Fermi National Accelerator Laboratory

TIFR-TH-85/
Fermilab-Pub-85/90-T
May,1985.

GAUGE AND GENERAL COORDINATE INVARIANCE IN STRING THEORIES

SUMIT R. DAS
Tata Institute for Fundamental Research,
Bombay 400005, India
and
Fermi National Accelerator Laboratory *
P.O.Box 500, Batavia, Ill.60510, U.S.A.

and

SPENTA R. WADIA
Tata Institute for Fundamental Research,
Bombay 400005, India.

ABSTRACT

We investigate how a single string propagates in a vacuum containing condensates of spin-1, spin-2 and antisymmetric tensor string modes. It is shown that such propagation is governed by gauge and general coordinate invariance in the weak field limit.

* Permanent Address

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

Recently, string theories have emerged as serious candidates for a unified description of all fundamental forces and matter [1]. The most promising models are the Type I superstring based on either O(32) gauge group [2] and the Heterotic String [3]. These models live in ten dimensional space-time. However, plausible compactifications to four dimensions have been proposed which lead to phenomenology consistent with low energy physics [4].

String theories are usually defined on flat Minkowski spacetime. The transverse oscillations of the string lead to an infinite tower of modes which may be thought of as particles constituting the string. At energies much lower than the string tension (the zero slope limit), only the zero mass modes can be excited - these constitute the particles of the low energy world. For the interesting models, the massless modes form a N=1 supergravity multiplet coupled to Yang-Mills fields.

One interesting feature of string theories is that principles like general coordinate invariance and gauge invariance are not postulated a priori. For example the "gluons" are simply spin-1 massless modes carrying group quantum numbers. In the zero slope limit, however, the interactions of these gluons amongst themselves as well as with other fields reduce to a form which is identical to one dictated by gauge invariance. The same happens for "gravitons" which are the spin-2 massless excitations of the closed string. Thus, in a certain sense gauge and general coordinate invariance emerge as consequences of interactions.

In this letter we shall ask the question as to how these invariance principles are present in string theories. Consider the string field $\Phi(x(\sigma))$ in the bosonic model. (This creates a string along the curve $X(\sigma)$). The string coordinate (for an open string) $X(\sigma)$ may be expanded into normal modes in the standard fashion:

$$X^\mu(\sigma,\tau) = X_0^\mu + \sum_{n=1}^{\infty} X_n^\mu(\tau) \frac{\cos n\sigma}{n} \qquad (1)$$

A general expansion for $\Phi(X)$ is then of the form [5]:

$$\Phi(X(\sigma)) = \varphi(X_0) + A^\mu(X_0) H_1(X_1{}^\mu) + \cdots \qquad (2)$$

where $H_1(X_1^\mu)$ is the standard harmonic oscillator wave function for the massless vector state of the first quantised string. Similar considerations are, of course, valid for the closed string exciations. In the full fledged second quantised string theory $A^\mu(X_0)$ is the "photon" field operator. The usual Fock vacuum of the theory contains no strings, i.e. it is annahilated by all operators appearing in the expansion of $\Phi(X)$. Presumably this is not the true vacuum of the theory. In particular six of the ten dimensions must compactify into generically non-flat manifolds. While it is not clear how to obtain the topology of the internal manifold a local curvature means that the vacuum contains a graviton condensate. And a background gauge field means that there is a condensate of vector modes. In fact the vacuum likely contains both massless and massive modes. We shall not ask how such a vacuum might arise - that requires a full understanding of the second quantised interacting theory. Instead we shall assume that suitable background fields are already present - and investigate how a string propagates in this background. This will be done within the first quantised framework.

Before considering string theories let us consider scalar quantum electrodynamics in the first quantised picture, following the original treatment of Feynman [6]. If the particle trajectory is denoted by $X(t)$ the amplitude for a free particle to go from the point X_1 to X_2 is given by :

$$\langle x_2 | x_1 \rangle = \int_{-\infty}^{+\infty} K(x_2, \tau_2 ; x_1, \tau_1)\, e^{-\frac{1}{2} i m^2 (\tau_2 - \tau_1)} d\tau_2 d\tau_1 \qquad (3)$$

where K is given by the path integral :

$$K(x_2, \tau_2 ; x_1, \tau_1) = \int \mathcal{D}x(\tau) \exp(i S_0^P) \qquad (4)$$

where

$$S_0^P = -\frac{1}{2} \int_{\tau_1}^{\tau_2} d\tau \left(\frac{dx^\mu}{d\tau} \cdot \frac{dx_\mu}{d\tau} \right) \qquad (5)$$

In (3) m is the mass of the Klein-Gordon particle. Once the free propagator is obtained from (3) and (4) the propagator in a background electromagnetic field may be obtained by the standard Feynman rules. The vertex for interaction of a photon of momentum k with the particle is simply:

$$V(k) = e \frac{dx^\mu}{d\tau} \epsilon_\mu(k) \exp(i k \cdot x(\tau)) \qquad (6)$$

where ϵ_μ is the polarisation vector for the photon. For a transverse photon $k.e = 0$. The amplitude $K(x_2, \tau_2 ; x_1 \tau_1)$ is now obtained by inserting the vertex (6) at various intermediate points between τ_1 and τ_2 and integrating over these points. Of course, all this is equivalent to adding the interaction term [6]:

$$S_I^P = e \int_{\tau_1}^{\tau_2} \frac{dx^\mu}{d\tau} \cdot A_\mu(x(\tau))\, d\tau \qquad (7)$$

to the free action S_o^P. The total action describes the gauge invariant interaction of the particle with the external field. Under the gauge transformation:

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \alpha(x)$$

the amplitude transforms as:

$$\langle x_2 | x_1 \rangle \rightarrow e^{i\alpha(x_2)} \langle x_2 | x_1 \rangle e^{-i\alpha(x_1)} \qquad (8)$$

as dictated by gauge covariance. For a non-Abelian external field the interaction is the same as (7), but A_μ is now a matrix. Consequently the path integral has to be time-ordered in the standard fashion. Consider the time slice between τ_i and τ_{i+1}. It may be easily checked that under a gauge transformation of the potential at i :

$$A_\mu^{(i)} \rightarrow U(x_i)\, A_\mu^i\, U^{-1}(x_i) + \frac{1}{i}\left(\partial_\mu U(x_i)\right) U^{-1}(x_i)$$

the part of the kernel belonging to the time slice transforms as

$$\langle x(\tau_{i+1}) | x(\tau_i) \rangle \rightarrow U(x(\tau_{i+1})) \langle x(\tau_{i+1}) | x(\tau_i) \rangle U^{-1}(x(\tau_i)) \qquad (9)$$

provided $(\tau_{i+1} - \tau_i) \rightarrow 0$. Thus in the naive continuum limit one recovers the usual gauge transformation property of the amplitude - in a way analogous to the behaviour of gauge transformation properties in lattice gauge theories.

Note that the above effective action has been deduced from perturbation theory. However, one takes (7) to be the general interaction term describing interactions with non-perturbative fields (like that of a monopole) as well.

Now consider the first quantised bosonic string; described by $X^{\mu}(\sigma,\tau)$. The covariant action in an orthonormal gauge is given by :

$$S_0^S = \frac{1}{2\pi}\int_0^{\pi} d\sigma \int_{\tau_1}^{\tau_2} d\tau \;\eta_{\mu\nu}\,\partial_\alpha X^{\mu}\partial^{\alpha}X^{\nu} \qquad (\alpha=\sigma,\tau) \qquad (10)$$

where $\eta_{\mu\nu}$ is a Minkowski metric in space-time. The transition amplitude between the states $|X_1(\sigma),\tau_1\rangle$ and $|X_2(\sigma),\tau_2\rangle$ is formally given by :

$$\langle X_2,\tau_2|X_1,\tau_1\rangle = \int \mathcal{D}X(\sigma,\tau)\,\exp\left[iS_0^S\right] \qquad (11)$$

First consider the interaction of the massless vector mode of an open string. For a "photon" with a momentum k the vertex function is (in a covariant gauge):

$$V_1(k) = g\,\frac{\partial X^{\mu}(0,\tau)}{\partial\tau}\,\epsilon_\mu(k)\,e^{ik\cdot X(0,\tau)} \qquad (12)$$

where $\epsilon_\mu(k)$ is the polarisation vector. This is the vertex when the photon hits the $\sigma=0$ end of the string. The vertex at the $\sigma=\pi$ end may be written down likewise. In a way entirely analogous to scalar electrodynamics the amplitude in the presence of a background field of spin-1 particles is obtained by adding to S_0^S the term :

$$S_I^{(1)} = g\int_{\tau_1}^{\tau_2} d\tau\int_0^{\pi} d\sigma\;\frac{\partial X^{\mu}(\sigma,\tau)}{\partial\tau}A_\mu(X(\sigma,\tau))\,\delta(\sigma) \qquad (13)$$

and using the full action $S_0^S + S_I^{(1)}$ in eqn.(11). To describe the full amplitude one has to, of course, integrate over all possible topologies of the world sheet. The string thus propagates with its ends having the usual gauge-invariant interaction with the background field. For non-Abelian fields, $A_\mu(x)$ is once again a matrix. Note that the form of the vertex (12) was obtained in the string theory from reparametrisation invariance and masslessness of the mode. This,however, leads to gauge invariance.

Let us now consider the interaction of a string with the massless spin-2 mode of the closed string. This interaction can occur anywhere along its extension. The vertex function is now :

$$\frac{\kappa}{\pi}\int_0^\pi d\sigma\, \epsilon_{\mu\nu}(k)\, \frac{\partial X^\mu}{\partial\tau_-}\frac{\partial X^\nu}{\partial\tau_+}\, e^{ik\cdot X(\sigma,\tau)} \qquad (14)$$

where $\tau_\pm = \tau \pm \sigma$. $\epsilon_{\mu\nu}$ is the transverse polarisation tensor.

$$k_\mu \epsilon_{\mu\nu} = k_\nu \epsilon_{\mu\nu} = 0$$

(14) may be rewritten as :

$$\frac{\kappa}{\pi}\int_0^\pi d\sigma\, \epsilon_{\mu\nu}(k)\, \partial_\alpha X^\mu \partial^\alpha X^\nu\, e^{ik\cdot X(\sigma,\tau)} \qquad (15)$$

If $h_{\mu\nu}(x)$ denotes the graviton field the propagation of the string in the given background is thus described by the total action:

$$S_0^S + \frac{\kappa}{\pi}\int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma\, h_{\mu\nu}(x)\, \partial_\alpha X^\mu \partial^\alpha X^\nu$$

$$= \frac{1}{2\pi}\int d\tau\, d\sigma\, g_{\mu\nu}(x)\, \partial_\alpha X^\mu \partial^\alpha X^\nu \qquad (16)$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \qquad (17)$$

appears as the total background metric in which the string propagates in a manner consistent with general covariance. However, the constraint equations for the orhonormal gauge are not generally covariant. The action (16) supplemented by a covariant gauge condition describes the motion of the string upto $O(h_{\mu\nu})$. Note that since $h_{\mu\nu}(x)$ is transverse, only the transverse components of the metric are non-flat. In fact, in a light cone gauge the effective action is:

$$\frac{1}{2\pi}\int_0^{\pi} d\sigma \int_{\tau_1}^{\tau_2} d\tau \; g_{ij}\, \partial_\alpha x^i \partial^\alpha x^j \tag{18}$$

where i,j denote transverse components. This defines a non-linear sigma model on a manifold with metric g_{ij}.

In an entirely analogous fashion an antisymmetric tensor field $A_{\mu\nu}(x)$ interacts with the string via an interaction term:

$$\int d\sigma d\tau \; A_{\mu\nu}(x)\, \partial_\alpha x^\mu \partial_\beta x^\nu \epsilon^{\alpha\beta} \tag{19}$$

which is, of course, the well-known Kalb-Ramond coupling [7], reflecting a U(1) gauge invariance.

The closed string also has a dilaton mode $\Phi(x)$. Fradkin and Tseytlin [9] have suggested that $\Phi(x)$ couples to the string via a term:

$$S_D = \int d^2\xi \sqrt{g}\, R^{(2)} \Phi(x) \tag{20}$$

where $R^{(2)}$ is the scalar curvature for the two-dimensional world sheet. (This is,of course, prior to fixing a conformal gauge). Thus if $\langle\Phi(x)\rangle = \phi_0$ (a constant),

$$S_D \sim \phi_0 \times \left(\begin{array}{c}\text{EULER CHARACTERISTIC} \\ \text{OF WORLD SHEET}\end{array}\right) \tag{21}$$

Thus e^{ϕ_0} behaves as a loop expansion parameter in the string theory. It is not clear how such a term arises in our approach.

The above considerations may be easily generalised to superstrings. Let us, for example, consider the motion of a Type I superstring in a background graviton field. The free superstring action in the light cone gauge is given by:

$$S_0 = \int d\sigma d\tau \left[-\frac{1}{2\pi} \partial_\alpha X^i \partial^\alpha X_i + \frac{i}{4\pi} \bar{S} \gamma^- \rho^\alpha \partial_\alpha S \right], \qquad (22)$$

Here S^{Aa} denotes a ten-dimensional Majorana-Weyl spinor which also transforms as a spinor under world sheet reparametrisations. For a closed string one has:

$$S^{Aa} = \begin{pmatrix} S^{1a} \\ S^{2a} \end{pmatrix} = \begin{pmatrix} S^a(\tau_-) \\ S^a(\tau_+) \end{pmatrix}$$

a being the space-time spinor index. The vertex for emission of a spin two particle with momentum p is given by: [1]

$$\frac{\mathcal{K}}{\pi} \int_0^\pi d\sigma \; B_c^i(p,\tau_-) \tilde{B}_c^j(p,\tau_+) \, e^{ik\cdot x(\sigma,\tau)} \qquad (23)$$

where

$$B_c^i(p,\tau_-) = \frac{\partial X^i}{\partial \tau_-} + \frac{1}{2} p^i R^{ij}(\tau_-) \qquad (24)$$

$$R^{ij}(\tau_-) = \bar{S}(\tau_-) \gamma^{ij-} S(\tau_-) \qquad (25)$$

and similarly for $\tilde{B}_c^j$. The resulting effective action is:

$$\begin{aligned} S = \int d\sigma d\tau \Big\{ & -\frac{1}{2\pi} g_{ij} \partial_\alpha X^i \partial^\alpha X^j + \frac{i}{4\pi} \bar{S} \gamma^- \rho^\alpha \partial_\alpha S \\ & - \frac{i\mathcal{K}}{\pi} (\partial_j h_{ik})(\partial_\alpha X^k) \bar{S} \gamma^{ij-} \rho^\alpha S \\ & - \frac{\mathcal{K}}{4\pi} (\partial_j \partial_\ell h_{ik}) R^{k\ell}(\tau_+) R^{ij}(\tau_-) \Big\} \end{aligned} \qquad (26)$$

This may be rewritten as:

$$S = \int d\sigma d\tau \Big\{ -\frac{1}{2\pi} g_{ij}\, \partial_\alpha x^i \partial^\alpha x^j + \frac{i}{4\pi} \bar{S} \gamma^- \rho^\alpha \partial_\alpha S$$
$$- \frac{i\mathcal{K}}{\pi} (\nabla_j h_{ik}) (\partial_\alpha x^k) \bar{S}\, \hat{\gamma}^{ij-} \rho^\alpha S$$
$$- \frac{\mathcal{K}}{4\pi} (\nabla_j \nabla_\ell h_{ik}) \hat{R}^{k\ell}(\tau_+) \hat{R}^{ij}(\tau_-) + O(h^2) \Big\} \tag{27}$$

where ∇_j is the covariant derivative with the metric g_{ij} and $\hat{\gamma}^i$ denote the curved space gamma-matrices. This action is clearly invaraint under general coordinate transformations in the transverse space. Note that $\partial_j h_{ik}$ may be replaced by Γ_{ikj}, the affine connection, and $\partial_j \partial_\ell h_{ik}$ may be replaced by the curvature $\mathcal{R}_{ijk\ell}$, thus obtaining the form quoted in Ref.[4]. The case of other background fields may be dealt with in a similar fashion.

String theories on curved background spaces have been discussed in detail [8]. In these papers the theory is defined on a background geometry to begin with. In this note we have indicated how such models may arise as the first quantised description of a string defined on a trivial background but living in a dynamically generated non-trivial vacuum. As a result of interactions whose forms are dictated by the reparametrisation invariance of the string theory, weak background spin-1 and spin-2 fields couple back to the string in a fashion which reflects gauge and general coordinate invariance. One expects this to hold for arbitrarily strong fields - but this probably requires a treatment in the full second quantised framework. Furthermore, just as in particle electrodynamics, one can adopt the form of effective action derived from perturbation theory as the general action describing strings propagating in non-perturbative fields as well.

We have not considered the case of the heterotic string. Application of our method to the case of the heterotic string shall throw valuable light on the nature of the stringy Kaluza-Klein mechanism.

ACKNOWLEDGEMENTS : One of us (S.R.D.) would like to thank the members of the Theory Group at Tata Institute for Fundamental Research for hospitality. S.R.D. also wishes to thank A.Sen and H.B.Nielsen for discussions in the early stages of this work.

Note Added : After completing this work we learnt that A.Sen [10] has also obtained similar results.

REFERENCES

1. For a recent review, see J.H.Schwarz, Phys.Rep. *89C* (1982) 223 ; An early review is J.Scherk, Rev.Mod.Phys. 47 (1975) 123.
2. M.B.Green and J.H.Schwarz, Phys.Lett. 149B (1984) 117; Caltech Preprints CALT-68-1194 & CALT-68-1224.
3. D.J.Gross, J.Harvey, E.Martinec and R.Rohm, Phys.Rev.Lett. 54 (1985) 502; Princeton Preprint, Jan. 1985. A similar idea is contained in P.G.O.Freund, Phys.Lett. 151B (1985) 387.
4. P.Candelas, G.T.Horowitz, A.Strominger and E.Witten, Santa Barbara Preprint NSF-ITP-84-170 (1984).
5. M.Kaku and J.Lykken, Paper presented at the Symposium on Anomalies, Geometry and Topology, Argonne National Laboratories, March 1985 (to appear in proceedings).
6. R.P.Feynman, Phys.Rev. 76 (1949) 769; Phys.Rev. 80 (1950) 440.
7. M.Kalb and P.Ramond, Phys.Rev. D9 (1974) 2273; Y.Nambu, Phys. Rep. 23C (1976) 250.
8. S.Jain, R.Shankar and S.R.Wadia, TIFR Preprint TIFR-TH-85-3; D.Nemeschansky and S.Yankelowicz, Phys.Rev.Lett. 54 (1985) 620; *E.Bergshoff, S.Ranjdbar-Daemi, A.Salam, H.Sarmadi and E.Sezgin,* ICTP Preprint (May 1985).
9. E.S.Fradkin and A.A.Tseytlin, Lebedev Institute Preprint (1984)
10. A.Sen, Fermilab Preprint Fermilab-Pub-85/60-T.