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THE LIMITING TEMPERATURE UNIVERSE AND SUPERSTRING

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ABSTRACT

Some cosmological implications of superstring theories are discussed. In particular, the possible role of the limiting temperature in the early universe is examined.

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Recent developments in superstring theory by Green and Schwarz[1] indicate that superstring is a prime candidate for the unified theory of all forces in nature. If the superstring is indeed the model of unified forces, it must provide solutions to (or ways to bypass) all the cosmological problems that we usually face. In particular, questions concerning the cosmological constant, the horizon and the flatness problems should be answered by the superstring theory. The latter problems are related to the early evolution of the universe. Here the discussion will be limited to the implications of the limiting temperature feature present in string theories on these cosmological problems in the early universe. It is well known that any string model has a spectrum whose degeneracy grows exponentially as a function of the mass[2]. Hagedorn first observed that such a spectrum implies a limiting temperature[3]. Consequences in the early universe due to such a limiting temperature coming from the dual string model was studied by Huang and Weinberg[4]. Here I shall investigate the issues in superstring models.

Let us write the number of species of particles with mass between m and m+dm as N(m)dm. For large m,

$$N(m)\, dm \to A\, m^{-B}\, e^{\beta_0 m}\, dm \qquad (1)$$

The thermodynamical partition functions would converge only if the temperature $T=1/\beta$ is below the value $T_0=1/\beta_0$. Therefore the limiting temperature T_0 is the maximum temperature for a system in thermal equilibrium. The Green and Schwarz (G-S) SO(32) superstring [1] includes both the open and the closed strings. Let p(n) be the level degeneracy, i.e. the number of states, of the open string at the level n, where $n= \alpha' m^2$ (

m=mass). Then $p(n) = 16\, C_n\, q_D(n)$, where $q_D(n)$ is given by

$$\sum_n q_D(n)\, x^n = \prod_k (1 + x^k)^D / (1 - x^k)^D$$

so that, for asymptotically large n,[5]

$$p(n) \to 16\, C_n\, (D/64)^{(D+1)/4}\, n^{-(D+3)/4}\, e^{\pi\sqrt{(Dn)}} \qquad (2)$$

Light-cone quantization shows that D is the number of transverse directions, i.e. D=8, since the superstring is in 10-dimensions. Here C_n=496 for even integer n and C_n=528 for odd n. Because of supersymmetry, p(n) includes equal numbers of bosonic and fermionic states for each n. In addition to the choice $c=k=\hbar=1$, it is convenient to choose the mass scale $M = 1/\sqrt{\alpha'}$ such that $\alpha' = 1$. Using eq.(1) and $q_8(n)$, the superstring has

$$\beta_0 = 1/T_0 = \pi(2\sqrt{2}), \qquad B = 9/2 \qquad (3)$$

We observe that the coefficient B is insensitive to the fermionic content of the string.The level degeneracy of the closed strings of the G-S superstring is given by $p(n) = 128\, q_8(n)^2$. Therefore it has the same limiting temperature with B=10. Also the coefficient A for the closed string is rather small in comparison to the open string. Hence, to a reasonable approximation, the closed string states in the G-S superstring can be ignored in the calculations of thermodynamical quantities.

On the other hand, the phenomenologically interesting $E_8 \times E_8$ Heterotic String [6] has only closed string states. Its level degeneracy is given as a product of the level degeneracies of the left and the right moving states.

$$p_H(n) = p_L(n+1)\, p_R(n)$$

where $p_R(n) = 16\, q_8(n)$ and $p_L(n)$ is given by[6,7]

$$p_L(n) = 480 \sum \sigma_7(n-m)\, d_{24}(m)$$

where $\sigma_7(n)$ is the sum of the 7th power of the divisors of n, with $\sigma_7(0)$ defined to be 1/480 and $d_D(n)$ is given by $\sum d_D(n)\, x^n = \Pi(1-x^k)^{-D}$. From the asymptotic form of the level degeneracy, its limiting temperature[6] and the corresponding B are

$$\beta_0 = \pi(2+\sqrt{2}), \qquad B=10 \qquad (4)$$

We note that, without compactification, the counting gives B=18. The compactification on a self-dual lattice decreases B to 10, while the limiting temperature is unchanged. The limiting temperature for both superstring models are around a tenth of M. The above formulae are evaluated for the free strings. To calculate the energy density and the pressure in thermal equilibrium, string interactions must be introduced to allow the string states to come to thermal equilibrium. It turns out that the above formulae are valid in the presence of tree level string interactions, since in this approximation, the string states have zero widths [8]. We shall discuss string interactions beyond the tree level later.

The compactification (from 10 dimensions to K+1=4 dimensions) scale is expected to be close to M . In this case, the limiting temperature will be unaffected while the coefficient B may be somewhat decreased or remains unchanged. On the other hand, the momentum integration in the evaluation of interesting quantities such as the energy density and the pressure of the massive states are effectively that of the final (K+1) dimensional space-time. The reason for this is quite simple. Due to the extreme rapid increase in the number of massive states with energy, most of the available energies are used to form new massive states instead of converting into kinetic energies of the less massive ones. Hence the momenta of the massive states are very small. Since the momenta in the compactified dimensions are dicrete, the contributions of the zero momentum modes in those dimensions dominate. Therefore, depending on the scale (or scales) of compactification, the effective space-time dimension may be 10 or 4 or any dimension in between. For effective (K+1) space-time dimensions, the pressure P is given by

$$P(K,\beta) = P_0(K,\beta) + P_1(K,\beta)$$

$$P_0(K,\beta) = 8F_K C_0\, \Gamma(K)\, \zeta(K+1)\beta^{-K-1} 2^{1-K} / \Gamma(K/2) \tag{5}$$

$$P_1(K,\beta) = 2\int^{\infty} dm\, N(m)\, (m/2\pi\beta)^{(K+1)/2}\, K_{(K+1)/2}(\beta m)$$

where it is convenient to separate the contribution of the massless particles to the pressure P, namely P_0 , from that of the massive particles, namely P_1. Here $\zeta(K+1)$ is the Riemann Zeta function, $\Gamma(K/2)$ the Gamma function and $K_{(K+1)/2}$ the modified Bessel function. F_K includes the bose-fermionic effect, $F_K = 1 + (1-2^{-K})$. For the massive states, the bose-fermionic difference is negligible. The lower integration limit is m=1 for open strings and m=2 for closed strings[9]. The energy density ρ is given by

$$\rho(K,\beta)=KP(K,\beta)+\rho_2(K,\beta) \tag{6}$$

$$\rho_2(K,\beta)=\pi^{-1}\int^{\infty} dm\, N(m)\, m^2 (m/2\pi\beta)^{(K-1)/2} K_{(K-1)/2}(\beta m)$$

As the temperature approaches the limiting temperature from below, the behaviors of P and ρ depend on both the value of B_K and the effective dimension K. For $T \to T_0$, we find

$$\begin{aligned} \rho &\propto (T_0-T)^{B_K-K/2-2} && \text{for } B_K < K/2+2 \\ &\propto |\ln(T_0-T)| && B_K = K/2+2 \\ &\to \text{finite} && B_K > K/2+2 \end{aligned} \tag{7}$$

where the pressure has similar behaviors if the value B in eq.(7) is replaced by B+1. The above property is similar to that first observed in Ref.[4]. For the Heterotic string where B=10, both ρ and P approach finite values as T approaches the limiting temperature in any dimension. For the G-S string, the critical space-time dimension is $K+1 = 2B_K-3 \leqslant 6$. If, for some as yet unknown reasons, the energy density must remain finite at $T=T_0$, then dimensional compactification is necessary to reduce the 10-dimensional space-time to 5 or less dimensions, (depending on the value of $B_{K,}$ which in turn depends on the compactification). This may suggest a dynamical reason for compactification.

Recent discussions[10] on the compactification of superstrings indicate that, among other properties: (1) the cosmological constant is zero or close to zero at that scale; (2) the Grand unification (GUT) symmetry breaking is probably due to expectation values of Wilson lines in the non-simply-connected manifold instead of the usual Higgs mechanism; and (3) topological objects such as cosmic strings, domain walls, monopoles etc. are formed at the GUT scale. Properties (1) and (2) seem to suggest that the necessary ingredients for the standard inflation [11,12] (which is introduced to solve the flatness problem, the horizon problem, and problems related to densities of heavy topological objects) are missing. If so , it is not clear how these cosmological problems are solved in the superstring theory. One expects tremendous entropy generation at the GUT scale to suppress the densities of the heavy objects while not affecting the baryosynthesis that should take place at a temperature close to but slightly lower than the GUT scale[11].

There are a number of possibilities, depending on the details of the dynamics. We shall discuss some of the possobilities here. Let us first consider the case of the G-S string with compactification to our 4-dimensional spase-time or the Heterotic string. Suppose the universe is in thermal equilibrium precisely at the limiting temperature at certain time. The finite energy desity ρ as well as the pressure P are at their maximum values, ρ_{max} and P_{max} , respectively. Given the fomulae for ρ and P, we can use the equation of state and the Einstein equations to study the evolution of the universe. As T begins to drop below T_0 , ρ decreases until it reaches three times the pressure while P_1 hardly changes (actually P_1 is quite negligible any way) . The entropy S, given as $S= (P + \rho) R^3 \beta$, (where R is the cosmic scale factor) cannot be conserved during the early part of this expansion period of the universe. This is somewhat like isobaric expansion where entropy is being generated. However, a straightforward calculation shows that the amount of entropy generated is rather modest. far from the amount necessary to solve the cosmological problems. The same picture emerges if the universe starts at a lower temperature.

Alternatively, the universe may start at a density and /or pressure above their maximum values allowed by thermal equilibrium. In this case, the universe will be in a non-equilibrium state until the expansion brings ρ and P down so that it can settle into thermal equilibrium at $T<T_0$. Unfortunately, such out-of -equilibrium expansion is difficult to study. If somehow the universe can remain in thermal equilibrium at T above T_0, we still do not see enough inflation taking place in a way consistent with observations.

Suppose the compactification has not yet taken place at T_0. One obvious possibility of large entropy generation is via dimensional compactification , similar to that proposed for Kaluza-Klein compactification[13]. Also the expectation values of Wilson lines on the non-simply connected manifold may help to stabilze the radii of the extra dimensions at the compactification scale. Here, the decay of an exponentially large number of massive states may be of some help.

Another possibility is that the limiting temperature is actually not an ultimate temperature, but rather it indicates the presence of a phase transition. It is well known that, in certain types of gauge theories, such as large-N gauge theory[14], there are two phases: below the critical temperature T_C , the theory is in the confining phase where the spectrum is essentially that of a string; while above T_C , the theory is in the deconfining phase. In the deconfining phase, the spectrum does not have the type of degeneracy as given in eq.(2)-(4) so that temperatures above the limiting temperature are well defined. It is natural to entertain the possibility that superstring theories as formulated in Ref[1,6] correspond to the theories in the confining phase. Above the critical temperature T_C, where $T_C < T_0$, the superstring theory enters the deconfining phase. It is likely that the limiting temperature is precisely the critical temperature. As T approaches T_C from below , string-interactions beyond the tree approximation may become important. This will modify the behavior of quantities such as ρ and P near T_C.

The existence or the absence of such a phase transition may be revealed by a numerical analysis. If it exists, the order of such a phase transition may also be determined. Questions concerning the massless states become crucial. Hopefully the phase transition does not affect the massless graviton (and the massless gauge fields) so that the gravitational interactions remain the same above the critical temperature. This assumption is reasonable since confined states are in general massive, with the possible exceptions of Goldstone-pion-like states and Goldstino-like states. If the phase transition is first order or if it is a slow one (as in the new inflatioary universe) it may provide the supercooling necessary for inflation.

Although the inflation necessary to solve the cosmological problems may be related to the limiting temperature feature, no obvious scenario is founded. A more likely place to obtain inflation in the superstring theory may come from the evolution of the universe around the time of the gluino condensation[15], which is necessary for both supersymmetry breaking and may be for the zero cosmological constant constraint. This inflationary scenario is quite different from the new inflationary universe. It will be discussed elsewhere[16].

To conclude, it seems that the superstring theories have properties quite different from conventional theories so that the standard inflation may not be applicable. However, it does offer a number of new possibilities for tackling the cosmological problems.

After the completion of this work, the work by M.J. Bowick and L.C.R. Wijewardhana (Yale Preprint 85-04) has come to my attention. Their work is similar to mine, although they have not considered the cosmological implications. I thank the authors for discussions.

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