_X^{(i)}(x)\epsilon_a{}^{\overline{n}\overline{l}}\Omega_{m\overline{n}\overline{l}}^{(i)}(y) \tag{9}$$

and

$$A_m^{(\overline{3},\overline{27})} = \sum_{i=1}^{b_{11}} \sum_{\substack{X,a \\ \overline{n},\overline{l}}} (T^*)_{aX} C'^{(i)X}(x) g^{a\overline{n}}(y) \Omega^{(i)}_{m\overline{n}}(y) \; . \tag{10}$$

$\Omega^{(i)}_{m\overline{n}\overline{l}}(y)$ and $\Omega^{(i)}_{m\overline{n}}(y)$ are respectively harmonic (1.2) forms and (1.1) forms, b_{11} and $b_{12} = b_{21}$ are Betti numbers of the manifold and $C^{(i)}_X(x)(C'^{(i)X}(x))$ are scalar fields in the $27(\overline{27})$ representation of E_6. The "stable" matter spectrum consists of $|\chi|/2 = |b_{11} - b_{12}|$ massless E_6 multiplets. From now on, we treat the case $b_{12} > b_{11}$ and suppress, in most cases, the index which labels generations or axions. One can readily read off, from the discussions below, the formulae for the other case $b_{12} < b_{11}$, and also for the case in which some of the $C^{(i)}$ and some of the $C'^{(i)}$ remain simultaneously massless after compactification. Let us now extract lower dimension operators which arise from (4), (5) and (6). First of all, it is worth mentioning that a dimension four operator

$$-\frac{c}{7200} \int \hat{\omega}_{3L}^{(d=6)} \hat{\omega}_{3Y_1}^{(d=6)} \left\{ tr_1 F^2 - tr_2 F'^2 - 5 tr R_4^2 \right\}$$

coming from S'_3 vanishes identically thanks to the construction of Ref. 4. The counter terms do not spoil the anomaly free property of the four dimensional theory which is achieved by the choice of fermion representation. Actually, this is a version of Witten's observation[9] that anomaly cancellation in ten dimensional sense ensures anomaly free four dimensional theories after compactification.[10]

The lowest dimension operators left after the compactification of the counter terms are dimension six operators. Let us exhibit these. After a tedious but straightforward algebra, we obtain reasonably simple interactions:

$$\begin{aligned} S'_1 + S_2 + S'_3|_{dim6} &= \frac{c}{240} \sum_{i=1}^{b_{11}} K^{(i)'} \int_{M_4} a^{(i)}(x) (5 tr_1 F_4^2 - tr_2 F'^2 + 15 tr R_4^2) \\ &+ \frac{c}{48} \sum_{i=1}^{b_{12}-b_{11}} K^{(i)} \int_{M_4} dB_4 C^{*(i)} \overleftrightarrow{\mathcal{D}} C^{(i)} \\ &+ c \sum_{i=1}^{b_{12}-b_{11}} K^{(i)} \int_{M_4} \left(\frac{1}{1440} \omega_{3L}^{(d=4)} - \frac{1}{7200} \omega_{3Y_1}^{(d=4)} + \frac{1}{7200} \omega_{3Y_2}^{(d=4)} \right) C^{*(i)} \overleftrightarrow{\mathcal{D}} C^{(i)} \\ &+ c \sum_{i=1}^{b_{12}-b_{11}} p^{(i)} \int_{M_4} \left(\frac{1}{288} tr R_4^2 - \frac{1}{3600} tr_2 F'^2 \right) C^{*(i)} \cdot C^{(i)} \; , \end{aligned}$$

$$(11)$$

where $c = \frac{i^6}{6!(2\pi)^6}$

$$
\begin{aligned}
K^{(i)} &\equiv \int_k \epsilon_a{}^{\overline{n}\overline{l}} \Omega^{(i)}_{m\overline{n}\overline{l}} \epsilon^{an'l'} \Omega^{(i)}_{\overline{m}'n'l'} dz^m{}_\wedge dz^{\overline{m}'} tr \hat{R}_6^2 \,, \\
K^{(i)\prime} &\equiv \int_k g^{a\overline{n}} \Omega^{(i)}_{m\overline{n}} \Omega^{(i)}_{\overline{m}'a} dz^{m'}{}_\wedge dz^{\overline{m}} tr \hat{R}_6^2 \,,
\end{aligned}
\tag{12}
$$

and

$$
p^{(i)} \equiv \int_k \epsilon^{bnl} \Omega^{(i)}_{\overline{m}nl} (\overleftrightarrow{\hat{D}})_b{}^a \epsilon_a{}^{\overline{n}'\overline{l}'} \Omega^{(i)}_{m'\overline{n}'\overline{l}'} dz^{m'}{}_\wedge dz^{\overline{m}} \hat{\omega}_{3L}^{(d=6)} \,.
$$

Also $(D)_x{}^y$ and $(\hat{D})_a{}^b$ are covariant derivatives acting on the 27 of E_6 and 3 of $SU(3)$ respectively. Observe that $K^{(i)}$ and $K^{(i)\prime}$ are purely imaginary.

The first term is a typical coupling of "model dependent" axions to gauge and gravitational fields discussed previously in the literature[9,11,12]. The strength of the coupling differs in general for each axion. The second term and some pieces of the third term are combined to give the coupling of the model independent axion to matter scalars $HC^*\overleftrightarrow{D}C$. This has some features to be discussed below. A main purpose of the rest of this letter is to see how (11) modifies Witten's truncated action[7,12,13] which fits into a standard $N = 1$ supergravity form.

The truncated action contains symmetries related to the classical scale invariance[7] in addition to $N = 1$ local supersymmetry and Peccei-Quinn-like symmetry. To be more explicit, the ten dimensional action is rescaled by λ^4 under the transformation $g_{MN} \to \lambda g_{MN}$, $\phi \to \lambda^{-1}\phi$ and some rescaling involving fermions. The truncated action inherits this symmetry. On the other hand, (4), (5), and (6) and the resultant (11) do not contain g_{MN} or ϕ. They do not rescale and violate the symmetry.

A similar argument also applies to a rescaling property[13,14] which is distinct from the above scale invariance: $\phi \to r^{\frac{1}{2}}\phi$,$\kappa_{10} \to r\kappa_{10}$ and some rescaling for fermions. This is also broken in (4), (5), and (6) and therefore in (11).

One consequence of the violation of the scale invariance is that the "coupling function" f_{AB} of the gauge kinetic term $\int d^2\theta f_{AB} W_A W_B$ is no longer restricted to be of the form $f_{AB} \propto S\delta_{AB}$: The fourth term in (11) proportional to $tr_2 F\tilde{F}C^* \cdot C$

is CP violating and amounts to a contribution to the imaginary part of f_{AB} unless the manifold is chosen to make the coefficients $p^{(i)}$ vanish, guaranteeing the CP invariance of the four dimensional theory. A dimension seven operator similar to this, namely, $tr_2 F\tilde{F}(W + W^{\dagger})$ appears too.

If the scale invariance is valid, it highly restricts the form of the Kähler potential:

$$G = -\log(S + S^{\dagger}) - 3\log(T + T^{\dagger}) - h\left(\frac{C^{*} \cdot C}{T + T^{\dagger}}\right) + \log|W|^{2} \tag{13}$$

where $h(C, S, T)$ is a priori an arbitrary function and generates D-type interactions between the matter fields and S and T given by

$$S = \phi^{-1}e^{3\sigma} - 3i\sqrt{2}D \tag{14}$$

and

$$T = \phi e^{\sigma} + i\sqrt{2}a\ , \tag{15}$$

where the model independent axion field D is related to $H_{\mu\upsilon\rho}$ by a duality transformation $H_{\mu\upsilon\rho} = \frac{1}{\sqrt{g_4}}(\phi e^{3\sigma})^2 \epsilon_{\mu\nu\rho\sigma}\partial^{\sigma} D$. The symmetries mentioned above tell us that the argument of the function must be the combination $x = \frac{C^{*} \cdot C}{T + T^{\dagger}}$[13] alone and, with Witten's truncation procedure, $h(x) = -3\log(1 - 2x)$.

Since the counter terms violate the above mentioned symmetry, one naturally expects that the argument of the function h be more general. In fact, the inclusion of the term $\frac{cK^{(i)}}{48} H C^{*}\overleftrightarrow{\mathcal{D}} C$ (in form language) changes the kinetic term for the D field from - $\frac{1}{2}\phi^2 e^{-6\sigma}(\partial_{\mu}D)^2$ into

$$\mathcal{L} = -\frac{1}{2}\phi^2 e^{-6\sigma}(\partial_{\mu}D + \frac{c}{24}K_1 C^{*}\overleftrightarrow{\mathcal{D}}_{\mu} C)^2\ . \tag{16}$$

This form is suggestive of a more general Kähler potential:

$$G = -\log(S + S^{\dagger}) - 3\log(T + T^{\dagger}) - h\left(\frac{C^{*} \cdot C}{T + T^{\dagger}}, \frac{C^{*} \cdot C}{S + S^{\dagger}}\right) + \log|W|^{2}\ . \tag{17}$$

We will not pursue this question further here. But one qualitative feature which is obvious even in a crude discussion is the appearance of the coupling of S and C_i superfields previously uncoupled.

The couplings $atrF\tilde{F}$ which we obtained in (11) are related to the issue of getting a vanishing cosmological constant: With $b_{11} = 1$,[12] there is only one holomorphic

(1,1) form $\Omega_{m\bar{n}}$ available, and therefore one model dependent axion. The above coupling invalidates the Ward identity, associated with Peccei-Quinn-like symmetry, which otherwise ensures $\Lambda = 0$[15]. In more general cases $b_{11} > 1$, more axions are available through the decomposition (7); One can find an appropriate linear combination. The associated Peccei-Quinn-like symmetry is not spoiled by the above coupling, promising $\Lambda = 0$

Closely related to the main result (11) is a certain non-renormalization theorem recently shown:[16] Nonderivative F terms relevant to superstrings are not renormalized to any finite order in σ model perturbaion theory. The proof goes as follows; Due to the gauge invariant coupling for the two form B, the fields $a^{(i)}$ have only derivative coupling. Nonderivative couplings cannot, therefore, depend on $a^{(i)}$. On the other hand, $a^{(i)}$ belongs to the lowest component of a chiral multiplet $T^{(i)} = b^{(i)} + ia^{(i)} + \cdots$. For some i, $b^{(i)-1}$ is the radius of the compact manifold[17], *i.e.*, the inverse of the coupling constant in σ model perturbation theory. The F terms are analytic functions of $T^{(i)}$. Hence, forbidding the dependence of the F terms on $a^{(i)}$ in a supersymmetric theory amounts to forbidding the dependence on $b^{(i)}$ as well and therefore on σ model perturbation theory to all orders.

Clearly, the premise of the gauge invariant $B_{m\bar{n}}$ coupling is violated in (4), (5) and (6). The theorem does not apply. This confirms that corrections given in (11) are outside the domain of the conventional σ model perturbation theory. They might contain some sources of destabilizations relevant to low energy phenomenology.

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