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Can The Hawking Effect Thaw a Broken Symmetry?

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Abstract

A spontaneously broken symmetry is shown not to be restored as witnessed by accelerating (Rindler) observers. To one-loop thermal contributions appear only in differences between Minkowski and Rindler vacuum matrix elements. All observers obtain (up to coordinate transformations) the same matrix elements of physical operators. Identical effective potentials and order parameters are obtained by all observers.

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Suppose a symmetry is spontaneously broken as defined by the vacuum state in Minkowski space. It is well known that the effects of thermal equilibrium can restore a broken symmetry(1). Typically, there exists a critical temperature, T_c, at which the effective thermal potential as a function of order parameter φ_c develops an absolute minimum at $\varphi_c=0$, the system undergoes a second order phase transition, and φ_c relaxes to this minimum at higher temperatures (the order of the phase transition is irrelevant presently, but we study φ^4 for which the above statements are true in 3+1 dimensions).

Furthermore, it is well known that accelerating observers in a Lorentz invariant groundstate witness a thermal energy-momentum distribution of particles of temperature $g/2\pi$, where g parameterizes a Rindler ensemble of accelerating observers.(2) The question then naturally arises, "if $g > 2\pi T_c$, will the accelerating observer witness a restoration of the broken symmetry?" Furthermore, will a stationary observer (Schwarzschild) around a black-hole of sufficiently small mass witness symmetry restoration?

Simple heuristic arguments suggests that the answer must be no. In the groundstate we have the order parameter, $\varphi_c = \langle 0|\varphi|0\rangle$ which is a scalar and must transform into itself in the Rindler coordinate system. Furthermore, if T_c corresponds to a second order phase transition temperature, then the infinite correlation length measured, e.g., transversely to the direction of acceleration, must become finite for $T > T_c$. However the general coordinate transformation relating the accelerating observers (defined by Rindler coordinates) to the stationary observer is nonsingular (except at the horizon and within a half-infinite range of the Minkowski coordinate system) and cannot, therefore, convert a finite proper length scale in one frame into an infinite one in another. Thus, $g=2\pi T_c$ cannot correspond to a second order phase transition.

These arguments raise paradoxes however. For example, if an accelerating observer truly observes radiation of temperature T_H, then he must find thermal corrections to operator matrix elements, e.g. φ^2 must display a thermal correction of $T_H^2/12$ which conflicts with the covariance requirement that all observers must agree that it have the same value. The energy density, which for a spontaneously broken theory is essentially the effective potential, must be the same for all observers up to covariant transformation factors. How then does the accelerating observer obtain the usual thermal corrections to this quantity which would normally lead to symmetry restoration?

We have resolved these paradoxes and find the results somewhat surprising. We shall presently state the results of our one-loop analysis, the full technical details to be presented elsewhere(3). We have made use of a functional Schroedinger formalism of the Hawking effect given previously(4) and which graphically reproduces the usual results of Fulling, Unruh and others(2,5,6). The analysis requires a 3+1 massive solution which has only been discussed previously in ref.(4) and in ref.(6). The actual analysis was somewhat tedious, involving momentum integrations with respect to order of modified Bessel functions(7) (these are Kontorovich-Lebedev transforms(8)).

The Hawking effect arises because of the ambiguity in the choice of the evolution surfaces of the theory, i.e. the ambiguity of the choice of Hamiltonian. Consider the d+1 dimensional Rindler space as defined by the coordinate transformations:(9)

$$\left.\begin{aligned} x_0 &= g^{-1} e^{g\xi_R} \sinh g\eta \\ x_1 &= g^{-1} e^{g\xi_R} \cosh g\eta \\ \vec{x}_\perp &= \vec{x}_\perp' \end{aligned}\right\} \begin{matrix} x_1 > 0 \\ (R) \end{matrix} \qquad \left.\begin{aligned} x_0 &= -g^{-1} e^{g\xi_L} \sinh g\eta \\ x_1 &= -g^{-1} e^{g\xi_L} \cosh g\eta \\ \vec{x}_\perp &= \vec{x}_\perp' \end{aligned}\right\} \begin{matrix} x_1 < 0 \\ (L) \end{matrix} \tag{1}$$

These define the L and R Rindler wedges.

The evolution of the wave-functional in Minkowski space can be given on a family of surfaces labelled by coordinate time x^0 and we thus have the Schroedinger equation:

$$H_M \Psi(\varphi, x^0) = i \frac{\partial}{\partial x^0} \Psi(\varphi, x^0) \tag{2}$$

Thus H_M admits a groundstate, Ψ_M, which we take to be the true physical groundstate of the world. All observers, accelerating or not, make measurements in Ψ_M.

However, the natural evolution for a Rindler observer occurs along surfaces labelled by the coordinate time η. We may construct a Hamiltonian which explicitly generates this evolution[4] and obtain the Schroedinger equation:

$$H_R \Psi(\varphi, \eta) = i \frac{\partial}{\partial \eta} \Psi(\varphi, \eta) \tag{3}$$

H_R propagates any state on the surfaces labelled by η. The groundstate of H_R is a wave-functional Ψ_R, which is inequivalent to Ψ_M, yet is the natural definition of the groundstate as defined by the accelerating observers. We see that at $t=\eta=0$ the states are defined on a common hypersurface and may be compared. Relative to Ψ_R the state Ψ_M appears to be excited and the particle number operator in Rindler momentum is that of a Bose gas of temperature $T_H = g/2\pi$ (2). The density matrix as seen by an observer on the R wedge is thermal upon integrating over the degrees of freedom on the L wedge.(5) Explicit representations of H_R, H_M, Ψ_R, Ψ_M are given in ref.(4) and the thermal energy distribution may be read off immediately when Ψ_M is expressed in terms of the Rindler mode coefficients.

In thermal equilibrium we typically wish to compute the expectation of an operator, $\mathcal{O}$, with the density matrix and subtract its expectation at T=0, i.e. compute:

$$\mathrm{Tr}(\rho_T \mathcal{O}) - \langle 0|\mathcal{O}|0\rangle \tag{4}$$

where ρ_T is the thermal density matrix. What is the analogous computation in the present case? Clearly, from the point of view of the accelerating observer we wish to compute the difference:

$$\langle \Psi_M|\mathcal{O}|\Psi_M\rangle - \langle \Psi_R|\mathcal{O}|\Psi_R\rangle \tag{5}$$

since Ψ_M is the thermally excited state in which the observer exists and Ψ_R is the (fictitious) relative groundstate of zero temperature. Indeed, we find that such differences as in eq.(5) do lead to the usual thermal results. The analogous problem in the massless case is implicit in the analysis of ref.(10).

Consider for example the expectation value of φ^2 as computed in an unbroken theory in Ψ_M (we compute a point-split quantity to regularize the infinities and we take, for technical reasons, the point-split in the direction of acceleration; this was found to be a convenient method of regularizing the longitudinal momentum integrals over the order of modified Bessel functions which occur in the solution of the Rindler wave-functional):

$$\langle \Psi_M | \varphi(x_1 + \tfrac{\varepsilon}{2}) \varphi(x_1 - \tfrac{\varepsilon}{2}) | \Psi_M \rangle$$
$$= \varphi_c^2 + \frac{\mu}{4\pi^2 \varepsilon} K_1(\mu\varepsilon) \qquad (6)$$

This is the usual Feynman propagator for spacelike separation ε. We have explicitly checked that this result obtains both in the Minkowski mode representation of Ψ_M and in the Rindler mode representation of Ψ_M. This constitutes an explicit verification of the general covariance of the functional Schroedinger formalism(4). Here μ is the mass parameter associated with the state as discussed above eq.(10) below.

We may carry out the same evaluation in the inequivalent Rindler groundstate, Ψ_R, to obtain:

$$\langle \Psi_R | \varphi(x_1 + \tfrac{\varepsilon}{2}) \varphi(x_1 - \tfrac{\varepsilon}{2}) | \Psi_R \rangle = \varphi_c^2 + \frac{\mu}{4\pi^2 \varepsilon} K_1(\mu\varepsilon) \qquad (7)$$
$$- \frac{1}{4\pi^2} \int_0^\infty \frac{d\omega}{\pi^2 + \omega^2} \frac{\sqrt{2}\mu}{x_1} (1 + \cosh\omega)^{-\frac{1}{2}} K_1\left(\sqrt{2}\, x_1 \mu (1 + \cosh\omega)^{\frac{1}{2}}\right)$$

We note the appearance of the third term on the rhs in which the limit ε --> 0 has been taken. This term arises in the Kontorovich-Lebedev transform of the form $\int_0^\infty \exp((\pi - \varepsilon')x) K_{ix}(\alpha) K_{ix}(\beta) dx$ which occurs in the difference of the expressions in Minkowski and Rindler space(3). This term always enters such expressions with a minus sign. The singular part of eq.(7) is identical to that of eq.(6) as it must be; we are in no way tampering with the short-distance part of the theory and thermal effects are purely infra-red. We now consider the difference of these results as in eq.(5). We take the ε --> 0 and, corresponding to a high temperature approximation, $x_1 \to 0$, making use of the small argument limit of the $K_\nu(z)$ functions:

$$\langle \Psi_m | \varphi(x_1)^2 | \Psi_m \rangle - \langle \Psi_R | \varphi(x_1)^2 | \Psi_R \rangle$$
$$\to \frac{1}{4\pi^3 x_1^2} \int_0^\infty \frac{du}{(1+u^2)(1+\cosh \pi u)} = \frac{1}{48\pi^2 x_1^2} \qquad (8)$$

Noting that the proper acceleration of a Rindler observer at $x = g^{-1}\exp(g\xi)$ is just $g\exp(-g\xi) = 1/|x| = g(x)$, we see that we must, in configuration space, adopt a local definition of the Hawking temperature, $T_H = g(x)/2\pi$ (this is well known; we have made use here of the analysis by Candelas and Deutsch[10] in which such a result is implicit. This does not conflict with the momentum space result which may be considered an average on the manifold). Thus, we see that:

$$\langle \Psi_m | \varphi^2(x_1) | \Psi_m \rangle - \langle \Psi_R | \varphi^2(x_1) | \Psi_R \rangle \Rightarrow \frac{T_H^2(x_1)}{12} \qquad (9)$$

which is precisely the correct result for a scalar field in thermal equilibrium at high temperature T_H. Thus the accelerating observer witnesses the correct thermal contribution to the difference in eq.(9), but <u>it occurs not because the Minkowski vacuum expectation value is enhanced by an amount $T^2/12$, but rather because the Rindler groundstate expectation value is depressed by an amount $-T^2/12$</u>. In actual fact, the accelerating observer measures $\langle \varphi^2 \rangle$ to be that given in eq.(6) which is interpreted as the thermally corrected value; the thermal terms have added to and cancelled the third term of eq.(7) and the general invariance of φ^2 is maintained.

Similar results follow for the energy density which we take to be T_{oo}. It is well known that in the massless case the expectation value of this quantity in the Rindler groundstate receives a negative correction of the form $-aT^4$ and similar results are known for black-holes in the Schwarzschild groundstate(10,9). To address the issue of symmetry restoration we must go beyond these massless field analyses.

In an unstable φ^4 theory we may choose a gaussian wavefunctional centered about φ_c which has a mass parameter μ minimizing the expectation value of the energy density. This may be obtained by a variational calculation. For an unbroken theory we have $\varphi_c=0$ and $\mu=m$, the Hamiltonian mass of the scalar field. If the theory is broken with the potential $-m^2\varphi^2/2 + \lambda\varphi^4/4!$ we have $\varphi_c \neq 0$ and μ^2 is the curvature of the potential at φ_c. We thus find for μ^2 and the expectation value for a point split(3) T_{oo}:

$$\langle \Psi_m | T_{oo} | \Psi_m \rangle = -\frac{1}{2} m^2 \varphi_c^2 + \frac{1}{4!} \lambda \varphi_c^4 - \frac{1}{2\pi^2} \frac{\mu^2}{\varepsilon^2} K_2(\mu\varepsilon)$$

$$\mu^2 = -m^2 + \frac{1}{2} \lambda \varphi_c^2 \qquad (10a,b)$$

The $\mu^2\varepsilon^{-2}K_2(\mu\varepsilon)$ is quartically divergent in $\varepsilon \to 0$. Upon appropriate cosmological constant, mass and coupling constant renormalization we are left with the usual effective potential to $O(\hbar)$.

We have computed the corresponding quantity in the Rindler Hamiltonian groundstate. Here there occurs an overall factor of $\exp(2g\xi)$ which at $t=\eta=0$ corresponds to the covariant transformation of T_{oo} from Rindler to Minkowski coordinates. The singular term of eq.(10)

appears and again we find finite additional negative terms involving K_1 and K_2 functions which are somewhat too lengthy to reproduce here. In the limit $x_1 \to 0$ we obtain the leading terms in the effective high temperature expansion in the form:

$$\langle \psi_M | T_{00} | \psi_M \rangle - e^{-2g\xi} \langle \psi_R | T_{00} | \psi_R \rangle$$

$$\rightarrow \; a T_H^4(x_1) + \frac{1}{24} \mu^2 T_H^2(x_1) \tag{11}$$

These are the usual high temperature contributions. We emphasize that the numerical value of the Stefan-Boltzmann constant depends upon the choice of stress-tensor and is the usual result, $\pi^2/30$, for "new-improved" stress-tensors only. We have studied the usual unimproved stress-tensor since interactions and mass-terms already break conformal invariance (i.e. we neglect conformal coupling to gravity in deriving $T_{\mu\nu}$). For the unimproved case we obtain $a=11\pi^2/240$ (the new-improved stress-tensor is conformally invariant and more closely resembles a radiation stress-tensor through its tracelessness[10]).

In true thermal equilibrium the second term on the rhs of eq.(11) is normally responsible for symmetry restoration at high temperature with μ^2 given by eq.(10). However, again the thermal corrections arise because the Rindler expectation value is depressed by the negative of the rhs of eq.(11). In actual fact, the accelerating observer measures the zero-temperature effective potential as obtained in eq.(10) in Minkowski space with its minimum at φ_c. Thus, the true physical minimum is always that of the broken symmetry as measured by all observers, and the invariance of the order parameter φ_c merely corroborates this fact.

Other corollaries follow immediately. For example, if the Minkowski vacuum is supersymmetric we expect the supersymmetry to remain intact for all observers, but perhaps be broken by the negative pseudo-thermal terms in the Rindler groundstate. Quark-deconfinement is also not expected to be witnessed by accelerating observers. The use of naive quark fragmentation ideas to treat the UHE black-hole spectrum thus becomes suspect. The present analysis casts doubt upon the naive interpretations of Hawking temperature effects in inflationary cosmologies as well.

Do these arguments apply to real evaporating black-holes? Here we must contend with the issue of non-zero curvature. If, for example, the scalar fields are conformally coupled to gravity then we may expect that the $\xi R\varphi^2$ term might lead to a restoration of a broken symmetry, but such would be common to Kruskal and to Schwarzschild observers alike and is not related to the Hawking effect. Insofar as the black-hole metric may be treated as static, the above arguments suggest that no restoration of symmetry can occur. But energy conservation dictates that the metric must relax, i.e. become time dependent as the radiation leaks across a surface boundary. The time dependence of the metric during evaporation is a further correction to the effective potential and we have no insight into its consequences. Thus, we believe that the restoration of symmetry for real black-holes, should it occur at all, would appear as a geometry effect and not in any sense a Hawking-thermal effect.

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