Fermi National Accelerator Laboratory

FERMILAB-Pub-85/88-A
June 1985

On the Relic Abundance of Stable Neutrinos

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ABSTRACT

The relic abundance of neutrinos with only the usual electroweak interactions is well known. Here, we calculate the relic neutrino abundance in models where neutrinos have additional 'stronger than weak' interactions and give numerical results for the model of Gelmini and Roncadelli. In models with new neutrino interactions, the relic abundance depends upon the strength of the new interactions and can take on virtually any value. As a consequence, the Universe can be neutrino-dominated for any neutrino mass in the range 30eV - 3GeV, provided that the new interaction has the appropriate strength. Furthermore, neutrinos in the mass range 30eV - 3GeV can be 'cosmologically safe' even if their lifetimes are greater than the age of the Universe.

Submitted to Phys. Lett. B.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

In the standard, hot big bang cosmology [1] neutrinos with the usual electroweak interactions decouple at a temperature of

$$T_{dec} \sim \begin{cases} few MeV & m_\nu \lesssim 100 MeV \\ m_\nu/20 & m_\nu \gtrsim 100 MeV. \end{cases} \tag{1}$$

As a result, light ($m_\nu \lesssim 10 MeV$) neutrinos have a relic abundance of $n_\nu/n_\gamma = 3/11$, since they decouple when relativistic; here, n_γ is the number density of photons and n_ν the number density of neutrinos plus antineutrinos. Heavy ($m_\nu \gtrsim 10 MeV$) neutrinos, on the other hand, track their equilibrium abundance, $n_\nu/n_\gamma \sim (m_\nu/T)^{3/2}\ \exp(-m_\nu/T)$, down to a temperature of about $T_f \sim m_\nu/20$, when their annihilations freeze out (annihilation rate Γ_{ann} becomes comparable to the expansion rate of the Universe H), leaving them with a relic abundance of [2-3].

$$n_\nu/n_\gamma \approx 10^{-7}(m_\nu/GeV)^{-3} \tag{2}$$

Thus for neutrinos with standard interactions there are two neutrino masses for which relic neutrinos dominate the mass density of the Universe: 30 eV and a few GeV. Intermediate masses result in $\Omega_\nu > 1$; if such neutrinos exist, they must be unstable [4].

In extended models, neutrinos can have additional interactions, e.g., annihilation of heavy neutrinos into light neutrinos mediated by a massive or massless (pseudo) scalar or $\nu\nu$ or $\nu\bar{\nu}$ annihilation into light (pseudo) scalars [5-6]. In such models, neutrinos remain in equilibrium to a lower temperature since their annihilations freeze out later, and thus they should have a smaller relic abundance today. In order to compute the relic abundance precisely one must integrate the Boltzmann equation [2, 3, 7, 8, 9];

$$\dot{n}_\nu + 3Hn_\nu = -\langle\sigma v\rangle(n_\nu^2 - n_{EQ}^2) \tag{3}$$

where overdot indicates a time derivative, n_{EQ} is the equilibrium number density for a particle of mass m_ν, and $\langle\sigma v\rangle$ is the thermally-averaged annihilation rate [F1].

In the remainder of this paper we will set up this problem in full generality and solve for the relic abundance approximately. We will then focus on the model of Gelmini and Roncadelli [5] and solve the Boltzmann equation numerically. Finally, we will briefly comment on the cosmological implications of our results.

By introducing the following dimensionless variables

$$\begin{aligned} Y &= n_\nu/n_\gamma \\ x &= m_\nu/T_\nu \\ \alpha &= T_\nu/T_\gamma \end{aligned} \tag{4}$$

the Boltzmann equation can be written in a more useful form [F2]

$$dY/dx = -(1.2\varsigma(3)/\pi^2\alpha g_*^{1/2})(\langle\sigma v\rangle m_\nu m_{pl})\; x^{-2}\;(Y^2 - Y_{EQ}^2) \tag{5}$$

where as usual $g_* \equiv \sum_{Bose} g_B + \frac{7}{8}\sum_{Fermi} g_F$ counts the effective number of relativistic degrees of freedom and $\varsigma(3) = 1.20206...$ We have assumed that the Universe is radiation-dominated so that the expansion rate $H = 1.67 g_*^{1/2} T_\gamma^2/m_{pl}$. The equilibrium abundance of neutrinos plus antineutrinos relative to photons is given by

$$\begin{aligned} Y_{EQ}(x) &= [2\varsigma(3)]^{-1}\alpha^3\int_0^\infty[\exp[(x^2+u^2)^{1/2}]+1]^{-1}u^2\,du, \\ &\approx \begin{cases} \frac{3}{4}\alpha^3 & x \ll 3 \\ \frac{1}{\varsigma(3)}\left(\frac{\pi}{8}\right)^{1/2}\alpha^3 x^{3/2} e^{-x} & x \gg 3 \end{cases} \end{aligned} \tag{6}$$

So long as the rate for interactions which create and destroy neutrinos is rapid compared to the expansion rate of the Universe ($\Gamma_{ann} > H$), the abundance of neutrinos will be the equilibrium abundance: $Y \approx Y_{EQ}$. When the interaction rate falls below the expansion rate, neutrinos effectively cease being created or destroyed and their abundance freezes out. A very good approximation to actually integrating the Boltzmann equation is to set the final abundance Y_f equal to Y_{EQ} at the 'freeze-out' temperature, where the 'freeze-out' temperature is defined by

$$\Gamma_{ann}(T_f) \;=\; n_{EQ}\langle\sigma v\rangle|_{T_f} \;\approx H(T_f).$$

By solving the equation $\Gamma_{ann} = H$, one obtains the freeze-out temperature and approximate relic abundance:

$$\begin{aligned} x_f &= \ln\left(\frac{0.077}{g_*^{1/2}} m_{pl} m_\nu (\sigma v)_0 \alpha^2\right) + (1/2-n)\ln\left[\ln\left(\frac{0.077}{g_*^{1/2}} m_{pl} m_\nu (\sigma v)_0 \alpha^2\right)\right], \qquad (7) \\ &= 17.1 + \ln[m_{GeV}(\sigma v)_{-10}] + (1/2-n)\ln\left\{17.1 + \ln[m_{GeV}(\sigma v)_{-10}]\right\} \end{aligned}$$

$$\begin{aligned} Y_f &= \frac{6.8 g_*^{1/2}\alpha}{m_{pl} m_\nu (\sigma v)_0}\left[\ln\left(\frac{0.077\alpha^2}{g_*^{1/2}} m_{pl} m_\nu (\sigma v)_0\right)\right]^{n+1} \qquad (8) \\ &= 7.3\times 10^{-9} m_{GeV}^{-1}(\sigma v)_{-10}^{-1}[17.1 + \ln(m_{GeV}(\sigma v)_{-10})]^{n+1}, \end{aligned}$$

where $\alpha = (4/11)^{1/3}$ and $g_* = 3.36$ have been used to evaluate Eqns (7,8) (these are the appropriate values for the temperature of interest: $T_f \lesssim few\ MeV$). We have parameterized the temperature-dependence of $\langle\sigma v\rangle$ by: $\langle\sigma v\rangle = (\sigma v)_0 x^{-n}$ ($n > -1$ to assure freeze-out) and $(\sigma v)_0 = (\sigma v)_{-10}\times 10^{-10} GeV^{-2}$. In Eq (8)we have assumed that the neutrinos are nonrelativistic at freeze-out: $x_f \gtrsim 3$; if they are not then $Y_f = 0.75\alpha^3$.

Assuming that the neutrino species in question is stable, or at least long-lived, its contribution to the present mass density is

$$\rho_\nu = Y_f m_\nu n_\gamma, \tag{9}$$

$$\left(\frac{\Omega_\nu h^2}{\theta^3}\right) = 38 Y_f m_{keV}, \tag{10}$$

where $\Omega_\nu = \rho_\nu/\rho_{crit}$, $\rho_{crit} = 1.05 \times 10^4 h^2 eV cm^{-3}$ is the critical density, $2.7\theta K$ is the present value of the photon temperature, and $H = 100h\ km\ \sec^{-1} Mpc^{-1}$ is the present value of the Hubble parameter. In Fig. 1 we show contours of $\frac{\Omega_\nu h^2}{\theta^3} = .01$ and 1.0 in the $(\sigma v)_0$-m_ν plane for $n = 0,1$, and 2. For neutrinos which only interact via the usual weak interactions, $<\sigma v>$ is given by (assuming Dirac neutrinos [F3])

$$<\sigma v> = \frac{G_F^2 m_\nu^2}{2\pi} \sum_i (C_V^2 + C_A^2)_i \tag{11}$$

where the sum is over particles with mass less than m_ν and C_V and C_A are related to I_3 and Q ($C_V = I_3 - 2Q\sin^2\theta; C_A = I_3$). In this case, a cosmologically-interesting relic abundance, i.e., $\Omega_\nu h^2/\theta^3 \sim .01$–$1.$, occurs only for the neutrino mass intervals: $m_\nu \approx 10$–$100 eV$ and $m_\nu \approx$ few GeV. However, if neutrinos have an additional means of annihilating, then a cosmologically-interesting abundance can occur for any neutrino mass between 10 eV and a few GeV, provided that the annihilation cross section has the appropriate value (see Fig. 1).

Consider, in particular, the model of Gelmini and Roncadelli [5]. In their model neutrinos can annihilate into massless Goldstone bosons, called majorons, and the annihilation cross section is [F4]:

$$
\begin{aligned}
<\sigma v> &= \frac{g^4}{256\pi} s\, [\beta^{-1} \ln \frac{1+\beta}{1-\beta} - 2] \qquad (12) \\
&= \frac{m_\nu^2 x^{-1}}{1024\pi v^4} \qquad\qquad (x>>1)
\end{aligned}
$$

where v on the rhs of Eq (12) is the vacuum expected value of the Higgs triplet, $\sqrt{s}$ is the c.m. energy, and β is the neutrino velocity in the c.m. frame. Substituting this cross section into Eqns (7, 8, 10), we find that

$$
\begin{aligned}
x_f &\approx 16.8+\ln(m_{keV}^3/v_{MeV}^4) - \frac{1}{2}\ln[1+\ln(m_{keV}^3/v_{MeV}^4)/18] \qquad (13) \\
Y_f &\approx 7.8\times 10^{-6}(v_{MeV}^4/m_{keV}^3)\ [1+\ln(m_{keV}^3/v_{MeV}^4)/18]^2 \qquad (14) \\
\Omega_\nu h^2/\theta^3 &\approx 3.5\times 10^{-5}(v_{MeV}^4/m_{keV}^2)\ [1+\ln(m_{keV}^3/v_{MeV}^4)/18]^2 \qquad (15)
\end{aligned}
$$

We have also integrated the Boltzmann equation using this cross section and the results are displayed in Table 1 and in Fig. 2. The numerical results agree well with the freeze-out approximation -- typically to within 30%.

The relic abundance of neutrinos in this model had been previously calculated by Georgi, Glashow, and Nussinov [11], and their results are also shown in Fig. 2. For a given mass, their estimate for $\Omega_\nu h^2/\theta^3$ is smaller than ours by a factor of $3\times 10^{-5}\ m_{keV}^{-1/2}$. In Fig. 2 we show the value of v required to obtain $\Omega_\nu h^2/\theta^3 = 0.01$ and 1.0 as a function of m_ν; since $\Omega_\nu h/\theta^3 \propto v^4$ the discrepancy here is only a factor of $14 m_{keV}^{1/8}$. The reason for the discrepancy between our results and their results is twofold: in computing the relic abundance they neglected the creation term in the Boltzmann equation [F5] and also assumed that the Universe was matter-dominated, thereby overestimating the age of the Universe at a given temperature. Both effects go in the direction of overestimating the importance of annihilations.

Georgi, Glashow and Nussinov also discussed the astrophysical bound on v based upon the emission of majorons from red giants and other stars: $v<75keV$. Using this bound and their results for the relic abundance, they concluded that massive neutrions in the majoron model model must necessarily be cosmologically uninteresting as $\Omega_\nu h^2/\theta^3$ must be less than 0.01. Fukugita, Watamura, and Yoshimura [12] have very carefully analyzed the emission of majorons (and other light Goldstone bosons) from stars and obtain the less stringent bound: $v<0.9MeV$. Using our results for the relic abundance and this bound on v, we find that there is a tiny bit of phase space for which $\Omega_\nu h^2/\theta^3$ is 0.01, for neutrino masses of around 1 eV.

If we consider a Higgs triplet model without a global B-L symmetry at high temperatures, neutrino annihilation proceeds through the emission or exchange of scalar particles, S, with mass m_S [6]. The masses of a heavy neutrino and a light neutrino are related to the vacuum expected value of the Higgs triplet, v, by $m_{\nu_H} = g_H v$, $m_{\nu_L} = g_L v$. If $m_S > m_{\nu_H}$, then ν_H annihilation will only occur through S exchange, $\nu_H \nu_H \rightarrow \nu_L \nu_L$, with cross section

$$<\sigma v> = \frac{3 g_L^2 g_H^2 m_\nu^2}{128\pi m_S^4}\ x^{-1} \qquad (x>>1). \tag{16}$$

Using Eqns (7, 8, 10), we find that for this model

$$\Omega_\nu h^2/\theta^3 = \tag{17}$$
$$\frac{1.7\times10^{-6}}{g_L^2 g_H^2}\ \frac{(m_S/MeV)^4}{m_{keV}^2}\ \left[1+\ln(m_{keV}^3 g_L^2 g_H^2/(m_S/MeV)^4)/21.4\right]^2.$$

In this model there is no "red giant" limit, so $m_S^4/g_L^2 g_H^2$ could easily be large enough to give interesting $\Omega_\nu h^2$ for any value of m_ν in the range 10eV - few GeV.

To conclude, if neutrinos have additional interactions, then as is clear from Fig. 1 any neutrino species of mass between 30 eV and a few GeV can dominate the mass density of the Universe, provided the annihilation cross section has the appropriate value. In particular, if the neutrino mass is greater than about 1 keV, then the relic neutrinos will behave as cold dark matter rather than hot dark matter, implying that structure in such a Universe would form in a hierarchical manner rather than larger structures forming first and fragmenting as is the case for hot dark matter [13]. Finally, we note that the addition of massive particles coupled strongly to neutrinos (or of massive neutrinos that annihilate into light neutrinos) affects primordial nucleosynthesis in a way distinct from simply increasing the number of massless neutrinos [14].

We thank Gary Steigman for helpful comments. This work was supported by the DOE (at Fermilab and The University of Chicago) and by the NASA (at Fermilab), and M.S. Turner's Alfred P. Sloan Fellowship.

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[F1] In deriving Eqn (3) we have assumed that $n_\gamma \propto R^{-3}$, i.e., that the number of photons per comoving volume remains constant. This is only strictly true in the absence of entropy production and when g_* is constant. For temperatures below $m_e/3 \sim 0.1 MeV$ -- which are the temperatures of interest here, g_* should remain approximately constant. If g_* is not constant, then one should use the entropy density $s = (2\pi g_*^2/45)T^3$ in place of n_γ as the fiducial, as in the absence of entropy production $s \propto R^{-3}$.

[F2] By writing $H = 1.66 g_*^{1/2} T_\gamma^2/m_{pl}$ we are assuming that the Universe is radiation-dominated throughout. In actuality the Universe is only radiation-dominated down to a temperature $T \sim 6eV(\Omega h^2/\theta^3)$. However, at freeze-out the relic neutrino mass density is always dominated by that of the photons since $\rho_{\nu EQ}/\rho_\gamma \sim (m/T)^{5/2}\exp(-m/T)$, and so if the relic neutrino density is to be cosmologically significant, $\Omega_\nu \sim 0.01-1$. the Universe must still be radiation-dominated at freeze-out.

[F3] For low-energy annihilation of Majorana neutrinos via Z boson exchange, there is a p-wave suppression factor as first pointed out by Goldberg [9]. For calculations of the relic abundance of Majorana neutrinos in this case, see ref. [10].

[F4] The cross section in Eqn (12) has been averaged over initial states (ν and $\overline{\nu}$), thermally-averaged, and includes the appropriate factor of 1/2 for identical particles in the final state. Since $s = 4m_\nu^2/(1-\beta^2)$, the thermal average of β^2 is: $<\beta^2> = <1-4m_\nu^2/s> \rightarrow (3/2x)$ in the limit of $x>>1$.

[F5] In the Boltzmann equation the n_ν^2 term takes into account annihilations, while the n_{EQ}^2 term accounts for pair creations. So long as $n_\nu \approx n_{EQ}$ (until $T \sim T_f$), these rates are equal -- as they must be by detailed balance. Thus one cannot neglect inverse annihilation reactions, even when $T \lesssim m_\nu$ and they are Boltzmann suppressed. For $T \lesssim T_f$, both annihilations and inverse annihilating can be neglected since their rates are less than the expansion rate.

TABLE 1 - RELIC NEUTRINO MASS DENSITY $\Omega_\nu h^2/\theta^3$

m_{keV}	v_{MeV}	10^{-1}	3×10^{-1}	1	3	10	30	100
10^{-3}		4.1×10^{-4}	4.6×10^{-3}	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}
10^{-2}		2.0×10^{-5}	6.7×10^{-4}	1.7×10^{-2}	8.5×10^{-2}	1.0×10^{-1}	1.0×10^{-1}	1.0×10^{-1}
10^{-1}		5.4×10^{-7}	2.5×10^{-5}	1.3×10^{-3}	3.5×10^{-2}	5.0×10^{-1}	1.0	1.0
1		1.1×10^{-8}	5.7×10^{-7}	4.1×10^{-5}	1.7×10^{-3}	7.9×10^{-2}	1.5	8.8
10		1.9×10^{-10}	1.1×10^{-8}	8.6×10^{-7}	4.5×10^{-5}	2.9×10^{-3}	1.1×10^{-1}	4.1

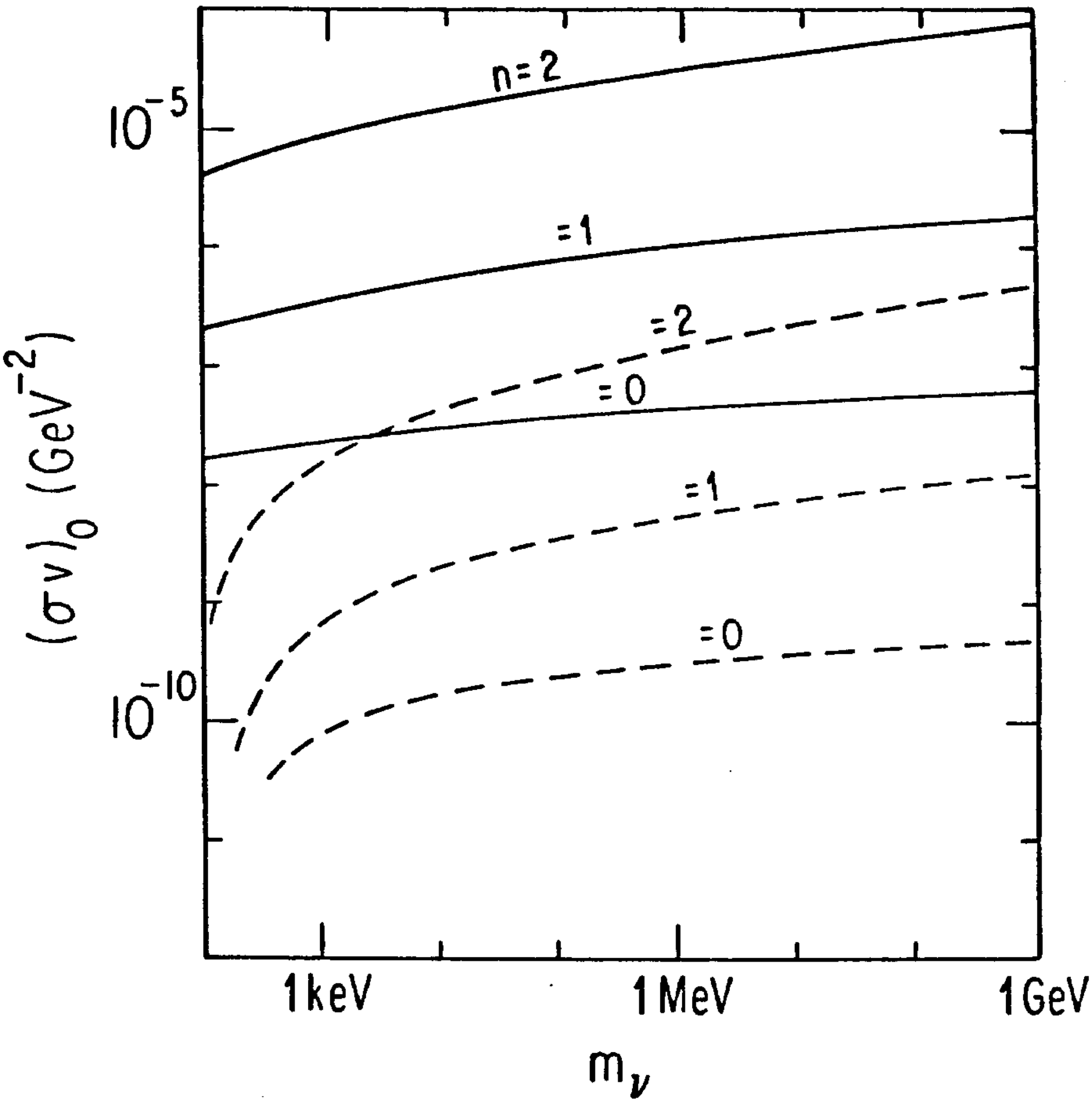

FIG. 1 - Contours of $(\Omega_\nu h^2/\theta^2) = 0.01$ (solid curves) and 1.0 (broken curves) in the m_ν - $(\sigma v)_0$ plane for n = 0, 1, and 2. The temperature dependence of the annihilation cross section has been parameterized by: $<\sigma v> = (\sigma v)_0 x^{-n}$ (for $n > -1$).

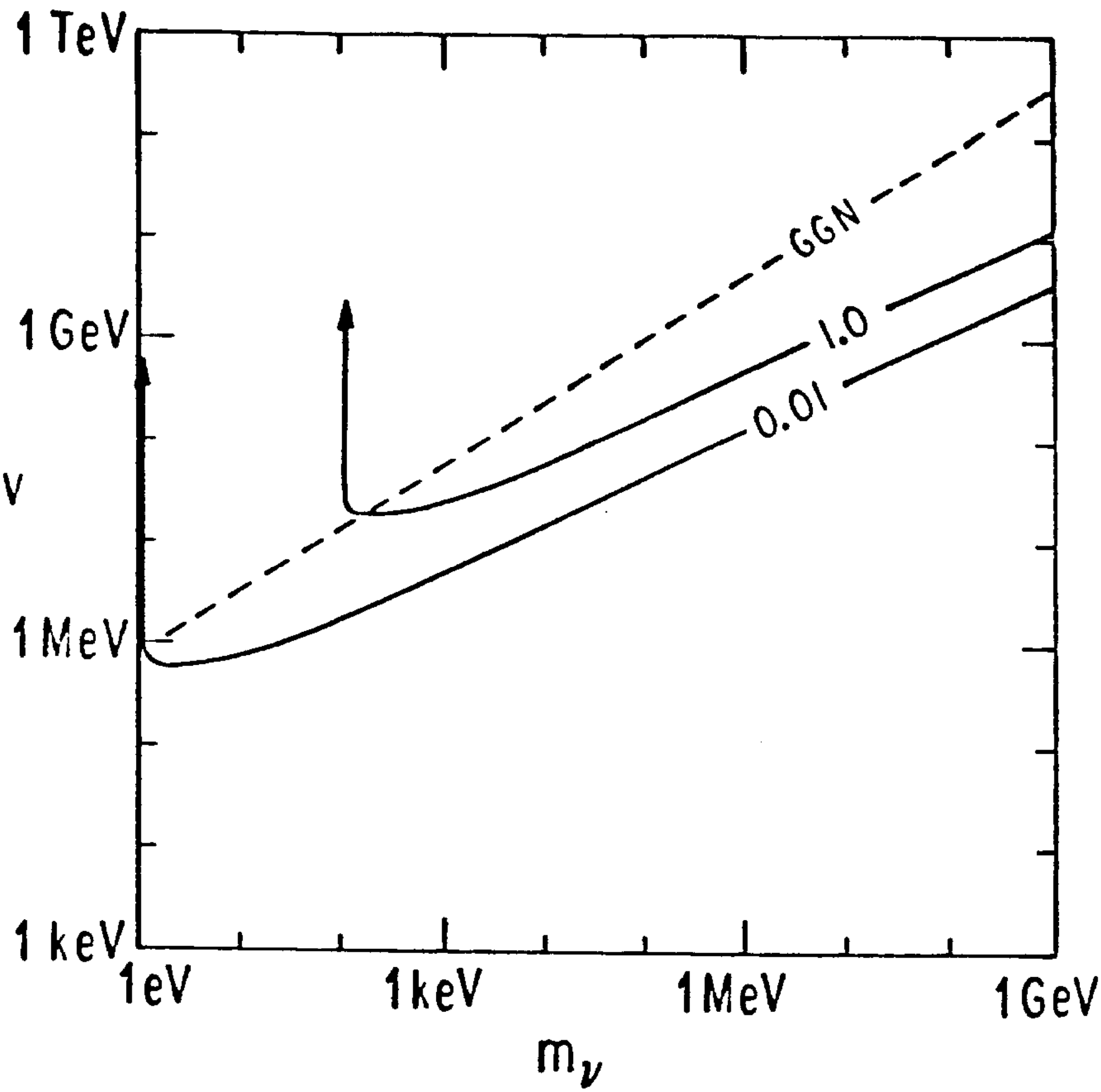

FIG. 2 - Contours of $(\Omega_\nu h^2/\theta^3) = 0.01$ and 1.0 in the v–m_ν plane for the Gelmini-Roncadelli model. The broken line labeled GGN shows the earlier results of ref. 10 for $\Omega_\nu h^2/\theta^3 = 0.01$. Majoron emission from stars places an upper limit to v of 0.9 MeV [12], and so there is a tiny bit of phase space where $\Omega_\nu h^2/\theta^3 = 0.01$ is possible.