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NEW ACTIONS FOR SUPERSTRINGS

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ABSTRACT

New covariant actions for both the Green-Schwarz Superstrings and the Heterotic Strings are presented. The construction of these new actions is based on a simple, intuitive, physical picture. The connection to the Green-Schwarz light-cone quantization is discussed.

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Developments in the superstring theory[1] clearly demonstrated that the superstrings have many desirable properties not found in usual quantum field theories. One of these properties is the natural incorporation of quantum gravity free from divergences. Recent analysis on the vacuum configurations of superstrings[2] showed that the $E_8 \times E_8$ Heterotic String[3] has the potential to be the ultimate unified model of all forces in nature. However, despite of the tremendous progress made since the Neveu-Schwarz-Ramond model[4] was first written down about fifteen years ago, a satisfactory action formulation is still missing. The best attempt so far is the Lorentz-covariant action proposed by Green and Schwarz[5]. In addition to global supersymmetry, the G-S action also has a local supersymmetry built in. In their action formulation, this local supersymmetry is needed to reduce by half the degrees of freedom of the Majorana-Weyl fermions. Unfortunately, this same local supersymmetry is also the obstacle to a covariant quantization of the superstrings[6]. It is clear that covariant quantization is most useful to the study of string interactions. Therefore it will be desirable if one can formulate an alternative Lorentz-covariant action which allows both the covariant and the light-cone quantizations.

In this letter, the approach taken in Ref {7} is used to construct new Lorentz-covariant actions for both the Superstrings and the Heterotic String. These actions have a very simple intuitive physical picture. Here, only the construction of the actions and the light-cone quantization are presented. Covariant quantization and other properties of this new action formulation are under investigation and will be discussed elsewhere[8].

Let us start with the relativistic string in D dimensions. Here the standard form[9] will be adopted:

$$S_1 = -C/2 \int d^2u^\delta \sqrt{-g}\; g^{\alpha\beta}\, \eta^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu \tag{1}$$

where $\mu,\nu,\rho = 0,1,\ldots,D-1$ and $\alpha,\beta,\delta = 0,1$; here $u^\delta = u^\delta(\tau,\sigma)$. $\eta^{\mu\nu}$ is the Minkowski metric, $\eta^{\mu\nu} = (-1,1,1,\ldots,1)$. The constant C is simply $C^{-1} = 2\pi\alpha'$.

Bosonic and fermionic fields are confined on this string so that they can only move along the string. In general, the fields on the string can interact among themselves. In the simple cases that we shall consider here, they are free fields trapped on the string. The string propagation generates a world sheet. At each point on the world sheet, the tangential directions are given by the two tangent vectors $t_{\alpha\mu} = \partial_\alpha X_\mu$. To confine a D-dimensional spin-1/2 fermion on the string, we must project the fermionic motions onto the tangential directions. These projections and similar projections for scalar fields give

$$S_F = C \int d^2u^\delta \sqrt{-g}\, g^{\alpha\beta} \left(i t_{\alpha\mu} \overline{\Psi}_A \gamma^\mu \partial_\beta \Psi_A \right) \tag{2}$$

$$S_B = -C/2 \int d^2u^\delta \sqrt{-g}\, g^{\alpha\beta}\, \partial_\alpha \Phi^I \partial_\beta \Phi^I \tag{3}$$

where I labels the boson fields, $I = 1,2,\ldots,J$; and A labels the fermion fields, $A=1,2,\ldots,K$. For massless free fields on a closed string, the left-moving (clockwise) and the right-moving (anti-clockwise) components of each field move independently and so can be separated. This separation depends on the particular fermion under consideration. For fermion fields in superstrings, this separation can be achieved by adding the following terms to the action:

$$S_{L,R} = C \int d^2u^\delta \sqrt{-g}\, e^{\alpha\beta} \left(\pm i\bar{\Psi} t_\alpha \cdot \gamma \partial_\beta \Psi \right) \tag{4}$$

where the antisymmetric tensor $e^{\alpha\beta} = \epsilon^{\alpha\beta}/\sqrt{-g}$, with $\epsilon^{01} = -\epsilon^{10} = 1$. In fact, they should be labelled as independent distinct fields. For scalar fields, this can be achieved by demanding each bosonic field to have the form:

$$\Phi^I_{L,R}\,(u^0, u^1) = \Phi^I\,(u^0 \pm u^1) \tag{5}$$

Alternatively, the same effect can be achieved by introducing a Lagrange multiplier term, as suggested by Siegel{3,10}:

$$S_\lambda = -C/2 \int d^2u^\delta \sqrt{-g} \sum \lambda^{I\alpha\beta} \partial_\alpha \Phi^I \partial_\beta \Phi^I \tag{6}$$

where the Lagrange multiplier $\lambda^{I\alpha\beta}$ is a gauge degree of freedom; λ^I is defined to have only one independent component: it is real, symmetric and satisfies the relations : $\det(\lambda^I) = 0$ and $g_{\alpha\beta}\, \lambda^{I\alpha\beta} = 0$.

Since interaction terms are not considered in this simple picture, any other terms such as four-fermion interaction terms are explicitly excluded. Therefore, the above formulation completes our construction of the string actions. At this stage, any number of fields, including gauge fields{7}, can be put on the string. However, to complete the construction of specific models, we must further define the properties of the fermion fields.

Let us now consider the Green-Schwarz (G-S) superstring in ten dimensions. The action is given by (with two fermions, A=1,2):

$$S = S_1 + S_F + C\int d^2u^\delta \sqrt{-g}\, e^{\alpha\beta} t_{\alpha\mu}\left(-i\bar{\Psi}_1 \gamma^\mu \partial_\beta \Psi_1 + i\bar{\Psi}_2 \gamma^\mu \partial_\beta \Psi_2\right)$$

$$= S_1 + 2iC\int d^2u^\delta \sqrt{-g}\, t_{\alpha\mu}\left(P_-^{\alpha\beta}\bar{\Psi}_1 \gamma^\mu \partial_\beta \Psi_1 + P_+^{\alpha\beta}\bar{\Psi}_2 \gamma^\mu \partial_\beta \Psi_2\right) \quad (7)$$

where $2P_\pm^{\alpha\beta} = g^{\alpha\beta} \pm e^{\alpha\beta}$ and $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$. Here S_1 and S_F are given in Eq.(1) and (2). Here the fermions are Majorana-Weyl (M-W) fields:

$$2h\Psi_A = (1 \pm \gamma_{11})\Psi_A = 0 \quad (8)$$

Ψ_1 and Ψ_2 can have the same or opposite handedness. For Type 1 superstring, they must have the same handedness. Since the fermions are spinors in 10-dimensional space-time while their equations of motion involve only tangential components, we can further specify the fermion fields. It is obvious that the constraints must be satisfied in such a way that the terms in the action (7) do not vanish. Let us start with the supersymmetry requirement. Under global supersymmetry transformation:

$$\delta X_\mu = i\bar{\epsilon}_A \gamma^\mu \Psi_A \quad (9a)$$

$$\delta \Psi_A = \epsilon_A \quad (9b)$$

we find that the action (7) is supersymmetric if $\delta S=0$; this follows if

$$\{\bar{\Psi}_B\gamma_\mu\partial_\beta\Psi_B\}\,\gamma^\mu\partial_\alpha\Psi_A = \gamma\cdot L_{B\beta}\partial_\alpha\Psi_A = 0 \tag{10a}$$

where B is not summed. Multiplying this by $\bar{\Psi}_A$, we see that the four vectors $L_{B\alpha\mu}$ (where B=1,2 and α=0,1) are light-cone vectors in Minkowski space; in fact, they are parallel to each other.

Although the fermion fields do not interact among themselves, they are moving on the string which is moving, rotating and vibrating. The distribution and motion of the fields affect the shape of the string, which in turn defines the path of the fields' motion. In general, this results in a coupled system which is difficult to solve. As we shall see in a moment, the equations of motion become uncoupled and hence can be easily solved if the following covariant terms also exactly vanish:

$$\gamma\cdot L_{B\alpha}\Psi_A = \gamma\cdot\{\partial_\beta L_{B\alpha}\}\Psi_A = 0 \tag{10b}$$

We observe that the covariant constraint (10a) automatically follows from the constraints (10b) and the derivative of the first term. We now further extend the above constraints by demanding the fermion fields to satisfy the following set of constraints:

$$\gamma\cdot\{\partial_{\beta_1}\partial_{\beta_2}\cdots\partial_{\beta m}L_{B\alpha}\}\Psi_A = 0 \qquad m = 0,1,2,3,\cdots \tag{11}$$

Eq.(10) is simply the m=0,1 terms of the whole set Eq.(11). Together with the

Majorana-Weyl property Eq.(8) and Eq.(11), the covariant action (7) gives the G-S superstrings. Appropriate boundary conditions for open (closed) strings must be included.

Using the Taylor expansion, it is straightforward to show that Eq.(11) allows the choice of a fixed (i.e., u^{α} independent) $L_{B\alpha\mu}$. This means that the following constraint can be derived from Eq.(11):

$$r\cdot\gamma\Psi_A = r_\mu\gamma^\mu\Psi_A = 0 \qquad (12)$$

where r_μ is a constant (i.e. u^{α} independent) light-cone vector, $r^2 = 0$. To see that the reverse is also true, i.e., that Eq.(12) implies Eq.(11), it is useful to write the Minkowski metric as $\eta_{\mu\nu} = n^i{}_\mu n^i{}_\nu - r_\mu q_\nu - q_\mu r_\nu$, where q_μ is the other light-cone vector and the vector $n^i{}_\mu$, $i=1,2,\ldots,8$, are the eight transverse orthonormal vectors; they are defined so that $n\cdot q = n\cdot r = q^2 = 0$. We also use the relation $r\cdot\gamma(q\cdot\gamma)+q\cdot\gamma(r\cdot\gamma) = -2\, r\cdot q = 2$. This means the set of constraint Eq.(11) and the constraint Eq.(12) are equivalent, at least classically. Therefore, the set of covariant (cubic in Ψ) constraints Eq.(11) may be replaced by the (linear in Ψ) constraint Eq.(12) at the price of explicit covariance. This explains why light-cone quantization of the superstring is so much easier than the covariant quantization.

We observe that the action (7) is obtained if all the four-fermi interaction terms in the G-S covariant action[5] are dropped. It is easy to see that the local supersymmetry present in the G-S covariant action[5] is absent here. Since $[\partial_1,\partial_2]\, X_\mu = 2i\bar{\epsilon}_1\gamma_\mu\epsilon_2$, it is clear that the action (7) has only ten-dimensional global supersymmetry. For the Heterotic String, we simply replace the left-moving Ψ_2 in the action (7) by a set of sixteen left-moving scalar fields Φ^I , I=1,2,...,16, as described above.

To be specific, let us consider the equations of motion for the superstring action (7); by varying $g^{\alpha\beta}$, we obtain the constraint equation :

$$g_{\alpha\beta}\left(-1/2\ \partial_\rho X^\mu \partial^\rho X_\mu + i\bar{\Psi}_A t_\rho \cdot \gamma \partial^\rho \Psi_A\right)$$
$$= -\ \partial_\alpha X^\mu \partial_\beta X_\mu + i\bar{\Psi}_A t_\alpha \cdot \gamma \partial_\beta \Psi_A + i\bar{\Psi}_A t_\beta \cdot \gamma \partial_\alpha \Psi_A \qquad (13)$$

Varying the action with respect to X_μ and Ψ_A, we get:

$$\partial_\alpha(\sqrt{-g}\, g^{\alpha\beta} \partial_\beta X_\mu) = 0 \qquad (14)$$

$$\sqrt{-g}\, P_-{}^{\alpha\beta}(\partial_\alpha X)\cdot\gamma\partial_\beta \Psi_1 = 0 \qquad (15)$$

$$\sqrt{-g}\, P_+{}^{\alpha\beta}(\partial_\alpha X)\cdot\gamma\partial_\beta \Psi_2 = 0 \qquad (16)$$

where the properties Eqs.(8) and (11) of the fermions are used to simplify the equations of motion of X_μ and Ψ. Fully covariant quantization is performed by treating Eqs.(11) and (13) as Gupta-Bleuler-like constraints. Alternatively, the system can be quantized with Eq.(12) by treating Eq.(13) as Gupta-Bleuler-like constraints. Since Eq.(12) is not covariant, this is different from both the covariant and the light-cone quantizations; we shall refer to this approach as the semi-covariant quantization.

Reparametrization invariance allows us to pick the orthonormal gauge $\sqrt{-g}\, g^{\alpha\beta} = (-1,1)$. In this gauge, we define $\dot{X} = \partial_\tau X$ and $X' = \partial_\sigma X$. The super-Poincare generators are obtained from the integrals (along the string) of the corresponding conserved currents :

$$P^\alpha{}_\mu = Ct^\alpha{}_\mu - 2iC(P_-{}^{\alpha\beta}\, \bar{\Psi}_1 \gamma_\mu \partial_\beta \Psi_1 + P_+{}^{\alpha\beta} \bar{\Psi}_2 \gamma_\mu \partial_\beta \Psi_2) \tag{17}$$

$$\begin{aligned} M^\alpha{}_{\mu\nu} = X_\mu P^\alpha{}_\nu - X_\nu P^\alpha{}_\mu &-(iC/2)\, P_+{}^{\alpha\beta} t_\beta{}^\rho \bar{\Psi}_1 \{\gamma_\rho \gamma_{\mu\nu} + \gamma_{\mu\nu}\gamma_\rho\} \Psi_1 \\ &-(iC/2)\, P_-{}^{\alpha\beta} t_\beta{}^\rho \bar{\Psi}_2 \{\gamma_\rho \gamma_{\mu\nu} + \gamma_{\mu\nu}\gamma_\rho\} \Psi_2 \end{aligned} \tag{18}$$

$$Q^\alpha{}_A = 2iCP_\pm{}^{\alpha\beta} t_\beta \cdot \gamma \Psi_A \tag{19}$$

where $2\gamma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]$. Besides the global supersymmetry, the action (7) is also invariant under the following local bosonic symmetry transformation{5}:

$$\delta_\lambda \Psi_1 = \sqrt{-g}\, P_-{}^{\alpha\beta} \lambda_\alpha \partial_\beta \Psi_1 \tag{20a}$$

$$\delta_\lambda \Psi_2 = \sqrt{-g}\, P_+{}^{\alpha\beta} \lambda_\alpha \partial_\beta \Psi_2 \tag{20b}$$

$$\delta_\lambda X_\mu = i\bar{\Psi}_A \gamma_\mu \delta_\lambda \Psi_A \tag{20c}$$

From the constraint equation (13) and the string equation (14), we see that, in general, the factors $(\dot{X} \pm X') \cdot \gamma$ and the corresponding factors $(\dot{X} \pm X')^2$ are not zero. Therefore the fermion equations of motion (15) and (16) can be simplified to become

$$\partial_\tau \Psi_1 + \partial_\sigma \Psi_1 = \partial_\tau \Psi_2 - \partial_\sigma \Psi_2 = 0 \tag{21}$$

When restricted to Eq.(21), the local bosonic symmetry given by Eq.(20) turns out not to have any additional consequences and hence can be disregarded in the solution and the quantization of the model.

The commutation relation of X_μ and the anti-commutation relation of Ψ can be obtained from the action (7) via the corresponding Poisson and Dirac brackets respectively. For the independent modes :

$$[\, X_\mu \, , P_{0\nu} \,] = [\, X_\mu(\tau,\sigma) \, , C\dot{X}_\nu(\tau,\sigma') \,] = i\delta(\sigma-\sigma')\eta_{\mu\nu} \tag{22}$$

$$2\{\, C(-\dot{X} \pm X')\cdot(\gamma\Psi_A)_a(\tau,\sigma) \, , \overline{\Psi}_{Ab}(\tau,\sigma') \,\} = \delta(\sigma-\sigma')\, \delta_{ab} \tag{23}$$

where the $\pm$ sign is for A=1,2 (A not summed). The subscript a, b run over the non-zero components of the fermion fields in a basis where γ_{11} and h are diagonal. Eq.(21) shows that $\Psi_1(\Psi_2)$ has only right-moving (left-moving) modes. This gives rise to the factor of 2 in Eq.(23), due to the difference between the Dirac and the Poisson brackets of Ψ. Note that Eq.(23) involves the string variables.

To be specific, let us consider the light-cone quantization of the open string with the constraint (12) and the appropriate open string boundary conditions[1] (i. e., N=1 supersymmetry and σ runs from 0 to π). Let us set $2\alpha' = 1$. The string variables X_μ can be easily solved and expanded into modes exactly as in the open bosonic string[1,11]. The remaining conformal symmetry[11] still allows us to choose the light-cone gauge, i.e. $\alpha^+_n = 0$. This means $X^+(\tau,\sigma) = x^+ + p^+\tau$. Then α^-_n are dependent variables determined by Eq.(13). The independent string modes have canonical commutators. We now make an explicit choice of r_μ so that $r\cdot\gamma=\gamma^+$; then Eq.(12) becomes $\gamma^+\Psi_A=0$. The fermion anti-commutator (23) can now be simplified by using γ^+ as a projection. Going back to a general Dirac matrix representation where γ_{11} is not diagonal, we must restore the projection h , so that the fermion anti-commutator (23) becomes

$$4p^+ \left\{ \Psi_{1a}(\tau,\sigma)\,,\ \overline{\Psi}_{1b}(\tau,\sigma') \right\} = \pi\delta(\sigma-\sigma')\,(\gamma^+ h)_{ab} \tag{24}$$

and similarly for Ψ_2 . Solving the fermion equations (21) with the open string boundary conditions gives (all integers n) :

$$\Psi^a{}_{2,1} = 1/(2\sqrt{2p^+}) \sum_{-\infty}^{+\infty} S^a{}_n\, e^{-in(\tau\pm\sigma)} \tag{25}$$

so that Eq.(24) gives

$$\{S_{ma}\,,\ \overline{S}_{nb}\} = (\gamma^+ h)_{ab}\, \delta_{m+n,0} \tag{26}$$

while the fermion modes S_{na} commute with the independent string modes.

We can now calculate the super-Poincaré generators given by Eq. (17), (18) and (19) in terms of the string and the fermion modes, where the N=1 supercharge $Q = \sqrt{2}\,(Q_1 + Q_2)$. A comparison shows that these super-Poincaré generators are precisely those of the G-S light-cone action formulation{1} so that the closure of the super-Poincaré algebra on the mass-shell immediately follows. It is intriguing to note that, although the supersymmetry transformation in the light-cone gauge of our action looks different from that of the G-S light-cone action formulation{1}, the results in the mode-expanded formalism and the spectrum agree. The light-cone quantization of the Type 2 Superstrings and the Heterotic String can be carried out in similar fashions.

In our covariant action formulation, both the free string picture and the 10-dimensional global supersymmetry are explicit. From the point of view of two-dimensional field theory, our formulation looks highly unusual; however, we believe it is the natural approach to formulate extended objects in real Minkowski space-time. Semi-covariant quantization, covariant quantization and BRS-invariant covariant quantization are under investigation and will be discussed elsewhere{8}.

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