Fermi National Accelerator Laboratory

FERMILAB-Pub-85/37-THY
March, 1985
Submitted to Nuclear Physics B

Functional Schroedinger Approach to Quantum Field Theory in DeSitter Space and Inflation

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Abstract

We give a formulation of quantum field theory in deSitter space in a functional Schroedinger picture. With conformal coupling to gravity in the scalar field action we find that we must add improvement terms at the functional action level as well, due to the nontrivial transformation properties of the wave-functional. This results in a Schroedinger equation built of the canonical stress-tensor. We obtain the Bunch-Davies vacuum wave-functional. We evaluate φ^2 and the stress-tensor as matrix elements in the wave-functional and confirm with Pauli-Villars regularization the usual one-loop results. An arbitrary initial wave-functional with the appropriate short-distance behavior leads asymptotically to the Bunch-Davies vacuum.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. Introduction

In this paper we develop a differential functional Schroedinger description of quantum field theory in classical curved spacetime by application to deSitter space. There are a great many treatments of field theory in deSitter space[1-8], but the present for matter fields is formally most akin to the Wheeler-DeWitt[9-11] equation for quantum gravitational fields.

In an earlier paper[12,13] we constructed a general formalism with application to the Hawking effect (via the Rindler problem). In the present paper we extend the formalism to cosmological settings and illustrate, in the Bunch-Davies wave-functional, the calculation of the stress tensor (and conformal anomaly) and φ^2. This calculation is exact in the present formalism in the absence of interactions other than with the classical background gravitation. Moreover, we confirm the usual one-loop results in the literature (without recourse to cumbersome point-splitting techniques; our method employs dimensional regularization and Pauli-Villars subtraction conditions).

We find that the correct Hamiltonian density is the canonical construction, which we refer to as the (0,0) component of the "canonical stress-tensor". This is distinct from the "gravitational stress-tensor", obtained by variation of the action with respect to the metric, in the presence of conformal coupling to gravity[14,15]. In the absence of conformal coupling the two constructions agree. We show how "tree-approximation" conformal invariance is derived in the conformal limit in d+1 dimensional deSitter space.

The reason for the use of the "canonical" stress-tensor as opposed to the "gravitational" one may be traced to the nontrivial transformation properties of the wave-functional under conformal transformations. We study this in section II(B and C) below. We give a functional action from which the Schroedinger equation follows by extremalization. If one constructs this from the "gravitational" (new-improved) stress-tensor then we must add functional improvement terms to the functional action. This leads to a "new-improved" Schroedinger equation which is identical to that obtained directly from the "canonical" stress-tensor.

What is the appropriate Schroedinger equation to adopt in a cosmological setting? The answer lies in a quantum mechanical generalization of the principle of equivalence. The statement that "freely falling test particles travel along geodesics" generalizes to the quantum mechanical statement "the vacuum state evolves by the Hamiltonian defined in a comoving inertial coordinate system". This yields asymptotically (in time) a unique definition of vacuum. That is to say, prescribe that we (1) construct a coordinate system in which test particles at fixed spatial coordinates x_i are inertial (e.g. Minkowski, global deSitter, or Kruskal coordinates have these properties) (2) construct the Hamiltonian in this coordinate system (e.g. by way of the formalism of ref.(12)) (3) solve the Schroedinger equation subject to arbitrary initial conditions. This system as viewed by observers comoving in noninertial coordinate systems (e.g. Rindler coordinates, static deSitter coordinates, or Schwarzschild coordinates corresponding to the above mentioned inertial systems) will appear thermally excited and lead to the Hawking effect.

This prescription leads to an unambiguous vacuum state in deSitter space after many e-foldings. Arbitrary deSitter breaking initial conditions after a long time relax into a "Bunch-Davies" vacuum. However, the asymptotically vanishing corrections are infinities associated with the definition of the state. We may employ Pauli-Villars regularization to remedy these ambiguities; one must assume here that the regulator fields evolve with the free physical fields, i.e. the subtractions are effectively comoving or time dependent although the conditions on the masses of the Pauli-Villars regulators are unambiguous and invariant. The use of Pauli-Villars regularization, it is shown presently, leads to equivalent expressions for the renormalized stress-tensor and φ^2 expectations as obtained by point-splitting with Schwinger-DeWitt subtractions.

We do not solve a deeper issue of the computability of inflationary models in the absence of a complete theory of quantum gravity. The solutions to the functional Schroedinger equation are gaussians in the oscillator amplitudes which define the theory. Each oscillator at time $t=0$ may be labeled by its momentum, k_i. At $t > 0$ the physical momentum of an oscillator of label momentum k_i is redshifted into $a(0)k_i/a(t)$, where $a(t)$ is the metric scale factor. If one presumes that the present vacuum is essentially free field theory on momentum scales of order M_W (or more conservatively, of order m_e in QED) then in minimal inflation these scales were once many orders of magnitude above the Planck scale, though we probably wish to presume that the theory is not free-field on such large scales (or at least, that the phase strucure is not the same above M_P as below). In fact, the length scale of 100Mpc is, prior to minimal inflation(16), equal to the Planck scale. Thus, all density fluctuations below that scale are expected to have been influenced by the physics of quantum gravity during the course of the evolution of the Universe.

There is no provision in any scenario of inflationary cosmology to account for the relaxation of the vacuum state as modes are drawn down from above the Planck scale to below. Naively we might expect particle production at this scale given by an energy density rate of $M^4_{Planck}H$, where H is the Hubble constant. Even today this is an incredible 10^{96}ergs/cm^3/sec! How the vacuum gracefully passes through the quantum gravity phase transition without attendant particle production is, to me, as great a mystery as the smallness of the cosmological constant. Equivalently, any fine tuning of the cosmological constant, $M_{Planck}{}^4$, must also control its time derivatives, $M_{Planck}{}^4H^n$.

II. Functional Schroedinger Picture

(A) Formulation

We begin with an invariant action defined through the Lagrangian density for a d+1 dimensional real scalar field theory:

$$S = \int d^{d+1}x \sqrt{g}\, \mathcal{L}$$
$$= \frac{1}{2}\int dx^0 d^d x\, |g|^{1/2}\left(g_{\mu\nu}\partial^\mu\varphi\,\partial^\nu\varphi - \mu^2\varphi^2 - \xi R\varphi^2\right) \quad (1)$$

where we've allowed for a conformal coupling to gravity specified by ξ. In general we may choose the metric "gauge" $g_{0i}=0$. This is not necessary and in ref.(12) the general formalism is given. We obtain modifications to this formalism in the presence of conformal coupling as discussed in II(C). With the metric gauge choice a canonical momentum to φ may be defined collinear with the time differential by:

$$\pi_0 = |g|^{-1/2}\,\delta^{(d+1)}S/\delta\partial^0\varphi = \partial_0\varphi \qquad (2)$$

We may construct from $\mathcal{L}$ and π_0 a "canonical" Hamiltonian density which may be viewed as the (0,0) component of the canonical stress-tensor:

$$\begin{aligned} T^{c}_{\mu\nu} &= \partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\mathcal{L} \\ &= \partial_\mu\varphi\,\partial_\nu\varphi - \tfrac{1}{2}g_{\mu\nu}\left(\partial_\rho\varphi\,\partial^\rho\varphi - \mu^2\varphi^2 - \xi R\varphi^2\right) \qquad (3) \end{aligned}$$

$T^{c}_{\mu\nu}$ is not covariantly conserved for arbitrary ξ, but upon quantization the canonical Hamiltonian will generate the time evolution of the wave-functional in a manner consistent with the usual Heisenberg picture formulation(5) and will lead to a conformally invariant theory in tree approximation in the limit $\mu^2 \rightarrow 0$ and $\xi \rightarrow (1-1/d)/4$.

Alternatively, we could derive the "gravitational" stress-tensor by variation of S with respect to $g_{\mu\nu}$:

$$T^{g}_{\mu\nu} = -2|g|^{-\frac{1}{2}}\,\delta^{(d+1)}S/\delta g^{\mu\nu} \tag{4}$$
$$= \partial_\mu\varphi\,\partial_\nu\varphi - \tfrac{1}{2}g_{\mu\nu}\left(\partial_\rho\varphi\,\partial^\rho\varphi - \mu^2\varphi^2\right) - \xi G_{\mu\nu}$$
$$- \xi\left(\varphi^2{}_{;\mu;\nu} - g_{\mu\nu}\varphi^2{}_{;\rho;}{}^{\rho}\right)$$

(the functional derivative denoted by (d+1) is d+1-dimensional, i.e. $\delta\varphi(x)^{(d+1)}/\delta\varphi(y) = \delta^{d+1}(x-y)$). Here $G_{\mu\nu}$ is the Einstein tensor. This form is equivalent to that given in ref.(1) upon use of equations of motion and explicit subtraction of trace. The above expression is traceless upon use of equations of motion. $T^{g}_{\mu\nu}$ is covariantly conserved; indeed, $T^{g}_{\mu\nu;}{}^{\nu}=0$ defines the equations of motion of $\varphi(x,t)$. We note that in, for example, 3+1 space-time in the conformal limit, $\mu^2 \to 0$, $\xi \to \frac{1}{6}$, and $G_{\mu\nu} \to 0$, we recover the usual "new improved" stress tensor:

$$T^{g}_{\mu\nu} \to \frac{2}{3}\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{6}g_{\mu\nu}\partial_\rho\varphi\,\partial^\rho\varphi - \frac{1}{3}\varphi\,\partial_\mu\partial_\nu\varphi \tag{5}$$

We see that the relationship between $T^{c}_{\mu\nu}$ and $T^{g}_{\mu\nu}$ is:

$$T^{g}_{\mu\nu} = T^{c}_{\mu\nu} - \xi R_{\mu\nu}\varphi^2 - \xi\left(\varphi^2_{;\mu;\nu} - g_{\mu\nu}\varphi^2_{;\rho;}{}^{\rho}\right)$$

$$\langle T^{g}_{\mu\nu}\rangle_D = \langle T^{c}_{\mu\nu}\rangle_D - \xi R_{\mu\nu}\langle\varphi^2\rangle_D \qquad (6a,b)$$

and where the expectation is taken in a deSitter invariant state (such as the Bunch-Davies vacuum) in which φ^2 is a constant.

In Section C we construct the Schroedinger action which follows upon using the (0,0) component of the gravitational stress tensor as a Hamiltonian density. This action does not lead to tree-approximation conformal invariance and is inequivalent to the canonical form. We attribute this to the nontrivial transformation properties of the wave-functional under a conformal transformation. It thus becomes necessary to add improvement terms to this action at the functional level. The result is the action constructed from the canonical stress-tensor.

We must postulate an equal time commutation relation between φ and π_0. This is defined on a space-like hypersurface and is essentially a global relationship; as the separated points approach one another we require that the commutator be locally covariant. Thus we demand:

$$[\varphi(x), \pi_0(y)] = i|g|^{-\frac{1}{2}} g_{00}\, \delta^{(d)}(x-y) \tag{7}$$

(this follows by the fact that $\delta^{d}(x-y)/|g|^{\frac{1}{2}}$ transforms as an upper index covariant 4-density time component since it is conjugate to $|g|^{\frac{1}{2}}d^{d}x$ or $d\Sigma_0$).

We pass to Schroedinger picture by introducing a d-dimensional functional derivative on the spacelike hypersurface and define:

$$\pi_0(x) = -i|g|^{-\frac{1}{2}} g_{00}\, \delta^{(d)}/\delta\varphi(x) \tag{8}$$

where now the fields $\varphi(x)$ are to be regarded as time independent configurations which are the fundamental degrees of freedom of the system.

We then postulate a covariant Schroedinger equation which propagates the state wavefunctional, $\Psi(\varphi,t)$, in the time variable $t=x^0$ built upon the canonical Hamiltonian:

$$H_0\Psi = \frac{1}{2}\int d^dx\, |g|^{\frac{1}{2}} g_{00} \Big\{ -\left(\frac{g_{00}}{|g|^{\frac{1}{2}}}\right)^2 \frac{\delta^2}{\delta\varphi^2} - g_{00}\big(g^{ij}\nabla_i\varphi\nabla_j\varphi$$
$$-\mu^2\varphi^2 - \xi R\varphi^2\Big\}\Psi = i\frac{\partial}{\partial t}\Psi(\varphi,t) \tag{9}$$

$\Psi(\varphi,t)$ is the amplitude to find field configuration $\varphi(x)$ at time t. Though this equation follows from a manifestly covariant construction, it describes a global object, $\Psi(\varphi,t)$, which requires a global initial surface boundary condition.

Now we specialize to the case of open deSitter space with the metric choice:

$$ds^2 = dt^2 - e^{2Ht}\, d\vec{x}^{\,2} \tag{10}$$

In d+1 dimensional deSitter space we have the following quantities:

$$\begin{aligned} R &= d(d+1)H^2 \\ G_{\mu\nu} &= g_{\mu\nu}\, d(1-d)H^2/2 \\ \xi &= (\text{in conformal limit}) = \tfrac{1}{4}\left(1-\tfrac{1}{d}\right) \end{aligned} \tag{11a,b,c}$$

We thus arrive at the functional Schroedinger equation:

$$H_0\Psi = \frac{1}{2}\int d^dx\, e^{dHt}\Big\{ -e^{-2dHt}\frac{\delta^2}{\delta\varphi^2} + e^{-2Ht}\nabla\varphi\cdot\nabla\varphi$$
$$+\mu^2\varphi^2 + \xi\, d(d+1)H^2\varphi^2\Big\}\Psi = i\frac{d}{dt}\Psi \qquad (12)$$

based upon eq.(9), which is conformally invariant. Generally we may view $\Psi(\varphi(x),t)$ as a path integral from $t=-\infty$ to the surface t if we are careful to define the path integral on some earlier surface with appropriate boundary conditions. Presently it is easier to solve directly for the wave-functional from the Schroedinger equation (12).

(B) Tree Approximation Conformal Invariance

It is interesting to inquire as to how the theory described by eq.(12) becomes conformally invariant. This is only "tree approximation" invariance since the conformal anomaly occurs in one-loop as the trace of the stress-tensor (spoiling conservation of the scale current). In Schroedinger picture the conformal terms occur because the field configurations must be time-independent. The time derivative on the rhs of eq.(12) is a total derivative. If the field configurations acquire an overall time dependent rescaling, then we reexpress the rhs in terms of a partial derivative with respect to time and one with respect to the field configurations. This leads to modifications of the Hamiltonian which cancel against the conformal coupling term.

To see how this goes consider the time dependent rescaling of the fields:

$$\varphi' = \exp\left\{\left(\frac{d-1}{2}\right)Ht\right\}\varphi . \tag{13}$$

This is the rescaling which leads to the usual conformally invariant action. The Schroedinger equation of eq.(12) becomes:

$$\begin{aligned} H\Psi = \frac{1}{2}\int d^d x\, e^{-Ht}\left\{-\frac{\delta^2}{\delta\varphi'^2} + \nabla\varphi'\cdot\nabla\varphi' + \mu^2\varphi'^2 e^{2Ht}\right. \\ \left. + \left\{d(1+d)H^2\varphi'^2 e^{2Ht}\right\}\right\}\Psi = \left(i\frac{\partial}{\partial t} - i\left(\frac{d-1}{2}\right)H\int d^d x\, \varphi'\frac{\delta}{\delta\varphi'}\right)\Psi \end{aligned} \tag{14}$$

where $\partial/\partial t$ acts upon the explicit time dependence in $\Psi(\varphi,t)$. The last term on the rhs follows by noting:

$$\frac{d}{dt}\varphi \equiv 0 = \frac{\partial\varphi'}{\partial t}e^{-\left(\frac{d-1}{2}\right)Ht} - \left(\frac{d-1}{2}\right)H\varphi' e^{-\left(\frac{d-1}{2}\right)Ht} . \tag{15}$$

We transfer the second term on the rhs onto the lhs and complete the square with the kinetic term:

$$\frac{1}{2}\int d^dx\, e^{-Ht}\Big\{\Big(i\frac{\delta}{\delta\varphi'} - \Big(\frac{1-d}{2}\Big)H\varphi' e^{Ht}\Big)^2 + \nabla\varphi'\cdot\nabla\varphi'$$

$$+ \Big(\mu^2 + \{d(1+d)H^2 - \Big(\frac{1-d}{2}\Big)^2 H^2\Big)\varphi'^2 e^{2Ht}\Big\}\Psi \qquad (16)$$

$$= \Big(i\frac{\partial}{\partial t} + i\Big(\frac{1-d}{4}\Big)H\,\delta^d(0)\Big)\Psi$$

The last term on the rhs arises from the ($\delta/\delta\varphi(x))\varphi(x) = \delta^d(0)$ in the completion of the square; similarly we pick up the last term of the lhs upon squaring.

The functional and time derivatives may be transformed by essentially functional gauge transformations. We define a new wave-functional, $\hat{\Psi}(\varphi',t)$ implicitly by:

$$\Psi(\varphi' e^{-(\frac{d-1}{2})Ht}, t) = e^{-(\frac{1-d}{4})Ht\,\delta^d(0)}\, e^{-i\int d^dx (\frac{1-d}{4})\varphi'^2 H e^{Ht}}\,\hat{\Psi}(\varphi', t)$$

$$(17)$$

and we arrive at:

$$\frac{1}{2}\int d^d x \left\{ -\frac{\delta^2}{\delta\varphi'^2} + \nabla\varphi' \cdot \nabla\varphi' + \mu^2 \varphi'^2 e^{2Ht} \right.$$

$$\left. + \left(\xi d(d+1) - \left(\frac{1-d}{2}\right)^2 + \left(\frac{1-d}{2}\right)\right) H^2 \varphi'^2 e^{2Ht} \right\} \hat{\Psi}$$

$$= e^{Ht}\, i \frac{\partial}{\partial t} \hat{\Psi} \equiv i \frac{\partial}{\partial \tau} \hat{\Psi}(\varphi, t(\tau)) \qquad (18)$$

(recalling that $\partial/\partial t$ acts only upon explicit time dependence in Ψ). We see that by choosing $\xi = (1-1/d)/4$ and $\mu^2=0$ that we arrive at the manifestly conformally invariant theory:

$$\frac{1}{2}\int d^d x \left[-\frac{\delta^2}{\delta\varphi'^2} + \nabla\varphi' \cdot \nabla\varphi' \right] \hat{\Psi}(\varphi', t(\tau)) = i \frac{\partial}{\partial \tau} \hat{\Psi}(\varphi', t(\tau)) \qquad (19)$$

This verifies the usual conformal invariance obtained by manipulation of the action in the framework of the Schroedinger picture.

(C) Functional Action; "New Improved" Functional Action

The Schroedinger equation may be viewed as arising from the functional action:

$$\tilde{S} = \int \mathcal{D}\varphi\, dx^0 \left[\int d^d x \sqrt{g} \left\{ \frac{1}{2} \frac{g_{00}}{|g|} \frac{\delta \overline{\Psi}}{\delta \varphi} \frac{\delta \Psi}{\delta \varphi} + \frac{1}{2} \overline{\Psi} (g^{ij} \nabla_i \varphi \nabla_j \varphi - \mu^2 \varphi^2 - \xi R \varphi^2) \Psi \right\} + i \overline{\Psi} \frac{\partial}{\partial x^0} \Psi \right] \quad (20)$$

by variation with respect to the wave-functional $\overline{\Psi}$. The canonical stress-tensor appears implicitly in this formula, and as we've seen in Section (C) it leads to the correct tree-approximation conformal invariance(14).

Why does the "gravitational" stress-tensor not generate the correct Schroedinger equation? These lead to identical theories in the $\xi=0$ limit, or in the flat space limit (in which the improvement terms are total derivatives and have a canonical structure).

The reason is evidently the non-trivial transformation properties of Ψ under conformal transformations. We see in eq.(17) that we must perform an effective "functional gauge transformation" upon Ψ to bring it into the form of a manifestly conformally invariant Schroedinger equation as in eq.(19). We may view this as the analogue of the transformation in eq.(13) which brings the new improved action into a conformally invariant form when $\xi=(1-1/d)/4$, and $\mu^2=0$. Hence, we expect that if we construct the functional action with the gravitational stress-tensor that we must allow for "functional improvement" terms.

Indeed, the functional action constructed with the gravitational stress-tensor in eq.(4) takes the form:

$$\tilde{S}_g = \int \mathcal{D}\varphi\, dx^0 \Big[\int d^dx \sqrt{g} \Big\{ \frac{1}{2} \frac{g_{00}}{|g|} \frac{\delta\bar{\Psi}}{\delta\varphi} \frac{\delta\Psi}{\delta\varphi} + \frac{1}{2} \bar{\Psi}\big(g^{ij} \nabla_i\varphi \nabla_j\varphi - \mu^2\varphi^2 + 2\xi G^{00}\varphi^2 - 2\xi g^{ij} D_i(\nabla_j\varphi^2)\big)\Psi \Big\} + i\bar{\Psi}\frac{\partial}{\partial t}\Psi \Big] \quad (21)$$

Here i and j are summed over the d-spatial dimensions. Thus, the new-improved functional action must be that in eq.(20) and we see that the relationship is:

$$\tilde{S} = \tilde{S}_g - \xi \int \mathcal{D}\varphi\, dx^0 \int d^dx \sqrt{g}\, \bar{\Psi} \Big\{ g^{00} R_{00} \varphi^2 - g^{ij} D_i(\nabla_j\varphi^2) \Big\} \Psi \quad (22)$$

The last terms in eq.(22) represent the functional improvement necessary to obtain the conformally invariant Schroedinger equation in eq.(19). The last term of eq.(22) is generally not a surface term in curved spacetime and involves a functional derivative through eq.(8).

(D) Diagonalization of H in Momentum Space

It is convenient to pass to momentum space (this effectively diagonalizes the Hamiltonian of eq.(12)):

$$\varphi(x) = \int \frac{d^d k}{(2\pi)^d}\, \alpha_k\, e^{ik\cdot x} \; ; \qquad \frac{\delta}{\delta\varphi(x)} = \int \frac{d^d k}{(2\pi)^d}\, e^{-ik\cdot x} \frac{\partial}{\partial \alpha_k}$$

$$\frac{\partial}{\partial \alpha_k}(\alpha_p) = \delta^d(p+k) \; ; \qquad \alpha_k = \bar{\alpha}_{-k} \tag{23}$$

and the Schroedinger equation becomes:

$$\frac{1}{2}\int \frac{d^d k}{(2\pi)^d}\, e^{dHt}\left\{ -e^{-2dHt} \frac{\partial^2}{\partial \alpha_k \partial \bar{\alpha}_k} + \left(e^{-2Ht} k^2 + \mu^2 + \xi R\right) \alpha_k \bar{\alpha}_k \right\} \Psi$$

$$= i \frac{\partial}{\partial t} \Psi(\alpha_k, t) \tag{24}$$

To solve this consider the anzatz:

$$\Psi(\alpha_k, t) = \mathcal{N} \exp\left\{ -\int \frac{d^d k}{(2\pi)^d} A(k,t) |\alpha_k|^2 - i\Omega(t) \right\} \tag{25}$$

Substitution into eq.(24) yields:

$$i\dot{A}(K,t) = -A(K,t)^2 e^{-dHt} + K^2 e^{(d-2)Ht} + (\mu^2 + \xi R)e^{dHt} \tag{26}$$

To solve eq.(26) it is useful to define:

$$\Gamma_K(t) = e^{-dHt} A(K,t) - \frac{idH}{2} \tag{27}$$

and we obtain (in the case of constant or very slowly evolving H):

$$-i\dot{\Gamma}_K = -\Gamma_K^2 + \mu^2 + \xi R - \frac{d^2H^2}{2} + K^2 e^{-2Ht} \tag{28}$$

With the substitutions:

$$U_k(\tau) = \exp\left\{ i \int_{t_0}^{t(\tau)} \Gamma_k(t)\, dt \right\} \quad ; \quad \tau = H^{-1} e^{-Ht} \tag{29}$$

we find that $U_k(\tau)$ satisfies Bessel´s equation:

$$\frac{1}{k^2}\frac{d^2}{d\tau^2}U_k(\tau) + \frac{1}{k^2\tau}\frac{d}{d\tau}U_k(\tau) + \left(1 - \frac{\nu^2}{k^2\tau^2}\right)U_k(\tau) = 0 \tag{30}$$

where the index is: $\nu = \left(\frac{d^2}{4} - \frac{\mu^2 + \xi R}{H^2}\right)^{1/2}$.

The general solution for U is given by:

$$U_k(\tau) = A_k H_\nu^{(1)}(k\tau) + B_k H_\nu^{(2)}(k\tau) \tag{31}$$

Clearly the particular solution is determined by the initial conditions. Presently in this section we shall adopt a simplifying assumption which is equivalent to the choice of vacuum made by Bunch and Davies[5] in their Heisenberg picture analysis. We choose: A_k=0, B_k=1, whence we have for A(k,t):

$$A(\kappa,t) = i(H\tau)^{-d}\left\{ \frac{H_\nu^{(2)\prime}(\kappa\tau)}{H_\nu^{(2)}(\kappa\tau)}\,\kappa H\tau + \frac{dH}{2} \right\} \tag{32}$$

which gives us immediately the momentum space representation of the vacuum wave-functional. We thus can compute the probability of finding a given field configuration in terms of the Fourier coefficients:

$$\Psi^*\Psi = \mathcal{N}^2 \exp\left\{ -\frac{2}{\pi}\int \frac{d^dk}{(2\pi)^d}\,|\alpha_\kappa|^2\,\frac{H\,e^{dHt}}{H_\nu^{(1)}(\kappa\tau)H_\nu^{(2)}(\kappa\tau)} \right\} \tag{33}$$

(apart from overall normalization) where use has been made of the Hankel function Wronskian. We have not obtained here the explicit form for the zero-point energy (it develops an imaginary part to maintain the norm of the wave-functional). The real part is the (0,0) component of the stress tensor which we presently evaluate.

We remark that to evaluate the stress-tensor we require the following simple identities which follow upon performing the (gaussian) functional integration in the wave-function of eq.(26):

$$\int \mathcal{D}\varphi\, \Psi^*(\varphi)\, \alpha_k \alpha_p \Psi(\varphi) = \frac{\delta^d(k+p)}{2\,\mathrm{Re}(A(k,t))}$$

$$= \frac{\pi}{2H} e^{-dHt}\, H_\nu^{(1)}(k\tau)\, H_\nu^{(2)}(k\tau)\ \delta^d(k+p) \qquad (34a)$$

$$\int \mathcal{D}\varphi\, \Psi^*(\varphi)\, \frac{\partial^2}{\partial\alpha_k \partial\alpha_p} \Psi(\varphi) = \delta^d(k+p)\, \frac{\overline{A(k,t)}A(k,t)}{2\,\mathrm{Re}(A(k,t))}$$

$$= \frac{\pi}{2H}\, e^{dHt} \Big\{ H_\nu^{(1)\prime}(k\tau)\, H_\nu^{(2)\prime}(k\tau)\, \vec{k}^2 e^{-2Ht}$$

$$+ \frac{HKd}{2}\, e^{-Ht} \left(H_\nu^{(2)\prime} H_\nu^{(1)}(k\tau) + H_\nu^{(1)\prime} H_\nu^{(2)}(k\tau) \right)$$

$$+ \frac{d^2H^2}{4} H^{(1)} H^{(2)}(k\tau) \Big\}. \qquad (34b)$$

(III) Evaluation of the Stress-Tensor

(A) Dimensionally Regularized Result

The Schroedinger wave-functional may be used directly to evaluate the stress-tensor in deSitter space. We work in d space dimensions and have from eq.(3) the formal expressions for the canonical stress tensor components (we will obtain the gravitational stress-tensor below):

$$T_{oo}^{c} = -\frac{1}{2} e^{-dHt} \frac{\delta^2}{\delta\varphi^2} + \frac{1}{2} e^{-2Ht} \vec{\nabla}\varphi \cdot \vec{\nabla}\varphi + \frac{1}{2}(\mu^2 + \xi R)\varphi^2 \quad (35a,b)$$

$$T_{ii}^{c} = -\frac{1}{2} e^{2-2d} \frac{\delta^2}{\delta\varphi^2} + \frac{1}{2} \nabla_i\varphi \nabla_i\varphi - \frac{1}{2} \vec{\nabla}_{\perp}\varphi \cdot \vec{\nabla}\varphi - \frac{1}{2}\mu^2\varphi^2 e^{2Ht}$$

(the T_{ij} for $i \neq j$ components vanish in a rotationally invariant state). where we have substituted the operator expression for the canonical momentum wherever it appears into eq.(3). Here $\nabla_{\perp}$ denotes the gradient over the d-1 dimensions orthogonal to index i. A direct evaluation of the expectation values of these operators in the general Schroedinger wave-functional of eq.(25′) gives:

$$\langle T_{oo}^{c} \rangle = \frac{1}{4} \int \frac{d^d K}{(2\pi)^d} \left\{ \frac{\overline{A(K,t)} A(K,t)}{\mathrm{Re}\, A(K,t)} + \frac{K^2 e^{-2Ht} + \mu^2 + \xi R}{\mathrm{Re}\, A(K,t)} \right\} \quad (36a)$$

$$\langle T_{ii}^{\ c}\rangle = \frac{1}{4}\int \frac{d^d k}{(2\pi)^d}\left\{ \frac{\bar{A}(k,t)A(k,t)}{\mathrm{Re}\,A(k,t)}\, e^{2(d-1)Ht} + \frac{(\mu^2+\xi R)e^{2Ht} - (1-\frac{2}{d})k^2}{\mathrm{Re}\,A(k,t)}\right\} \tag{36b}$$

We may substitute eq.(34a and b) into eq.(36a and b) Here we may rescale the momentum, $k'=ke^{-Ht}$, and we see that the formal time dependence disappears everywhere and we thus obtain:

$$\langle T_{00}^{\ c}\rangle = \frac{\pi}{8H}\int \frac{d^d k'}{(2\pi)^d}\Big\{ \vec{k}'^2 H_\nu^{(1)\prime} H_\nu^{(2)\prime} + \left(H_\nu^{(2)\prime}H_\nu^{(1)} + H_\nu^{(1)\prime}H_\nu^{(2)}\right)\frac{Hd|\vec{k}|}{2} + \Big(\vec{k}^2+\mu^2 + \xi R + \frac{H^2 d^2}{4}\Big) H_\nu^{(1)} H_\nu^{(2)}\Big\}; \tag{37a}$$

$$\langle T_{ii}^{\ c}\rangle = \frac{\pi}{8H}\, e^{2Ht}\int \frac{d^d k}{(2\pi)^d}\Big\{ \vec{k}^2 H_\nu^{(1)\prime} H_\nu^{(2)\prime} + \frac{Hd|\vec{k}|}{2}\left(H_\nu^{(1)\prime}H_\nu^{(2)} + H_\nu^{(2)\prime}H_\nu^{(1)}\right) + \Big(\Big(\frac{2-d}{d}\Big)\vec{k}^2 + \frac{H^2 d^2}{4} - \mu^2 - \xi R\Big) H_\nu^{(1)} H_\nu^{(2)}\Big\}. \tag{37b}$$

Now the arguments of the Hankel functions are all $k'H^{-1}$. This rescaling of momentum is a general feature of cosmological metrics. It implies that a momentum label k refers to a physical momentum ke^{-Ht} at time t subsequent to initialization. The physical meaning of this is clear, but the implication for interacting theories, such as gravity at the Planck scale is not obvious.

To evaluate the integrals indicated in eq.(37a and b) we made use of the discontinuous integral of Weber and Schafheitlin(17,18) to obtain:

$$\int \frac{d^d k}{(2\pi)^d} k^\rho H_\nu^{(1)}(k) H_\nu^{(2)}(k) = \left\{ \frac{2^{\rho+1} \pi^{-\frac{d}{2}-1} \Gamma(1-\rho-d) \Gamma\left(\frac{\rho+d}{2}\right)}{\Gamma\left(\frac{d}{2}\right) \Gamma\left(1-\frac{\rho+d}{2}\right) \Gamma\left(\frac{1}{2}+\nu\right) \Gamma\left(\frac{1}{2}-\nu\right)} \right\} \cdot \Gamma\left(\frac{\rho+d}{2}+\nu\right) \Gamma\left(\frac{\rho+d}{2}-\nu\right) \qquad (38)$$

In the dimensionally regulated expressions we may integrate the first terms of eq.(37a,b) by parts (symmetrically) and use the Bessel equation to obtain an expression in $H_\nu^{(1)} H_\nu^{(2)}$ and a term in $H_\nu'^{(1)} H_\nu^{(2)} + H_\nu^{(1)} H_\nu'^{(2)}$ which cancels the second similar terms. This leaves the result:

$$\langle T_{00}^{c}\rangle = \frac{\pi}{4H}\int\frac{d^dK}{(2\pi)^d}\left(\vec{K}^2+\mu^2+\xi R\right) H_\nu^{(1)}(\kappa H^{-1})\, H_\nu^{(2)}(\kappa H^{-1})$$

$$\langle T_{ii}^{c}\rangle = \frac{\pi}{4H}\int\frac{d^dK}{(2\pi)^d}\, d^{-1}\vec{K}^2 H_\nu^{(1)}(\kappa H^{-1})\, H_\nu^{(2)}(\kappa H^{-1}) \qquad (39a,b)$$

Evaluating eq.(39) using eq.(38) yields:

$$\langle T_{00}^{c}\rangle = d e^{-2Ht}\langle T_{ii}^{c}\rangle + \frac{\pi}{4}\left(\mu^2+\xi R\right)\int\frac{d^dK}{(2\pi)^d} H_\nu^{(1)}(\kappa H^{-1})\, H_\nu^{(2)}(\kappa H^{-1})$$

$$= \pi^{-d/2} H^{d-1}\left\{\frac{\Gamma(-1-d)\Gamma(1+d/2)\Gamma(1+d/2-\nu)\Gamma(1+d/2+\nu)}{\Gamma(d/2)\Gamma(-d/2)\Gamma(\nu+\frac{1}{2})\Gamma(\frac{1}{2}-\nu)} H^2 + \right.$$

$$\left. \frac{(\mu^2+\xi R)}{4}\frac{\Gamma(1-d)\Gamma(d/2-\nu)\Gamma(d/2+\nu)}{\Gamma(\nu+\frac{1}{2})\Gamma(\frac{1}{2}-\nu)\Gamma(1-d/2)}\right\}$$

$$= \frac{H^{1+d}\pi^{-d/2}}{2}\frac{\Gamma(1-d)\Gamma(1+\frac{d}{2}-\nu)\Gamma(1+\frac{d}{2}+\nu)}{(1+d)\Gamma(1-\frac{d}{2})\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{1}{2}+\nu)} = e^{-2Ht}\langle T_{ii}^{c}\rangle \qquad (40)$$

We thus obtain the final dimensionally regulated result in d spatial dimensions:

$$\langle T^{c}_{\mu\nu}\rangle = g_{\mu\nu}\,\frac{H^{1+d}\,\pi^{-d/2}}{2}\,\frac{\Gamma(1-d)\,\Gamma(1+\frac{d}{2}-\nu)\,\Gamma(1+\frac{d}{2}+\nu)}{(1+d)\,\Gamma(1-\frac{d}{2})\,\Gamma(\frac{1}{2}-\nu)\,\Gamma(\frac{1}{2}+\nu)} \qquad (41)$$

which is deSitter invariant with the metric (here the metric is of course the deSitter metric defined through eq.(10)).

Similarly, we have implicitly evaluated the quantity $\langle\varphi^2\rangle$ in the above analysis:

$$\langle\varphi^2\rangle = \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{\mathrm{Re}\,A(k,t)}$$

$$= \frac{H^{d-1}\,\pi^{-d/2}}{2}\,\frac{\Gamma(1-d)\,\Gamma(\frac{d}{2}-\nu)\,\Gamma(\frac{d}{2}+\nu)}{\Gamma(\frac{1}{2}+\nu)\,\Gamma(\frac{1}{2}-\nu)\,\Gamma(1-\frac{d}{2})}\,. \qquad (42)$$

(B) Renormalization

The formal expressions of eq.(41) and eq.(42) may be subtracted either by the method of ref.(2-5) or by direct application of Pauli-Villars regularization. Pauli-Villars regularization is a misnomer as the method leads both to a regularized as well as renormalized result in an unambiguous way.

In using Pauli-Villars subtraction we must specify a definite space-time dimensionality for the scale dimensions of eq.(41,42). For example, if we choose to evaluate the stress-tensor in 1+1 space-time we want d=1 and we must multiply the rhs of eq.(41) by the factor $(m^2)^{((1-d)/2)}$ which fixes the scale dimensionality of the bare expression to be $(\mathrm{mass})^2$. Here $m^2=\lambda\mu^2$ is an arbitrary mass scale proportional to the theory mass scale μ^2. This is because converting a dimensionally regularized expression to a Pauli-Villars regularized expression cannot exploit the arbitrariness of dimensional continuation. Technically, if these operations are not performed, the Pauli-Villars subtractions are spoiled by logarithms.

We imagine taking linear combinations of the unrenormalized expressions of eq.(41) and eq.(42) with a set of similar expressions for large mass regulator fields, e.g. for an operator $\mathcal{O}(\mu^2)$:

$$\langle \mathcal{O}(\mu^2)^{ren} \rangle \equiv \langle \mathcal{O}(\mu^2) \rangle - \sum_{i=1}^{N} \alpha_i \, \mathcal{O}(M_i^2). \qquad (43)$$

We further impose for the N regulator fields N moment conditions on the coefficients α_i:

$$(\mu^2)^M - \sum_{i=1}^{N} \alpha_i (M_i^2)^M; \quad M = (1, 2, \ldots N). \qquad (44)$$

We then define the renormalized quantity by taking the limit $M_i^2 \to \infty$ in eq.(43).

Let us exhibit the procedure in 1+1 dimensions for the stress-tensor. We require the large mass limit of eq.(41) which requires the large argument limit of the digamma functions and the expansion of the radical ν. First we write, in $d=1-\varepsilon$ space dimensions:

$$\langle T_{\mu\nu}{}^{c}\rangle = g_{\mu\nu}\,\frac{\mu^2+\xi R}{4\pi}\left[\frac{1}{\varepsilon}-\frac{1}{2}\ln\frac{H^2}{\lambda\mu^2}-\frac{1}{2}\left(\psi\left(\tfrac{1}{2}-\nu_0\right)+\psi\left(\tfrac{1}{2}+\nu_0\right)\right)\right.$$
$$\left.+\ \text{constant}\right] \qquad (45)$$

and the large mass limit of the expression for the stress tensor in $1-\varepsilon$ dimensions becomes:

$$\langle T_{\mu\nu}{}^{c}\rangle \rightarrow g_{\mu\nu}\left(\frac{M^2+\xi R}{4\pi}\right)\left(\frac{1}{\varepsilon}-\frac{1}{2}\ln\frac{H^2}{\lambda M^2}+\frac{1}{2}\ln\left(\frac{M^2}{H^2}\right)+\frac{H^2}{6M^2}\right.$$
$$\left.+\ \text{constant}\right) \qquad (46)$$

where ν_0 is $\left(\frac{1}{4}-\frac{\mu^2+\xi R}{H^2}\right)^{1/2}$.

Taking now the linear combination with the regulator fields shows that the $1/\varepsilon$ singularity and arbitrary λ parameter have cancelled and we obtain the finite, renormalized canonical stress-tensor:

$$\langle T_{\mu\nu}^{c\ ren}\rangle = \langle T_{\mu\nu}(\mu^2)\rangle - \Sigma' \alpha_i \langle T_{\mu\nu}(M_i^2)\rangle \qquad (47)$$

$$= \frac{g_{\mu\nu}}{8\pi}\left[(\mu^2+\xi R)\ln\left(\frac{2\mu^2}{R}\right) + (\mu^2+\xi R)\left(\Psi\left(\tfrac{1}{2}+\nu_0\right)+\Psi\left(\tfrac{1}{2}-\nu_0\right)\right) + \frac{R}{6}\right]$$

Similarly, applying the Pauli-Villars subtraction conditions to the φ^2 matrix element yields for d=1:

$$\langle \varphi^{2\ ren}\rangle = \frac{1}{4\pi}\left[\ln\left(\frac{2\mu^2}{R}\right) - \left(\Psi\left(\tfrac{1}{2}+\nu_0\right)+\Psi\left(\tfrac{1}{2}-\nu_0\right)\right)\right] \qquad (48)$$

To evaluate the gravitational stress-tensor we make use of eq.(47) and eq.(48) to obtain:

$$\langle T_{\mu\nu}^{g}\rangle = \langle T_{\mu\nu}^{c}\rangle - \xi R_{\mu\nu}\langle\varphi^2\rangle$$

$$= \frac{g_{\mu\nu}}{8\pi}\left[\mu^2 \ln\frac{2\mu^2}{R} + \mu^2\left(\Psi\left(\tfrac{1}{2}+\nu_0\right) + \Psi\left(\tfrac{1}{2}-\nu_0\right)\right) + \frac{R}{6}\right] \qquad (49)$$

which agrees with the usual result(5). In the conformal limit, $\xi \to 0$ and $\mu^2 \to (0)$ we obtain:

$$\langle T_{\mu\nu}{}^{\gamma}\rangle = \langle T_{\mu\nu}^{c}\rangle = g_{\mu\nu}\,\frac{R}{48\pi} \qquad (50)$$

which is the familiar trace anomaly in deSitter space-time of dimensionality 1+1

This result establishes that our vacuum state is the correct representation of the Bunch-Davies vacuum in terms of Schroedinger wave-functionals. We have further verified the usual results in 3+1 given previously in the literature.

IV. Implementing Physical Boundary Conditions

In the preceding analysis we have adopted a particular solution for the vacuum state which technically facilitates our evaluation of matrix elements and is thus identical to the choice made in the earlier literature[1-5]. Essentially, the choice that $U_k(t)$ be just $H_\nu^{(2)}(k\tau)$ associates the asymptotic phases $\exp(-iEt)$ with destruction operators. We've obtained in the present formalism results identical to the previous literature and the results are required to be deSitter invariant since the action of the global deSitter charges presumeably annihilate the state. Counterterms are generally covariant since they probe the short distance structure of the theory where we've engineered in local coordinate invariance. This, however, does not imply that the state constructed above is physically interesting. Indeed, we can imagine that at time $t<0$ the Universe is actually in a Robertson-Walker or even Minkowskian ground state. At a subsequent $t=0$ we switch on a deSitter metric (this occurs naturally in a sense in inflation). Then we must require that the state in the deSitter phase match up with the vacuum from the earlier phase. We show presently that any arbitrary initial conditions will lead after several e-foldings to the Bunch-Davies vacuum. Similar conclusions are inherent in the models studied by Vilenkin and Ford[22].

Consider the quantity $U_k(t)$ as defined in eq.(28). We can impose at $t=0$ the initial conditions:

$$u_K(0) = A_K H_\nu^{(1)}(KH^{-1}) + B_K H_\nu^{(2)}(KH^{-1}) = 1$$

$$\dot{u}_K(0)\, u_K^{-1}(0) = iA_0 + \frac{d}{2}H = -K\left(A_K H_\nu^{(1)\prime}(KH^{-1}) + B_K H_\nu^{(2)\prime}(KH^{-1})\right) \tag{51a,b}$$

By definition $U_k(t)=1$ at $t=0$ and the second result follows from eq.(**27**). Here $A_0(K)$ is the initial width of the Schroedinger wave-functional.

Solution of eq.(51) for the coefficients, A_k and B_k yields:

$$A_K = \frac{i\pi}{4H}\left[K H_\nu^{(2)\prime}(KH^{-1}) + \left(iA_0 + \frac{dH}{2}\right) H_\nu^{(2)}(KH^{-1})\right]$$

$$B_K = \frac{-i\pi}{4H}\left[K H_\nu^{(1)}(KH^{-1}) + \left(iA_0 + \frac{dH}{2}\right) H_\nu^{(1)}(KH^{-1})\right] \tag{52a,b}$$

and the vacuum wave-functional width is:

$$A(k',t) = ie^{dHt}\left\{ \frac{k'(A_k H_\nu^{(1)\prime}(k'H^{-1}) + B_k H_\nu^{(2)\prime}(k'H^{-1}))}{A_k H_\nu^{(1)}(k'H^{-1}) + B_k H_\nu^{(2)}(k'H^{-1})} + \frac{id}{2}H \right\} \quad (53)$$

where $k' = k\exp(-Ht)$. This is an unwieldy expression and involves cross-terms of the form $H_\nu^{(i)}(kH^{-1})H_\nu^{(j)}(k'H^{-1})$. To proceed we adopt an "inflation approximation" in which we regard $\exp(-Ht)$ as infinitesimally small. We thus use asymptotic forms for large arguments in $H_\nu^{(i)}(kH^{-1})$ and we ultimately drop terms with rapid phase oscillation, e.g. $\cos(kH^{-1})H_\nu^{i}(k'H^{-1})$.

Let $A_o = (k^2 + m^2)^{1/2}$ for arbitrary m (this is the form appropos Minkowski space for m constant). Here $m(k^2)$ is a rotationally invariant function of the three momentum $\vec{k}$. The short distance behavior of this function must be constant, and $m(\vec{k}^2)$ can be represented by a Laurent series in descending powers of $\vec{k}^2$. In the inflationary approximation we have:

$$A_K \to \frac{1}{4}\sqrt{\frac{2K\pi}{H}}\left[(1-A_0K^{-1}) + \frac{idH}{2K}\right]\exp(-i\chi)$$
$$B_K \to \frac{1}{4}\sqrt{\frac{2K\pi}{H}}\left[(1+A_0K^{-1}) - \frac{idH}{2K}\right]\exp(i\chi) \qquad (54\,a,b)$$

where $\chi = kH^{-1} - \frac{1}{2}\nu\pi - \frac{1}{4}\pi$. The real part of A becomes:

$$\left[\mathrm{Re}\, A(\kappa,t)\right]^{-1} \Rightarrow \frac{\pi}{2H} e^{-dHt}\left\{\frac{K'^2 + \frac{1}{8}d^2H^2e^{-2Ht} + \frac{1}{2}m^2e^{-2Ht}}{|\kappa'|\left((\kappa')^2 + m^2e^{-2Ht}\right)^{1/2}}\right\} \cdot H_\nu^{(1)}(\kappa'H^{-1})\, H_\nu^{(2)}(\kappa'H^{-1}) \qquad (55)$$

Curiously, if we expand in m^2 we see that these corrections are suppressed, occuring in order m^4H^{-4}:

$$\left[\mathrm{Re}\, A(\kappa,t)\right]^{-1} \to \frac{\pi}{2H} e^{-dHt}\left\{1 + \frac{d^2H^2}{8\kappa'^2}e^{-2Ht} + \mathcal{O}\left(\frac{m^4}{\kappa'^4}e^{-4Ht}\right)\right\} \cdot H_\nu^{(1)}(\kappa'H^{-1})\, H_\nu^{(2)}(\kappa'H^{-1}) \qquad (56)$$

For large t we see that eq.(56) approaches the result for the Bunch-Davies vacuum:

$$\left[\mathrm{Re}\, A(k,t)\right]^{-1} \longrightarrow \frac{\pi}{2H} e^{-dHt}\, H_\nu^{(1)}(k'H^{-1})\, H_\nu^{(2)}(k'H^{-1}) + \mathcal{O}\left(\frac{H^2}{k^2} e^{-2Ht}\right) \tag{57}$$

The term involving $d^2H^2/8$ leads to logarithmic infinities in φ^2 and $T_{\mu\nu}$. These can be subtracted by application of the Pauli-Villars scheme. The initial choice of A_0 is thus irrelevant for the asymptotic behavior of the vacuum. This establishes that the vacuum state asymptotically approaches the Bunch-Davies vacuum in the inflation limit and we recover the renormalized expressions as in eq.(49). Moreover, by writing down an arbitrary initial (e.g.thermal) density matrix at t=0 (see the explicit Schroedinger representation of ref.(12) for the thermal density matrix) we find that in the inflation approximation we obtain a coherent density matrix which corresponds to the Bunch-Davies vacuum (essentially the thermal cross-terms approach zero, i.e. the density matrix becomes factorizeable into the product of the BD wavefunction and its complex conjugate). This result is remarkable and somewhat diminishes the problem of choice of the initial cosmological vacuum: all vacua lead to Bunch-Davies! Does the same hold in Robertson-Walker? We suspect by analogy that the answer is yes; the fact that we live in an approximate vacuum state may be a consequence of the expansion of the Universe.

V. Concluding Remarks

A general feature of the vacuum state in deSitter space or any Robertson-Walker metric is the association of redshifting factors, $\exp(-Ht)$, with the momentum of any mode. One can show that the physical momentum of an excitation of label momentum k becomes $k\exp(-Ht)$. As mentioned in the introduction, if at $t=0$ we know the structure of the vacuum state only up to some scale of order $k=M_X$ (X is surely "Planck", at least), then our knowledge of the short distance behavior of the theory below M_X becomes worse and worse as $t \rightarrow \infty$.

The subtraction schemes used to renormalize $T_{\mu\nu}$ and φ^2, etc., always presume that regularization occurs by some large, constant (in time), mass scale, e.g. the M_i of the Pauli-Villars scheme. Above M_i we effectively suppress the zero-point fluctuations of the theory (this may occur in reality by way of e.g. supersymmetry). However, in cosmological vacua we are always replenishing modes immediately below M_i. If we did not replenish these modes then $T_{\mu\nu}$ would redshift like ordinary radiation, $\propto \exp(-4Ht)$. This replenishment allows constant $T_{\mu\nu}$. But it must in reality be associated with some physical process, e.g. particle production, which is not accounted for in our free field theory (hence our previous remark about the existence of derivatives of the cosmological constant).

We have not given a detailed discussion of the asymptotic behavior of arbitrary initial states. We anticipate that it is straightforward to recover the results of Vilenkin and Ford[19] in the infra-red divergent limit.

We thank W. Bardeen, R. Brandenberger, K. Freese, L. McLerran, M. Mueller, R. Pisarski, and especially J. Traschen for useful discussions.

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