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A CASE AGAINST BARYONS IN GALACTIC HALOS

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Abstract

The possibility that galactic halos are composed of baryonic matter is discussed in detail. Specifically, halos composed of snowballs, dust and rocks, planets, stars, dead stellar remnants, and hot and cold gas are considered. The serious problems that would arise for each of these types of matter lead us to the conclusion that halos cannot plausibly contain substantial amounts of such matter. This conclusion is considered in relation to big bang nucleosynthesis. Various non-baryonic candidates for halos are also discussed.

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I. INTRODUCTION

Evidence supporting the existence of halos around spiral galaxies (Faber and Gallagher 1979) is quite compelling. It is known that halos contain a large amount of mass, several times the mass contained in the luminous parts of galaxies, and this mass is either sub-luminous or non-luminous. On the other hand, the nature of the halo mass is quite unclear.

An obvious choice for the halo matter is that it is ordinary nucleonic matter which at the present time is not contained in nuclear burning stars. It is precisely because halos are so dark that it is very difficult to rule out this hypothesis. Instead, in the following, we will try to show that it is very implausible for galactic halos to be made of ordinary baryonic matter (Hegyi and Olive 1983).

Similar arguments to several of those that we will present here were first raised by Burbidge and Hoyle (1963) in their efforts to understand the existence of non-thermal radio emission from a spherical volume surrounding the Galactic disk; this emission was interpreted as originating from a galactic halo (Baldwin 1955). To compensate for the difficulty of maintaining a galactic halo of hydrogen gas Burbidge and Hoyle proposed that the halo might be a transient phenomena.

Today, there are a number of reasons for believing that spiral galaxies possess massive halos. One of the earliest discussions of massive halos surrounding spiral galaxies was given by Hohl (1973, 1975). He found his models of spiral disks to be unstable with respect to the growth of long wavelength modes, and as a result, the disks tended to develop into bar-shaped structures within about two revolutions. Hohl was able to stabilize his models by adding a fixed central force which he identified with a halo population of stars and the central core of the galaxy. Kalnajs (1972), considering only exact solutions for infinitely thin spiral disks, explored ways of stabilizing the initially cool rotational state. Perhaps his most interesting result was that by embedding the spiral disk in a uniform density halo, stability could be obtained.

The possibility that spiral galaxies might be surrounded by massive halos was emphasized by Ostriker and Peebles (1973). Using a 300-star galactic model they studied the instability of spiral structure to the development of bar-like modes. The onset of instability was reached when t, the ratio of the kinetic energy of rotation to the total gravitational energy, increased to a value ~ 0.14. From a literature survey, the authors concluded that for systems ranging from fluid MacLaurin spheroids to flat galactic systems with 10^5 stars, the critical value for the onset of instability appears to be $t \approx 0.14$. Two different ways were suggested to stabilize the spiral structure: a hot disk population with radial orbits and a hot spherical halo. From a variety of arguments, it is now known that the halo mass distribution is spherical.

The strongest evidence for the existence of halos is based on the rotation curves of spiral galaxies. Rotation curves have been measured by both optical and radio techniques (Rogstad and Shostak 1972, Roberts and Rots 1973, Haschick and Burke 1975, Roberts 1975, Sancisi 1977, Rubin, Ford and Thonnard 1978, Krumm and Salpeter 1979). The data obtained on more than 50 spiral galaxies indicate symmetric rotation curves which must satisfy the equilibrium condition that the gravitational force is balanced by the centrifugal force so that

$$M_r = (K/G)v^2 r \tag{1.1}$$

where K is a constant ranging from $2/\pi$ for a thin disk to unity for a sphere, G is the gravitational constant, v is the rotational velocity at radius r, and M_r is the mass contained within r. The observations clearly show that v is independent of r at large radii. Thus we see that $M_r \propto r$. This is quite unlike the distribution of luminous matter. To explain the rotation curves it is necessary to postulate the existence of a dark massive halo surrounding the galaxy.

Throughout the following, we shall take a halo containing 10^{12} $M_\odot$ within 100 kpc radius as our standard halo. This yields an average halo density

$$\rho_H = 1.6\times10^{-26} \text{ gcm}^{-3} \tag{1.2}$$

We estimate the fraction of the critical density contained in halos by means of mass-to-light ratios. On the scale of binary galaxies, the mass-to-light ratio in halos, $(M/L)_H$, is

$$(M/L)_H > 70 \; h_0 \; , \tag{1.3}$$

(Faber and Gallagher 1979) where h_0 is defined by

$$h_0 = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \tag{1.4}$$

and H_0 is the present value of the Hubble parameter. We take $1/2 < h_0 < 1$. The fraction of closure density in halos, Ω_H, is then derived from

$$\Omega_H = \frac{(M/L)_H}{(M/L)_c} \tag{1.5}$$

where

$$(M/L)_c = 1400 \; h_0 \tag{1.6}$$

is the critical mass to light ratio in these units. Thus we will take $\Omega_H >$ 0.05.

In the sections which follow, we examine the possible types of baryonic matter. We begin in section 2 by considering that the halo is composed primarily of hydrogen. We consider first that the hydrogen is trapped in very cold electrostatically bound objects (snowballs) and find that such objects (if they could form) would quickly evaporate. A halo of hydrogen gas would quickly heat up to establish hydrostatic equilibrium. However, constraints from the X-ray background lead to substantial restrictions on the amount of mass which could be in the form of hot gas.

In section 3, we consider the possibility that low mass stars or Jupiters make up the halo. Using optical and infrared observations which set limits on the mass-to-light ratios of halos, we find limits on the slope of the initial mass function for a halo of stars and Jupiters. We then show that this slope is incompatible with observations of existing stellar systems.

In section 4, more massive stars which would have evolved to white dwarfs and neutron stars are considered. The problem here is that massive halo stars would lead to substantial heavy element contamination in later generations of stars. A non-stellar metallic halo would also lead to contamination of later generations of stars and so may also be excluded.

In section 5, we discuss black holes even though they do not have well defined baryon numbers. If they are able to accrete their ejecta very efficiently, black holes remain a possibility for the halo matter.

In section 6, we compare these results to what we would expect from big bang nucleosynthesis. In order that the computed abundances of the light elements agree with observation one can derive a lower limit to the nucleon abundance (Yang et al. 1984) indicating that the portion of closure density in the form of baryons is $\Omega_B > 0.01\ h_o^{-2}$. For $h_o \sim 1/2$, this means that $\Omega_B \gtrsim 0.04$, or comparable to the density of halos. For h_o as low as 1/2, to make nucleosynthesis compatible with our results, assuming that the baryons are trapped in halos, we claim that the baryons must be trapped in black holes. Finally, in section 7, we discuss the non-baryonic possibilities for galactic halos focusing on aspects of galaxy formation.

II. HYDROGEN

a) Snowballs

We begin our discussion by considering the possibility that halos consist of snowball-like objects, i.e. cold condensed hydrogen bound by electrostatic forces, with binding energies ~1 eV. We will assume that the average density of the snowballs is about that of solid hydrogen $\rho_s \sim 7 \times 10^{-2}$ g-cm^{-3}. Given that these objects would have typical halo velocities ~250 kms^{-1} or kinetic energies of ~600 eV per molecule, it is clear that a single collision between them would be destructive. We must then require that the time between collisions, Γ_c^{-1}, be longer than the present age of the universe, τ_u. Thus we require

$$\Gamma_c \tau_u < 1 \tag{2.1}$$

where $\tau_u \sim 1.5 \times 10^{10}$ yrs. and the collision rate is

$$\Gamma_c = n\ \sigma\ v \tag{2.2}$$

where

$$n = \frac{\rho_H}{m_s} \tag{2.3}$$

is the number density of snowballs of mass m_s needed to provide the overall density in the halo. The cross-section, σ, is just the geometric one

$$\sigma = \pi r_s^2 = \pi \left(\frac{3m_s}{4\pi\rho_s}\right)^{2/3} \tag{2.4}$$

Thus we have that

$$m_s \gtrsim \rho_H^3 v^3 \tau_u^3 \rho_s^{-2} \approx 1 \text{ g} \tag{2.5}$$

corresponding to snowballs with radii $r_s \gtrsim 2$ cm.

Under these conditions (collisionless formation) snowballs must have condensed out at a time when the overall density was equal to the present halo density ρ_H. In an $\Omega = 1$ Universe, we can compute the redshift at which $\rho = \rho_H$. This will occur when the cosmic time, τ, is equal to twice (allowing for the expansion from the big bang) the collapse timescale of the halo, τ_c, where

$$\tau_c = \left(\frac{3\pi}{32G\rho}\right)^{1/2} \tag{2.6}$$

and the redshift for the formation of snowballs is

$$1 + z = \left(\frac{2}{3H_o\tau}\right)^{2/3} \approx 3.5h_o^{-2/3} . \tag{2.7}$$

We can at best expect that the snowballs form as late as $1 + z = 3.5$ at a temperature

$$T = T_o(1 + z) \approx 9.5°K . \tag{2.8}$$

where T_o is the present temperature of the cosmic background radiation.

At these temperatures, one must now ask whether or not solid hydrogen can remain in equilibrium with the gaseous state. In fact, the vapor pressure of solid hydrogen can be found to be .93 mm Hg (Johnson 1960) which is sufficiently high so that there is no equilibrium between the two phases. In equilibrium, all the hydrogen is gaseous. Using standard two phase thermodynamic equilibrium methods (see eq. Sears 1950), one can show that the halo density would have to increase by a factor of more than 10^{21} before any equilibrium could exist between the gaseous and solid phases.

Having established that snowballs will eventually evaporate, we now calculate the rate of evaporation. The timescale for the evaporation of an H_2 molecule is,

$$\tau_{ev} \sim \nu_0^{-1} \exp(b/kT) \ , \tag{2.9}$$

where $\nu_0 \sim 10^{12}$ s^{-1} is the characteristic vibration frequency of a solid hydrogen lattice and b is the binding energy of an H_2 molecule to the surface of the solid. Using the principle of corresponding states, we estimate b by taking the depth of a Leonard-Jones potential, and find that $b \sim .77kT_c$. The critical temperature, T_c, is 33.3 °K for H_2. With these values of ν_0 and b in eq. (2.9) it may be seen that the evaporation timescale is very short, $\tau_{ev} \sim 10^{-11}$ s at T = 9.5 °K. We note that even today at T = 2.7 °K the evaporation timescale is only $\tau_{ev} \sim 10^{-8}$ s. To see that such a timescale will totally evaporate any snowball, let us compute the minimum radius, R, of a snowball which survives a gravitational orbital period of about 10^{16} s. Using an evaporation timescale at 2.7 °K, i.e. $\tau_{ev} \sim 10^{-8}$ s, it may be seen that $R \gtrsim 10^{16}$ cm. Clearly, such an object is not electrostatically bound but instead gravitationally bound. This type of object will be discussed below.

b) Cold Gas

If we suppose that the halo is composed of cold gas we must demand that either the collapse timescale for the gas is very long to avoid heating or the gas is in hydrostatic equilibrium and its temperature is sufficiently low to avoid the observed upper limits on the background X-ray flux, (see next subsection). It is easy to see that the gas must be in hydrostatic equilibrium; the collapse timescale at halo density ρ_H is just $\tau_c \sim 5 \times 10^8$ yrs. which is much less than the age of the galaxy.

Let us now see whether a hydrogen gas halo in equilibrium can be cold. For the equation of state, we use the ideal gas form for a ionized hydrogen halo,

$$P_r = (2\rho_r/m_p)kT \tag{2.10}$$

where P_r and ρ_r are the pressure and mass density at radius r from the center of the halo and m_p is the proton mass. In equilibrium,

$$dP_r/dr = - G M_r \rho_r/r^2 . \tag{2.11}$$

Solving equations (2.10) and (2.11) for the equilibrium temperature we find that

$$T_{eq} = (Gm_p/4k)(M_r/r) \approx 1.3 \times 10^6 \text{ °K} \tag{2.12}$$

for our standard halo with $M_r \approx 10^{12} M_\odot$ and $r \approx 100$kpc. We note, however, that various cooling processes will be operative which may be expected to cool the hot halo on a timescale which is short compared to H_0^{-1}. Therefore, unless the halo has substantial energy pumped into it to maintain the equilibrium temperature, it will collapse and star formation may be expected to occur. Thus it is not possible to consider a cold gaseous halo.

c) Hot Gas

We will now show that even if it were possible to maintain the halo at $T \sim 10^6$ °K, it would emit an X-ray flux which is in conflict with limits on the observed X-ray background. Since the presumed halo gas is only contained within halos and does not extend into the space between galaxies, it is convenient to define a clumpiness parameter C,

$$C \equiv \langle\rho^2\rangle/\langle\rho\rangle^2 \tag{2.13}$$

where ρ is the average mass density of the universe. The product of C and Ω can be put into the useful form (Field 1972)

$$C\Omega = \rho_H/\rho_c \tag{2.14}$$

where ρ_c, the critical density, is $\rho_c = 1.88\times10^{-29}\ h_o^2 g\text{-cm}^{-3}$. Silk (1973) has compiled constraints on the quantity $\delta C\Omega^2$ where $\delta \approx h_o^3$. Using $C\Omega \approx 900\ h_o^{-2}$ and $\Omega \geqslant \Omega_H = 0.05$, we have for halos with $h_o > 1/2$,

$$\delta C\Omega^2 \gtrsim 22\ . \tag{2.15}$$

Silk's (1973) constraints based on the observed x-ray background (adjusted for our normalization of h_o) limit this quantity at $T = 10^6$ °K to

$$\delta C\Omega^2 \lesssim 3\ . \tag{2.16}$$

For higher temperatures this constraint becomes more severe and thus we find an inconsistency with the supposition that the halo is made of hot gas.

Suppose instead of a single gaseous cloud, the halo consisted of several smaller clouds. These clouds could be cooler and perhaps they would avoid the above constraint. However, there is a lower limit to the density of the clouds in order that they avoid cloud-cloud collisions over the age of the galaxy. Once again we will use a geometric cross-section for collisions so that the rate for collisions will be

$$\Gamma = \pi R_c^2 \, \rho_H \, v/M_c \tag{2.17}$$

where R_c is the average cloud radius, $v \sim 3 \times 10^7$ cm/sec is the relative velocity between clouds and ρ_H/M_c is the number density of clouds. The requirement that $\Gamma^{-1} > 1.5\times10^{10}$ yrs implies that

$$R_c^2 \lesssim 1 M_c(g) \ \mathrm{cm}^2 \tag{2.18}$$

or

$$\rho_{cloud} \gtrsim 3 \times 10^{-18} \, M_c^{-1/2}(M_\odot) \ \mathrm{g\ cm}^{-3} \tag{2.19}$$

where $M_c(g)$ and $M_c(M_\odot)$ are the cloud masses in grams and in solar mass units, respectively. The temperature of each cloud is again found from the assumption of hydrostatic equilibrium (2.12)

$$T = 2 \times 10^{-16} \, [M_c(g)/R_c(cm)] \tag{2.20}$$

If we let N be the number of clouds in the halo, we see that for $N < 50$ the temperature of each cloud is $> 10^6$ °K and this case may be excluded. The case of gaseous clouds in clusters of galaxies has been considered by Goldsmith and Silk (1972) and Tartar and Silk (1974) and we will draw on some of their arguments. From (2.18) and (2.19) we see that the clumpiness of the gas clouds is greatly increased relative to the single cloud halo. In this case $C\Omega = \rho_{cloud}/\rho_c$ and we can write

$$\delta C\Omega^2 \gtrsim 4\times10^3 \, N^{1/2} \, . \tag{2.21}$$

The case of $N < 500$, for example, may also be excluded. For $N = 500$, $T \approx 4\times10^5$ °K and $\rho_{cloud} \approx 7\times10^{-23}$ g-cm^{-3}. Though the X-ray limits are less stringent by a factor of $\exp[1.3\times10^6/4\times10^5] = 12$ relative to the case of N=1, the clumpiness increases by a factor of $\sim1.4\times10^4$ which is enough to rule out this case.

However, for a sufficiently large number of clouds, the cloud temperature will be low enough so that the X-ray observations will not provide any constraint. As indicated above, these clouds may be expected to cool quickly unless some energy source can be found to keep them heated. In a similar discussion on the missing mass in the Coma cluster, Tarter and Silk (1974) considered a population of M8 dwarfs contained within the clouds to provide the necessary energy flux. Although this cannot be ruled out, we agree with their conclusion that it is very unlikely that only M8 stars formed without any more massive brighter ones as well.

III. LOW MASS STARS AND JUPITERS

a) Mass Ranges

The most interesting candidate (and the one we will discuss in the most detail) for a baryonic halo is low mass stars or Jupiters (Dekel and Shaham 1979, Gott 1981, Hegyi 1981, Hohfeld and Krumm 1981). We will distinguish two mass scales for these stars or planets. The maximum mass for a Jupiter is determined by the onset of nuclear burning; that is, for $m > m_0$ the object is a luminous star. We take $m_0 = 0.08\ M_\odot$. The lower limit on the mass of Jupiters, m_{min}, is much less certain. Our definition for m_{min} will be that it is the lowest mass object which can collapse gravitationally.

Our choice for m_{min} comes only from theoretical arguments. Because of this, we will test our results using different values of m_{min}. Calculations for the lower limit to the Jeans mass by Low and Lynden-Bell (1976) and Rees (1976) give $m_{min} = 0.007\ M_\odot$. Silk (1982) finds $m_{min} = 0.005\ M_\odot$ and Palla, Salpeter and Stahler (1983) claim $m_{min} > 0.004\ M_\odot$ based on a treatment of optically thin clouds.

Although we have discussed m_{min} as a lower limit to the Jean's mass, it is not clear whether such objects actually form. Indeed it has been argued (Tohline 1980) that the first generation of stars had masses greater than 1500 $M_\odot$. Being as conservative as possible, we will adopt the value $m_{min} = 0.004\ M_\odot$.

Given this mass range for Jupiters, we are now in a position to place a limit on the halo mass in Jupiters. The most important assumption we will make is the use of a single power law for the initial mass function for low mass stars and Jupiters. In fact, it would be surprising if the initial mass function became discontinuous in any way at m_0; m_0 is primarily determined by the lowest temperature at which nuclear burning occurs. The initial mass function, on the other hand, should not be determined by nuclear physics but rather by gravity and atomic physics since no nuclear interactions occur during the fragmentation and collapse stages of the formation of the halo. Consequently, we feel justified in assuming a single power law. The usefulness of this assumption is that by placing constraints on the luminous portion of the initial mass function we are able to constrain the Jupiters as well.

The initial mass function is defined by

$$\phi(m) = Am^{-(1+x)} \tag{3.1}$$

where $\phi(m)$ is the number of stars formed per unit volume per unit mass. In the following we will not assume any value for x, but will set limits on it from the observations. As we will see, in order to put the missing mass in the form of Jupiters, we require a slope parameter $x > 1.7$. However, observations of stellar systems suggest a much smaller slope.

Let us now define the quantities which are calculable in terms of the initial mass function $\phi(m)$. The total mass density in Jupiters and low mass stars is given by

$$\begin{aligned} \rho_m &= \int_{m_{min}}^{m_G} m\phi(m)dm \\ &= \frac{A}{1-x}\left[m_G^{1-x} - m_{min}^{1-x}\right] \end{aligned} \tag{3.2}$$

where $m_G \approx 0.75\ M_\odot$ is taken to be the mass of a giant (the results are not very sensitive to this choice) and m_{min} is the above described minimum mass for a collapsed object. The second quantity that we will need is the luminosity density due to these low mass stars and it is defined by

$$\rho_L = \int_{m_o}^{m_G} L(m)\phi(m)dm + \rho_G \tag{3.3}$$

where $L(m)$ is the luminosity of a star of mass m, m_o, the cutoff for nuclear burning is taken to be $m_o \approx 0.08\ M_\odot$ and ρ_G is the luminosity density due to giants.

To determine the form of $L(m)$ we have used spectral data of Tinsley and Gunn (1976) for dwarf stars in the range 0.091-0.69 $M_\odot$. We find that to a good approximation, this data can be fit by the power law form

$$L(m) = cm^D \; . \tag{3.4}$$

Since the observations to which we will compare these calculations were made in the I and K Johnson spectral bands, we have computed c and D for these two cases. In the I band, $c = 1.49\times10^{-2}$ and $D = 2.71$, and in the K band, $c = 3.12\times10^{-2}$ and $D = 2.11$. The normalizations were chosen so that an $L(m)$ equal to unity corresponds to zero magnitude and m is expressed in solar masses. Using (3.4) the luminosity density (3.3) becomes

$$\rho_L = \frac{Ac}{D-x}\left[m_G^{D-x} - m_o^{D-x}\right] + \rho_G \quad . \tag{3.5}$$

To calculate the contribution to the luminosity due to the giants, we use the method and data of Tinsley (1976) and Tinsley and Gunn (1976). Then, we have

$$\rho_G = \ell N_G \tag{3.6}$$

where ℓ is the luminosity of a giant and N_G is the number density of giants. From Tinsley and Gunn (1976) we find $\ell_I = 0.692$ and $\ell_K = 2.64$. The number of giants, N_G, can be expressed as

$$N_G = -\tau_G\left(\frac{dN}{dm}\right)\left(\frac{dm}{dt}\right) \tag{3.7}$$

where $\tau_G = 7.3\times10^8$ yrs is the stellar lifetime as a giant and dm/dt follows from (Hejlesen et al. 1972)

$$m = at^{-\theta} \tag{3.8}$$

where t is the age of the star as it becomes a giant. We have taken (Tinsley 1976) $a = 366.4$ and $\theta = 0.265$. The value of a has been corrected using the calculations of Sweigart and Gross (1978) for giants of 0.7 $M_\odot$ in order to take into account a lower metal abundance. Tinsley (1976) has used $Z = 0.01$ and we are interested in $Z \approx 10^{-5}$. We take the correction to the lifetime to be proportional to exp[28.6Z-.286] for small z. The lifetimes of stars with $Z < 10^{-5}$ are not further affected.

We have used $m_G = 0.75\ M_\odot$ for the mass of a giant which by (3.8) corresponds to $t = 1.4 \times 10^{10}$ yrs. In terms of the initial mass function, (3.7) becomes

$$N_G = A\tau_G \theta\ a^{-x}\ t^{\theta x - 1} \qquad (3.9)$$

The comparison of our calculations to observations will be in terms of a mass-to-light ratio which is normalized to solar units in the I and K bands. In these bands we have $(L/M)_{I\odot} = 2.47 \times 10^{-2}$ and $(L/M)_{K\odot} = 4.32 \times 10^{-2}$. The quantity we will calculate therefore is the following,

$$Q = (\rho_m/\rho_L)(L/M)_\odot \qquad (3.10)$$

As one can see, Q depends only on x and our choice of m_{min}. (The slight dependence on m_G is irrelevant here). In Figs. 1 and 2, we plot the value of log Q as a function of x for different values of m_{min} in the I and K bands, respectively.

The observations to which we compare these calculations were made on NGC 4565 (Hegyi 1981 and Boughn, Saulson and Seldner 1981). To calculate M/L for the Halo of NGC 4565, we shall use $M/L = \rho_m/\rho_L = \sigma_m/\sigma_L$, where σ_m and σ_L are the projected mass and luminosity densities. It is necessary to evaluate the projected halo mass density in terms of the 21 cm rotational velocity 253 Kms^{-1} (Krumm and Salpeter 1979) and the maximum extent of the halo, R_{max}. This may be seen to be

$$\sigma_m = \frac{v^2}{2\pi G}\frac{1}{r}\tan^{-1}\sqrt{\left(\frac{R_{max}}{r}\right)^2 - 1} \qquad (3.11)$$

at galactic radius r. The distance to NGC 4565 is unlikely to be larger than 24 Mpc, and since the rotation curve has been observed out to 11.6', $R_{max} > 81$ Kpc.

We now turn to the observational data on the surface brightness of the halo of NGC 4565. Data taken with the annular scanning photometer (Hegyi and Gerber 1977) in the Kron I band has been discussed by Hegyi (1981). That data has been transformed to the Johnson system and expressed in solar units. A least squares fit to that data using the functional form $\sigma_L = a/r + b$ has been performed. (This functional form assumes that R_{max} is large compared to r so that the $\tan^{-1}$ function in eq. 3.17 reduces to $\pi/2$.) A 2σ lower limit to σ_m/σ_L can be related to a 2σ upper limit on a. Shown in Fig. 3 is the I band surface brightness data (Hegyi 1981) plotted as a function of galactic radius. The solid line is the 2σ upper limit to the halo surface brightness. The photometric conversion factor is 10^{-5} counts/scan/arc sec^2 is equal to 25.34 mag/arc sec^2 I_{Kron}. The zero on the scale is an estimate of the sky brightness and does not affect the upper limits obtained. When the 2σ upper limit on a is converted into a lower limit on the mass-to-light ratio in solar units in the Johnson I band (the transformation is insensitive to color for M type stars) it is

$$Q_I = M/L_I > 76\ M_\odot/L_{\odot,I}\ . \tag{3.12}$$

Observations in the K band have been made by Boughn, Saulson, and Seldner (1981) using a chopping secondary. Their 2σ lower limit is

$$Q_K = M/L_K > 38\ M_\odot/L_{\odot,K} \tag{3.13}$$

We now ask what value of x is consistent with the observational data. The strongest constraints on x come from observation of spectral features in elliptical galaxies (Tinsley and Gunn 1976, Tinsley 1978, Gunn, Stryker and Tinsley 1981) and indicate that $x < 1$. A fit to observations in the range (0.1-1) $M_\odot$ by Miller and Scalo (1979) yields a much lower value of x for low mass stars, x=0.4. Photometric data ranging from globular clusters to elliptical galaxies can be fit by the weaker constraint $x < 1.35$ using a single free parameter, the metal abundance (Aaronson et al. 1978 and Frogel et al. 1980). From a theoretical point of view Silk and Takahashi (1979) claim that $0 < x < 1$.

Let us now look at the results. In Table 1, we list the lower limits on x in I and K bands for m_{min} = 0.004, 0.005 and 0.007 $M_\odot$. As one can see, in each case we have $x > 1.7$ from which we conclude that a halo filled with Jupiters is not consistent with the available observational data on the initial mass function.

It is also possible to choose a value for x and determine the constraints on m_{min}. For $x < 1$ there is no solution for m_{min} consistent with the observations summarized by (3.12) and (3.13). For x=1.35 we need $m_{min} \lesssim 10^{-4} M_\odot$ which is about a factor of 40 lower than the calculated lower limits.

TABLE 1

Lower Limits on x

$m_{min}/M_\odot$	I	K
.004	1.74	1.79
.005	1.80	1.84
.007	1.88	1.92

The major assumption that we have made was to use a single power law initial mass function for both the stars and Jupiters. Without this assumption, we cannot draw our conclusions. For example, if the initial mass function was a delta function peaked at Jupiter masses, we could not rule it out. However, such a proposal was made by Kashlinsky and Rees (1983) in which the halo was formed from a number of massive objects which interacted tidally imparting angular momentum to each other. Subsequently, they cooled down to form a thin disk and fragmented to Jupiters. But it appears (Hegyi 1984) that cooling to the extent necessary to form a thin disk does not satisfy the Toomre (1964) stability criteria. A disk which is hot enough for stability is too hot to allow such low mass gravitational condensations to develop.

Our arguments on the slope of the initial main sequence are weakened only if the slope becomes steeper at lower masses. A softening of the slope or a cutoff would strengthen them. Indeed one might expect a softening for Population III objects since the lower metal abundance allows for less efficient cooling and fragmentation. Probst and O'Connell (1982) argue in favor of a softening or a cutoff in the solar neighborhood below 0.1 $M_\odot$. In conclusion we find that given our assumptions on the initial mass function, present observations appear to rule out the possibility of a Jupiter filled halo.

IV. LARGE MASS STARS

Next, we consider a halo composed of stars with an initial mass greater than 2 $M_\odot$. Such stars with masses greater than 2 $M_\odot$ will have evolved from the main sequence leaving end products of either white dwarfs, neutron stars or black holes (see following section). Since both white dwarfs and neutron stars have masses around 1.4 $M_\odot$ (Chandrasekhar 1935 and Taylor and Weisberg

1982) at least 40% of the present halo mass must have been ejected from these stars. Although neutron stars could have larger masses, their formation, if due to supernovae, still ejects a sizeable fraction of the initial mass of the star. Our arguments against stars with $M > 2\ M_{\odot}$ are based on the problem of hiding this ejected material.

The ejecta will be a problem if it is in the form of gas, as we saw in section II. Hot gas is ruled out by the X-ray upper limits and cold gas will either form stars in the halo or collapse onto the disk adding too much mass there. A more serious problem arises because of the content of the ejecta. A substantial fraction (> 10%) of the ejected mass will be in the form of helium and metals (Arnett 1978). With metals in the ejecta one would expect that conditions for star formation would be similar to those in the solar neighborhood thus producing a similar initial mass function with luminous stars.

Whether or not star formation occurs, metals seem to pose an additional problem. There appear to exist some very old low mass stars in the nuclear bulge with very low metalicity, $Z \sim 10^{-5}$ (Bond 1981 and Hills 1982). This metal abundance is so low that virtually any ejecta from massive halo stars should contaminate these low metalicity stars since they form after the halo.

Let us estimate the extent to which we can accept metal contamination. If even one percent of the ejecta is in the form of metals and 0.4 of the star's mass is ejected, to account for the dark matter by dead remnants we would have in ejecta $\Omega_{metals} \simeq (0.4)(0.01)\Omega_H = 2\times10^{-4}$. The minimum metalicity of stars in the nuclear bulge will then be

$$Z = \Omega_{metals}\ \epsilon/\Omega_D \qquad (4.1)$$

where $\Omega_D \lesssim .005$ for the disk and nuclear bulge of spiral galaxies. We define ϵ to be that fraction of the ejecta which mixes with the disk and nuclear bulge. Because of the existence of the low metalicity stars we must have $Z < 10^{-5}$ or

$$\epsilon < 10^{-5}\ \Omega_D/\Omega_{metals} < 3\times10^{-4}\ . \tag{4.2}$$

We know of no way to prevent such low levels of mixing.

One can see from the above argument that a halo containing more metals would be even less likely. If, for example, we tried to argue that the halo consisted primarily of metals in the form of rocks, dust, grains, etc., we would have $\Omega_{metals} > 0.05$ and hence

$$\epsilon < 1\times10^{-6}\ . \tag{4.3}$$

V. BLACK HOLES

Strictly speaking, as black holes do not have well defined baryon numbers they need not enter into our discussion here. Cosmologically, however, we can distinguish between two types of black holes; those which are primordial (i.e. those which have formed before the period of big bang nucleosynthesis) and those which are not. Post-nucleosynthesis black holes were presumably in the form of baryons at the time of nucleosynthesis and would have contributed to the baryon to photon ratio $\eta \sim (3\text{-}5)\times10^{-10}$ (Yang et al. 1984). We discuss briefly any constraints on their likelihood. A more thorough treatment is in preparation (Hegyi, Kolb and Olive 1984).

As it is very hard to constrain black holes by observations, we will only be able to make some comments on their formation. If the black holes are in the range 1-50 $M_\odot$, we must require a very special initial mass function. But, with no metals, this is a possibility. Massive stars, however, are unstable to mass loss (Dearboarn et al. 1978) and it is believed that their formation is accompanied by large amounts of ejected material as in the formation of neutron stars. Hence similar arguments regarding gas in the halo would apply to the non-metallic material and our arguments regarding metals would apply to the rest of the ejecta.

It may be possible to avoid these problems for massive ($M > 200\ M_\odot$) black holes if one assumes that nearly all (see limits in the previous section) of the ejected material was subsequently accreted onto the black holes or if the black holes were formed by gravitational instabilities with no ejected material (Bond et al. 1984, Carr et al. 1984, Ober et al. 1983). Also, upper limits on the masses of accreting black holes have been determined by Carr (1979).

VI. BIG BANG NUCLEOSYNTHESIS

One of the chief successes of the standard hot big bang model is the prediction of the abundances of the light elements through big bang nucleosynthesis (BBN) (Peebles 1966; Wagoner, Fowler and Hoyle 1967; Schramm and Wagoner 1977; Olive et al. 1981; Yang et al. 1984). The abundance of the light elements, in particular ^{4}He, depends primarily on three parameters: the neutron half-life which fixes the weak interaction rates, the number of light particle species (or neutrino flavors), and of interest to us here, the baryon to photon ratio η,

$$\eta \equiv n_B/n_\gamma \ , \tag{6.1}$$

at the time of nucleosynthesis. In this section we would like to look at the expected value of η from BBN and its implications for the dark matter in galactic halos.

The baryon to photon ratio enters into the BBN calculations by serving as a bottleneck to the formation of deuterium. At high temperatures $T > 10^{11}$°K, neutrons and protons were kept in thermal equilibrium through the weak interactions so that the ratio of neutrons to protons is just

$$(n/p) \sim \exp[-\Delta m/T] \tag{6.2}$$

where Δm is the neutron-proton mass difference. At $T \sim 10^{10}$°K, these interactions freeze out, meaning that they cannot keep up with the overall expansion rate of the universe. Subsequently (n/p) only changes due to free neutron decays.

At these high temperatures, although the rate for neutron-proton collisions to form deuterium is rapid

$$n+p \rightarrow D+\gamma \ , \tag{6.3}$$

the abundance of deuterium is kept small by the very high density of photons. In equilibrium, the process (6.3) yields a deuterium to nucleon ratio

$$n_D/n_N \sim \eta \exp(E_0/kT) \tag{6.4}$$

where $E_0 = 2.2$ MeV and $\eta \sim (10^{-10})$. It is only when the temperature falls to about $T \simeq 10^9$°K when

$$\eta^{-1} \sim \exp(E_0/kT) \tag{6.5}$$

that nucleosynthesis actually begins.

One must remember that because ^{4}He is the most abundant isotope (after hydrogen) it is the most sensitive test of BBN calculations. Nearly all of the deuterium produced and hence nearly all of the neutrons get incorporated into ^{4}He. The ^{4}He abundance can be estimated in terms of (n/p) by

$$Y_p = 2(n/p)/(1 + (n/p)) \tag{6.6}$$

where Y_p is the primordial mass fraction of ^{4}He. One can see now that for larger values of η, nucleosynthesis can begin earlier, i.e. at a higher temperature. This leaves less time for neutrons to decay and hence implies a large value of Y_p. Similarly for smaller η, Y_p is also smaller. If we use $\tau_{1/2}$ = 10.6 min for the neutron half-life and assume that there are at most three flavors of light neutrinos, then the BBN calculations are consistent with the observed abundances of D, ^{3}He, ^{4}He and ^{7}Li for (Yang et al. 1984)

$$3\times10^{-10} < \eta < 10^{-9} . \tag{6.7}$$

The value of η can be converted to the fraction of closure density ρ_c in the form of baryons,

$$\begin{aligned}\Omega_B &\equiv \rho_B/\rho_c = m_N\ \eta\ n_\gamma/\rho_c \\ &=3.53\times10^7\ \eta\ h_o^{-2}\ (T_o/2.7K)^3\end{aligned} \tag{6.8}$$

where m_N is the nucleon mass and we have taken the present number density of photons to be

$$n_\gamma = 400(T_o/2.7K)^3 \ . \qquad (6.9)$$

T_o is the present temperature of the microwave background radiation. Using $1/2 < h_o < 1$, $2.7 < T_o < 3$ and the limits on η from (6.7) we find that Ω_B is restricted to the range

$$0.01 < \Omega_B < 0.19 \ . \qquad (6.10)$$

We note that if $h_o = 1/2$, the lower limits on η and T_o imply $\Omega_B > 0.04$. We are then left with the following question: Where are all the baryons?

From the discussion above, we see that for $\Omega_H = .05$, at least 20% of the halo must be in the form of baryons if the baryons ae uniformly distributed. In all of our previous arguments we consider baryons as making up the entire halo. We must now ask what fraction of the halo <u>can</u> be in baryons. The main issue is the fraction of Ω_H which can be in the form of Jupiters. If we maintain that the slope of the initial mass function is $x < 1$ and $m_{min} = 0.004\ M_\odot$, we see from Figs. 1 and 2 that $Q_I < 7$ and $Q_K < 3.4$ which is roughly a factor of 10 below the lower limits from observations. Hence if only 10% of the halo were in low mass stars (and hence only 10% of the luminosity) we would be within the observed limits. Thus perhaps we can live with $\Omega_B = 0.005$ in the form of baryons. Relaxing the constraint $x < 1$ to $x < 1.35$ implies that $Q_I \lesssim 19$ and $Q_K \lesssim 8.8$ indicating that we may be able to tolerate Ω_B as high as 0.013, which is above the lower limit from BBN with $h_o = 1$.

It would also be possible to hide an $\Omega_B \approx 0.0067$ in the form of hot gas. The Silk (1973) limit on hot gas at $T = 10^6$°K implied that

$$\Omega_B < 3/\delta C\Omega = 3/(900\ h_o) \qquad (6.11)$$

or another 13% of Ω_H. The portion of Ω_H in cold gas is more difficult to estimate. Cold gas would be expected to cool and form stars. We have already taken into account that portion of Ω_H which can be in low mass stars (high mass stars would be directly observed). Trying to be conservative, we might have another 10% in cold gas which had not yet formed low mass stars. This brings our total contribution in baryons to the halo to $\Omega_B \sim 0.017$. We need not consider the dead remnants since the limits based on the metal abundance arguments are very strong.

We see, therefore that for $h_0 \sim 1$, the lower limit on Ω_B from BBN can be accommodated in galactic halos. This still leaves more than 60% of the halo which must be in black holes or in non-baryonic matter. For $h_0 \sim 1/2$ we found $\Omega_B > 0.04$ from BBN which is more than double the amount for which we can allow. In this case, we must have a sizeable contribution to the halo in the form of black holes or the dark matter is not uniformly distributed on a cosmological scale.

VII. NON-BARYONIC HALOS

In this concluding section we would like to briefly look at the various non-baryonic possibilities for galactic halos (for a review see Primack and Blumenthal 1984). The non-baryonic candidates may be classified into three categories (Bond and Szalay 1983): hot, warm and cold particles. Hot particles are neutrino-like which are relativistic when they decouple from the thermal background. Warm particles are those which decouple earlier and at a higher temperature so that their present temperature (and density) is lower than hot particles. Cold matter is defined to be non-relativistic at decoupling.

Without going into any detail on each candidate specifically, we only point out that each has its advantages and its problems. Neutrinos are the classic example of hot particles. They are very good for getting the very large scale structure of the universe (Melott 1983, Klypin and Shandarin 1982; Bond, et al. 1983; White, et al. 1983). The mass scale at which clumping occurs is given by (Bond, et al. 1980)

$$m_J = 3\times10^{18}\ M_\odot/(m_\nu(ev))^2 \tag{7.1}$$

Thus, even for $m_\nu < 100$ eV, clumping occurs at mass scales of $\gtrsim 10^{14}\ M_\odot$. This presents a problem for producing galactic scale structure (Bond, et al. 1984).

The most natural candidate for warm particles might be something similar to right-handed neutrinos. Any particle whose interactions are weaker than the typical weak interaction will decouple earlier and thus may have a larger mass (Olive and Turner 1982). For mass up to 1 keV, we see that 7.1 gives clumping on galactic scales. Some supersymmetric particles such as the gravitino have also been raised as candidates for warm particles; (Pagels and Primack 1982; Bond, et al. 1982). Another possibility from supersymmetry may be the partner of the axion or the axino (Kim 1983, Kim et al. 1984, Olive et al. 1984).

Cold particles seem to do very nicely on galactic scales and yield good fits to galaxy correlation functions (Peebles 1980). In this case it is the large scale that is a mystery. Indeed if one needs $\Omega > 0.1$ or 0.2 on the largest scales or $\Omega = 1$ in the inflationary scenario (Guth 1981) one would have a similar value on galactic scales. There are of course several candidates for cold matter such as heavy neutrinos (Gunn et al. 1978), supersymmetric particles such as the photino (Goldberg 1983, Ellis et al. 1984) or the sneutrino (Hagelin, Kane and Raby 1983, Ibanez 1984), axions (Abbot and Sikivie 1983; Dine and Fischler 1983; Preskill, et al. 1983; Ipser and Sikivie 1983; Turner, et al. 1983), and primordial black holes (Carr 1977; Crawford and Schramm 1982; and Freese, et al. 1983). Finally, magnetic monopoles would only be interesting if one could avoid the limits due to galactic magnetic fields (Parker 1960; Turner, et al. 1982).

As we can see, there is clearly no lack of non-baryonic candidates for galactic halos. The question we have tried to address is whether or not they are necessary. Inflationary models which predict $\Omega = 1$ clearly require non-baryonic matter at least on the largest scales. Big bang nucleosynthesis limits the baryonic component of Ω to $\Omega_B < 0.19$. On galactic scales we have tried to show that the dark matter in halos cannot be in the form of baryons. By allowing some hot gas and low mass stars we found $\Omega_B < 0.017$ (< 0.03 for a Salpeter initial mass function). Either the remainder of the halo material is in black holes or it is non-baryonic.

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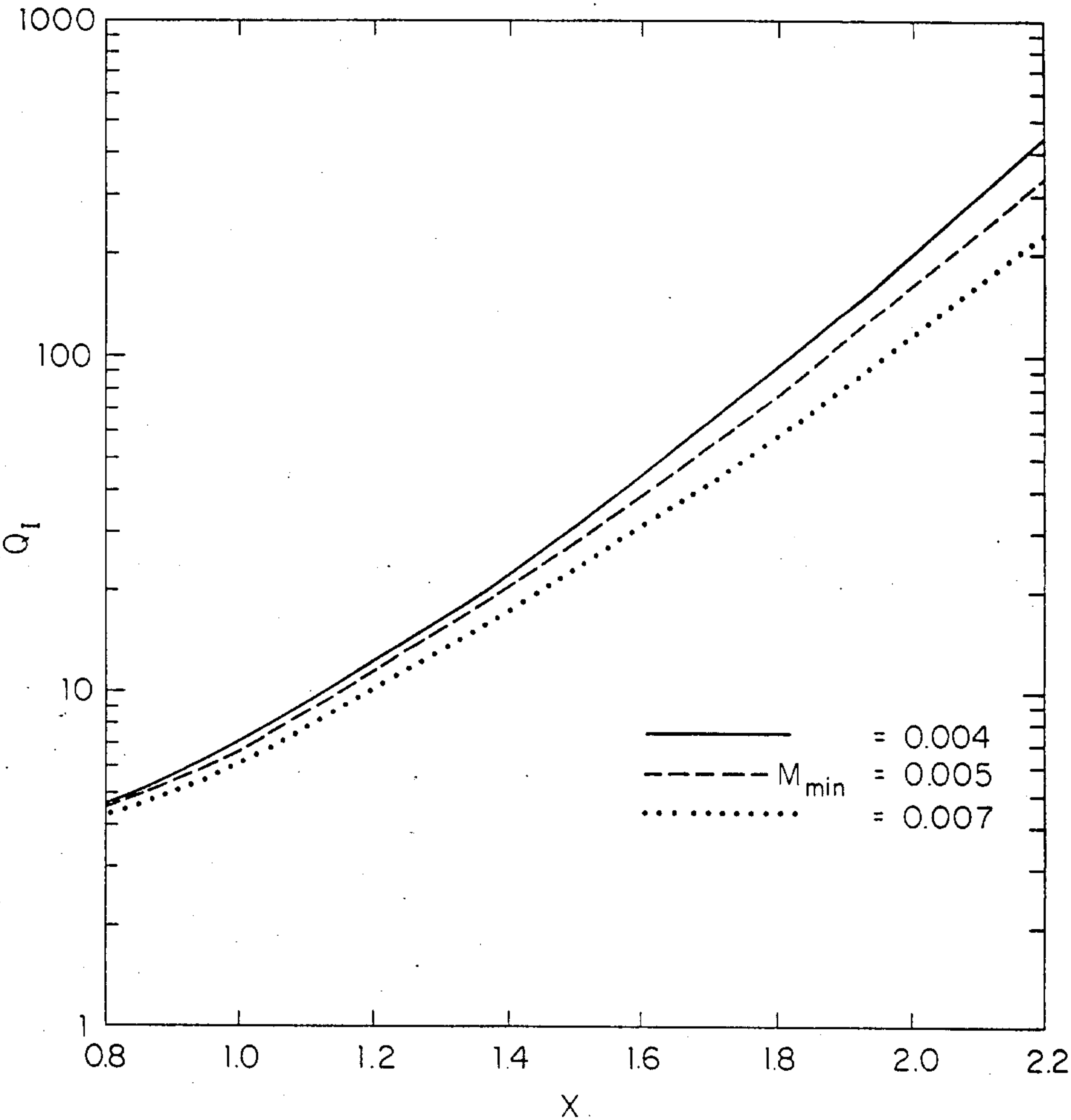
Figure Captions

Fig. 1. Q_I, the mass-to-light ratio in the I spectral band calculated in solar units, is plotted as a function of the slope of the initial mass function. The three curves drawn show the effect of varying the lower limit to the Jeans mass, plotted in solar masses.

Fig. 2. Q_K, the mass-to-light ratio in the K spectral band calculated in solar units, is plotted as a function of the slope of the initial mass function. The three curves drawn show the effect of varying the lower limit to the Jeans mass.

Fig. 3. The measured surface brightness in the I_{Kron} band of the halo of NGC 4564 versus galactic radius. The line drawn in the figure is the 2σ upper limit of the surface brightness. The photometric calibration is 10^{-5} counts/scan/arc sec^2 = 25.34 mag I_{Kron}.



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M_{min} = 0.004
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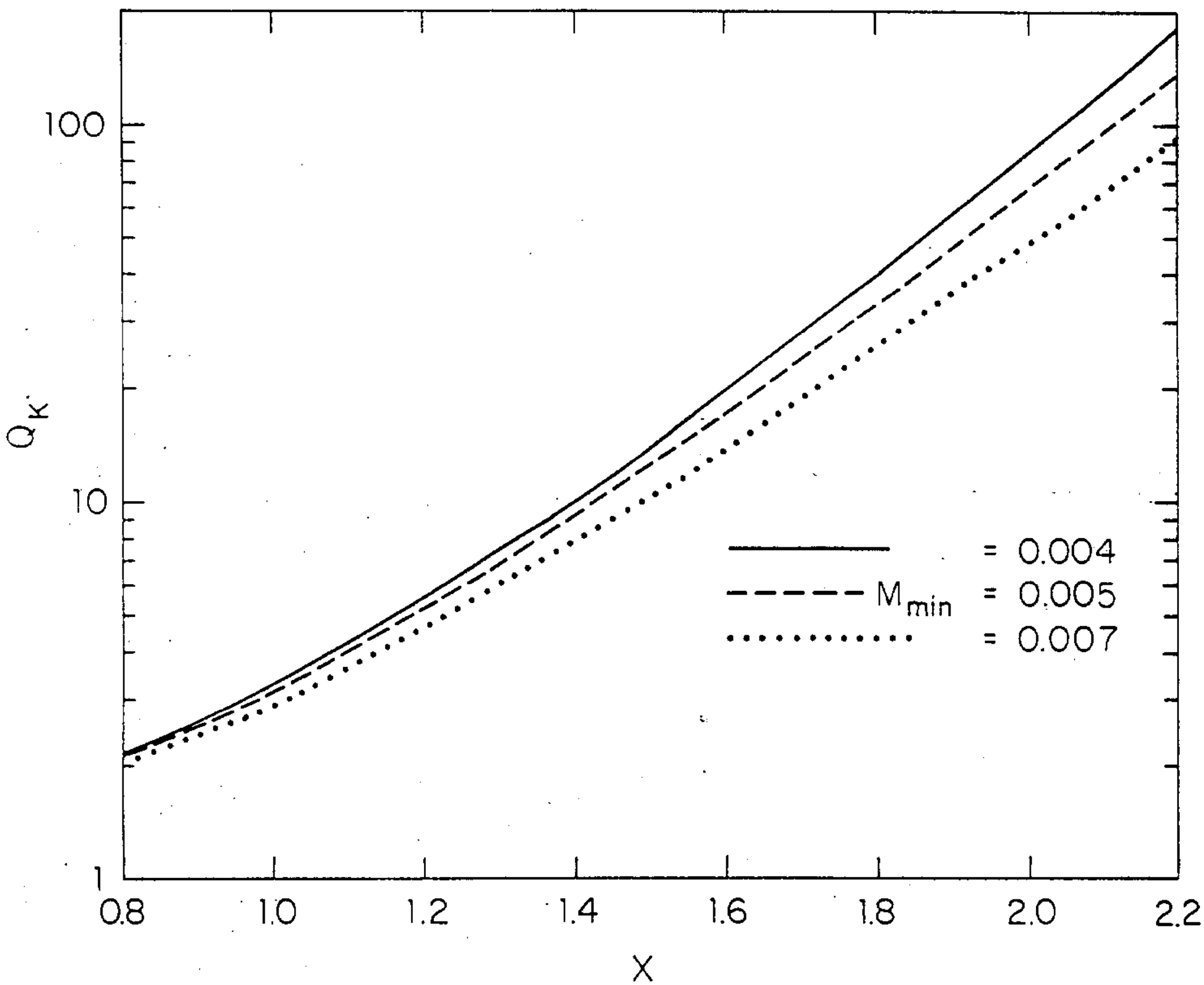

Q_K
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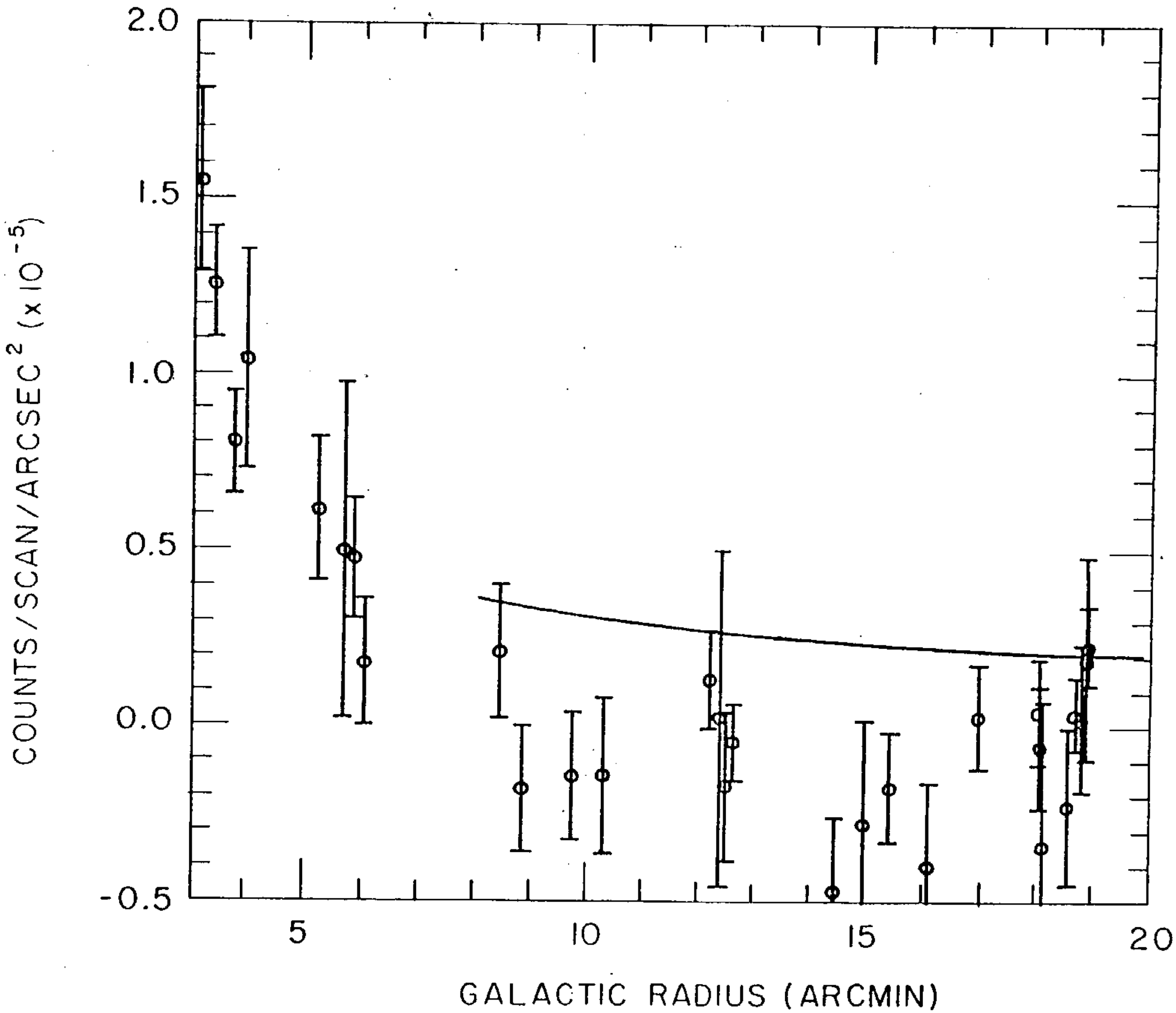
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GALACTIC RADIUS (ARCMIN)