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More on the Realization of New Inflation

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Abstract

We generalize a recent analysis by Albrecht and Brandenberger regarding the realization of inflation. In particular, we are concerned with general potentials for an SU(5) singlet scalar field (the inflaton) which may or may not possess non-renormalizable interactions.

The attractiveness of an inflationary[1] period in the early Universe has sparked the production of a great number of models[2] and recently some questions[3] as to whether inflation is cosmologically feasible. The basic idea behind the new inflationary scenario[4] is that if the potential describing the self-interactions of a scalar field ϕ, is sufficiently flat at say $\phi \approx 0$ and if the high temperature potential has a minimum at $\phi \approx 0$ then inflation will occur during the slow roll-over from $\phi \approx 0$ to the global minimum at $\phi = v$, at the same time producing density fluctuations $\delta\rho/\rho \sim 10^{-4}$.

In the original new inflationary models[4] of the Coleman-Weinberg type, the minimum at high temperature is at $\phi = 0$. In more general models, this is not guaranteed and must be imposed as an additional constraint.[5] Indeed in models of inflation in the context of minimal N=1 supergravity[6],[7] it was shown[8] that this constraint could not be satisfied using a single chiral superfield. However if one employs N=1 supergravity with non-minimal kinetic terms one can satisfy this constraint[9] and in particular a very simple model is found[10] in the context of SU(n,1) supergravity theories[11] which may naturally fall out of certain superstring theories.[12] In what follows we shall assume that this condition has been imposed and is satisfied by the scalar potential $V(\phi)$.

These same thermal effects which supply a high temperature minimum at $\phi = 0$ have been held suspect[3] in that at high temperatures, the field is not expected to be localized about that minimum but rather fluctuate out to distances $\phi \sim T$. Thus at high temperatures $T > T_c$, where T_c is the critical temperature for the phase transition, domains will form with $\phi = v$ and in those regions where $\phi \approx 0$, kinetic energy terms will dominate the Lagrangian over the potential energy. In a response to these questions, Albrecht and Brandenberger,[13] have determined conditions under which the standard inflationary scenario follows. In particular they looked at a Coleman-Weinberg potential and a simple ϕ^4 potential. In this letter, we will generalize their results for an arbitrary scalar potential of the form

$$V(\phi) = \sum_n \lambda_n \phi^n \tag{1}$$

where the non-renormalizable terms ($n>4$) are expected to be cut off by inverse powers of the Planck mass $M_p = 1.2 \times 10^{19}$ GeV, (e.g. when they arise in supergravity theories) so that below M_p, we can treat this as an effective theory. Our results confirm their conclusion that primordial inflation[14] is preferred.

In ref. 13, Albrecht and Brandenberger have shown that if one could neglect the scalar self interactions, the redshifting of the spatial gradient and kinetic terms allow the Universe to enter an inflationary period. In fact it is only the interaction terms which present difficulties of the type in ref. 3. Thus if one can show that interactions remain negligible for a timescale sufficient for inflation to solve the cosmological problems, the problems of ref. 3 are not realized.

Wherever possible, we will use the notation of ref. 13. We start by considering a pure scalar theory whose self interactions are described by the potential given by eq. (1). For $V(\phi)$ to be suitable for inflation we must have a flat potential so that the energy density becomes dominated by V(0). During inflation, the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \tag{2}$$

will have the de Sitter-like behavior $a(t) \sim a_o e^{Ht}$ with $H^2 = 8\pi V(0)/3M_p^2$. We can rewrite eq. 2 in terms of a conformal time η

$$ds^2 = -a^2(\eta)(d\eta^2 - d\vec{x}^2) \tag{3}$$

with $a^2(\eta) = (H\eta)^{-2}$ and $\eta = -e^{-Ht}/H$. We can expand[15] ϕ in terms of its normal modes

$$\phi = \frac{1}{(2\pi)^{3/2}} \int d^3k\; q_k(\eta) e^{ik\cdot x} + \text{h.c.} \tag{4}$$

The equation of motion for the q_k's is

$$q''_k - 2\eta^{-1} q'_k + k^2 q_k = I \tag{5}$$

and

$$I = \sum_n n\lambda_n \frac{1}{(2\pi)^{3(n-1)/2}} \int d^3k_1 \ldots d^3k_{n-2}\, q_{k_1} \cdot\cdot q_{k_{n-2}} q_{k-k_1-\ldots-k_{n-2}} \tag{6}$$

where the prime refers to a derivative with respect to η and I is due to the interactions derived from $\partial V/\partial\phi$.

If we for the moment neglect interactions (I=0) then we can solve[15] eq. 5 for $q_k(\eta)$,

$$q_k(\eta) = d_1\eta^{3/2}H_\nu^{(1)}(k\eta) + d_2\eta^{3/2}\, H_\nu^{(2)}(k\eta) \tag{7}$$

where $H_\nu^{(1),(2)}$ are the Hankel functions of the first and second kinds of order ν. For a scalar field which is not coupled to curvature, the index $\nu = 3/2$ in the massless limit (inflationary cosmology requires that $m \ll H$). We can write the solutions of q_k as

$$q_k^{(1)} = c_1(k^{-3/2}\sin(k\eta) - k^{-1/2}\eta\,\cos(k\eta)) \tag{8a}$$

$$q_k^{(2)} = -c_2\,(k^{-3/2}\cos(k\eta) + k^{-1/2}\eta\,\sin(k\eta)) \tag{8b}$$

and c_1, c_2 are typically of order H.

If the interactions are small we can treat them as a perturbation and calculate the perturbed solution following ref. 13

$$q_k^I(\eta) = -q_k^{(2)}(\eta)\int_{-1/H}^{\eta} d\tau I(\tau)\varepsilon(\tau)q_k^{(1)}(\tau) + \qquad (1\leftrightarrow 2) \tag{9}$$

where

$$\varepsilon(\eta) = (q_k^{\prime(1)}q_k^{(2)} - q_k^{(1)}q_k^{\prime(2)})^{-1} = (c_1c_2\eta^2)^{-1} \tag{10}$$

(At late times, at the end of inflation, when $k\eta<<1$ only the first term in eq. (9) contributes.) We begin the evaluation of the integral in equation (9) by finding an upper bound for the interaction term I from eq. (6). To estimate I, we take for upper and lower bounds in the integral $H<k<T_c$, i.e. k is limited by the horizon size and the temperature at the onset of inflation. For k in this range, $|q_k| \leq k^{-1/2}$ (see eq. (8)). Substituting $q_k = k^{-1/2}$ in I and using the random phase approximation we find that I must satisfy the following inequality

$$I \lesssim \sum_n \frac{\sqrt{n}}{(2\pi)^n} |\lambda_n| \, T_c^{n-5/2} = I_{max} \tag{11}$$

The perturbed solution q_k^I is then bounded by

$$q_k^I \leq q_k^{(2)}(\eta) \, I_{max} \int_{-1/H}^{\eta} \varepsilon(\tau) q_k^{(1)}(\tau) \, d\tau \tag{12}$$

$$\approx I_{max}/T_c^2$$

The perturbed solution for q_k^I will remain valid so long as $q_k^I \lesssim q_k^{(1),(2)}$ for a typical value of $q_k^{(2)} \sim H/T_c^{3/2}$ at late times. Thus we have our condition on the source term and hence the couplings λ_n,

$$\sum_n \frac{\sqrt{n}|\lambda_n|}{(2\pi)^n} \, T_c^{\,n-4} \lesssim \left(\frac{H}{T_c}\right) \tag{13}$$

This is the generalization of the result in ref. 13. Given a scalar potential V suitable for inflation, eq. (13) becomes the requirement that inflation actually takes place. If eq. (13) is not satisfied, the interaction terms become important too soon and the problems discussed in ref. 3 become serious.

It is now relatively straightforward to see that eq. (13) implies the preference for primordial inflation in connection with the $n \geq 4$ terms. Recall that primordial inflation is characterized by a large symmetry breaking expectation value v compared with the critical temperature $T_c \sim V(0)^{1/4} \sim (HM_p)^{1/2}$ $(v \gg T_c)$ whereas Coleman-Weinberg type inflation has $v \sim T_c$. For very flat potentials $(\lambda_1 = \lambda_2 = 0)$, the high temperature potential takes the form

$$V_T = \lambda_o + \lambda_3 \phi^3 + \frac{1}{12} \frac{\partial^2 V}{\partial \phi^2} T^2 + \ldots \tag{14}$$

Hence our bound (eq. (13)) then imposes the following constraints on the couplings λ_i. Starting with the cubic term we see that

$$\lambda_3 \lesssim H \tag{15}$$

The quartic coupling is bounded by

$$\lambda_4 \lesssim (H/T_c) \tag{16}$$

This result differs from that in ref. 13 in the power of H/T. Finally, for $n \geq 4$, if we suppose that these couplings are cut off by powers of v, the global minimum for ϕ, $\langle\phi\rangle = v$, so that $\lambda_n = \tilde{\lambda}_n v^{4-n}$, eq. (13) becomes

$$\frac{\sqrt{n}\tilde{\lambda}_n}{(2\pi)^n}\left(\frac{T_c}{v}\right)^{n-4} \lesssim \frac{H}{T_c} \tag{18}$$

Clearly for $v \gg T_c$, the constraints on $\tilde{\lambda}_n$ become much less severe.

In conclusion, we have solved for the criteria that inflation occurs for general scalar potentials whose T=0 form satisfies the ordinary inflationary constraints.[16)]

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