ANL-HEP-PR-85-21
FERMILAB-Pub-85/47-T

ULTRALIGHT DIRAC NEUTRINOS IN A LEFT-RIGHT SYMMETRIC MODEL CONTAINING MIRROR FERMIONS

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Abstract

We propose a low energy, left-right symmetric gauge model incorporating mirror fermions that naturally produces ultralight Dirac neutrinos (< 100 eV). The numbers of standard and mirror generations must be equal to prevent neutrino masses in the cosmologically-disfavored range 100 eV < M_ν < 2 GeV. Our model is consistent with the experimental limits on rare processes.

*Research supported in part by Grant No. PHY-8304880 from the National Science Foundation and a Presidential Research Professor Award from NIU.

The masses of fermions represent convincing evidence that (apparently) unnatural hierarchies exist in nature. The neutrinos, being among the lightest fermions, embody this problem in its extremest form, but their exceptional lightness may be amenable to an exceptional and perhaps simple explanation. For example there might be no righthanded neutral fermions, so the neutrinos are strictly massless. However, it is plausible, on the basis of unified theories, that neutrinos do have small masses and, in fact, several experiments[1] indicate a finite electron neutrino mass. If so, either it is a Dirac mass or some cancellation mechanism[2] is at work, because the nonobservation of neutrinoless double beta decay[3] would otherwise set strong limits on the neutrino's Majorana mass. One might also expect that neutrinos are Dirac particles by analogy with the other fermions. This is the point of view we shall take; the problem is then to understand why they are distinguished in having such small masses.

Traditional accounts[4] of small neutrino masses have the disadvantage that they invoke physics at an ultra high energy scale ($> 10^9$ GeV). They assume that standard and ultramassive neutrinos are intimately connected, transforming under the same global symmetry, and that there are no intermediate energy corrections to the neutrino masses. Models[5] that predict ultralight Dirac masses introduce additional ultraheavy gauge-singlet fermions transforming under the global symmetry. Furthermore all such mechanisms have failed for Dirac neutrinos in the case of left-right symmetric theories[6,F1]—theories that are attractive in their own right, and particularly so if the neutrinos are Dirac particles.

We describe here a left-right model incorporating mirror fermions that does produce naturally light Dirac neutrinos. Our model is a low energy one, valid for $E \leq 1$ TeV, and avoids awkward assumptions about higher energy physics. It succeeds through an unusual skewed form of the neutrino mass matrix, which arises upon the imposition of a Z_6 discrete symmetry. Other important features of our mechanism, which we motivate below, are conventionally assumed in left-right theories so as to accord with phenomenology. We will consider here just the lepton sector--our innovations do not affect the quark sector. Our model in the lepton sector is consistent with experimental constraints[7] on rare processes involving flavor-changing neutral currents, unlike many left-right models. The rho parameter is automatically unity at the tree level, and small $W_L - W_R$ mixing is built into our model. The ultralight righthanded neutrinos do not violate the cosmological bound[8] on the number of neutrinos since these extra degrees of freedom do not reach thermal equilibrium during nucleosynthesis in the absence of Majorana contributions.[9]

The mirror fermions introduced in our model are fermions with the same quantum numbers but opposite chirality to the normal fermions. A number of attractive theoretical ideas indicate their existence.[10,11,12,13] Also the mirroring of all fermions produces automatic anomaly cancellation in the quark and lepton sectors separately and ensures baryon number conservation in the low energy theory, in contrast to the standard model. However, some composite models require that the number of mirror and standard generations differ in order to provide anomaly matching at the composite and preon levels.[14] We find the mass spectrum of neutrinos generated in our model favors an equal number of mirror and standard generations in order to circumvent the cosmological argument[15] against neutrino masses in the 100 eV - 2 GeV range.

Recent work has shown how to avoid a former difficulty with mirror fermions. Senjanovic, Wilczek and Zee[12] have demonstrated that a discrete symmetry, e.g. a Z_6 symmetry, can persist down to low energies as a relic of a GUT symmetry and protect the mirror and standard fermions from condensing together and developing a large mass, with no domain wall problem.[16]

We now describe our model, which is a left-right symmetric gauge theory containing mirror lepton families. The gauge symmetry is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.[17] The electromagnetic charge is $T_{3L} + T_{3R} + 1/2Y_{B-L}$. We write down the lepton fields as lefthanded two-component Weyl spinors. They form doublets under the gauge groups according to

$$L_i = \begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}_L \sim (2,1;-1) \qquad L_i^c = \begin{pmatrix} \ell_i^c \\ -\nu_i^c \end{pmatrix}_L \sim (1,2;+1)$$
$$M_i = \begin{pmatrix} E_i^+ \\ N_i \end{pmatrix}_L \sim (2,1;1) \qquad M_i^c = \begin{pmatrix} N_i^c \\ -E_i \end{pmatrix}_L \sim (1,2;-1) \; . \tag{1}$$

Here L_i and M_i are the normal and mirror lepton fields, respectively, where the index i runs over the number of generations of each type, which at this stage we allow to be unequal. Different generations will in general have different quantum numbers under the Z_6 symmetry.

In order to break the symmetry and give masses to the leptons we introduce the following Higgs representations

$$\chi_L = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix} \sim (2,1;1) \qquad \chi_R = \begin{pmatrix} \bar{\chi}^+ \\ \bar{\chi}^0 \end{pmatrix} \sim (1,2;1)$$
$$\phi_i = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & \phi'^0 \end{pmatrix} \sim (2,2;0) \qquad \Phi_i = \begin{pmatrix} \Phi^+ & \Phi^{++} \\ \Phi^0 & \Phi'^+ \end{pmatrix} \sim (2,2:2) \; . \tag{2}$$

The χ_R field is needed to break the $SU(2)_R$ symmetry at a scale larger than the electroweak scale, so as to minimize righthanded weak interactions. This is accomplished via the assumption that $\langle \chi_R \rangle \gg \langle \chi_L \rangle$ as is standard in left-right models.[18] The ϕ and Φ fields, transforming as doublets under both $SU(2)_{L,R}$, generate Dirac mass terms for the leptons by mixing $SU(2)_L$ fields with $SU(2)_R$ fields. No Majorana contributions occur in our model, even when radiative corrections are taken into account.

Although our Higgs sector is more complex than in the standard model, this is not necessarily a disadvantage. The possibility exists that the symmetry breaking is dynamical in origin. It could well be a complex process, which perhaps we can simulate using several Higgs fields in low representations of the gauge group. A positive feature of our Higgs sector is that all Higgs field are doublets or singlets under $SU(2)_L$, so that the ρ-parameter is unity at the tree level.

With these Higgs fields the most general Yukawa Lagrangian one can then write down (in a condensed notation) is

$$L_Y = g_{LL}\, L\phi L^c + \tilde{g}_{LL}\, L\tilde{\Phi}L^c + g_{MM}\, M\phi M^c + \tilde{g}_{MM}\, M\tilde{\phi}M^c + g_{LM}\, L\Phi M^c + \tilde{g}_{ML} M\tilde{\Phi}L^c + \text{h.c.} \quad (3)$$

where, for example,

$$g_{LL}L\Phi L^c \equiv \sum_{ijk} g_{LL}^{ij;k}\, L_i^T\, (-i\sigma_2)\, \phi_k L_j^c$$

and the several fields of each type are summed over. We have employed the notation $(\tilde{\phi},\tilde{\Phi}) \equiv \sigma_2\, (\phi^*,\Phi^*)\sigma_2$. $\tilde{\phi}$ transforms under $SU(2)_L \times SU(2)_R$ as $\tilde{\phi} \to U_L\tilde{\phi}U_R^+$, just as ϕ does. As mentioned above, we impose a Z_6 symmetry (to be discussed below) that restricts the Yukawa Lagrangian to the skewed form:

$$L_Y = g_{LL} L\phi L^c + \tilde{g}_{MM}\, M\tilde{\phi}M^c + \tilde{g}_{ML} M\, \tilde{\phi}L^c + \text{h.c.} \tag{4}$$

Note that we have eliminated the g_{LM} couplings, and that the terms involving ϕ and $\tilde{\phi}$ are now segregated. This segregation is, in any case, necessary in most left-right models in order to avoid flavor-changing neutral currents (FCNC) due to Higgs exchange.[19]

After symmetry breaking we obtain the lepton mass matrix

$$\mathcal{M}_{\pm,0} = \begin{pmatrix} 0 & M \\ M^T & 0 \end{pmatrix}_{\pm,0} \tag{5}$$

in the basis $[(M,L), (M^c,L^c)]$. The diagonal submatrices are null because there are no Majorana mass contributions[20] in our model. The off-diagonal submatrix in the charged sector is

$$M_{\pm} = \begin{array}{cc} & \begin{array}{cc} E^c & \ell^c \end{array} \\ \begin{array}{c} E \\ \ell \end{array} & \begin{bmatrix} \tilde{g}_{MM}\langle\phi^{o*}\rangle & 0 \\ 0 & g_{LL}\langle\phi^{o}\rangle \end{bmatrix} \end{array} \tag{6a}$$

while the neutral lepton submatrix has off-diagonal terms and is skewed:

$$M_0 = \begin{array}{cc} & \begin{array}{cc} N^c & \nu^c \end{array} \\ \begin{array}{c} N \\ \nu \end{array} & \begin{bmatrix} \tilde{g}_{MM} \langle \phi'^{o*}\rangle & \tilde{g}_{ML} \langle \phi^{o*}\rangle \\ 0 & g_{LL} \langle \phi'^{o} \rangle \end{bmatrix} \end{array} \tag{6b}$$

Each entry in (6a) and (6b) represents a submatrix in the standard and mirror generations spaces. We neglect CP violation in the present work and take all vacuum expectation values (VEV) to be real. Note that the VEV $\langle \phi^o \rangle$ appears only in the charged matrix $M_{\pm}$ while $\langle \phi'^{o} \rangle$ appears only in the neutral matrix, as a result of the segregated form of the Yukawa Lagrangian in Eq. (4). Also, there is no standard/mirror mixing for the charged leptons unlike the situation for the neutral leptons. This allows the possibility that ultralight neutrino masses can be generated by standard/mirror mixing.

To compute the lepton masses, we calculate the eigenvalues of $M_{\pm}M_{\pm}^T$ and $M_0M_0^T$ which appear in

$$\mathcal{M}^2_{\pm,0} = \begin{bmatrix} MM^T & 0 \\ 0 & M^TM \end{bmatrix}_{\pm,0} \quad . \tag{7}$$

For the charged lepton case the result is straightforward, with the standard and mirror masses set by $g_{LL} \langle \phi^o \rangle$ and $\tilde{g}_{MM} \langle \phi^{o*} \rangle$, respectively. Experiment reveals that the former are in the range $.5 \times 10^{-3}$ – 1.8 GeV, while the charged mirrors must be heavier than 22 GeV to have avoided detection at PETRA.[21]

The results in the neutral sector are affected additionally by the standard/mirror mixing. We will assume that the scale of this mixing is the natural one, which is just the electroweak breaking scale of approximately 250 GeV. The block diagonal entries in the neutrino mass matrix are proportional to the same Yukawa couplings of the ϕ fields that fix the charged lepton masses; however, the neutrino and charged lepton matrices depend on distinct components of those fields. It turns out that the ratio of the VEV's of the different components, $\langle\phi'^{o}\rangle/\langle\phi^{o}\rangle$, can be naturally small, i.e. the ratio is proportional to couplings which when taken to zero increase the symmetry, as will be seen below. We will take $\langle\phi'^{o}\rangle/\langle\phi^{o}\rangle \sim 10^{-3}$. That $\langle \phi'^{o} \rangle \ll \langle \phi^{o} \rangle$ has usually been assumed without justification in left-right theories.[18] Since the mixing of W_L and W_R is proportional to $\langle \phi'^{o} \rangle \langle \phi^{o} \rangle$, the experimental constraint that it be small requires that at least one of these two VEV's is small. Thus small W_L-W_R mixing is automatic in our model.

Scaling the block diagonal entries by 10^{-3} relative to $M_\pm$, M_0 has entries of the following order of magnitude:

$$M_0 \equiv \begin{bmatrix} I & B \\ 0 & S \end{bmatrix} \sim \begin{bmatrix} > 22 \times 10^{-3} & 10^2 \\ 0 & 10^{-3} - 10^{-6} \end{bmatrix} \text{GeV} \quad . \tag{8}$$

Let N_m represent the number of mirror generations and N_s the number of standard generations of leptons. The characteristic equation associated with $M_0 M_0^T$ for $\lambda = m^2$ and $N = N_m + N_s$ is given by

$$\lambda^N + C_{N-1}\,\lambda^{N-1} + \ldots + C_0 = 0 \quad , \tag{9}$$

where $C_{N-1} = -\sum_{i,j} (M_{0ij})^2$, $C_{N-2} = \Sigma(\mathrm{Det}_{ij,k\ell}\ M_0)^2, \ldots, C_0 = (-1)^N\ (\mathrm{Det}\ M_0)^2$

Here C_{N-2} involves the sum of squares of determinants of all 2x2 submatrices of M, C_{N-3} the sum of squares of determinants of all 3x3 submatrices of M, etc. The eigenvalues are approximately given by

$$\lambda = |C_{N-1}|,\ |C_{N-2}/C_{N-1}|, \ldots, |C_0/C_1|. \tag{10}$$

Barring accidental cancellations, there are N_m eigenvalues on the order of $B^2 \sim (10^2\ \text{GeV})^2$, N_m eigenvalues on the order of $\frac{I^2 S^2}{B^2} \sim (10^{-7} - 10^{-10}\ \text{GeV})^2$, and $N-2N_m$ eigenvalues on the order of $S^2 \sim (10^{-3} - 10^{-6}\ \text{GeV})^2$.

Masses in the ultralight (1-100 eV) and massive (> 2 GeV) ranges are cosmologically acceptable, but the intermediate (100 eV - 2 GeV) mass range is cosmologically unacceptable unless very rapid decay modes exist[15]. In the absence of very rapid neutrino decay modes for the intermediate mass neutrino, our model favors an equal number of standard and mirror generations.

Let us now consider a concrete example of our model in which there are two standard and two mirror generations of leptons, i.e. $N_s = 2$, $N_m = 2$. The mass matrices of (6a) and (6b) assume the form

$$M_\pm = \begin{array}{cc} & \begin{array}{cccc} (E^c_\mu)_{-2} & (E^c_e)_2 & (\mu^c)_0 & (e^c)_1 \end{array} \\ \begin{array}{c} (E_\mu)_1 \\ (E_e)_2 \\ (\mu)_2 \\ (e)_0 \end{array} & \begin{bmatrix} \tilde{g}^{11}_{MM}\langle\phi^{o*}\rangle_1 & 0 & 0 & 0 \\ 0 & \tilde{g}^{22}_{MM}\langle\phi^{o*}\rangle_2 & 0 & 0 \\ 0 & 0 & g^{11}_{LL}\langle\phi^{o}\rangle_{-2} & 0 \\ 0 & 0 & 0 & g^{22}_{LL}\langle\phi^{o}\rangle_{-1} \end{bmatrix} \end{array} \tag{11a}$$

$$M_0 = \begin{array}{cc} & \begin{array}{cccc} (N^c_\mu)_{-2} & (N^c_e)_2 & (\nu^c_\mu)_0 & (\nu^c_e)_1 \end{array} \\ \begin{array}{c} (N_\mu)_1 \\ (N_e)_2 \\ (\nu_\mu)_2 \\ (\nu_e)_0 \end{array} & \begin{bmatrix} \tilde{g}^{11}_{MM}\langle\phi'^{o*}\rangle_1 & 0 & \tilde{g}^{11}_{ML}\langle\phi^{o*}\rangle_{-1} & 0 \\ 0 & \tilde{g}^{22}_{MM}\langle\phi'^{o*}\rangle_2 & 0 & \tilde{g}^{22}_{ML}\langle\phi^{o*}\rangle_3 \\ 0 & 0 & \tilde{g}^{11}_{LL}\langle\phi'^{o}\rangle_{-2} & 0 \\ 0 & 0 & 0 & g^{22}_{LL}\langle\phi'^{o}\rangle_{-1} \end{bmatrix} \end{array} \tag{11b}$$

We have explicitly introduced a discrete Z_6 symmetry under which every field f transforms like $f \to e^{(2\pi i n_f/6)} f$ with $-3 < n_f < 3$. The n_f corresponding to the fields and VEV's in Eq. (11) are indicated by subscripts. In this simple $N_s = N_m = 2$ case no mixing occurs among the standard generations nor among the mirror generations. The skewed form of the neutrino mass matrix is a direct consequence of the discrete symmetry imposed.

The charged lepton masses are just the diagonal entries in (11a). For the sake of illustration we set the masses of E_μ, E_e, μ, and e equal to 100, 50, 0.1, and 10^{-3} GeV ,respectively. Scaling the corresponding M_0 entries by 10^{-3}, and making an appropriate choice for the off-diagonal entries, we obtain:

$$M_0 = \begin{bmatrix} .1 & 0 & 200 & 0 \\ 0 & .05 & 0 & 50 \\ 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix} \text{GeV} \tag{12a}$$

yielding neutrino masses

$$\begin{array}{ll} m_1 \simeq 200 \text{ GeV} & m_3 \simeq 50 \text{ eV} \\ m_2 \simeq 50 \text{ GeV} & m_4 \simeq 1 \text{ eV} \end{array} \tag{12b}$$

On the other hand, if $N_s = 2$ but $N_m = 1$, so that the numbers of standard and mirror generations are unequal, the neutrino mass matrix is of the form

$$M_0 = \begin{bmatrix} .1 & 200 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix} \tag{13}$$

and generates a neutrino of mass 1 keV, which is cosmologically unfavored as noted before.

We discuss briefly the Higgs sector to clarify the origin of the small ratio $\langle\phi'^{o}\rangle/\langle\phi^{o}\rangle$. As indicated in (11), it includes two ϕ fields and two Φ fields

$$\phi_{-1,-2} \sim (2,2;0)_{-1,-2}$$

and (14)

$$\Phi_{1,3} \sim (2,2;2)_{1,3}$$

as well as the χ fields of Eq.(2) which are unimportant in the following discussion. By introducing a third Φ_{-1} field, we ensure that the most general Lagrangian consistent with the Z_6 symmetry does not respect any additional continuous global symmetry. (The Yukawa interactions are unchanged.) The terms in the Lagrangian violating what would otherwise be the U(1) symmetry

$$\phi_{-1} \to e^{i\theta}\phi_{-1},\ \phi_{-2} \to e^{-i\theta}\phi_{-2},\quad \Phi_i \to \Phi_i \tag{15}$$

are the mixing terms

$$\mathrm{Tr}\left(\Phi_3^\dagger\Phi_1\phi_1^\dagger\tilde{\phi}_1\right),\ \mathrm{Tr}\left(\Phi_3^\dagger\Phi_1\tilde{\phi}_{-2}^\dagger\phi_{-2}\right),\ \mathrm{Tr}\left(\Phi_3^\dagger\Phi_{-1}\tilde{\phi}_{-1}^\dagger\phi_{-1}\right),\ \text{etc.} \tag{16}$$

These are the terms involving both Φ and ϕ fields in a non-trivial way. Since setting the corresponding couplings to zero increases the symmetry from Z_6 to $Z_6 \times U(1)$, it is natural to take these couplings to be small. Furthermore, for a range of the parameters of the Higgs potential, the VEV's $\langle\phi'^{o}\rangle_{-1}$ and $\langle\phi'^{o}\rangle_{-2}$ would vanish in the absence of these terms. Upon their inclusion these VEV's become non-zero, in magnitude proportional to the above couplings:

$$\langle\phi'^{o}\rangle \sim g\,\frac{\langle\Phi\rangle^2}{\langle\phi^{o}\rangle} \sim g(250\ \text{GeV})\ , \tag{17}$$

assuming $\langle\phi^{o}\rangle \sim 250$ GeV. Thus $\langle\phi^{o\prime}\rangle \ll \langle\phi^{o}\rangle$ can be achieved naturally in the sense of 't Hooft.[22] The small explicit breaking of the U(1) due to the terms in Eq.(16) gives the associated would-be Goldstone boson a small mass. This scalar can be made invisible by slightly complicating the Higgs sector, for instance by introducing a second Higgs field transforming like χ_R.

Having demonstrated that we can produce ultralight neutrinos, we turn to the experimental constraints[7] on our model. We must estimate the mixing of the normal and mirror leptons so as to compute the effects of the latter on rare processes involving flavor-changing neutral currents. Consider the $N_s = N_m = 2$ mass matrix in (11b). The physical neutral lefthanded fields are given in terms of the electroweak fields approximately by

$$\begin{bmatrix} N_\mu \\ N_e \\ \nu_\mu \\ \nu_e \end{bmatrix}_{\text{phys}} = \begin{bmatrix} 1 & 0 & \varepsilon_1 & 0 \\ 0 & 1 & 0 & \varepsilon_2 \\ -\varepsilon_1 & 0 & 1 & 0 \\ 0 & -\varepsilon_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} , \tag{18}$$

where the mixings ε_i are extremely small, $\varepsilon_i = g^{ii}_{LL} < \phi'^{o} >/ (\tilde{g}^{ii}_{ML} <\phi^o>)$ $\lesssim \frac{m_\mu}{(250\ \text{GeV})} \frac{< \phi'^{o} >}{< \phi^o >} \sim 10^{-6}$. On the other hand, the mixing of the righthanded conjugate neutrinos with their mirrors is a factor of m_E/m_ℓ greater, i.e. $O(10^{-3})$. But since these neutrinos interact via the righthanded gauge boson W_R, this mixing will not affect rare processes so long as W_R is sufficiently heavy. Also the multiplicity of Higgs particles coupling to fermions does not produce flavor changing neutral currents since the Z_6 symmetry renders this coupling diagonal in flavor space. Thus our $N_s = N_m = 2$ model is completely consistent with observed limits on rare processes.

With $N_s = 3$ and $N_m = 3$, Z_6 quantum numbers must be repeated for two of the three generations of leptons, which results in mixing among two of the standard and two of the mirror generations. Again we find that three of the masses can be made large, while three of the masses become ultralight and below the cosmological limit of 100 eV. This case of $N_m = N_s = 3$ with intergenerational mixing does run into problems with FCNC due to Higgs particles, as do many left-right theories.[19] A successful model would require an extended discrete symmetry that would enforce flavor-diagonal Higgs couplings. Surprisingly, however, FCNC effects other than those due to Higgs particles remain safely small even when some flavor mixing is present.

In conclusion, we have proposed a new means of naturally obtaining ultralight Dirac neutrinos, consistent with the nonobservation of neutrinoless double beta decay. Our model is a left-right gauge theory and contains mirror fermions which are prevented from condensing with the standard ones by means of a discrete symmetry that is unbroken down to low energies. This discrete symmetry enforces a skewed mass matrix for the neutral leptons; with an assumed natural hierarchy $< \phi'^{o} >/< \phi^{o} > \sim 10^{-3}$ consistent with small $W_L - W_R$ mixing, some of the neutrinos are rendered ultralight ($<$ 100 eV) and others massive ($\sim$ 100 GeV) in a manner reminiscent of the see-saw mechanism.[4] The discrete symmetry also avoids the generic problem of left-right theories with FCNC's in the Higgs sector. The smallness of the ratio m_ν/m_ℓ is explained as $m_\nu/m_\ell \sim (< \phi'^{o} >/< \phi^{o} >)^2\, m_E/m_W \sim 10^{-6}\, m_E/m_W$.

The cosmological bounds on neutrino masses can also be satisfied if an equal number of mirror and standard generations of leptons appear, so that neutrinos in the dangerous mass range (100 eV – 2 GeV) are avoided. Although the righthanded neutrinos are equally ultralight or massive as their lefthanded counterparts, difficulties with the cosmological bound for four ultralight two-component neutrinos are avoided even for the physically interesting case of 3 standard and 3 mirror generations, since no Majorana contributions are present which could thermalize the righthanded neutrinos during nucleosynthesis.

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Footnotes

F1. The contrary claim of S. Panda and U. Sarkar, Ref. 6, is negated due to an error in the manuscript.