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A GENERAL ANALYSIS OF THE CHIRAL PHASE TRANSITION*

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ABSTRACT

The universality classes applicable to the chiral phase transition are studied. The ε-expansion predicts that the chiral transition should always be of first order for three or more flavors. If fluctuations with topological charge evaporate at temperatures well below the chiral transition point, a new phase of hadronic matter could occur, one in which there would be massive violations of isospin.

1. INTRODUCTION

In this note I review some work done by Frank Wilczek and myself [1]. My treatment will be brief, as I will emphasize only those matters which we did not have the opportunity to discuss originally. In particular, I point out two relevant problems which should be answerable directly with current Monte Carlo techniques, without having to solve the full problem of QCD with dynamical fermions.

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2. THE ORDER OF THE CHIRAL PHASE TRANSITION

Take N_f flavors of quarks in the fundamental representation of a $SU(N_c)$ color gauge group. For now, I assume that the bare masses of the quarks all vanish.

Classically, the global flavor symmetry of the quarks is

$$G_{cl} = U_A(1)\times SU_L(N_f)\times SU_R(N_f)$$

G_{cl} includes (non-abelian) rotations of left- and right-handed quarks, and the abelian $U_A(1)$ for axial fermion number. The abelian U(1) for total fermion number can be ignored; it is always conserved in QCD-like theories, so this U(1) decouples from G_{cl} transformations. For instance, any effective theory which described the chiral transition is constructed from mesonic fields, which have zero fermion number.

The conservation of the $U_A(1)$ symmetry is violated quantum mechanically by the axial anomaly. If the quarks fields are denoted as $q_i(i=1\ldots N_f)$, the $U_A(1)$ current is $J^5_\mu \sim \bar{q}_i\gamma_5\gamma_\mu q_i$, and

$$\partial_\mu J^5_\mu = \frac{(g^2N_c)}{16\pi^2}\frac{N_f}{N_c}\,\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \quad . \tag{1}$$

The anomaly vanishes in the quenched approximation, $N_f=0$, or in the limit of infinite colors [2] (remember that as $N_c\to\infty$, g^2N_c is held fixed and $\sim O(1)$). In these cases, the flavor symmetry is just G_{cl}. Otherwise, the continuous axial symmetry is replaced by a discrete one, and the true flavor symmetry is

$$G_{fl} = Z_A(N_f)\times SU_L(N_f)\times SU_R(N_f) \quad .$$

I now make several assumptions: that the flavor symmetry, be it G_{cl} or G_{fl}, breaks spontaneously at zero temperature to $SU(N_f)$; that this symmetry is restored at a finite temperature T_{ch}; and finally, that the symmetry breaking can be parametrized by a field $\underline{\phi}$ in the vector representation of G_{fl} (or G_{cl}), $\phi_{ij} \sim \langle \bar{q}_i(1+\gamma_5)q_j \rangle$. Of these assumptions, the last is the most notable. It is certainly possible to imagine theories in which the important chiral order parameter lies not in the vector, but a higher representation of G_{fl} (G_{cl}). Such theories do not include QCD, however, for otherwise PCAC would not work as well as it does.

Given these assumptions, to understand the chiral transition is "merely" a matter of knowing the universality classes of systems in the vector representation of G_{fl} (G_{cl}). In the usual way, non-zero temperatures freeze out one direction, so the classes of significance, for systems at finite temperature in 3+1 dimensions, are those in three dimensions.

Unfortunately, in general there are no systems in condensed matter physics with G_{fl} symmetry, so these universality classes are unknown. To investigate, we must resort to a drastic but familiar approximation — an expansion in 4-ϵ dimensions.

A vector $\underline{\phi}$ transforms as

$$\underline{\phi} \to e^{i\alpha} U_L \phi U_R \tag{2}$$

under G_{cl}. In Eq. (2), α is the $U_A(1)$ phase; $U_{L,R}$ are $SU_{L,R}(N_f)$ rotations. A Lagrangian invariant under Eq. (2) is

$$L_{cl} = \tfrac{1}{2}\mathrm{tr}(\partial_\mu \underline{\phi}^\dagger)(\partial_\mu \underline{\phi}) + \tfrac{1}{2}m^2 \mathrm{tr}(\phi^\dagger \phi) + g_1(\mathrm{tr}(\phi^\dagger\phi))^2 + g_2 \mathrm{tr}(\underline{\phi}^\dagger \underline{\phi})^2. \quad (3)$$

There are two relevant couplings in L_{cl}, g_1 and g_2. If g_2 is set to zero, the symmetry is enlarged to $O(2N_f)$.

To incorporate the effects of the axial anomaly, we add a term L'_{fl} to L_{cl},

$$L'_{fl} = c_{inst}(\det\underline{\phi} + \det\underline{\phi}^\dagger) \quad . \quad (4)$$

We see from Eq. (4) that it is only invariant under the transformation of Eq. (2) if $\alpha = 2\pi n/N_f (n=1\ldots N_f)$, which is a $Z_A(N_f)$ symmetry.

The coupling of L'_{fl}, c_{inst}, is proportional to the density of fluctuations with topological charge,

$$c_{inst} \sim \left\langle (\mathrm{tr}F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right\rangle \quad . \quad (5)$$

One type of such fluctuations are instantons; e.g., a single instanton generates a $2N_f$-point interaction between quarks that has precisely the symmetry of Eq. (4), at least over distances far from the instanton [3]. In a misuse of language, I shall refer to any fluctuation which carried topological charge as an instanton.

Following the usual Landau type of analysis, to study the chiral transition we would allow the mass term of Eq. (3) to vary with temperature ($m^2<0$ at $T=0$, $m^2 \to 0$ as $T \to T_{ch}$), but neglect the variation of any couplings with temperature. While this is probably valid for g_1 and g_2, it is certainly <u>not</u> true for c_{inst}. This extraordinary phenomenon is only possible because the $U_A(1)$ symmetry breaking, which

c_{inst} represents, is entirely of quantum mechanical origin.

At high temperatures, the dominant contribution to $\langle(\mathrm{tr}F\tilde{F})^2\rangle$ is from (true) instantons [4]; as $T\to\infty$, this quantity, and so c_{inst}, vanish. Thus we can speak of the effective restoration of the $U_A(1)$ symmetry. This restoration is only approximate, since for any finite T, there are always some instantons around, $\langle(\mathrm{tr}F\tilde{F})^2\rangle \neq 0$.

Somewhat arbitrarily, I define a temperature T_{inst} as that for which $\langle(\mathrm{tr}F\tilde{F})^2\rangle$ is ~1% of its value at zero temperature. If $T_{inst} \gtrsim T_{ch}$, T_{inst} is of limited interest, but if $T_{inst} \ll T_{ch}$ (e.g., $T_{inst} \sim \frac{1}{2}T_{ch}$), then the relevant symmetry which is restored at T_{ch} is not G_{fl}, but, to a very good approximation, G_{cl}. Thus in our analysis of the chiral transition, we must distinguish between these two possibilities.

In 4-ε dimensions, the universality classes are found to be: [5]

Table 1

N_f	G_{fl} ($T_{inst} \gtrsim T_{ch}$)	G_{cl} ($T_{inst} \ll T_{ch}$)
0	-	"2cd"-O(0)
1	-	"2cd"-O(2)
2	"2cd"-O(4)	FI 1st
3	1st	FI 1st
≤4	FI 1st	FI 1st

Wherever in this table I have written "2cd", it means that the transition may be second order; if so, the critical exponents are those of a $O(2N_f)$ vector. This caveat is necessary because the dynamics can always be such so as to choose a first order transition. On the other hand, if 1st is written, then the transition is unavoidably first order.

To explain the table, let me begin with the case of G_{cl}. In $4-\varepsilon$ dimensions, for $N_f < \sqrt{2}$, the infrared stable fixed point has $g_1^* \sim O(\varepsilon)$, $g_2^* = 0$, so for $N_f=0$ and 1, the critical exponents are those of $O(2N_f)$. For $N_f \geq \sqrt{2}$, there is no infrared stable fixed point, resulting in a "fluctuation induced" (FI) first order transition.

A FI first order transition is the direct extension of the Coleman-Weinberg phenomenon from four [6] to below four dimensions [7-10]. What happens is the following. One can always get a first order transition if the quartic couplings are negative, assuming higher point couplings stabilize the system (hence the caveat above). If the bare quartic couplings are positive, mean field theory predicts a second order transition. For a FI transition, however, as $|m^2|$ decreases, even if the quartic couplings start in a region in which they are positive, they will evolve, by the action of the renormalization group, into regions where they are negative — so the transition remains first order [9,10].

This evolution of the quartic couplings occurs because there is no (infrared) stable fixed point towards which the couplings can flow as $m^2 \to 0$. For instance, in the G_{cl} theory, the fixed point with $g_1^* \sim O(\varepsilon)$, $g_2^* = 0$ is always (infrared) stable in the g_1 direction, but it is only so in the g_2 direction if $N_f < \sqrt{2}$.

The case of G_{fl} symmetry, $T_{inst} \gtrsim T_{ch}$, follows directly.

a. $N_f=0$: The axial anomaly vanishes, so the symmetry is never G_{fl}, but G_{cl}.

b. $N_F=1$: Instantons alone will generate a (quark) mass: L'_{fl} is linear in ϕ for $N_f=1$, so $\langle\phi\rangle \sim c_{inst}$. Hence there is really no chiral transition to speak of — as the temperature increases, $\langle\phi\rangle$, and so the instanton induced mass term, turn off smoothly.

c. $N_f=2$: $G_{fl} = Z(2)\times SU(2)\times SU(2) = O(4)$; for a second order transition, the critical exponents are those of $O(4)$.

d. $N_f=3$: The operator $L'_{fl} \sim \det \underline{\phi}$ is cubic in ϕ_{ij} for $N_f=3$, so L'_{fl} <u>itself</u> drives the transition first order.

e. $N_f\geq 4$: In the technical sense of the renormalization group, the operator L'_{fl} is either marginal ($N_f=4$) or irrelevant ($N_f>4$). In either case, the critical behavior is determined by what happens for G_{cl} symmetry, which is a FI first order transition.

One lesson is clear from Table 1: for the physically interesting case of two or three flavors, the nature of the chiral transition is very sensitive to the presence of the operator L'_{fl}. In numerical simulations with $N_f=2$ or 3, in order to get the chiral transition right, it is essential to get the temperature evolution of the η' right.

The greatest drawback to these results is that they are based on an ϵ-expansion, while we need $\epsilon=1$!. As a practical matter, there are many systems in which the prediction of a (FI) first order transition in $4-\epsilon$ dimensions is seen experimentally in three dimensions [8]. Even so, for systems with G_{cl} symmetry, the crossover from second to FI first order behavior occurs at $N_f^c = \sqrt{2} \pm O(\varepsilon)$ — but is N_f^c still <2 when $\varepsilon=1$?

It would be invaluable to have numerical simulations of spin systems with G_{cl} symmetry, for $N_f \geq 2$, in three dimensions. This raises an obvious question — since there is nothing universal about a first order transition, how does one know that it is FI per se?

To answer this, let me first review the (tricritical) phase diagram of a system which can undergo a second order transition in three dimensions, Fig. (1a). In this figure, the axes β_1 and β_2 are two relevant (bare) couplings. In a linear model, such as L_{cl}, one of these would be m^2, and the other, g_1 or g_2; as renormalizable couplings in three dimensions, positive six-point terms are added to L_{cl}. In a non-linear model, β_1 and β_2 could be nearest and next to nearest neighbor couplings.

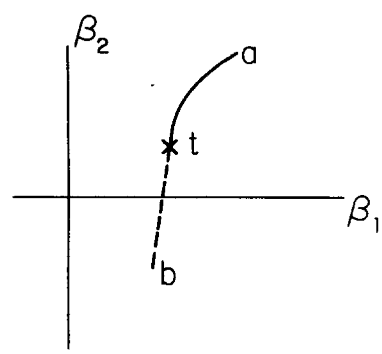

Fig. (1a)

In the space of β_1 and β_2, there is a line of second order transitions ("at", the solid line), and a line of first order transition ("bt", the dotted line), which meet at the tricritical point "t". The exponents along "at" are those of the whatever spin system we are considering, while the latent heat vanishes as "t" is approached along "bt". Precisely at the tricritical point, the transition is of second order, but the exponents are mean field like, up to logarithmic corrections.

Whatever microscopic dynamics we start with, we can view this diagram through an effective linear model. In this effective model, the quartic couplings are positive along "at", and negative (for at least some) on "bt"; the $(\text{mass})^2$ is zero on "at", and positive on "bt". Exactly at the tricritical point, the mass and all quartic couplings vanish, with logarithmic corrections generated by the six-point couplings [11].

The tricritical phase diagram of a system with a FI transition will be characteristically different. The crucial point is that the transition must remain of second order at the tricritical point, "t".

Why? At "t", the quartic couplings vanish, so the only fluctuations we need to worry about are due to the six-point terms. Let these couplings be η_1, $\eta_2 \ldots$; for simplicity, I assume that all of the η_i are positive. For example, in our model with G_{cl} symmetry, there will be three such couplings — for $\left(\mathrm{tr}(\underline{\phi}^\dagger \underline{\phi})\right)^3$, $\mathrm{tr}(\underline{\phi}^\dagger \underline{\phi})\, \mathrm{tr}(\underline{\phi}^\dagger \underline{\phi})^2$, and $\mathrm{tr}(\underline{\phi}^\dagger \underline{\phi})^3$. To leading order in small coupling, the β-function for the coupling η_i, $\bar{\beta}_i$, is

$$\bar{\beta}_i = \frac{\partial \eta_i}{\partial \ln \mu} = M_i^{jk} \eta_j \eta_k + O(\eta^3) \quad . \tag{6}$$

The matrix M_i^{jk} generates mixings between the η_i; its elements depend on the symmetry group, and the representation the scalar field lies in. Whatever the M_i^{jk} are, however, it is easy to show that each and every element is positive.

Now, quartic couplings drive a transition FI first-order if and only if there is no infrared stable fixed point. But for the η_i, there is always the trivial fixed point, $\eta_i^*=0$. Since each element of M_j^{jk} is positive, arguments from four dimensions [12] can be used to establish that the η_i must flow into the origin in the infrared limit. The η_i will vanish logarithmically in this limit, in a manner determined by the values of the M_i^{jk}.

The tricritical phase diagram of a FI system must then look like Fig. (1b). There are two first-order lines, "at" and "bt", which meet at the tricritical point "t". Since the transition is of second order at "t", the latent heat must vanish as "t" is approached from either direction. Of course, from the phase diagram alone, one will not be able to tell whether the first order transitions along "at" or "bt" are mean field like or FI. Nevertheless, it should certainly be possible to differentiate between Fig. (1a) or Fig. (1b).

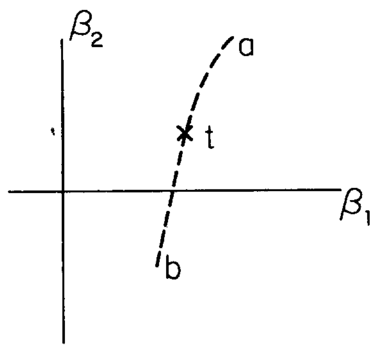

Fig. (1b)

The purpose of this long discussion is to emphasize that it is not sufficient to establish a FI transition by varying only one (relevant) coupling. For instance, if one varied β_1, keeping $\beta_2=0$, one could not tell if the phase diagram was like Fig. (1a) or (1b).

In this light, I note the numerical simulations of systems with G_{fl} symmetry by Kogut, Snow, and Stone [13]. While they did find a first-order transition for $N_f\geq3$, since they varied only a single coupling, it is not clear that the transition they found was FI.

A final caution is that the tricritical point need not lie in the

β_1-β_2 plane; it might be necessary to search in the space of higher couplings $\beta_3, \beta_4, \ldots$. On the other hand, while it would be interesting to find a FI tricritical phase diagram like Fig. (1b), if all one ever found in the theory were first order transitions, with no second order lines (surfaces, etc.), that alone would be fairly convincing evidence for a FI transition.

I conclude this section with two technical asides.

The first concerns a lattice theory with staggered fermions. For N_s types of staggered fermions, the full G_{fl} symmetry, with $N_f=4N_s$, is recovered only in the continuum limit. At finite lattice spacing, the exact chiral symmetry is not G_{fl}, but G_s, where $G_s = U(1)\times SU(N_s)\times SU(N_s)$ [14] (as before, the U(1) symmetry for total fermion number does not enter into G_s). Thus, a chiral transition observed in numerical simulations might be one characteristic of G_s, and not G_{fl}, symmetry. This is particularly true for $N_s=1$: then $G_s = U(1) \simeq O(2)$, and the transition could be second order, while from G_{fl} symmetry, a FI first order transition would be expected.

Secondly, I note the unusual nature of the "chiral" phase transition in three, instead of four, space-time dimensions. A single, massless, two-component spinor does not have a chiral symmetry in three dimensions, but by properly pairing up an even number, we can obtain a flavor symmetry G [15]; for our purposes, all we need to know about G is that it is a continuous symmetry. Suppose then that these massless fermions are coupled to a gauge group, and that at zero temperature, G is spontaneously broken.

How is the flavor symmetry restored at a finite temperature $T\neq 0$?

Let us imagine constructing an effective theory to describe the infrared properties of our model. For momenta $\ll T^{-1}$, the effective theory will be two-dimensional, while if G is broken at $T\neq 0$, it must include Goldstone bosons; but one cannot have interacting massless particles in two-dimensions [16]! The only way out is to conclude that while G is broken at T=0, it is restored at any temperature $T\neq 0$. A similar phenomenon is known to happen in the Gross-Neveu model [17].

3. T_{ch} VERSUS T_{inst}

As we saw in the last section, the nature of the chiral phase transition can change dramatically, depending on the relation of T_{ch} and T_{inst}; this is especially true for two or three flavors. Aside from prejudice, it is not clear what the relation between T_{ch} and T_{inst} is in real QCD.

To discuss the effects which are possible, we concentrate on the (nearly) Goldstone bosons of chiral symmetry. For the neutral bosons, π^0, η, and η', we parametrize their mass matrix as [1]

$$M^2_{\pi^0\pi^0} = \left(m_u+m_d\right) \frac{v^3}{f_\pi^2} \quad ,$$

$$M^2_{\pi^0\eta} = \frac{\left(m_u-m_d\right)}{\sqrt{3}} \; \frac{v^3}{f_\pi^2} \quad , \text{ etc.,} \tag{7}$$

$$M^2_{\eta'\eta'} = \frac{2\left(m_u+m_d+m_s\right)}{3} \; \frac{v^3}{f_{\eta'}^2} + K_{inst} \; .$$

Except for the term K_{inst}, the form of M^2 is standard from current algebra. K_{inst} is inserted to represent the contribution of the anomaly term to the η' mass, $K_{inst} \sim \langle(\mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu})^2\rangle$.

In the matrix, the π^0, η, and η' start out as SU(3) eigenstates. Because M^2 contains off diagonal elements, they can, and will, mix with each to form the mass eigenstates we observe. At zero temperature, the familiar result of this is a small mixing of the η and η'. If at finite temperature $T_{inst} \ll T_{ch}$, much more dramatic mixing can occur, Sec. 3.2.

For the bare quark masses, I take [18]:

$$m_u \sim 6\ \mathrm{MeV},\ m_d \sim 11\ \mathrm{MeV},\ m_s \sim 215\ \mathrm{MeV}\ . \tag{8}$$

In Eq. (7), I have assumed that $f_\eta = f_\pi = 93$ MeV, but leave $f_{\eta'}$ free of f_π. This is because SU(3) symmetry implies that $f_\eta = f_\pi$, but not that $f_{\eta'} = f_\pi$. The value of the chiral condensate is $\langle\bar{q}q\rangle \sim v^3$.

The eigenstates of M^2 agree with the observed masses of the π^0, η, and η' if

$$\begin{aligned} v &\sim 212\ \mathrm{MeV}\ , \\ K_{inst} &\sim (293\ \mathrm{MeV})^4/f_\pi^2\ , \\ f_{\eta'} &\sim 1.95\ f_\pi\ . \end{aligned} \tag{9}$$

The values of v and K_{inst} are typical of what one might expect (K_{inst} is written the way it is because of Eq. (10)). It does appear odd, though, that $f_{\eta'}$ turns out to be almost twice f_π.

K. Johnson has pointed out to me that this result is probably misleading. Suppose that we keep everything else the same, but vary m_s:

Table 2

m_s (MeV)	$f_{\eta'}/f_\pi$
200	16
215	1.95
225	1.56
250	1.27
300	1.15

As m_s changes over this range, v and $K_{inst}^{1/4}$ are almost constant. With this fit, $f_{\eta'}/f_\pi$ is very sensitive to the ratio of m_u or m_d to m_s. For slightly higher values of m_s, $f_{\eta'}/f_\pi$ is nearly the same as $f_K/f_\pi \sim f_\eta/f_\pi$ is observed to be, ~1.25 [18].

One can go further, by trying to fit K_{inst} to the known instanton density [3,4]. While a very dubious thing to do, it should serve to illustrate what could happen in QCD. I begin with a relation for K_{inst} derived by Witten, in the limit of a large number of colors:

$$K_{inst} = \frac{4N_f}{f_\pi^2} \left\langle \left(tr\ F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 \right\rangle \quad . \tag{10}$$

The instanton contribution to $\langle(tr\ F_{\mu\nu}\tilde{F}^{\mu\nu})^2\rangle$ is only calculable for instantons of small scale size ρ. Inserting a cutoff on scale size in by hand (I used $\rho\mu < 0.685$), the resulting instanton density has a nice bell-shaped distribution as a function of ρ - see, <u>e.g.</u>, Fig. (4) of Ref. [4]. For $N_c = N_f = 3$, from Eq. (6.15) of Ref. [4] I find, after integrating over ρ, that

$$K_{inst} = 9.85 \frac{m_u m_d m_s}{f_\pi^2} \mu , \tag{11}$$

where μ is the Pauli-Villars renormalization mass scale. The factors of m_u, m_d, and m_s enter from the (almost) zero modes of the fermions.

Using the reasonable value of $\mu \sim 200$ MeV, Eq. (11) gives a value of K_{inst} that is smaller than that of Eq. (9) by about four orders of magnitude.

Equation (11) gives such a bad estimate because it has been calculated from the single instanton density, assuming instantons and anti-instantons are uncorrelated. For zero bare quark mass, Eq. (11) gives $K_{inst} = 0$, which is absurd -- there are surely still fluctuations with topological charge in the QCD vacuum.

To get a better estimate of K_{inst}, consider the diagram by which instantons contribute to the η' mass, Fig. (2). For three flavors, the instanton induced quark interaction has six legs [3]: four tie onto η' 's, leaving two legs left over. Since these last two legs carry net chirality, they cannot attach to each other. They can tie into the vacuum by an insertion of the operator $\bar{q}q$, which results in a single factor for the chiral condensate, $\langle\bar{q}q\rangle$, in K_{inst}.

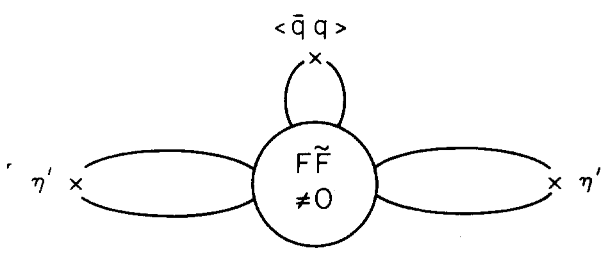

Fig. (2)

This can also be seen from the form of L'_{fl}, Eq. (4). For three flavors, $L'_{fl} \sim \det \underline{\phi}$ is cubic in the elements of $\underline{\phi}$, ϕ_{ij}. If two of these ϕ'_{ij}s attach to η' 's, one is left over; as $\langle\underline{\phi}\rangle \sim \langle\bar{q}q\rangle$, we find that $K_{inst} \sim c_{inst}\langle\bar{q}q\rangle$.

To represent this factor of $\langle\bar{q}q\rangle$ in K_{inst}, in Eq. (11) I replace each bare mass by a sum of a bare and a "constituent" mass, m_c, leaving everything else unchanged:

$$K_{inst} = 9.85 \, \frac{(m_u+m_c)(m_d+m_c)(m_s+m_c)}{f_\pi^2} \, \mu \; . \tag{12}$$

Since $\langle\bar{q}q\rangle \sim (\text{mass})^3$, this is at least dimensionally correct.

Taking $\mu = 200$ MeV, K_{inst} agrees with Eq. (9) if I set $m_c \sim 100$ MeV.

The above applies to zero temperature. At finite temperature, v, f_π, $f_{\eta'}$, and K_{inst} will all depend on T in a complicated way. To show what might happen, I will employ the fits of Eqs. (7) and (12) in two drastic approximations.

3.1 $T_{inst} \gtrsim T_{ch}$

To model this, I assume that all dimensional parameters vary in a simple way with the constituent mass, m_c:

$$v \sim 2.12 \, m_c \;,$$
$$f_\pi \sim 0.93 \, m_c \;,$$
$$f_{\eta'} \sim 1.95 \, f_\pi \;;$$

I take K_{inst} to depend on m_c as in Eq. (12). This is something like mean field theory for the chiral phase transition: as $T \to T_{ch}$, m_c

should decrease. For a first-order transition, as expected for three flavors from Sec. 2, m_c will jump to zero for some $m_c' \neq 0$, but this m_c' is surely <100 MeV.

As a function of m_c, the eigenstates of M^2 are given in Fig. (3). Even as $m_c \to 0$, nothing very interesting happens: the mass eigenstates are always nearly those of SU(3), with $m_{\pi^0}^2 \sim m_{\pi^\pm}^2$.

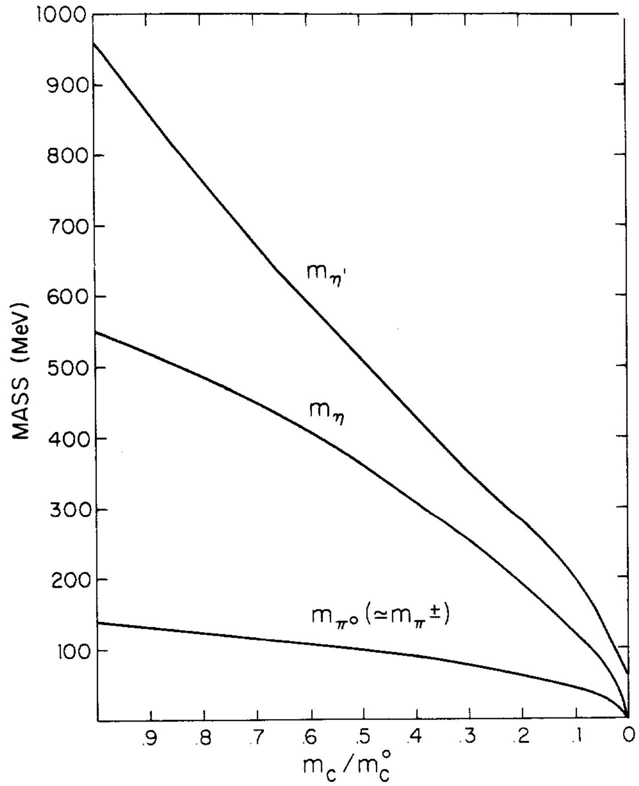

Fig. (3)

3.2 $T_{inst} \ll T_{ch}$

In this case, for temperatures well below T_{ch}, I take v, m_c, f_π, and $f_{\eta'}$ to be constant, and equal to their value at zero temperature; only K_{inst} is varied, $K_{inst} = K_{inst}(T)$.

If $K_{inst}(T) \sim 0$, for $T \ll T_{ch}$, something surprising occurs -- the eigenstates of M^2 are those of flavor, and not SU(3):

$$
\begin{aligned}
\eta' &\sim \bar{s}s , & m_{\eta'} &\sim 600 \text{ MeV} , \\
\eta &\sim \bar{d}d , & m_\eta &\sim 140 \text{ MeV} , \\
\pi^0 &\sim \bar{u}u , & m_{\pi^0} &\sim 80 \text{ MeV} .
\end{aligned}
\tag{13}
$$

While not obvious from the form of M^2, Eq. (7), this can be easily understood. *In a linear model like L_{cl}, the quark bare masses can be* represented by a term linear in $\underline{\phi}$, ~ tr($\underline{m}\underline{\phi}$). $\underline{m}$ is a diagonal matrix, with elements m_u, m_d, and m_s. Without the anomaly term L_{fl}', the would-be Goldstone bosons along the diagonal of $\underline{\phi}$ — the π^0, η, and η' — respond to this external field $\underline{m}$ by lining up directly along it; hence they are essentially flavor eigenstates.

The would-be Goldstone bosons from the off-diagonal elements of $\underline{\phi}$ -- the charged pions, and the kaons -- are already flavor eigenstates, so they are insensitive to changes in the anomaly. This means that while m_{π^0} ~ 80 MeV, $m_{\pi^\pm}$ is still ~138 MeV -- which is a phase of hadronic matter with massive violations of isospin!

Current algebra consistently estimates that m_u/m_d is far from one [18]. Normally, it is very difficult to detect the violations of isospin which should follow [19], since both m_u and m_d are much smaller than any other QCD mass scale. If T_{inst} is $\ll T_{ch}$ in QCD, it might afford a unique opportunity to observe large effects from $m_u/m_d \neq 1$.

How could these isospin violations be detected in, e.g., heavy ion collisions? Since π^0's are lighter than $\pi^\pm$'s in this phase, there should be more π^0's than expected from isospin invariance, $N_{\pi^0}/(N_{\pi^+}+N_{\pi^-}) > 1/2$. However, this is only true for pions generated in the hot regions, which may be swamped by those generated in colder ones. A more direct probe would be from the dilepton spectra. At $T = 0$, isospin invariance prohibits $\phi \to \eta'\pi^0$, so $\phi \to KK$; kinematically, ϕ is right above the KK threshold, *so its width is extremely narrow*, ~5 MeV. For $T_{inst} < T < T_{ch}$, $\phi \to \eta'\pi^0$ would be allowed, and the width of the ϕ would jump to a value like that of the ρ, ~100 MeV. This assumes that thermal broadening of the ϕ peak is not significant.

If the eigenstates of M^2 are plotted as a (linear) function of K_{inst}, as in the figure of Ref. [1], things appear rather discouraging. The only significant change in m_{π^0} occurs when $K_{inst}(T)$ is very small--$K_{inst}(T)<0.01\ K_{inst}(0)$; it is this criterion I use to define T_{inst}.

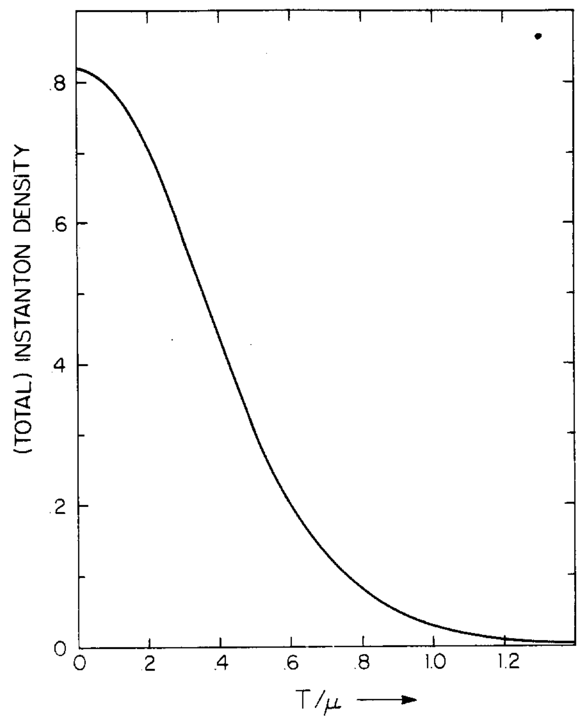

Fig. (4)

This is rather misleading, however, for the dependence of K_{inst} on temperature is certainly highly nonlinear. For example, with $N_c = N_f = 3$, the total instanton density depends on temperature as in Fig. (4) [4]. At $T \neq 0$, one can use the same cutoff on the instanton scale size as at $T = 0$. (This is similar to Fig. (5) of Ref. [4]. That figure was for $N_c = 2$, $N_f = 0$, and falls off less rapidly with T than the present Fig. (4).)

From before, $K_{inst} \sim \langle\bar{q}q\rangle\ c_{inst} \sim \langle\bar{q}q\rangle\ \langle tr(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\rangle$. Since $\langle tr(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\rangle \sim$ the total instanton density, I can estimate $K_{inst}(T)$ by multiplying Eq. (12) by the ratio of the total instanton density, at a temperature T, to that at $T = 0$; the variation of m_c, etc., with T is ignored.

With these approximations, the eigenstates of M^2 can be found just by folding Fig. (4) with that of Ref. [1]. This gives Fig. (5). In this diagram, the possibility of significant isospin violation appears plausible.

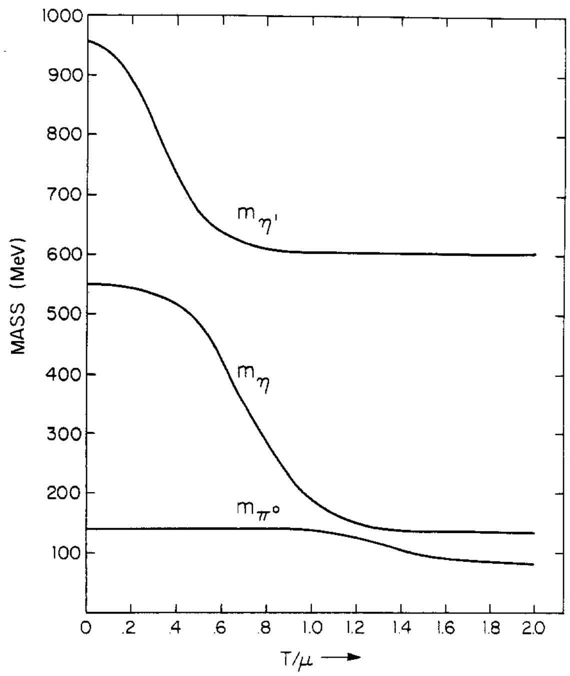

Fig. (5)

To test these ideas, it would be well worth studying the temperature dependence of $\langle \mathrm{tr}\,(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \rangle$ by numerical simulations [20]. If one knew that T_{inst} were much less than the deconfining transition temperature — for even the pure SU(2) lattice gauge theory — it would greatly encourage the suspicion that $T_{inst} \ll T_{ch}$ in QCD.

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