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LOW ENERGY SUPERSTRING COSMOLOGY: I

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ABSTRACT

We study possible cosmological solutions to N = 1 D = 10 supergravity with a constant dilaton mode and show that it is possible to integrate Einstein's equations exactly for flat physical and internal spaces. We then present a detailed analysis of the possible trajectories in the phase plane of the Hubble factors and find the allowed regions for a physically acceptable cosmology, which are remarkably small.

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In the quest for the unification of the fundamental interactions, Superstring theories[1] are presently considered as the best candidate. Indeed, while the alliance of local supersymmetry with higher dimensional theories offered the possibility of obtaining the full N=8 supergravity action in four dimensions via dimensional reduction of N=1 supergravity in 11 dimensions[2], the ultra violet divergences that were already present in four dimensional calculations of graviton and matter loop corrections did not get appreciably milder when going to higher dimensional supersymmetric theories. Thus, the proof that superstrings are anomaly free at 1 loop for $\mathbf{SO(32)}$ or $\mathbf{E_8} \times \mathbf{E_8}$ [3]; plus their finiteness[4] to at least that order, has triggered a great deal of action in the complete formulation of the correct theory with acceptable phenomenological predictions[5].

A related question of great importance is the cosmological implications of string theories. Beyond the Planck scale, the drastic changes that are brought up by the inclusion of the massive modes as intermediaries of gravitational interactions are surely going to have important consequences in our understanding of the initial singularity, as some recent work on the subject has shown[6], although still in a superficial way. As we lower the energy, the massive modes are frozen out of equilibrium and the string is thought to collapse to a point, with the massless modes being related to massless fields described by a local field theory.

The type of field theory obtained is related to the way the string theory is formulated, i.e. being it an open or closed string theory or by the number of supersymmetry generators (if any) involved.

In this letter, we propose to study possible cosmological solutions arising from the bosonic sector of N=1 D=10 supergravity theory[7] (or equivalently the Chapline-Manton action[8] with the Yang-Mills field strength set to zero) which describes the massless bosonic sector of the type I superstrings and of the phenomenologically more promising heterotic string[9]. As a simplifying first step, we are going to take a constant dilaton mode, thus effectively concentrating on the graviton and Kalb-Ramond fields. The complete model with a more general dilaton mode and a non-vanishing Yang-Mills field is presently under study[10].

The action can be written as

$$S = -\int d^{10}z\sqrt{-g^{10}}\left\{\frac{R}{16\pi G} - \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - \frac{1}{6}e^{\phi}F_{MNP}F^{MNP}\right\} \tag{1}$$

Setting $8\pi G = 1$ and taking the dilaton field ϕ to be constant, the field equations, after some convenient rescaling take the form

$$R_{MN} - \frac{1}{2}g_{MN}R = -T_{MN} \tag{2.1}$$

with

$$T_{MN} = F_{MPQ}F_M^{PQ} - \frac{1}{6}F_{PQR}F^{PQR}g_{MN} \tag{2.2}$$

$$\partial_M\left[\sqrt{-g^{10}}F^{MNP}\right] = 0 \tag{3}$$

where capital latin indices run from 0 to 9, greek indices run from 0 to 3 and small latin indices from 4 to 9.

In order to obtain a compactified solution of the field equations, we follow Myung and Kim[11] and write the Kalb-Ramond field components as

$$F_{\mu\nu\rho} = \varepsilon_{\mu\nu\rho\sigma}\Phi^{\sigma} \tag{4.1}$$

$$F_{mnp} = F_{\mu\nu p} = F_{\mu np} = 0 \tag{4.2}$$

where $\Phi^{\sigma} = \partial^{\sigma}\eta(x)$ can be thought of as being the gradient of a space-time dependent scalar field $\eta(x)$. This ansatz is consistent with the requirement of having a torsion free internal manifold if we want to preserve **SU(3)** holonomy.. It is easy to check that the Kalb-Ramond field equation (eq. (3)) is trivially satisfied with the above ansatz.

We note that the no-go theorem for 10 into 4 compactification[12] will not apply here since the four dimensional space-time is not maximally symmetric and the dilaton plus the Kalb-Ramond fields can, in principle, be functions of time.

In order to obtain a suitable cosmological description of the above model, we assume, as usual, that the line element is given by the generalized Robertson-Walker metric[13],

$$ds^2 = -dt^2 + R_3(t)^2 \tilde{g}_{ij}(x^l) + R_6(t)^2 \tilde{g}_{mn}(y^p) \tag{5}$$

If we additionally impose the field $\eta(x)$ to be only time dependent, we obtain the following field equations, for flat 3 and 6 dimensional spaces,

$$3\frac{\ddot{R}_3}{R_3} + 6\frac{\ddot{R}_6}{R_6} = -\frac{1}{2}\dot{\eta}^2 \tag{6.1}$$

$$\frac{d}{dt}\left(\frac{\dot{R}_3}{R_3}\right) + \left(3\frac{\dot{R}_3}{R_3} + 6\frac{\dot{R}_6}{R_6}\right)\frac{\dot{R}_3}{R_3} = \frac{3}{2}\dot{\eta}^2 \tag{6.2}$$

$$\frac{d}{dt}\left(\frac{\dot{R}_6}{R_6}\right) + \left(3\frac{\dot{R}_3}{R_3} + 6\frac{\dot{R}_6}{R_6}\right)\frac{\dot{R}_6}{R_6} = -\frac{1}{2}\dot{\eta}^2 \tag{6.3}$$

We note that in choosing a flat internal manifold (and not only Ricci flat as in Calabi-Yau) the model will not describe chiral fermions. We believe though, that at this point flatness is more crucial than chirality.

The authors of ref. (11) claimed to have found a particular solution to equation (6) where the physical 3-space would expand exponentially while the internal space would oscillate in time. However, the solutions were obtained under the assumption of having constant expansion rates (H_3 and H_6 constants), which is a strong constraint in the dynamical system. Furthermore, we would like to point out that their solution is inconsistent with the field equations, as these do not accept an oscillating solution for the internal manifold.

It is also important to mention the work by Freund and Oh on cosmological solutions for N = 1 D = 10 supergravity coupled to Yang-Mills[14]. The authors looked for power law solutions with a four dimensional anti de-Sitter space-time (which automatically constraints the physical radius to go as $R_3 \sim t$) and the Kalb-Ramond and Yang-Mills fields taking values in the internal space, and found both spaces expanding but with a slower rate for the internal space. It would be interesting to apply our method developed in the following to their dynamical system.

Here, we shall integrate eqn.(6) in an exact way and later analyse the dynamical behaviour of the system. If we introduce the logarithmic variables $H_3 = \frac{d(\ln R_3)}{dt}$ and $H_6 = \frac{d(\ln R_6)}{dt}$ we can rewrite eqn.(6) as a first order quadratic differential system as follows,

$$3\dot{H}_3 + 3H_3^2 + 6\dot{H}_6 + 6H_6^2 = -\frac{1}{2}\dot{\eta}^2 \tag{7.1}$$

$$\dot{H}_3 + 3H_3^2 + 6\dot{H}_3\dot{H}_6 = \frac{3}{2}\dot{\eta}^2 \tag{7.2}$$

$$\dot{H}_6 + 6H_6^2 + 3\dot{H}_3\dot{H}_6 = -\frac{1}{2}\dot{\eta}^2 \tag{7.3}$$

substituting (7.1) into (7.2) and (7.3) we get

$$\dot{H}_3 = \frac{3}{2}H_3^2 + 21H_3H_6 + \frac{45}{2}H_6^2 \tag{8.1}$$

$$\dot{H}_6 = -\frac{3}{2}H_3^2 - 12H_3H_6 - \frac{27}{2}H_6^2 \tag{8.2}$$

In order to solve eqn. (8.1) and (8.2) exactly, we write H_3 and H_6 in terms of the new variables r and θ as

$$H_3 = r sin\theta \tag{9.1}$$

$$H_6 = r cos\theta \tag{9.2}$$

After some algebra, eqn.(8) reduces to

$$\dot{r} = \frac{1}{2}r^2(3sin^3\theta + 39sin^2\theta\cos\theta + 21sin\theta\ cos^2\theta - 27\cos^3\theta) \tag{10.1}$$

$$r\dot{\theta} = \frac{1}{2}r^2(3sin^3\theta + 27sin^2\theta\cos\theta + 69sin\theta\cos^2\theta + 45\cos^3\theta) \tag{10.2}$$

Upon introduction of a new variable $x = \tan\theta$, eqn.(10) can be writen as a single equation relating r and x as follows

$$\frac{dr}{r} = \left(\frac{x^3 + 13x^2 + 7x - 9}{x^3 + 9x^2 + 23x + 15}\right)\frac{dx}{1 + x^2} \tag{11}$$

By using partial fractions, eqn.(11) can be easily integrated to give

$$r = r_0(x+3)^A(x+1)^B(x+5)^C(1+x^2)^D exp(Etan^{-1}x) \tag{12}$$

with

$$A = -\frac{3330}{987}\ ;\ B = \frac{907}{987}\ ;\ C = \frac{2911}{987}\ ;\ D = \frac{103}{987}\ ;\ E = \frac{213}{987} \tag{13}$$

As is well known from the theory of non-linear differential equations, the zeros of eqn(10.2) give the invariant lines in the phase plane $H_3 \times H_6$ [15]. These invariant lines are solutions of eqns. (8.1) and (8.2) where $H_3 = \alpha H_6$ ($\alpha = constant$), which in our case will be simply

$$H_3 = -H_6 \;;\; H_3 = -3H_6 \;;\; H_3 = -5H_6 \tag{14}$$

These lines are shown in fig. 1 with the labels α_1, α_2 and α_3 for $\alpha = -1, -3, -5$ respectively.

Fig. 1

Another important property of our dynamical system is that $H_3 = H_6 = 0$ is a critical point. Using this information together with the invariant lines and the solution (12), we can construct the phase plane diagram $H_3 \times H_6$ as shown in fig.1. Note that the lines α_2 and α_3 were not drawn on scale to allow an easier reading of the trajectories.

In what follows we will try to extract all the possible information from the dynamical analysis of the phase plane diagram.

First we note that along the invariant lines we can obtain the exact solution for the scale factors as functions of time;

i) $\alpha_1 = -1$, we find that

$$H_3(t) = \frac{H_{30}}{1 - 3H_{30}t} \;;\; H_6(t) = \frac{H_{60}}{1 + 3H_{60}t} \;;\; R_3 \sim t^{-\frac{1}{3}} \;;\; R_6 \sim t^{\frac{1}{3}} \tag{15.1}$$

ii) $\alpha_2 = -3$, we find that

$$H_3(t) = \frac{H_{30}}{1 + 3H_{30}t} \;;\; H_6(t) = \frac{H_{60}}{1 - 9H_{60}t} \;;\; R_3 \sim t^{\frac{1}{3}} \;;\; R_6 \sim t^{-\frac{1}{9}} \tag{15.2}$$

iii) $\alpha_3 = -5$, we find that

$$H_3(t) = \frac{H_{30}}{1 + \frac{9}{5}H_{30}t} \;;\; H_6(t) = \frac{H_{60}}{1 - 9H_{60}t} \;;\; R_3 \sim t^{\frac{5}{9}} \;;\; R_6 \sim t^{-\frac{1}{9}} \tag{15.3}$$

where H_{30} and H_{60} are the initial values for H_3 and H_6 respectively.

The arrows for the critical lines are obtained by analysing the sign changes in $H_3(t)$ and $H_6(t)$ for each solution. The reader can easily verify that only solutions (15.1) and (15.3) are compatible with the field equations for $\dot{\eta} = 0$. It is not sufficient to find the power law solutions from the critical lines since the consistency condition, eqn. (6.1) has to be satisfied together with eqns. (6.2) and (6.3). Alternatively, we could see that the whole region *between* the α_1 and α_3 lines is excluded (shadow region in fig.1) since it would produce a negative radius.

As a second step we have to follow the allowed trajectories for all possible initial conditions for H_3 and H_6. We can do this by studying each of the quadrants of the phase plane, taking special care in quadrants II and IV for the various possible subregions. We will concentrate in the region $H_3 \geq 0$ since we consider that only an initially expanding 3-space is of physical relevance. Nevertheless, once the behaviour for quadrant II is understood, the behaviour in quadrant IV follows from symmetry arguments, while trajectories starting at quadrant III are bound to produce a shrinking 3-dimensional and internal space-time.

If the initial conditions are such that that both H_3 and H_6 are positive (quadrant I), the physical radius undergoes an ever decreasing expansion rate until it eventually alternates a brief contraction (top of quadrant IV) with a brief expansion (bottom of quadrant II), while the internal radius grows at a rate that is increasingly faster, reaching a maximum and then slowing down until it eventually starts to contract at a rate that is initially faster but slows down until it stops asymptotically at a constant value.

For the region IIa, the physical radius undergoes a decreasing expansion rate as before, until it reverses expansion into contraction towards an asymptotic singularity (at the bottom of quadrant IV), while the internal radius will do exactly the opposite. This kind of model may remind us of a closed Friedmann universe (but without the bounce at the singularity) with the interesting feature that the internal dimensions will eventually grow enough to be observed after the physical radius has entered the contracting phase.

The next possible region of interest are the lines α_1 and α_3, which were shown to represent power law solutions for the scale factors. In fact, this method of integration, when applicable, produces a powerful way of finding power law solutions with due care taken. For example, we have seen that the whole region between these two lines, including the α_2 line, has to be excluded from the analysis since for this region eqn. (12) would give a negative radial coordinate.

We are thus left with region IIb. In this case, all trajectories will fall into the third (III) quadrant, which unfortunatelly does not provide a very hospitable universe for life to develop, as we mentioned earlier.

Going back through all the analysis, we find a surprisingly small region of the whole phase plane to be of possible physical relevance. If we assume a strictly conservative point of view, only the α_3 line in the II^{nd} quadrant provides an expanding open universe, with a power law behaviour very similar to the radiation dominated scenarios of conventional 4-dimensional cosmologies, and a shrinking internal space, as desired from observations. We expect that when the internal radius approaches the singularity, quantum effects will play a balancing role, stopping the collapse.

Although this is a highly simplified model, it is reasuring to know that this behaviour is not completly ruled out a priori. Relaxing our point of view, we may considere regiow IIa of some interest since we could well be living in the allowed portion of the diagram, i.e., before the shrinking of the physical radius and related expansion of the internal radius has started.

In any case, it does not seem possible to obtain an exponential inflation in this model. If we start with an unaturally large internal space, assume adiabatic expansion for the whole 10-dimensional space-time and wait long enough (since a power law expansion is relatively slow), the shrinking of the internal space may generate the correct amount of entropy, as we intent to show in a forthcoming work.

It must be said that quantum effects coming from one loop corrections on the matter fields, or the Casimir forces, will probably play an important role at these energies, and were neglected for simplicity. We intend to gradually add more structure to the field equations to understand precisely the function of each of the extra terms. For example, with the inclusion of the Yang-Mills field plus a gaugino condensate as an effective cosmological constant[16], the internal radius will acquire a potential that may lead to an inflationary phase in the physical space-time. If this is correct, superstrings will be as viable cosmologically as they are as candidates for a finite theory of gravity. And, surely, these two aspects should be manifestly unified in a coherent theory of physical interactions.

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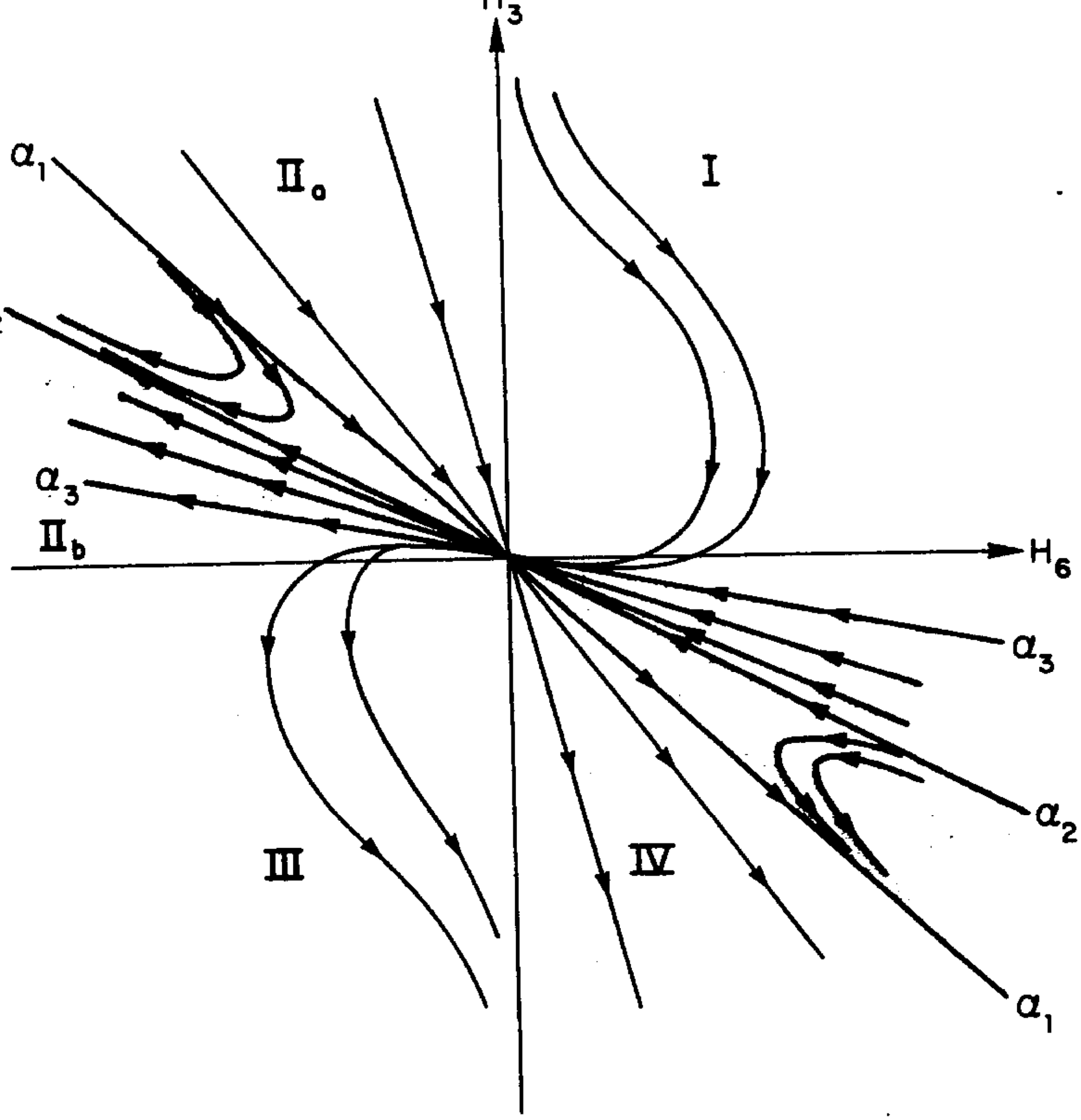
Figure Caption

Fig. 1 The Phase Plane trajectories are shown for a variety of possible initial values for H_3 and H_6. Note that the shadow region is excluded from analytical considerations.



H_3
H_6
α_1
α_2
α_3
α_3
α_2
α_1
I
II_a
II_b
III
IV

Fig. 1