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A Tomonaga-Schwinger-Dirac formulation for string theories.

SUMIT R. DAS*

and

MARK A. RUBIN

Fermi National Accelerator Laboratory
P.O.Box 500, Batavia, Ill.60510, U.S.A.

ABSTRACT

We present a Tomonaga-Schwinger-Dirac formulation of the first quantized free bosonic string theory in which all the Virasoro operators have simple geometric meanings. In a corresponding second quantized version, the "chordal" gauge transformations on the free string field, which generate linearized gauge transformations of spacetime fields, become natural transformations in an extended loop space. The geometrical nature of these transformations may allow them to be more easily generalized to the nonlinear transformations of the interacting string field theory.

* Address after November 1 : Department of Physics, California Institute of Technology, Pasadena, Ca. 91125.

Recently there has been some progress in attempts to understand the origin of gauge and general covariance in string theories [1-3]. In the usual formulations, the string field $\Phi[x^{\mu}(\sigma)]$ is a functional of the string coordinates $x^{\mu}(\sigma)$. Here σ labels the points along the string. In a first quantized formulation $\Phi[x^{\mu}(\sigma)]$ is the wave functional. A "gauge invariant" action for the free string is given by [1,2]:

$$S = \int \mathcal{D}x^{\mu}(\sigma)\ \Phi^{\dagger} P (L_0 - 1) P \Phi \tag{1}$$

where P is a projection operator which projects out fields satisfying the Virasoro conditions, i.e.:

$$L_n P \Phi = 0 \ , \ n > 0 \tag{2}$$

for any arbitrary Φ. Here L_n are the standard Virasoro operators expressed in the Schrodinger representation of the first quantized theory. In a gauge

$$L_n \Phi = 0 \ , \ n > 0 \tag{3}$$

the equation of motion following from (1) is:

$$(L_0 - 1) \Phi = 0 \tag{4}$$

In the first quantized picture (4) is the equation for the wave function which also satisfies the orthonormal gauge constraint given by (3).

The field theory defined by equation (1) has a huge invariance group -- the group of "chordal" gauge transformations [1,2] :

$$\Phi[x(\sigma)] \longrightarrow \Phi[x(\sigma)] + i\sum_{n>0} \epsilon_{-n} L_{-n}\, \Omega[x(\sigma)] \qquad (5)$$

where $\Omega[x(\sigma)]$ is an arbitrary functional and ϵ_{-n} are arbitrary parameters. The string field may be expanded in terms of component space-time fields which correspond to the various modes of the string. In terms of these component fields which represent the massless modes, the above transformations for n = 1 become linearized gauge and general coordinate transformations [1,2].

The Virasoro operators L_n generate reparametrisations of the parameters σ and τ labelling the world sheet in the first quantized theory [4]. One might wonder whether the L_n's also have a natural geometric meaning in loop space. To study this question we pass to the Schrodinger representatives of the L_n's-- i.e. their representatives as operators acting on the wave functionals $\Phi[x(\sigma)]$ (and hence on string fields in the second quantized theory).For the open string ,one has [4] :

$$L_n = -\frac{1}{4}\int_{-\pi}^{+\pi} d\sigma\, e^{in\sigma}\left[-i\frac{\delta}{\delta x^{\mu}(\sigma)} + \partial_\sigma x^{\mu}\right] \qquad (6)$$

(where in (6) the original interval $0 \le \sigma \le \pi$ has been extended to $-\pi \le \sigma \le \pi$ in the standard fashion :

$x^\mu(\sigma) = x^\mu(-\sigma)$; $\partial_\sigma x^\mu(\sigma) = -\partial_\sigma x^\mu(-\sigma)$). From (6) it is clear that "half" of the L_n's indeed have a simple geometrical meaning in loop space [5]. These are :

$$i(L_n - L_{-n}) = i\int_0^\pi d\sigma \sin n\sigma \; \partial_\sigma x^\mu \frac{\delta}{\delta x^\mu(\sigma)} \qquad ..(7)$$

which simply generate transformations on $\Phi[x(\sigma)]$ induced by reparametrisations of σ of the form :

$$\sigma \rightarrow \sigma + \varepsilon \sin n\sigma$$

This is satisfying, in the first quantized theory $i(L_n - L_{-n})$ do generate σ- reaparmetrisations [4]. The "other half" :

$$L_n + L_{-n} = \int_0^\pi d\sigma \cos n\sigma \left[-\frac{\delta^2}{\delta x^2} + (\partial_\sigma x^\mu)^2 \right] \qquad ..(8)$$

do not have any such geometric meaning. This is not surprising; $(L_n + L_{-n})$ generate σ-dependent reparametrisations of τ in the first quantized theory - and in our Schrodinger picture based on a wave functional $\Phi[x(\sigma)]$, τ has completely disappeared.

In this letter we propose a Tomonaga-Schwinger-Dirac type formulation of the free bosonic string theory in which all the Virasoro generators have natural geometric meanings.

First, let us briefly recall the usual formulation of the string theory [4]. The Nambu action is :

$$S = -\int d\tau d\sigma \left\{ \left(\frac{\partial x^\mu}{\partial \tau} \cdot \frac{\partial x_\mu}{\partial \sigma}\right)^2 - \left(\frac{\partial x}{\partial \tau}\right)^2 \left(\frac{\partial x}{\partial \sigma}\right)^2 \right\}^{1/2} \qquad (9)$$

where $X^{\mu}(\sigma,\tau)$ denotes the string coordinate. The action (9) is invariant under arbitrary reparametrisations of σ and τ. This allows to pick a class of coordinate systems on the world sheet defined by the orthonormal gauge :

$$\frac{\partial X^{\mu}}{\partial \sigma}\cdot\frac{\partial X_{\mu}}{\partial \tau} = 0$$
$$\left(\frac{\partial X^{\mu}}{\partial \sigma}\right)^2 + \left(\frac{\partial X^{\mu}}{\partial \tau}\right)^2 = 0 \qquad \ldots (10)$$

In this gauge the action becomes :

$$S = -\frac{1}{2}\int d\sigma\, d\tau \left\{\left(\frac{\partial X}{\partial \tau}\right)^2 - \left(\frac{\partial X}{\partial \sigma}\right)^2\right\} \qquad \ldots (11)$$

τ may be treated as a time variable - we shall refer to it as the parameter time. The momenta P^{μ} which follow from the action (9) satisfy first class constraints which may be written as :

$$L_n = \int_{-\pi}^{+\pi} d\sigma\, e^{in\sigma}\left(P^{\mu} + \frac{\partial X^{\mu}}{\partial \sigma}\right)^2 \qquad \ldots (12)$$

In the orthonormal gauge the hamiltonian is :

$$H = L_0 \qquad \ldots (13)$$

In covariant quantisation one imposes usual commutators and a subset of the constraints (12) are imposed as subsidiary conditions on the physical states, given by equation (3).

In the Schrodinger picture, the momenta are represented as:

$$P^{\mu}(\sigma) \rightarrow -i\frac{\delta}{\delta X^{\mu}(\sigma)} \qquad \ldots (14)$$

so that the L_n's are given by equation (6). The states are described by wave functionals $\tilde{\Phi}[x(\sigma),\tau]$ which obey the Schrodinger equation:

$$i\,\frac{\partial\tilde{\Phi}}{\partial\tau} = L_0\,\tilde{\Phi} \tag{15}$$

Since L_0 is independent of τ one may make a Fourier transform in τ:

$$\tilde{\Phi}[x(\sigma),\tau] = \int dm^2\, e^{-im^2\tau}\,\Phi_{m^2}[x(\sigma)] \tag{16}$$

so that equation (15) becomes:

$$(L_0 - m^2)\,\Phi_{m^2}[x(\sigma)] = 0$$

Equation (4) is obtained for $m^2 = 1$.

In the above formalism the parameter time τ plays a special role. The state of a string is described by picking a constant-τ slice on the world sheet and specifying the wave functional $\tilde{\Phi}[x(\sigma),\tau]$. Given the initial data on any such constant-τ line, the wave functional at any other later constant-τ line may be obtained by integrating the Schrodinger equation (15). The restriction to straight lines of constant τ -- i.e. the same τ for each value of σ -- prevents one from making a σ-dependent reparametrisation of τ.

Nambu [6] and Hosotani [7] have proposed string equations in which σ and τ are treated in a symmetrical manner. The relationship between these string equations and the standard one discussed above is not very clear. Here we adopt a quite different approach. We shall obtain a more symmetric situation by considering string wave functionals $\psi[X(\sigma), \tau(\sigma)]$ which are defined on any arbitrary space-like line denoted by $\tau(\sigma)$ on the world sheet. The new wave equation then relates this to the corresponding functional on any other space-like line $\bar{\tau}(\sigma)$. This question has been addressed in the context of particle field theories by Tomonaga [8], Schwinger [9] and Dirac [10]. Here we shall obtain a similar formalism for bosonic strings starting from the standard Nambu action and following a canonical method discussed by Kuchař in the context of quantum gravity [11].

Consider the bosonic string theory in an orthonormal gauge, described by equations (10) and (11). σ and τ will be referred to as "flat" coordinates on the world sheet. Introduce curvilinear coordinates ξ^α (α = 0,1):

$$\xi^\alpha = \xi^\alpha(\sigma, \tau) = \xi^\alpha(\sigma^i)$$

where we have used the notation:

$$\sigma^0 = \tau \quad ; \quad \sigma^1 = \sigma$$

In these coordinates the action (11) is

$$S = -\frac{1}{2}\int d^2\xi \sqrt{g}\, g^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X_\mu \tag{17}$$

where

$$g^{\alpha\beta} = \frac{\partial\xi^\alpha}{\partial\sigma^i}\frac{\partial\xi^\beta}{\partial\sigma_i} \tag{18}$$

$$g_{\alpha\beta}\, g^{\beta\gamma} = \delta^\gamma_\alpha \quad ; \quad g \equiv \det(g_{\alpha\beta})$$

Treating ξ^0 as the parameter time, the canonical momenta conjugate to X^μ are:

$$\pi^\mu = \sqrt{g}\, \frac{\partial \xi^0}{\partial \sigma^i} \frac{\partial \xi^\beta}{\partial \sigma_i} \partial_\beta X^\mu \tag{19}$$

and the hamiltonian density is :

$$h = \left(\sqrt{g}\, \frac{\partial \xi^0}{\partial \sigma^i} T^i{}_j \right) \frac{\partial \sigma^j}{\partial \xi^0} \tag{20}$$

where T^i_j is the energy-momentum tensor in flat coordinates :

$$T^i_j = \partial^i X^\mu \partial_j X_\mu - \frac{1}{2} \delta^i_j \left(\partial^k X^\nu \partial_k X_\nu \right) \tag{21}$$

The crucial point is that the quantity within brackets in equation (20) depends only on the coordinates X^μ, their conjugate momenta π^μ, their "spatial" derivatives $\partial x^\mu/\partial \xi'$ and the spatial derivatives of σ^i, $\partial \sigma^i/\partial \xi'$ - but <u>not</u> on $\partial \sigma^i/\partial \xi^0$. The hamiltonian density is thus linear in the quantities $\partial \sigma^i/\partial \xi^0$. The form of the Lagrangian density ,

$$\begin{aligned} \mathcal{L} &= \pi^\mu \frac{\partial X_\mu}{\partial \xi^0} - h \\ &= \pi^\mu \frac{\partial X_\mu}{\partial \xi^0} - \left(\sqrt{g}\, \frac{\partial \xi^0}{\partial \sigma^i} T^i{}_j \right) \frac{\partial \sigma^j}{\partial \xi^0} \end{aligned} \tag{22}$$

suggests that $\sigma^i(\xi)$ may be regarded as dynamical variables with their own conjugate momenta :

$$P_j \equiv -\sqrt{g}\, \frac{\partial \xi^0}{\partial \sigma^i} T^i_j \tag{23}$$

The resulting dynamical system where both σ and τ are dynamical variables has a zero hamiltonian. However, all the momenta are not independent, but subject to the constraints given by equation (23).

The momenta for X^{μ} are also not all independent, because of the orthonormal gauge conditions (10). These may be rewritten in terms of our new dynamical variables as :

$$T_{\sigma\sigma} \pm T_{\sigma\tau} = \frac{1}{(\partial\tau/\partial\xi') \pm (\partial\sigma/\partial\xi')}\left[P_{\sigma} \pm P_{\tau}\right] = 0 \qquad (24)$$

where T is the energy momentum tensor defined in equation (21). Since the hamiltonian is zero the entire dynamics of the system is contained in the constraints (23) and (24).

The curves of constant ξ^0 denote a family of one-dimensional spacelike surfaces on the world sheet. Given the initial data on any such surface, equations (23) and (24) have to be integrated to yield the dynamical variables at another surface at a later time $\bar{\xi}^0$. It is convenient to express equation (23) in terms of components tangential and normal to these constant- ξ^0 surfaces which are, respectively :

$$\frac{\partial\sigma}{\partial\xi'} P_{\sigma} + \frac{\partial\tau}{\partial\xi'} P_{\tau} + \frac{\partial X^{\mu}}{\partial\xi'} \cdot \Pi_{\mu} = 0 \qquad (25)$$

$$\frac{1}{\sqrt{(\partial_{\xi'}\sigma)^2 - (\partial_{\xi'}\tau)^2}} \left\{ \frac{\partial\sigma}{\partial\xi'} P_{\tau} + \frac{\partial\tau}{\partial\xi'} P_{\sigma} + \frac{1}{2}\left(\Pi_{\mu}\Pi^{\mu} + \left(\frac{\partial X}{\partial\xi'}\right)^2\right)\right\} = 0 \qquad (26)$$

Canonical quantisation now proceeds by imposing the commutators :

$$\begin{aligned} \left[X^\mu(\xi_1,\xi_0), \Pi^\nu(\xi_1',\xi_0)\right] &= i\eta^{\mu\nu}\delta(\xi_1-\xi_1') \\ \left[\sigma^i(\xi_1,\xi_0), P^j(\xi_1',\xi_0)\right] &= i\delta^{ij}\delta(\xi_1-\xi_1') \end{aligned} \tag{27}$$

In the Schrodinger picture the states are described by a wave functional :

$$\tilde{\Psi}\left[X^\mu(\xi_1), \sigma(\xi_1), \tau(\xi_1), \xi_0\right] \tag{28}$$

which gives the probability amplitude on the spacelike lines ξ_0 =constant. The momenta are represented by:

$$\Pi_\mu \to -i\frac{\delta}{\delta X^\mu(\xi^1)} \quad ; \quad P_j \to -i\frac{\delta}{\delta\sigma^j(\xi^1)} \tag{29}$$

The equations (25) and (26) are imposed as conditions on the wave functional :

$$\left[\frac{\partial\sigma}{\partial\xi_1}\frac{\delta}{\delta\sigma(\xi_1)} + \frac{\partial\tau}{\partial\xi_1}\frac{\delta}{\delta\tau(\xi_1)} + \frac{\partial X^\mu}{\partial\xi_1}\frac{\delta}{\delta X^\mu(\xi_1)}\right]\tilde{\Psi} = 0 \tag{30}$$

$$\left[\frac{\partial\sigma}{\partial\xi^1}\frac{\delta}{\delta\tau(\xi_1)} + \frac{\partial\tau}{\partial\xi^1}\frac{\delta}{\delta\sigma(\xi_1)} + \frac{1}{2}\left(-\frac{\delta^2}{\delta X^2} + \left(\frac{\partial X}{\partial\xi^1}\right)^2\right)\right]\tilde{\Psi} = 0 \tag{31}$$

These equations are of the general form:

$$\mathcal{a}\tilde{\Psi} = \mathcal{A}\tilde{\Psi}$$

where the operators $\mathcal{a}$ act on the variables in $\tilde{\Psi}$ which specify the spacelike surface in question, while $\mathcal{A}$ acts on the real dynamical variables. These operators may be shown to obey the set of consistency conditions derived by Dirac [10].

The equation (30) simply states that $\tilde{\psi}$ is invariant under arbitrary reparametrizations of ξ^1. This allows us to pick a parametrization without any loss of generality by setting :

$$\xi^1 = \sigma$$

Denote the resulting wave functional by $\psi[x(\sigma), \tau(\sigma)]$. Solving for $\delta\psi/\delta\sigma$ from (30), equation (31) becomes :

$$i\frac{\delta\psi}{\delta\tau(\sigma)} = \left\{ \frac{1}{2(1-\partial_\sigma\tau)^2}\left[-\frac{\delta}{\delta x^\mu(\sigma)} - (\partial_\sigma\tau)(\partial_\sigma x^\mu)\right]^2 + \frac{1}{2}(\partial_\sigma x^\mu)^2 \right\}\psi \tag{32}$$

The wave functional is now a functional of $x(\sigma)$ and $\tau(\sigma)$ only : it does not depend on ξ_0 since the hamiltonian vanishes. $\psi[x^\mu(\sigma), \tau(\sigma)]$ is precisely the type of string wave functional we have been looking for. It gives the amplitude for a string to lie along the curve $x^\mu(\sigma)$ on the spacelike line denoted by $\tau(\sigma)$. Equation (32) gives the amplitude on a slightly displaced spacelike line $\tau(\sigma) + \delta\tau(\sigma)$ - and may be integrated to obtain ψ on any other spacelike line.

We now return to the constraints (24). For the moment let us impose them on the wave functional. In the $\xi^1 = \sigma$ parametrisation one has:

$$\left\{ \frac{\delta}{\delta\tau(\sigma)} \pm \left[(\partial_\sigma\tau)\frac{\delta}{\delta\tau(\sigma)} + (\partial_\sigma x^\mu)\frac{\delta}{\delta x^\mu(\sigma)}\right] \right\}\psi = 0 \tag{33}$$

(The prefactor in (24) has been ignored since it can never be zero if $\tau(\sigma)$ denotes a spacelike line). We shall not, however, impose the full equation (33), but only its "positive frequency" part. This is similar to conventional string quantisation where the subsidiary condition (3) is imposed only for $n > 0$ [4]. For the open string we define the operator :

$$\begin{aligned} R_n &= 2\int_0^{\pi} d\sigma \left\{ \cos n\sigma \frac{\delta}{\delta\tau(\sigma)} - i \sin n\sigma \left(\partial_\sigma \tau \frac{\delta}{\delta\tau(\sigma)} + \partial_\sigma x \frac{\delta}{\delta x(\sigma)} \right) \right\} \\ &= \int_{-\pi}^{+\pi} d\sigma\, e^{in\sigma} \left\{ \frac{\delta}{\delta\tau(\sigma)} - \left(\partial_\sigma \tau \frac{\delta}{\delta\tau(\sigma)} + \partial_\sigma x^\mu \frac{\delta}{\delta x^\mu(\sigma)} \right) \right\} \quad \ldots (34) \end{aligned}$$

(with the standard extension to $-\pi \leq \sigma \leq \pi$). For the closed string, we define in addition:

$$\tilde{R}_n = \int_{-\pi}^{+\pi} d\sigma\, e^{in\sigma} \left\{ \frac{\delta}{\delta\tau(\sigma)} + \left(\partial_\sigma \tau \frac{\delta}{\delta\tau(\sigma)} + \partial_\sigma x^\mu \frac{\delta}{\delta x^\mu(\sigma)} \right) \right\} \quad (35)$$

We shall then impose the following condition on Ψ :

$$R_n \Psi = 0 \quad , n > 0 \qquad (36)$$

(with the additional condition $\tilde{R}_n \Psi = 0$ for the closed string).

The conditions (36) are the analogs of the Virasoro conditions. Together with the dynamical equation (32) they determine the wave functional.

In a second quantized theory based on the functional $\Psi[X(\sigma), \tau(\sigma)]$ the operators R_n now play the role of L_n in the conventional theory. In fact, it may be easily verified that the R_n's satisfy the standard Virasoro algebra (without the central charge). (It may be noted that instead of the R_n's defined above one might consider operators obtained by including the prefactor $1/(1 \pm \partial_\sigma \tau)$ of equation (24) in the integrands of (34) and (35). These operators do not, however, form a Virasoro algebra. This, in fact, was the motivation behind dropping the prefactor). However, unlike the L_n's , the R_n's have a clear geometric meaning in the extended loop space spanned by $X^\mu(\sigma)$ and $\tau(\sigma)$. The action of R_n on Ψ contains two pieces. The first piece involves

$$\frac{\delta}{\delta \tau(\sigma)} \Psi \tag{37}$$

which is the rate of change of Ψ under a change of $\tau(\sigma)$ – a σ-dependent reparametrisation of τ ! The second piece

$$\left[(\partial_\sigma \tau) \frac{\delta}{\delta \tau(\sigma)} + (\partial_\sigma X^\mu) \frac{\delta}{\delta X^\mu(\sigma)} \right] \Psi \tag{38}$$

relates to the change of Ψ under a reparametrisation of σ . The operators R_n thus implement σ and τ reparametrisations on the wave functional, and hence on the string field in a second quantized theory.

When the space-like lines on the world sheet are taken to be

$$\tau(\sigma) = \tau \quad (constant)$$

our formalism reduces to the standard one. The wave functional is now

$$\Psi[x^\mu(\sigma), \tau(\sigma) = \tau] = \tilde{\Phi}[x^\mu(\sigma), \tau]$$

The rate of change of $\tilde{\Phi}$ under a change of τ is given by:

$$\frac{\partial \tilde{\Phi}}{\partial \tau} = \int d\sigma \, \frac{\partial \tau(\sigma)}{\partial \tau} \, \frac{\delta \tilde{\Phi}}{\delta \tau(\sigma)} \qquad (39)$$

which becomes, using equation (32) and putting $\tau(\sigma) =$ a constant :

$$\frac{\partial \tilde{\Phi}}{\partial \tau} = \frac{1}{2} \int d\sigma \left[- \frac{\delta^2}{\delta x^2(\sigma)} + (\partial_\sigma x)^2 \right] \tilde{\Phi} \qquad (40)$$

which is precisely equation (15). The operators $\mathcal{R}_n$ similarly become L_n .

$\mathcal{R}_n$'s are derivative operators in loop space. A gauge invariant free string theory based on $\Psi[x(\sigma), \tau(\sigma)]$ would have chordal invariances of the type :

$$\Psi[x(\sigma), \tau(\sigma)] \rightarrow \Psi[x(\sigma), \tau(\sigma)] + \sum_n \epsilon_{-n} \mathcal{R}_{-n} \Omega[x(\sigma), \tau(\sigma)] \qquad (41)$$

which is a string analog of linearized gauge transformation in Yang-Mills theories:

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha$$

In analogy with Yang-Mills theories, it is not unreasonable to speculate that the full non-linear chordal gauge transformations - which translate into the non-linear gauge anmd general coordinate transformations of the massless modes - are obtained by replacing the $\mathcal{R}_n$'s in (41) by suitably defined "covariant" derivatives. These non-linear chordal invariances would then provide symmetry principles for constructing interacting string field theories. Given the simple geometrical meaning of the $\mathcal{R}_n$'s, the corresponding covariant extensions might not be too difficult to construct. Once the field theory is constructed with the fields $\psi[x(\sigma), \tau(\sigma)]$ one may pass to the more standard (and probably more practical) formulation based on fields $\Phi[X(\sigma)]$ by restricting to flat spacelike lines as outlined above.

It is also encouraging to note that the operators $\mathcal{R}_n$ or $\tilde{\mathcal{R}}_n$ do not require the existence of a flat spacetime metric $\eta_{\mu\nu}$ for their construction. They should therefore be of use in fashioning a string field theory whose formulation does not depend on the flat background space-time.

The formulation presented in this paper may be extended to include fermionic strings. This, together with various other related issues will be reported in a future communication [12].

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