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BLACK HOLES, PREGALACTIC STARS, AND THE DARK MATTER PROBLEM

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ABSTRACT

We review the different ways in which black holes might form and discuss their various astrophysical and cosmological consequences. We then consider the various constraints on the form of the dark matter and conclude that black holes could have a significant cosmological density only if they are of primordial origin or remnants of a population of pregalactic stars. This leads us to discuss the other cosmological effects of primordial black holes and pregalactic stars.

1. BLACK HOLES IN COSMOLOGY

1.1 Introduction

One of the most exciting predictions of general relativity theory is that there can exist regions of space-time in which gravity is so strong that nothing, not even light, can ever escape. As shown by the spherically symmetric Schwarzschild solution, this happens whenever a mass M is concentrated within a radius

$$R_S = \frac{2GM}{c^2} \simeq 3\left(\frac{M}{M_\odot}\right)\ \text{km}. \tag{1}$$

Of course, even though a black hole may exist mathematically, it is not obvious that the enormous compression required to form it can arise in nature: eqn (1) implies that the density of a region which has entered its Schwarzschild radius is

$$\rho_S = \frac{3c^6}{32\pi G^3 M^2} \simeq 10^{18} \left(\frac{M}{M_\odot}\right)^{-2} \text{ g cm}^{-3} \qquad (2)$$

and, for a solar mass object, this is a thousand times greater than nuclear density. Nevertheless, the discovery of neutron stars in 1967 (with radius only ten times larger than that required for collapse) forced astrophysicists to realize that such extreme conditions are not necessarily implausible [1]. Indeed our understanding of stellar evolution now suggests that sufficiently massive stars almost inevitably leave black hole remnants.

The realization that black holes could exist in the real Universe prompted a renewed interest in their mathematical properties and the last twenty years have seen some remarkable developments in this respect. Spherically symmetric gravitational collapse is well understood [2] and exhibits two crucial features. Firstly, an <u>event horizon</u> forms when the radius of the object falls below R_S; this is the boundary of the black hole and events inside it can never be seen by an outside observer. Secondly, having formed such an event horizon, the infalling matter collapses to a point of infinite density called a <u>singularity</u>; at such a point all known laws of physics break down. It is not obvious that these same features would be exhibited in a more general (non-spherical) collapse. However, powerful theorems show that a singularity must always occur somewhere once a body gets sufficiently compressed [3]. It would be embarrassing if this singularity (with its associated impredictability) could influence the outside world, so the <u>cosmic censorship</u> hypothesis (still not rigorously proved) states that an outside observer will always be shielded from its effects by an event horizon [4].

Another remarkable theorem shows that, however messy the collapse, the black hole will always settle down to a stationary state which depends only on the mass, angular momentum, and charge of the original object [5-7]. All other information about the object is lost and any irregularities are radiated away as gravitational and electromagnetic radiation[8]. This is called the No Hair Theorem and it makes the study of black holes remarkably simple: unlike all other astrophysical objects (which display a wide variety of properties), black holes are described by only three parameters. The stationary solution to which black holes evolve is called the Kerr-Newman solution[9] and, in the limit in which the angular momentum and charge are zero, it just becomes the Schwarzschild solution.

A third theorem shows that the surface area of a black hole never deceases[10]. This Area Theorem implies that black holes can never bifurcate, even though two of them can merge. However, the proof of this theorem applies only in classical theory and it can be violated by quantum effects. This was demonstrated by Hawking's discovery in 1974 that black holes are not black at all but radiate due to quantum effects with a temperature[11]

$$T_{BH} = \frac{hc^3}{8\pi GkM} = 10^{-7} \left(\frac{M}{M_\odot}\right)^{-1} K. \tag{3}$$

Since the holes lose energy in this way, they must shrink and eventually disappear altogether, even though this contradicts the Area Theorem. For holes of stellar origin, however, the temperature given by eqn (3) is tiny and the evaporation timescale is much longer than the age of the Universe, so the classical laws are still effectively valid.

1.2 How Black Holes Form

1.2.1 Stellar remnants. The most plausible mechanism for black hole formation invokes the collapse of stars which have completed their nuclear burning. However, this can only happen for sufficiently massive ones. Stars smaller than $4\ M_\odot$ are supposed to leave white dwarfs because the collapse of their remnants can be halted by electron degeneracy pressure[12], while stars in the mass range 4-$8\ M_\odot$ probably explode due to degenerate carbon ignition[13]. Stars larger than $8\ M_\odot$ but smaller than about $10^2 M_\odot$ are supposed to burn stably until they form an iron/nickel core[14]. At this stage no more energy can be released by nuclear reactions and so the core collapses. If the collapse can be halted by neutron degeneracy pressure, a neutron star will form and a reflected hydrodynamic shock then ejects the envelope of the star, giving rise to a type II supernova. If the core is too large, however, it necessarily collapses to a black hole, in which case it is not clear whether envelope ejection occurs. We do not know for certain what circumstances give rise to the formation of black holes rather than neutron stars, but it is probably reasonable to assume that a black hole will result if the initial stellar mass exceeds some critical value M_*.

It is difficult to predict the value of M_* theoretically. Indeed one could not be sure from numerical calculations that <u>any</u> stars in the $8 - 10^2 M_\odot$ range undergo collapse. However, there is good evidence that at least a few stellar black holes exist. For even though black holes can never be seen, one can still see their effects on surrounding objects. In particular, one can infer their presence in binary systems, especially when they are able to accrete material from the companion star and thereby generate X-rays. The first candidate for such an object was Cygnus X1 [15]. It was discovered by the X-ray satellite UHURU in 1972 and is still the best case of its kind, even though there are now several other ones (Circinus X-1 [16], LMCX-3 [17], GX339-4 [18]). The existence of stellar black holes is therefore likely on both theoretical and observational grounds, and M_* probably lies between 20 $M_\odot$ and 50 $M_\odot$.

1.2.2 <u>VMO remnants</u>. Stars larger than $10^2 M_\odot$ are radiation-dominated and therefore unstable to nuclear-energized pulsations during their hydrogen and helium burning phases [19]. It used to be thought that the resulting mass loss would be so rapid as to preclude the existence of such Very Massive Objects (VMOs). However, it is now thought that the pulsations will be dissipated as a result of shock formation [20] and this could reduce the mass loss enough for VMOs to survive for at least their main-sequence time [21]; this is just a few million years, independent of the VMO's mass. In fact, there is evidence for the existence of a few VMOs even at the present epoch (in particular, 30 Doradus [22], η Carina [23], and SN1961 [24]). However, VMOs encounter the much more serious "pair instability" as soon as they commence oxygen core burning; this is because the temperatures attained in this phase are enough to generate electron-positron pairs [25]. (This applies only for an oxygen core mass exceeding about 40 $M_\odot$, which is why ordinary stars are able to burn stably until they form an iron/nickel core.) This instability has two effects: sufficiently large cores collapse to black holes, while smaller ones explode [26]. Semi-analytical calculations [27], as well as numerical results[28], indicate that the critical dividing mass is $M_{oc} \approx 100\ M_\odot$ to an accuracy of 10% if there is no rotation, though the figure could rise to 500 $M_\odot$ if rotation is as large as possible [29]. The critical mass (M_c) for the initial hydrogen stars depends upon the amount of mass loss in the hydrogen and helium burning phase, but it would have to be at least 200 $M_\odot$. There is no direct evidence for any VMO holes in the real Universe, but all the tentative VMO candidates would appear to exceed the critical mass.

1.2.3 <u>SMO remnants</u>. Stars in the mass range above $10^5 M_\odot$ are unstable to general relativistic instabilities [30]. Such Supermassive Objects (SMOs) may collapse directly to black holes without any nuclear burning at all, at least if they have zero metallicity and no angular momentum. The presence of either metals or rotation may permit SMOs to explode in some mass range above $10^5 M_\odot$ but sufficiently massive ones will still collapse [31]. Although there is no definite evidence for the existence of SMOs, one could plausibly envisage their formation through relaxation at the centres of dense star clusters: the stars would be disrupted through collisions and a single supermassive star could then form from the newly released gas [32]. Indeed SMOs were originally invoked as an explanation for the violent activity associated with quasars, although accretion by their black hole remnants is now regarded as a more plausible explanation [33]. The holes would need to have a mass of about $10^8 M_\odot$, although it should be noted that supermassive holes would not necessarily derive from supermassive stars; they might also derive from the coalescence of smaller holes [34] or from accretion onto a single smaller hole [35].

Since quasars are probably precursors of galaxies, one might expect many galaxies to contain giant black holes in their nuclei even today[36]. In the last ten years considerable evidence has accumulated to support this view. Velocity dispersion and light curve measurements at the centre of M87 suggest[37] that it could contain a hole of $2\times10^{9} M_\odot$ and rotation curve measurements at the centre of our own galaxy indicate that it may house a $3\times10^{6} M_\odot$ hole[38]. The violence associated with some galactic nuclei could also result from the presence of a giant black hole: for example, the activity and X-ray emission of Centaurus A may be accounted for by a $10^{8} M_\odot$ hole[39].

1.2.4 Primordial remnants. Equation (2) shows that the formation of giant black holes does not involve the same extreme conditions as arise in stellar collapse: an object of $10^{9} M_\odot$ would only have the density of water on falling inside its event horizon. On the other hand, the formation of black holes smaller than a solar mass would require even more compression. Such conditions are unlikely to arise at the present epoch. They may, however, have arisen naturally in the first few moments of the Big Bang and this has led to the suggestion[40] that primordial black holes may have formed with mass much less than $1 M_\odot$. Such primordial holes could have formed from initial inhomegeneities if the Universe started off "semi-chaotic" or they could have formed spontaneously at a cosmological phase transition. We will be discussing these formation mechanisms in detail in Section (3). For the present it is sufficient to point out that primordial holes are expected to have a mass of order that of the particle horizon at their formation epoch. They could thus span an enormous mass range: from 10^{-5} g for those forming at the Planck time to $10^{5} M_\odot$ for those forming at 1 s. Although there is no conclusive evidence that primordial holes ever formed, they are of great theoretical interest since they are the only holes small enough for quantum effects to be important.

1.3 When Black Holes Form

A population of black holes could form at a variety of cosmological epochs, as indicated in Fig.1. The epoch of formation is not necessarily related to the mechanism of formation, although primordial holes form during the very early Universe by definition. Fig.1 also associates a "probability" with each scenario. The P estimates are necessarily subjective since it is difficult to assess the likelihood of any scenario in a field as prone to changing fashions as cosmology. Nevertheless, the probabilities are supposed to be a fair reflection of current trends, i.e., if one took a poll, the probabilities quoted might correspond to the fraction of cosmologists who would have credence in each scenario!

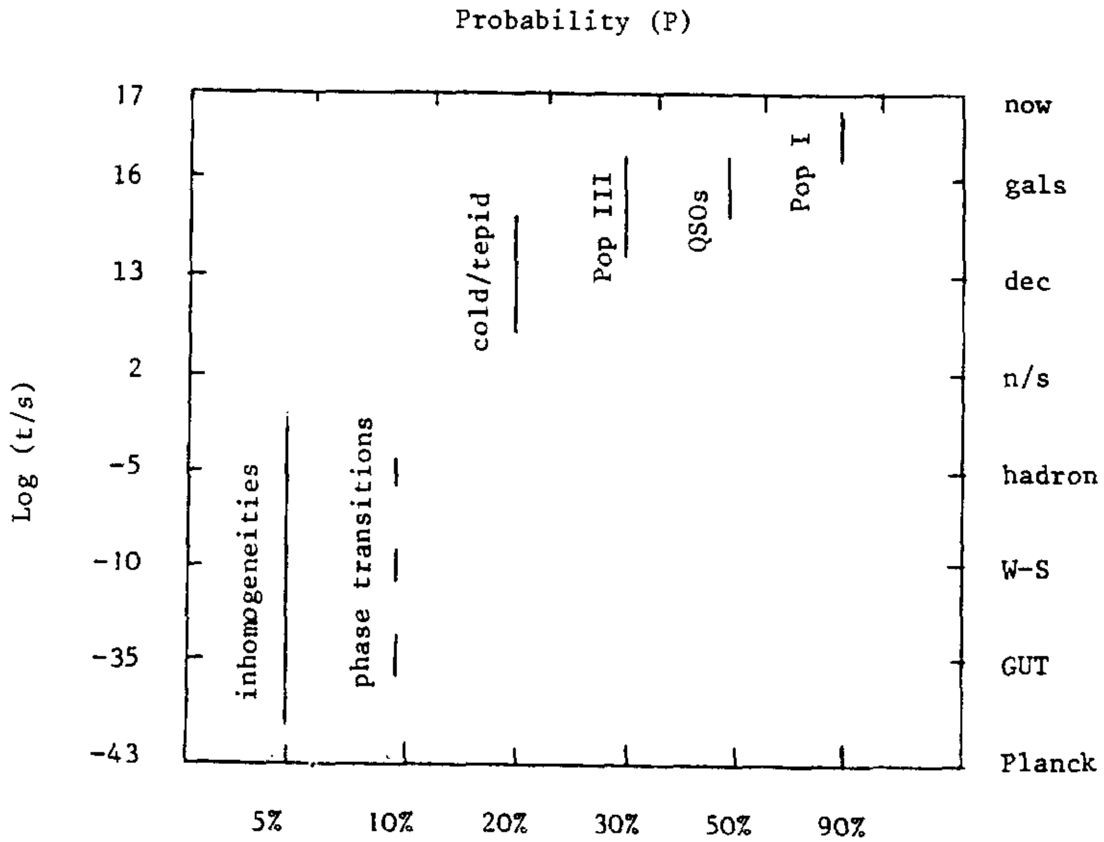


Figure (1): Probability of black holes forming at various epochs

1.3.1 Protogalactic holes. The holes most likely to exist are those which derive from ordinary Population I stars (i.e. stars which form in the discs of spiral galaxies). The number of such holes depends on the uncertain mass M_* but it would be very surprising if there were none of them at all. The probability of 90% is really an estimate of the likelihood that the prime candidate, Cygnus X-1, is a black hole. Since galactic discs probably do not form until a fairly recent epoch ($z \sim 1$), Population I holes are assumed to form at around 10^{10}y.

A probability of 50% has been assigned to the hypothesis that supermassive black holes power quasars and active galaxies. If the holes form through relaxation processes at the centres of protogalaxies, then they would arise shortly after the protogalaxies themselves. Since protogalaxies are expected to bind at about $z=10$, and since quasars are observed back to $z=4$, this probably corresponds to a time of order 10^9y. However, one cannot exclude the alternative hypothesis that the holes actually formed before the galaxies.

1.3.2 Pregalactic holes. As discussed in Section 4.2, Population III stars would be expected to form before galaxies providing the density fluctuations surviving at decoupling extended down to subgalactic scales. This is expected if the initial fluctuations were isothermal[41] and it may also apply with adiabatic fluctuations if the Universe is dominated by "cold" particles like axions[42]. The time at which such pregalactic stars would form depends on the amplitude of the density fluctuations: it would necessarily exceed the time of decoupling (10^6y) in a hot Universe and it would have to be before 10^9y. On the other hand, even if galaxies are the first objects to form, one could still postulate that a protogalactic generation of Population III stars formed. Indeed, in the sense that they have zero metallicity, the first stars to form are necessarily Population III. Thus the probability of the Population III scenario per se might be regarded as 100%. However, what is not certain is the characteristic mass of the first stars. The probability of 30% indicated in Fig.1 is supposed to specify the likelihood that some of these stars will be large enough to leave black hole remnants. If some of the Population III stars were actually SMOs, they could even produce the holes required to power quasars.

1.3.3 Primordial holes. The probability of primordial black hole formation has been estimated as only 5%. This is because, if the holes derive from initial inhomogeneities, then the amplitude of those inhomogeneities has to be very finely tuned if primordial holes are to be produced in a sufficient number to be interesting without being overproduced[43]. If the holes form at a phase transition, the situation is somewhat better because one does not need to depend on the prior existence of density fluctuations.

This conclusion assumes that the early Universe has a radiation equations of state (i.e. it presupposes the conventional hot Big Bang scenario). If the Universe started off "cold" (without any radiation content), the situation would be very different because the equation of state would go soft after the hadron era at 10^{-4}s. In this case, bound objects could form very prolifically on scales larger than 10 $M_\odot$ at very early times [44]. The same would apply if the Universe started off "tepid"[45] (with a photon-to-baryon ratio S much less than its present value of 10^9) because, in this case, the equation of state would go soft after $10^{-5} S^2$s. However, in both the cold and tepid scenarios the first objects would not be expected to form black holes directly; unless they were very large, they would produce primordial stars, in which case the black holes themselves would not arise until a much later cosmological epoch.

The consequences of this scenario are therefore rather similar to those of the Population III scenario, at least as far as black hole formation is concerned. However, there is an important difference in that one still has to generate the 3K background. The most natural way to accomplish this is through the stars themselves [46]. If most of the Universe is processed through VMOs, then one can shown that the photon-to-baryon ratio generated by their nuclear burning should be

$$S \approx \left(\frac{Gm_p^2}{\hbar c} \right)^{-1/4} \approx 10^{10}. \tag{4}$$

Thus the observed value of S is explained rather naturally. However, as discussed in Section 4.4.2, it turns out to be very difficult to thermalize starlight efficiently unless one invokes rather exotic grains [47]. An alternative scheme is to generate the radiation by black hole accretion at such an early time $(t < 10^6$ y) that light can be thermalized by free-free processes[48]. However, only SMOs larger than $10^6 M_\odot$ collapse on a timescale less than 10^6y. Fig.1 assigns a low probability of 20% to the cold and tepid scenarios, not only because of the problems involved in thermalizing the 3K background, but also because of the problems in generating the light elements whose abundances are so naturally explained by cosmological nucleosynthesis in the hot Big Bang picture[49].

1.4 How Many Black Holes

The sorts of holes which are most likely to exist would not be expected to contribute appreciably to the cosmological density. There could be a billion stellar holes in our own galactic disc but they would still only comprise a small fraction of the mass of the visible galaxy. For example, if one assumes that the mass distribution of large stars is described by the Scalo-Miller[50] spectrum and that a fraction ϕ_B of the mass of all stars larger than M_* ends up as a black hole, then the density of these holes (in units of the critical density) is only

$$\Omega_B \simeq \Omega_* \phi_B \left(\frac{M_{min}}{M_*}\right)^{1.2} \simeq 10^{-5} \left(\frac{M_*}{20M_\odot}\right)^{-1.2} \left(\frac{\phi_B}{0.1}\right) \qquad (5)$$

Similarly, even if every galaxy contains a $10^8 M_\odot$ black hole in its nucleus, the associated cosmological density would only be

$$\Omega_B \simeq \Omega_{gal} \left(\frac{M_{BH}}{M_{gal}}\right) \simeq 10^{-6} \qquad (6)$$

The question therefore arises of whether the other sorts of holes discussed above might not make a more significant contribution to the density, even though the probability of their existence may be a priori smaller. Could they make up in Ω for what they lack in P?

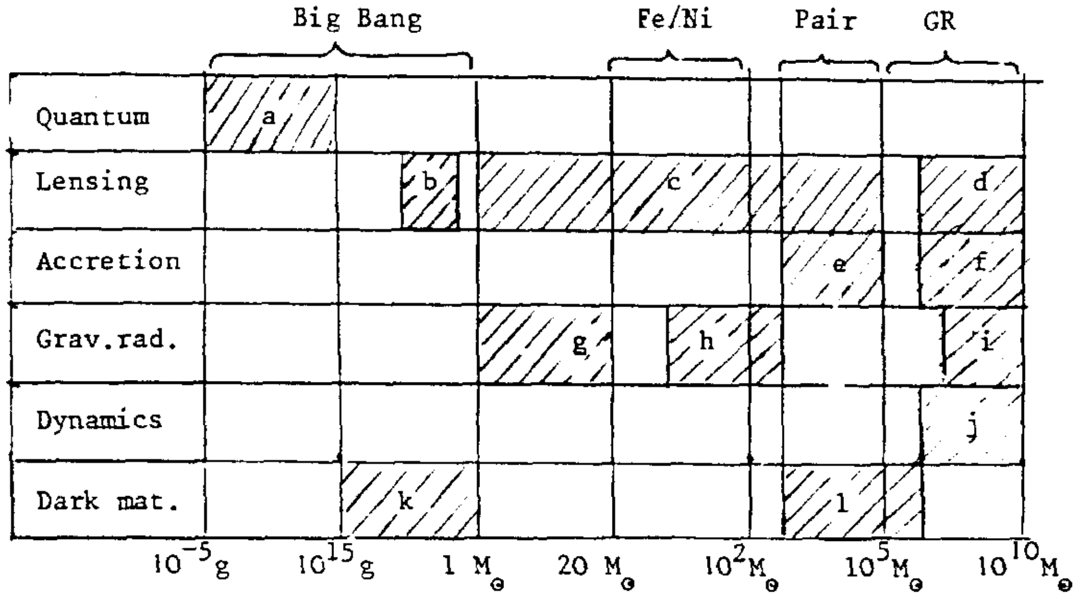
As we discuss in Section 2.2, there is now considerable evidence that a large fraction of the Universe's mass is dark[51]. There seem to be several dark components and, since darkness is such a pronounced property of objects which have undergone gravitational collapse, one naturally wonders whether black holes could provide one of them. This is certainly not an inevitable conclusion since there are several other viable candidates (in particular, some kind of elementary particle). Nevertheless, we will argue that black holes could at least be a plausible explanation for the dark matter in galactic halos. If this is indeed the case, one naturally wonders how such a large fraction of the Universe could have gone into black holes in the first place. There are probably only two possible answers: either they are the remnants of a first generation of Population III stars or they formed primordially but with a mass sufficiently large to have avoided evaporation by the present epoch.

We will be examining these options in turn in Sections (3) and (4). However, our discussion will not only focus on the dark matter issue; it turns out that primordial black holes and Population III stars can have important cosmological consequences even if they are not numerous enough to explain galactic halos. The general theme of these lectures might therefore be regarded as the cosmological consequences of black holes or their precursors. However, before scrutinizing primordial and Population III holes in particular, it may be useful to summarize some of the more general cosmological effects of black holes.

1.5 What Black Holes Do

Black holes could have a variety of astrophysical and cosmological consequences, even if their density is rather modest. The nature of these consequences will depend on the mass of the holes, as indicated in Table (1). Some of the effects will be discussed in greater detail in later sections but we summarize them below for completeness.

Table (1): Formation mechanisms for black holes and their consequences



(a) See Table (3)
(b) Intensity change
(c) Line/continuum
(d) Image-doubling
(e) IR background?
(f) X-ray background?
(g) Bar detection
(h) Laser detection
(i) Doppler detection
(j) Disc heating
(k) Encounters?
(l) See Table (4)

1.5.1 Dynamical effects. We will find that black holes could provide a lot of dark matter only if they are primordial with a mass between 10^{16}g and 1 $M_\odot$ or of Population III origin with a mass between $10^2 M_\odot$ and $10^6 M_\odot$. In the second case, the holes could have important dynamical effects. For example, if they reside in galactic halos, they could puff up the galactic disc, disrupt star clusters, and sink into the galactic nucleus through dynamical friction [52]. In general, these effects are significant only for holes larger than $10^5 M_\odot$ (i.e. only at the upper end of the permitted mass range). In the first case, however, the holes would be virtually undetectable by dynamical means. Even a minihole encountering the Earth would have no appreciable effect unless it were bigger than 10^{25} g and such an encounter could occur at most once every 10^5y. Thus the suggestion that the Tunguska event was induced by a small black hole[53] is rather implausible.

1.5.2 Lensing effects. Since the gravitational field associated with a black hole will bend light, anomalous effects are likely to occur whenever it traverses the line of sight of any light source. Both the shape and intensity of the source may change. In order for this traversal to occur with a reasonable probability, the source usually has to be an object like a quasar at a cosmologicial distance. Of course, this lensing effect is not specific to black holes (any object would do it) and so we will discuss the effect in Section 2.3.4 in the general context of the dark matter problem. However, in anticipation of that discussion, we note that gravitational lensing could permit the detection of black holes over the entire mass range from $10^{-4} M_\odot$ to $10^6 M_\odot$. It is thus the only effect which can probe both of the black hole scenarios for the dark matter mentioned above.

1.5.3 Quantum effects. The cosmological consequences of the evaporation of holes smaller than 10^{15} g will be discussed in detail in Section (3). We will argue that these holes could never have made a significant contribution to the cosmological density. Their evaporations may nevertheless have produced observable consequences. These are summarized in Table (2). In particular, they may have generated a detectable background of gamma-rays, positrons, and antiprotons; they may even have produced the cosmic deuterium abundance. The final explosive phase of evaporation could be particularly dramatic, the energy of a billion megaton bomb being released from a region whose size is only one thousandth of a fermi!

1.5.4 Gravitational radiation. Black holes should be the most efficient generators of gravity waves in nature. Most of the radiation would appear as a burst at the time of the original collapse [54]. The efficiency ϵ with which radiation energy is generated from the original rest mass depends on the asymmetry of the collapse. To ensure a high value of ϵ, the collapsing object has to undergo bounces or fragment when rotational effects become important: in the optimal case ϵ might be as high as 0.1 but, in general, it would be much less[55]. A variety of types of gravitational wave detectors should be operative within the next few years and, between them, they could seek for bursts from almost all the types of hole discussed in Section 1.2: bars could detect ordinary stellar collapses, laser interferometers could detect VMO collapses, and doppler tracking of interplanetary spacecraft could detect SMO collapses. If a large number of holes formed at some epoch, one would expect their bursts to overlap and form a background of gravitational waves. The characteristics of this background are discussed further in Section 4.4.8.

1.5.5 The clustering effect. The formation of black holes could generate large-scale density fluctuations. This is because one would expect there to be statistical ($\sqrt{N}$) fluctuations in their number density, at least over scales sufficiently large for their formation to be uncorrelated [56]. This number density fluctuation will not necessarily produce a growing fluctuation in the total density (because each hole would be expected to be initially surrounded by an underdense region), but it may do so in two circumstances: (1) if the holes form primordially when the Universe has a hard equation of state [57]; and (2) if the holes are born with large peculiar velocities [58]. In the first case, the density fluctuations are of the $\sqrt{N}$ form on all scales but only begin to grow at decoupling or when the holes dominate the density at $t_B = 10^{10}\Omega_B^{-2}$ s (whichever is earlier). This effect could conceivably generate the fluctuations required to make galaxies, providing the holes have a mass of at least $10^5 M_\odot$. In the second case, the $\sqrt{N}$ fluctuations are only set up on the scale over which the black holes' distibution can be randomized by their peculiar velocities. In order for this effect to explain galaxies, the holes must have a mass of at least $10^6 M_\odot$ and they need velocities of order 10^3 km s^{-1} . Such velocities might conceivably be produced by the gravitational radiation recoil effect[59-61].

1.5.6 Black hole accretion. Black holes may generate radiation through accretion [62]. At the present epoch, this may be the chief hallmark of their existence, since the resulting luminosity can be very large for black holes in binary systems or galactic nuclei. However, it should be emphasized that the luminosity generated by black hole accretion at pregalactic epochs could also be significant (providing, of course, the holes exist then). If we assume that the holes accrete gas at the Bondi rate [63] and that the accreted material is converted into radiation with an efficiency η , then this will exceed the Eddington limit for some period after decoupling providing the hole mass exceeds $10^{3}\eta^{-1}\ M_{\odot}$. Calculating how long the Eddington phase persists is complicated because the radiation generated will heat the background gas and thus suppress the accretion[64]. In general, however, one expects the accretion to have important consequences: besides affecting the thermal history of the Universe, it will also generate a significant background radiation density. If the holes are sufficiently large ($M \simeq 10^{8} M_{\odot}$), it has been proposed that their pregalactic accretion could explain the hard X-ray background [65,66]. On the other hand, the accretion of somewhat smaller holes ($M \approx 10^{6} M_{\odot}$) has been invoked to produce an infrared background[67]. Neither of these proposals should be taken too seriously (since they both presume specific accretion models) but it is certainly plausible that pregalactic black holes may have generated a lot of radiation in some waveband.

In concluding this section, it must be emphasized that we cannot be sure that any of the black holes whose formation mechanisms are indicated in Table (1) actually exist. They could exist in theory, but that is no guarantee that the conditions for their formation ever arose in practice (cf. the probabilities indicated in Fig.1). Accordingly, all the cosmological consequences discussed above are equally speculative. Nevertheless, even the possibility that black holes could have such a variety of effects emphasizes how important it is to study them.

2. THE DARK MATTER PROBLEM

In this section we will first review the evidence for the existence of different types of dark matter and we will then discuss the various candidates for explaining it. By collecting together all the different constraints on the mass of the dark objects, we will argue that it is unreasonable to expect all the dark matter problems to have a single solution. We suggest that black holes would be a plausible explanation for at least the dark matter in galactic halos.

2.1 Evidence For Four Types Of Dark Matter

2.1.1 The galactic disc. It has been known for many years that the local density of material in the galactic disc, as inferred from its velocity dispersion, exceeds the density observed in gas and visible stars. The most recent calculations [68] indicate that 50% of the disc mass is dark and this corresponds to a density parameter $\Omega \simeq 0.01$. Although this is a fairly modest density (compared to that associated with the other dark matter problems), it is the dark component for which the evidence is most unambiguous. The observations also indicate that the disc dark matter must have a velocity dispersion of less than 50 km s^{-1}. Thus it must itself be confined to the disc and cannot be associated with any of the other dark components discussed below.

2.1.2 Galactic halos. In several dozen spiral galaxies, the rotation curve measurements indicate that the rotation velocity is constant as far as the visible stars extend [69]. This corresponds to a density $\rho(R) \propto R^{-2}$, and hence to a mass $M(R) \propto R$, whereas the density of visible stars falls off as R^{-3}. In many cases the rotation velocity can be measured out to distances well beyond the visible stars (eg. by making 21 cm observations of neutral hydrogen) and yet still remains constant [70]. This indicates that spirals have a dark component which extends further than the visible material and contains considerably more mass. For our own galaxy, the rotation velocities of giant molecular clouds [71] and the velocity dispersion of globular clusters [72] suggest that the dark material extends to at least 30 kpc, while the dynamics of our satellites [73] may increase this to 60 kpc. Independent evidence for the existence of galactic halos may come from the persistence of warps in discs [74] and from the fact that an extended halo may be required to stabilize discs against bar formation [75]. Both these features would also require that the halo have a spheroidal distribution [76]. The mass-to-light ratio and mass of a typical spiral halo depend on the radius R_H to which it extends. If $R_H \approx 50$ kpc, we would require $M/L \approx 100$ and $M_H \approx 10^{12} M_\odot$; this corresponds to a density parameter $\Omega \approx 0.1$. It is unclear whether elliptical galaxies have halos [77], though there is some evidence that dwarf spheroidals do[78].

2.1.3 Clusters of galaxies. Measurements of the velocity dispersion in rich clusters of galaxies indicate [79] that their total mass exceeds the mass in their visible galaxies by at least a factor of 10. This corresponds to a typical mass to light ratio $M/L \approx 300$ and a density parameter $\Omega \sim 0.3$. The dark matter in clusters cannot be gas since, having a virial temperature of 10^8K, it would produce far more X-ray emission than is observed. However, some X-rays are seen and this suggests that the gas density is at least comparable to that in galaxies [80]. The dark matter in clusters could in principle be the the same as that in galactic halos. Indeed, in the hierarchical clustering picture [81], one would expect all the galaxies inside a cluster to be stripped of their individual halos, thus forming a collective dark component. However, this would explain the amount of dark matter in clusters only if the original galactic halos had a sufficiently large value of R_H.

2.1.4 The closure density. There are various theoretical reasons for expecting that that the Universe should have at least the critical density required for it to eventually stop expanding. For example, if the early Universe underwent an inflationary phase (thereby explaining the isotropy of the 3K background and the flatness problem [82]), one would expect the total density parameter to be 1 to at least 60 places of decimal! The isotropy of the Universe may also indicate that $\Omega = 1$, even if inflation never occurred.[83] Finally there may be aesthetic reasons for wanting the Universe to be closed (eg. to satisfy Mach's principle[84]). According to all these arguments, there must be a dark component with a mass-to-light ratio of at least $M/L \simeq 10^3$, corresponding to $\Omega \simeq 0.99$. This would not necessarily contravene dynamical observations, providing the component is distributed much more smoothly than galaxies themselves[85]; to avoid clustering, it would need to have a velocity dispersion exceeding $10^3 \mathrm{km\ s}^{-1}$. Thus, if the closure density dark matter does exist, it must be distinct from the preceding types of dark matter.

2.2 Candidates For The Dark Matter

There are many possible explanations for the various forms of dark matter discussed above. This is hardly surprising since most objects in the Universe are dark; for this reason, several of the explanations may turn out to be correct. The candidates may be grouped into two categories: non-baryonic types (in which the dark object is some sort of elementary particle) and baryonic types (in which it is something astrophysical). In the first case, the existence of the dark object (which may degenerately be termed an "ino") goes back to the very early Universe. In the second case, the dark object forms out of the background gas at a relatively late stage (viz. 10^6-10^9 y after the Big Bang); this may be termed the "Population III" scenario. The two possibilities are illustrated qualitatively by the first two diagrams in Fig. 2 and the candidates are listed explicitly in Table (2). Note that primordial black holes will be included in the non-baryonic category (even though the holes may be large enough to be regarded as astrophysical) since they form at a time when the baryons only comprise a tiny fraction of the Universe's total density.

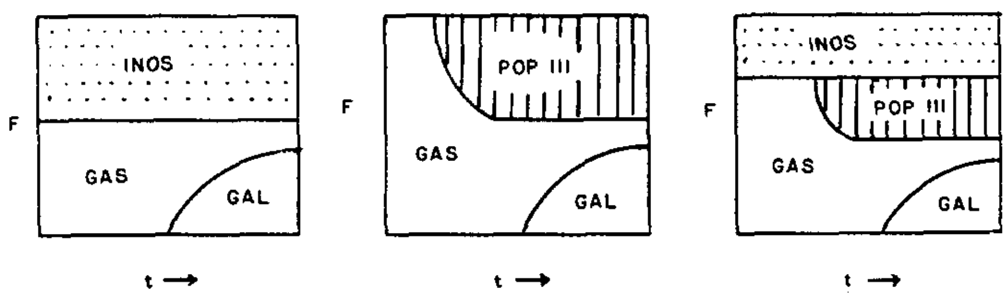


Figure (2): Three scenarios for dark matter and galaxy formation

2.2.1 <u>Elementary particle candidates.</u> In the conventional hot Big Bang picture any particle should exist in thermal equilibrium with all other particles at times sufficiently early that the various interaction rates exceed the cosmological expansion rate. When this condition first fails (at some temperature T_F), the particle concerned will "freeze out"; providing it is relativistic at this time (i.e. providing its rest mass m_x is less than kT_F) and providing it survives until the present epoch (i.e. providing it does not annihilate or decay), its present number density should be [86]

$$n_x \simeq g\, n_\gamma \simeq 10^2 g \text{ cm}^{-3}, \quad g = \left(\frac{T_x}{T_\gamma}\right)^3. \tag{7}$$

Here n_γ is the number density of the 3K photons and the factor g arises because the annihilation of other particle species after the freeze-out time will increase the relative photon density. Thus, as T_F increases, $g(T_F)$ decreases in a series of steps, each step corresponding to the rest mass of some particle. Since the particles are assumed to be non-relativistic today, eqn (7) implies that their present density is

$$\Omega_x \simeq g \left(\frac{m_x}{100\text{eV}}\right) \tag{8}$$

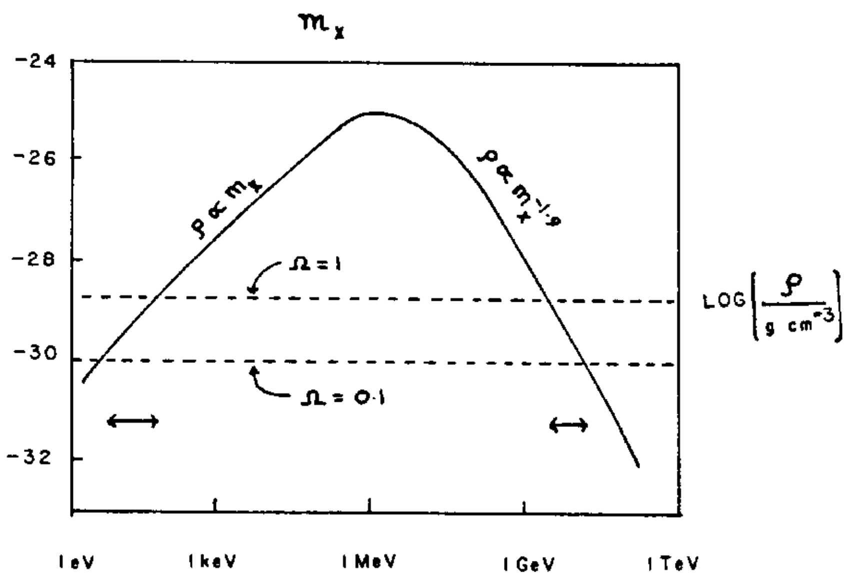


Figure (3): Density of "relict" inos as a function of their mass

If the particle is non-relativistic at freeze-out, the value of n_x is reduced by a Boltzmann factor[87] and this implies that the Ω_x decreases as a power of m_x for $m_x > kT_x$. The overall dependence of Ω_x on m_x is thus as indicated in Fig.3. In order to explain the halo or cluster dark matter problems without having more density than would be consistent with observations of the cosmological deceleration parameter, m_x must lie in one of two narrow bands, as shown by the arrows.

The original elementary particle candidate was the neutrino[88-91]. This freezes out when the weak interaction rate falls below the expansion rate at $T_F = 1$ MeV and $g = 3/11$. Thus eqn (8) shows that the neutrino could be of cosmological significance if its rest mass exceeds about 10 eV. Attention was focussed on this possibility as a result of tritium decay experiments, which appeared to indicate[92] a neutrino rest mass in the range $14\ \mathrm{eV} < m_\nu < 46\ \mathrm{eV}$. Independent evidence may come through the detection of neutrino oscillations: electron and muon neutrinos of energy E_ν should transform into each other over a distance[93]

$$L = \frac{E_\nu}{m_{\nu e}^2 - m_{\nu\mu}^2} \approx 10 \left(\frac{E_\nu}{10\mathrm{MeV}} \right) \left(\frac{m_\nu}{10\mathrm{eV}} \right)^{-2} \mathrm{m}. \tag{9}$$

At one stage there were claims to have found such an effect over a distance of 10 metres, though later experiments have not confirmed this. Indirect evidence for neutrino oscillations may come from the solar neutrino experiment[94]: the observed flux of electron neutrinos appears to be about a third that expected from nuclear reactions within the core of the Sun and this may be explained rather naturally if there are three neutrino species which oscillate into each other. However, eqn (9) implies that the oscillation lengthscale is less than the distance to the Sun providing m_ν exceeds 10^{-6}eV, so the solar neutrino problem does not in itself imply that m_ν is large enough to be of cosmological significance.

2.2.2 Warm and cold elementary particles. Another possibility is to invoke a particle whose mass is larger than 10 eV but which decouples earlier (reducing g) so that Ω_x is not too large. Candidates for such a particle arise in the supersymmetry theories (for which $T_F \approx 10^6$GeV and $g \simeq 0.01$) and include the gravitino[95], the right-handed neutrino[97], the photino[98,99], and sneutrino[100]. The mass of these particles ranges from 1 keV to 1 GeV and, if m_x is very large, one may need to invoke annihilations as well as an decrease in g to avoid an excessive value of Ω_x. Since the present temperature of these particles should should just be $g^{1/3}$ times the temperature of the microwave background, their velocity dispersion - prior to any clustering - should be

$$\langle v^2 \rangle^{\frac{1}{2}} \approx \frac{kT_x}{m_x c} \approx 20 \left(\frac{m_x}{10\text{eV}} \right)^{-1} g^{1/3} (1+z) \text{ kms}^{-1} \tag{10}$$

This decreases with increasing m_x and so particles much heavier than 10 eV are termed "warm" to distinguish them from particles like the neutrino which are termed "hot".

It should be stressed that the free-streaming of the particles will erase any density fluctuations on scales less than they can traverse by the time they go non-relativistic. This corresponds to a mass-scale[90]

$$M \approx \frac{m_{\text{planck}}^3}{m_x^2} \approx 10^{15} \left(\frac{m_x}{10\text{eV}} \right)^{-2} M_\odot \tag{11}$$

For hot particles like neutrinos, this scale is very large and so it may be very difficult to explain how the observed large-scale structure could have evolved by the present epoch. For warm particles like the gravitino, the damping scale could be as small as a galaxy and the problem is less pronounced. However, a currently more popular solution is to invoke a "cold" particle like the axion which is not subject to free-streaming at all[101-104]. The axion is associated with an extra symmetry introduced so that QCD does not exhibit CP violation. This symmetry is spontaneously broken at an energy scale $f_a \approx 10^{12}$GeV. At a lower temperature, $T_I \approx 1$ GeV, the axion developes a mass $m_a \approx 10^{-5}$eV due to QCD instanton effects and the associated density is

$$\rho_a \simeq \left(\frac{f_a^2}{M_p T_I} \right) m_a T^3 \tag{12}$$

The value of m_a is constrained by various astrophysical observations but Ω_a could certainly be large enough to be of cosmological significance.

Doubtless theorists will concoct many more types of elementary particles in the years to come, many of which might in principle have survived as relicts of the Big Bang, so it is to difficult at this stage to assess the front runner. The latest candidate, for example, is "shadow matter". This is matter which resembles ordinary matter but only interacts with it gravitationally[105]; its existence may be predicted by superstring theory. It is probably premature to lay bets on any particular candidate. However, with such a large zoo of "inos", it is not implausible that at least one of them will turn out to be cosmologically significant. Note that certain types of elementary particles can already be excluded: for example, magnetic monopoles could not have a significant cosmological density without contravening the Parker limit.[106] The only remaining non-baryonic candidate, primordial black holes, will be disussed in detail in Section (3). Suffice it to say that only non-evaporating ones in the mass range 10^{15}g to 1 $M_\odot$ could possibly be relevant to any of the dark matter problems discussed above.

2.2.3 Population III candidates. It is very difficult to predict a priori the mass of the Population III objects since it depends on the mass-scale at which fragmentation of the first bound clouds ceases. (This is discussed in detail in Section 4.2.) The suggestions range from objects as small as snowballs to objects as large as supermassive stars. We will discuss each of the possibilities in turn, although we will find later that many of them can be rejected on empirical grounds.

Snowballs of condensed hydrogen have been proposed but can be excluded immediately[107]. In order to have avoided collisions within the age of the Universe, they must have a size of at least 1 cm but they would then have been evaporated by the 3K background radiation. In any case, it is rather unlikely that any fragmentation scenario in the hot Big Bang picture could produce fragments as small as this. One might, on the other hand, envisage the fragments being as small as Jupiters (i.e. objects in the mass range $M < 0.08\ M_\odot$ which are too small to ignite their nuclear fuel). We will see later that such objects could only be detectable by their gravitational lensing effects.

There would be a better chance of detecting Population III objects which derive from nuclear-burning stars. Stars smaller than $1 M_\odot$ would still be burning but they would have to be at least as small as $0.1\ M_\odot$ in order to have a mass-to-light ratio large enough to explain any of the dark matter problems[51]. Stars larger than $1\ M_\odot$ would no longer exist but they could still have produced dark remnants. For example, we have seen that white dwarfs and neutron stars could derive from stars in the mass ranges $1 - 4\ M_\odot$ and $8 - 100\ M_\odot$, respectively, and that black holes could possibly derive from stars at the upper end of this mass range. However, as discussed in Section 1.1, only VMOs in the mass range above $M_c \approx 200\ M_\odot$ and SMOs in the mass range above $10^5 M_\odot$ could collapse to black holes without ejecting a large fraction of their initial mass first. In any case, a priori, all Population III objects could produce dark matter except those in the mass ranges $0.1 - 1\ M_\odot$ (which are too bright), $4 - 8\ M_\odot$ (which explode due to degenerate carbon ignition), and $100 - 200\ M_\odot$ (which explode due to the pair instability).

2.3 Constraints On The Dark Matter

2.3.1 Cosmological nucleosynthesis. In the standard hot Big Bang picture, the neutron-proton ratio freezes out at a temperature $T_F \simeq 10^{10}$K (i.e. at a time $t_F \simeq 1$ s), when the rate for the for the weak interactions $p + e^- \to n + \nu$, $n + e^+ \to p + \bar{\nu}$ falls below the expansion rate. At this point the ratio has a value

$$\left(\frac{n}{p}\right)_F \approx \exp\left[-\left(\frac{m_n - m_p}{kT_F}\right)\right] \approx \frac{1}{8} \qquad (13)$$

Since all neutrons (except the small fraction which are lost through β-decay) burn first into deuterium and then into helium at about 10^2s, the resulting helium abundance is[49]

$$Y \approx 2\left(\frac{n}{p}\right)_F \Big/ \left[1 - \left(\frac{n}{p}\right)_F\right] \approx \frac{1}{4} \qquad (14)$$

There are also small residual abundances of deuterium, helium-3, and lithium-7. These depend very sensitively on the total baryon density Ω_b, but it is a remarkable triumph of the standard model that the predicted abundances of all these elements[108] is consistent with observation providing $\Omega_b \simeq 0.1$. In particular, the observed deuterium abundance of 10^{-5} can only be explained by cosmological nucleosynthesis if $\Omega_b \leq 0.1(H_o/50)^{-2}$. Thus the dark matter in galactic halos (and conceivably even clusters) could be of baryonic origin, but a critical density certainly could not be unless one sacrifices the hot picture altogether[109]. We infer that Population III candidates could not have $\Omega \simeq 1$. This conclusion could be circumvented only for primordial black holes that formed before the neutron-proton freeze-out time; such holes would necessarily be smaller than $10^5 M_\odot$.

2.3.2 Enrichment constraints. The existence of Population I stars with metallicity as low as 10^{-3} excludes any of the dark matter problems being solved by stars with produce an appreciable metal yield. If the fraction of the initial stellar mass which is is left as a remnant is ϕ_r and the fraction returned as metals is Z_{ej}, the maximum density of the remnants is

$$\Omega_r = 10^{-3} \phi_r Z_{ej}^{-1} \Omega_g , \qquad (14)$$

where Ω_g is the gas density before the stars form. This constraint is disussed in more detail in Section 4.3.3. In the present context, it is sufficient to point out that it already excludes neutron stars as an explanation of anything except the local dark matter problem. Since these can only derive from stars in the mass range 8 - 60 $M_\odot$, for which $Z_{ej} > 0.2$ and $\phi_r < 0.2$, Ω_r could be at most $10^{-3}\Omega_g$ for neutron stars. A similar argument does not work for white dwarfs since these derive from stars in the range 1 - 4 $M_\odot$ and these return helium (for which the constraints are rather weak) rather than metals. However, even in this case Ω_r could be at most $\phi_r \Omega_g$ and this could only be large enough to explain the local dark matter.

2.3.3 Source count constraints. We have seen that stars with mass around 0.1 $M_\odot$ might in principle have a sufficiently high mass-to-light ratio to explain any of the dark matter problems. However, such stars could still be detectable as high velocity infrared sources[110] and searches already indicate[111] that their number density near the Sun can be at most

0.01 pc^{-3}. This is a hundred times too small to explain the local dark mattter problem and ten times too small to explain the halo problem, so M-dwarfs would seem to be excluded. A similar conclusion is indicated by infrared observations of some other spiral galaxies; these suggest that the mass of the halo objects must be less than 0.08 $M_{\odot}$, which would preclude any main-sequence stars[112]. Indirect arguments may even exclude Jupiters[107]: for it would be surprising if any fragmentation scenario could lead to fragments smaller than (say) 0.004 $M_{\odot}$ and, unless the fragments have a very steep spectrum, this implies there would still be too many light-producing stars above 0.1 $M_{\odot}$.

2.3.4 Lensing constraints. Both high and low mass Population III objects could produce interesting gravitational lens effects. If one has a population of objects with mass M and density Ω_r, then the probability that one of them will lie close enough to the line of sight of a quasar to image-double it is about Ω_r and the separation between the images is[113]

$$\theta \simeq 10^{-6} \left(\frac{M}{M_{\odot}} \right)^{\frac{1}{2}} \text{arcsec.} \qquad (15)$$

Thus the VLA - with a resolution of 0.1 arcsec - could search for lenses as small as $10^{10} M_{\odot}$ (indeed it has already found them), and the VLBI - with a resolution of 10^{-3} arcsec - could search for ones as small as $10^{6} M_{\odot}$. This effect is important only for very large Population III objects. However, a galaxy itself can act as a lens and, if one is suitably positioned to image-double a quasar, then it can be shown that there is also a high probability than an individual halo object will traverse one of the lines of sight. This will give appreciable intensity fluctuations in one but not both images.[114] This effect would be observable for stars larger than $10^{-4} M_{\odot}$ but the timescale of the fluctuations, being or order $40(M/M_{\odot})^{1/2}$y, would only be detectable over a reasonable period for $M < 0.1\ M_{\odot}$. However, yet another kind of lensing effect could permit the detection of objects with $M > 0.1\ M_{\odot}$. This derives from the fact that such objects could modify the ratio of the line to continuum output of the quasar[115]; the fluxes are affected differently because they come from regions which act as extended and pointlike sources, respectively, unless M is very large. This already excludes a critical density of objects with $0.1 < M/M_{\odot} < 10^{5}$, though not necessarily the tenth critical density required for halos. SMOs with $M > 10^{5} M_{\odot}$ are not excluded by this effect because, for them, the whole quasar nucleus would act as a point source. However, we have seen that holes only slightly larger than this might be detected by their direct image-doubling. Thus lensing constrains the Population III mass spectrum over nearly the entire range above $10^{-4} M_{\odot}$.

2.3.5 Dynamical constraints. A variety of dynamical effects constrains the masses of all four types of dark matter. For example, the survival of binaries in the galactic disc already requires [116] that the objects which comprise the local dark matter be smaller than $2\ M_{\odot}$. On the other hand, if the dark matter is dissipationless, it must necessarily form in the disc and this immediately excludes any ino solution. The requirement that the disc should not be puffed up too much by the heating effect of traversing halo objects implies [117] that the halo objects must be no larger than $10^{6} M_{\odot}$, an effect which is discussed in detail in Section 4.4.8. Even if the dark matter in clusters is different from the halo dark matter, the absence of unexplained tidal distortions of visible galaxies in (for example) the Virgo cluster implies [118] that the dark objects must still be smaller than $10^{9} M_{\odot}$. The fact that any dark matter which contains the critical density must avoid clustering like galaxies implies that it must have a velocity dispersion of at least 10^{3} km s^{-1}. This would seem to exclude any Population III candidates: even black holes could not be expected to born with recoil velocities this large. However, this argument would not exclude inos from having the closure density. In fact, phase space considerations - together with eqn (10) - imply that inos can form clusters of velocity dispersion σ and radius R only if [119]

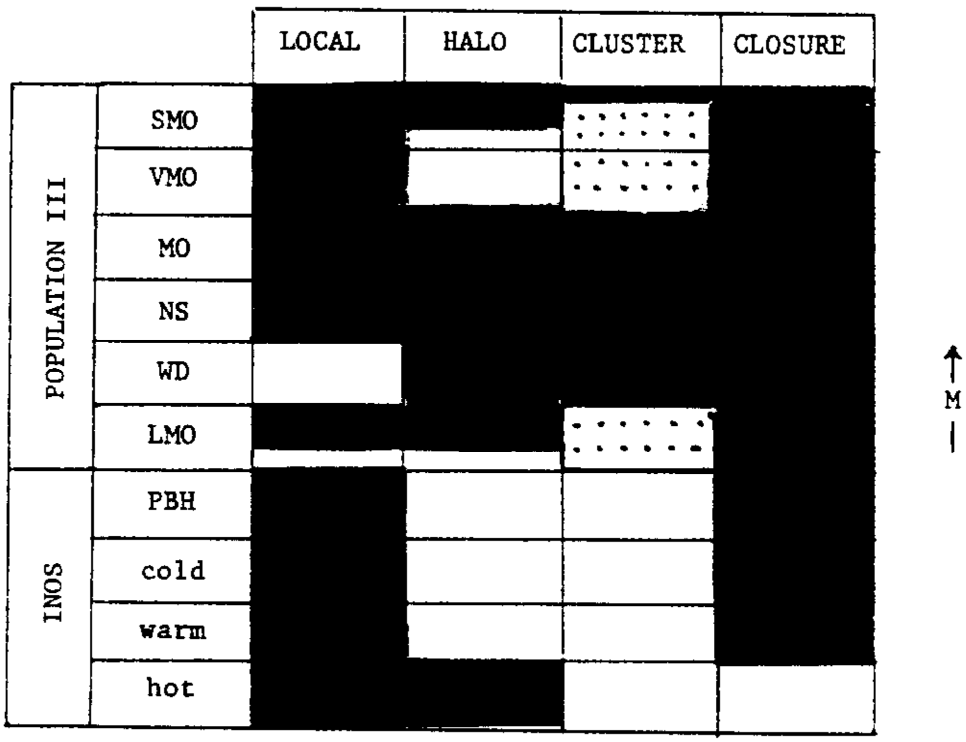
$$M_x > 30 \left(\frac{R}{10\text{kpc}} \right)^{-1/2} \left(\frac{\sigma}{200\text{kms}^{-1}} \right)^{-1/4} \text{eV} \qquad (16)$$

Thus inos can explain the dark matter in clusters only if $m_x > 4$ eV and they can explain dark halos only if $m_x > 20$ eV; in the latter case, eqn (8) implies $\Omega_x > 0.5$, which is probably too large. Thus the same feature which makes inos an attractive explanation of the closure dark matter detracts from their attractiveness as an explanation of the halo dark matter.

2.4 Conclusion

The various constraints discussed above are brought together in Table (2), which indicates which sorts of dark matter could explain each of the four dark matter problems. The shaded regions in this figure are excluded by at least one of the arguments given above. Whether the dotted region is excluded depends on whether the cosmological nucleosynthesis constraint demands that the dark matter in clusters be non-baryonic; this is marginal.

Table (2): Constraints on types of dark matter



The prime message of Table (2) is that one could not expect any single dark component to resolve all four dark matter problems. This should be of little surprise since most things in the Universe are dark. On the other hand, the figure does give some indications of what the best solutions might be: (i) the best candidate for the local dark matter would seem to be white dwarfs or Jupiters; (2) a possible solution for the halo dark matter would seem to be the black hole remnants of VMOs or low mass SMOs, though one cannot exclude primordial black holes or warm or cold inos; (3) the dark matter in clusters would need to be primordial black holes or inos if one adopts the cosmological nucleosynthesis limit in its strongest form but the other halo candidates would be viable if one adopts the weaker form; (4) the closure dark matter could only be hot inos.

Our analysis does not provide a unique answer to the four dark matter problems but it at least narrows down the range of possibilities. After all, a priori, there was an uncertainly of 10^{80} in the mass scale of dark matter, ranging from the 10^{-5} eV axion to the $10^{10}\ M_{\odot}$ SMO. Lacking a unique answer, each of us will doubtless assess the likelihood of the various candidates according to our own individual prejudices. Thus presumably particle physicists will prefer ino solutions, while astrophysicists will prefer Population III solutions. In a spirit of compromise, however, it is perhaps worth stressing that both ino and Population III solutions may be relevant: inos could provide the closure density and perhaps the dark mass in clusters, while black holes may provide the dark matter in halos. It is in this spirit of compromise that the third picture in Fig.3 is offered!

3. PRIMORDIAL BLACK HOLES

In this section we study the different ways in which primordial black holes (PBHs) may have formed in the early Universe and we derive their likely mass spectrum. We then consider their cosmological consequences, with particular emphasis on the evaporation of very small PBHs. Such holes are unlikely to have had an appreciable cosmological density but they may nevertheless have observable consequences.

3.1 The Formation of PBHs From Initial Inhomogeneities.

It was first pointed out by Hawking[40] that black holes could have formed in the first few moments of the Big Bang if the early Universe contained density inhomogeneities. Overdense regions would stop expanding with the background and undergo collapse providing they were larger than the Jeans length at maximum expansion, the Jeans length being $\sqrt{\gamma}$ times the horizon size if the background equation of state is $p = \gamma\rho$. The condition for this can be derived as follows. Consider a region with mass M which is overdense by a factor δ_o at some initial time t_o. When that region falls within the particle horizon at time t_H, the density fluctuation will have grown to

$$\delta_H = \delta_o \left(\frac{M}{M_o} \right)^{2/3} \tag{17}$$

(independent of γ) where M_o is the horizon mass at t_o. In this equation δ_H and δ_o represent the gauge-invariant energy density perturbations measured with respect to the comoving spatial hypersurface. After t_H the scale of the region and its overdensity evolve as

$$R \propto t^{\frac{2}{3(1+\gamma)}} , \qquad \delta \propto t^{\frac{2(1+3\gamma)}{3(1+\gamma)}} . \tag{18}$$

Thus the region binds (i.e. δ has grown to about 1) at a time

$$t_B = t_H \delta_H^{-\frac{3(1+\gamma)}{3(1+3\gamma)}} \tag{19}$$

and its scale is then

$$R_B = c t_B \delta_H^{1/2} \tag{20}$$

(i.e. it is smaller than the particle horizon by a factor $\sqrt{\delta_H}$). In order to collapse against the pressure at t_B we therefore need

$$\delta_H \geq \gamma, \quad \delta_o \geq \gamma \left(\frac{M}{M_o} \right)^{-2/3}. \tag{21}$$

On the other hand, a region cannot be larger than the particle horizon at maximum expansion without forming a closed Universe, topologically disconnected from our own.[43] One way to see this is to consider the spatial hypersurface containing the region at t_B: its 3-curvature is $(G\rho) \sim (ct_B)^{-2}$, so it will close up on itself if the overdensity extends beyond a scale $\sim ct_B$. Thus we require $\delta_H < 1$. Note that δ_H is a measure of the metric perturbation at all times prior to t_H. Thus a region with $\delta_H > 1$ is <u>always</u> disconnected from our Universe; it does not evolve to that state.

Equation (20) implies that, unless the equation of state is soft ($\gamma = 0$), PBHs must have of order the horizon mass M_H at formation. More precisely, holes forming at time t should have an initial mass

$$M(t) \approx \gamma^{3/2} M_H(t) \approx \gamma^{3/2} \left(\frac{c^3 t}{G} \right) \approx 10^{38} \gamma^{3/2} \left(\frac{t}{s} \right) g. \tag{22}$$

Thus holes forming at the Planck time ($t_p \sim 10^{-43}$s) should have the Planck mass ($M_p \sim 10^{-5}$g), those forming at 10^{-23}s should have a mass of 10^{15} g, and those forming at 1 s should have a mass of $10^5 M_\odot$. This means that PBHs could span an enormous mass range, encompassing both the ones which are small enough to evaporate and the ones which are large enough to have significant astrophysical effects.

Equation (22) is roughly confirmed by the detailed hydrodynamical calculations of Nadejin <u>et al</u>.[120] , who model PBH formation by a patching part of a k = +1 Friedmann universe onto a k = 0 Friedmann universe via a vacuum transition region. The evolution is found to depend on two parameters: the ratio of the size of the region (R) to the size of a k = +1 universe with the same density (R_{max}) and the ratio of the width of the transition region (Δ) to R. The first parameter indicates how close the overdense region is to being a separate universe; the second parameter determines the pressure gradient. As shown in Fig. 4, one needs a minimum value for R/R_{max} if a black hole is to form and, as the ratio Δ/R decreases (so that the pressure gradient

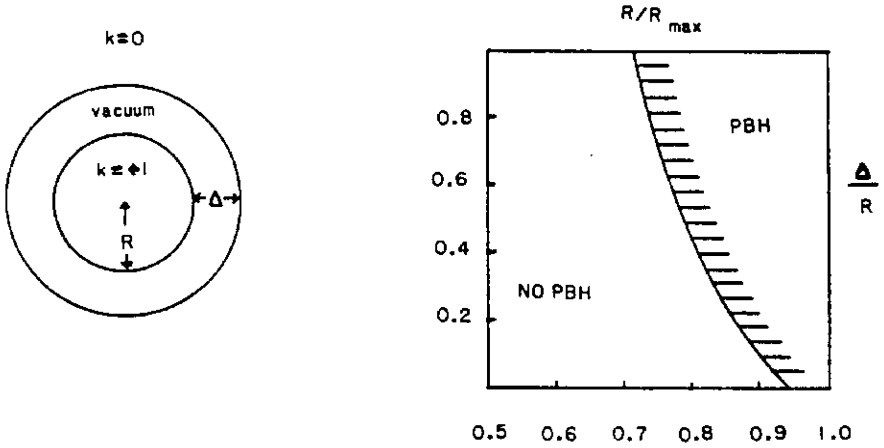


Figure (4): Condition for PBH formation from primordial inhomogeneities

increases, thereby making black hole formation more difficult), the value of R/R_{max} required increases. For example, as Δ/R goes from 1.0 to 0.1, the value of R/R_{max} required goes from 0.8 to 0.9. Perturbations with R/R_{max} less than the critical value turn into sound waves when they fall inside the Jeans length and are dissipated; only perturbations with R/R_{max} larger than the critical value grow large enough for gravitational collapse to ensue. The mass of the resulting black hole turns out to lie in the range 0.01 to 0.06 times the horizon mass at the formation epoch, which is somewhat smaller than the naive estimate given by eqn (22). The precise value depends on the equation of state.

3.2 The Growth and Mass Spectrum of PBHs

One issue which has attracted a lot of attention is the question of how much a PBH, once formed, can grow through accretion. A simple Newtonian argument of Zeldovich and Novikov[121] suggests that the black hole mass should evolve according to

$$M = M_H(t)\left[1 + \frac{t}{t_1}\left(\frac{M_H(t_1)}{M_1} - 1\right)\right]^{-1} \qquad (23)$$

where M_1 is the mass when the hole forms at time t_1. This implies that holes much smaller than the horizon cannot grow much at all, whereas holes of size comparable to the horizon could continue to grow at the same rate as the horizon ($M \propto t$) throughout the radiation-dominated era. Since eqn (22) indicates that a PBH must be of order the horizon size at formation, this suggests that all PBHs could grow to have a mass of order $10^{15} M_\odot$ (the horizon mass at the end of the radiation era). There are very strong observational limits on how many such giant holes could exist in the Universe[52], so the implication would be that very few PBHs ever formed. However, the Zeldovich-Novikov argument is clearly questionable since it neglects cosmological expansion effects and these are presumably going to hinder a black hole's growth. Indeed the notion that PBHs can grow at the horizon rate was disproved by Carr and Hawking[122], who showed that there is no spherically symmetric similarity solution which contains a black hole attached to a $k = 0$ Friedmann background via a pressure-wave. Since it must therefore soon become smaller than the horizon, at which stage cosmological effects become unimportant and the Zeldovich-Novikov argument does pertain, one concludes that a PBH cannot grow much at all.

An interesting special case arises if the equation of state is stiff ($p = \rho$) when the holes form. This possibility cannot be excluded, especially for holes smaller than 10^{15} g which form in the first 10^{-23}s after the big bang. Lin *et al.*[123] have argued that a similarity solution containing a black hole attached to a $k = 0$ Friedmann universe does exist in this situation, basically because the sound-wave propagates out at the same rate as the particle horizon. However, Bicknell and Henriksen[124] show that this solution contains a rather unphysical feature at the sound surface: one needs the incoming matter to be transformed into an ingoing null fluid. This feature actually permits growth at the horizon rate even if $p = \rho/3$[125]. Unless one grants this possibility, growth will be limited even in the stiff situation. Indeed the most likely consequence of a stiff equation of state would be the suppression of PBH formation.

Another important question concerns the mass spectrum of PBHs. This is related to the form of the density fluctuations fairly straightforwardly[43]. If the size of a region is to exceed the Jeans length at maximum expansion, we have seen that the amplitude of the density fluctuation on entering the horizon must exceed γ. If one assumes that the density fluctuations on a mass-scale M are spherically symmetric and have a Gaussian distribution with a root-mean-square value $\langle\delta_H(M)^2\rangle^{1/2} \equiv \varepsilon(M)$, then the probability of a region of mass M forming a black hole is

$$\beta(M) \sim \varepsilon(M) \exp\left[-\frac{\gamma^2}{2\varepsilon(M)^2}\right] \tag{24}$$

Providing $\beta \ll 1$, so that the probability of the hole being in a region which collapses later is small, this is directly related to the fraction of the Universe's mass which ends up in holes of mass M. Although one might question the necessity of a Gaussian distribution for the density fluctuations, none of our qualitative conclusions will depend on this except the exponential sensitivity of β on ε. Lindley has presented a more sophisticated treatment, allowing for deviations from a Gaussian distribution and for the effects of holes being swallowed by larger holes,[126] while Marochnik _et al._[127] and Barrow and Carr[128] have discussed the effects of deviations from spherical symmetry.

For "constant curvature" fluctuations (in which δ_o scales as $M^{-2/3}$), ε is scale-independent and the present number density of PBHs in the mass range M to M+dM can be shown to be $\Pi(M)dM$ with

$$\Pi(M) \propto \beta M^{-\alpha}, \quad \alpha = \left(\frac{1+3\gamma}{1+\gamma}\right) + 1. \tag{25}$$

The integrated mass density goes as $M^{2-\alpha}$. The dependence of the exponent α on the equation of state parameter γ stems from the fact that a region's mass is reduced by redshift effects before it undergoes collapse but not thereafter. For $\gamma = 1$, $\alpha = 3$; for $\gamma = 1/3$ (the most likely value), $\alpha = 5/2$; and as $\gamma \to 0$, $\alpha \to 2$. In fact, equations (24) and (25) do not apply if γ is very small because β may then be of order 1; in this case lots of holes are forming all the time and the effective value of α is 1 as a result of swallowing[44]. In any other situation, α exceeds 2, so most of the mass in PBHs will be in the smallest ones.

If the initial fluctuations go as $\delta_o \propto M^{-n}$ with $n > 2/3$ (i.e. if they fall off faster than constant curvature fluctuations), then $\varepsilon(M)$ decreases with M and

$$\pi(M) \propto \exp\left[-\frac{\gamma^2}{2\varepsilon_o^2}\left(\frac{M}{M_o}\right)\right]^{\frac{(n-2/3)(1+3\gamma)}{(1+\gamma)}} \tag{26}$$

where δ_o is the value of δ on the scale M_o. Thus the PBH spectrum is exponentially cut off above a mass

$$M_{max} \approx 10^2\, \varepsilon_o^{\left[\frac{2(1+\gamma)}{(1+3\gamma)(n-2/3)}\right]} M_o . \tag{27}$$

For example, if $M_o \approx 10^{-5}$ g and $\gamma = 1/3$, this gives $M_{max} \approx 10^{-3}\varepsilon^{4/(3n-2)}$ g. If $n < 2/3$, so that the initial fluctuations fall off less steeply than $M^{-2/3}$, ε increases with scale. However, this seems rather implausible since one necessarily gets separate closed universes on a sufficiently large scale. Thus only constant curvature fluctuations can generate PBHs with an extended mass spectrum.

3.3 The Formation of PBHs at a Phase Transition

Even if ε is very small, black holes might still form prolifically at any epoch when the Universe goes pressureless ($\gamma << 1$). For example, this might occur if the Universe's mass is ever channeled into particles[130] which are massive enough to be non-relativistic or if the equation of state softens[129] at some sort of cosmological phase transition. The Universe might also go pressureless after the nuclear density epoch ($t \sim 10^{-4}$ s) if it starts off "cold" (i.e. without any background radiation). In all these situations the important parameter determining the collapse probability is not ε but the asymmetry of the collapsing region[132]. For example, even if $p = 0$, turbulent effects might still prevent the formation of PBHs much smaller than the particle horizon. In this case, the mass spectrum would be given by eqn (25) with $\gamma = 0$.

So far we have assumed that the PBHs form from initial inhomogeneities. However, it is possible that black holes could form spontaneously at a cosmological phase transition even if the Universe starts off perfectly smooth. For example, bubbles of broken symmetry might arise at a spontaneously broken symmetry epoch and it has been suggested that black holes [133] could form as a result of bubble collisions. In fact, this only happens if the bubble formation rate is finely tuned: if it is too large, the entire Universe undergoes the phase transition immediately; if it is too small, the bubbles never collide. In consequence, the black holes should have of order the horizon mass at formation. Thus PBHs forming at the GUT epoch (10^{-35}s) would have a mass of order 10^{3}g, whereas those forming at the Weinberg-Salam epoch (10^{-10}s) would have a mass of order $10^{-5}\,M_\odot$.

Another possibility, suggested by Crawford and Schramm, is that PBHs could form spontaneously at the quark soup to hadron phase transition (10^{-6}s) as a result of the fact that the potential between two quarks increases with their separation.[135] The idea is that, when a hadron forms, neighbouring quarks will be able to feel the colour charge of quarks farther away across the hadron "gap" than in other directions (where colour screening reduces the interaction range). This means that hadron formation is more likely where a hadron already exists, resulting in spontaneous density fluctuations and black hole formation. The PBHs should have a mass of up to 0.1 $M_\odot$ in this case. It is hard to envisage a phase transition occurring later than this in a hot Universe, so one could not expect PBHs larger than 0.1 $M_\odot$ to form in the absence of initial inhomogeneities.

3.4 The Cosmological Effects of PBHs

Even if PBHs have a significant density today, they can only have comprised a tiny fraction of the cosmological density at early times. This is because the 3K background radiation density ($\Omega_R \approx 10^{-4}$ in units of the critical density), though small today, increases as $(1+z)^{-4}$ as one goes back in time, whereas the PBH density increases only as $(1+z)^{-3}$. Thus the fraction of the Universe in PBHs at time t is

$$\beta(t) = \left(\frac{\Omega_{PBH}}{\Omega}\right) (1+z)^{-1} \cong 10^{-6}\, \Omega_{PBH} \left(\frac{t}{s}\right)^{1/2} \qquad (28)$$

where the t dependence applies before the time $t_{eq} \approx 10^{10}\Omega^{-2}$s at which the Universe is matter dominated. We assume $\Omega_{PBH} \lesssim 1$ since this is the maximum value consistent with observations of the cosmological deceleration parameter. Using eqn (22), we thus have

$$\beta(M) < 10^{-8} \left(\frac{M}{M_\odot}\right)^{1/2} = 10^{-17} \left(\frac{M}{10^{15}g}\right)^{1/2} \qquad (29)$$

where 10^{15}g is the mass above which $\Omega_{PBH}(M)$ will have been unaffected by evaporations. Equation (24) therefore implies $\varepsilon(M) < 0.05$ on all scales above 10^{15}g. This has the important consequence that the early Universe cannot have been "chaotic" (with $\varepsilon \sim 1$) after 10^{-23}s.

This conclusion must be qualified. If the early Universe were truly chaotic, one would expect it to exhibit large anisotropies as well as inhomogeneities; in this case the assumption of spherical symmetry, on which eqn (24) is based, would fail. Barrow and Carr argue that PBH formation would be suppressed in an anisotropy-dominated Universe because the shear provides an effectively "stiff" equation of state.[128] In this case, one would not necessarily contravene limit (29), although one would still anticipate too many PBHs forming once the anisotropy became dynamically insignificant if $\varepsilon \sim 1$. However, the Barrow-Carr argument is rather simplistic; in another paper they argue for the opposite conclusion - that PBHs will be produced more abundantly in an shear-dominated Universe.[136] Therefore it seems likely that the exclusion of chaotic cosmologies is justified.

Another situation could invalidate eqn (29): the limit assumes that the Universe always has a radiation equation of state before t_{eq} but this assumpion would fail if the Universe started off cold (without any background photons). For example, if the 3K background were generated at some time t_R, eqn (28) would be replaced by

$$\beta(t) \cong \left(\frac{\Omega_{PBH}}{\Omega_R}\right) \min \left[1, \left(\frac{t_R}{t_{eq}}\right)^{1/2}\right] \qquad (30)$$

for $t_R > t > 10^{-4}$s. Thus, if t_R exceeds t_{eq}, one could in principle permit most of the Universe to go into PBHs in the mass range above 1 $M_\odot$. On the other hand, the equation of state should still be hard before 10^{-4}s because of strong interactions, so one still has a strong limit on the fraction of the Universe going into evaporating PBHs:

$$\beta(M) < 10^{-10} \left(\frac{M}{10^{15}g} \right)^{\frac{1}{2}} \min \left[1, \left(\frac{t_R}{t_{eq}} \right)^{\frac{1}{2}} \right] . \qquad (31)$$

Cold scenarios are now rather out of vogue because of the problems they face in generating and thermalizing the 3K background; one also has to give up the cosmological nucleosynthesis explanation for the light element abundances. In any case, one expects the first bound regions to form primordial stars and so, even though these stars may eventually give rise to black holes, the resulting scenario resembles the Population III picture more than the usual PBH picture. For the rest of this section we will therefore confine attention to the standard hot Big Bang scenario, reverting to a discussion of thecold scenario in Section (4).

The fact that $\beta(M)$ must be small in the hot scenario should occasion no surprise in view of the exponential sensitivity of β on ε. Indeed the striking feature of eqn (28) is that Ω_{PBH} could be significant even if $\beta(M)$ is tiny. Thus PBHs could be associated with all of the sorts of cosmological effects indicated in Table (1). Of course, it requires very fine tuning of ε if Ω_{PBH} is to have an interesting value: if ε is slightly larger than 0.05, the PBHs are grossly overproduced; whereas, if ε is slightly less than 0.05, their density is negligible. (For example if ε has the sort of value ~ 0.01 required to explain galaxy formation, Ω_{PBH} would only be 10^{-220}!) Therefore it might be regarded as a priori unlikely that PBHs would form prolifically enough to be interesting. This at least applies if the PBHs derive from primordial inhomogeneities. If they derive from a phase transition, there would still be a danger of violating eqn (29) but the exponential dependence of β upon ε would no longer apply, so an interesting value of Ω_{PBH} would not be a priori so unlikely.

The cosmological consequences of PBHs larger than 10^{15}g are discussed in Sections (1) and (2), so for the rest of this section we will confine attention to the consequences of the quantum mechanical evaporation of those smaller than 10^{15}g. Note that these holes form before 10^{-23}s, when the density of the Universe exceeds 10^{54}g cm^{-3}. At such extreme densities, we cannot necesssarily assume that the simple gravity-dominated scheme for PBH formation discussed in Section 3.1 is realistic, so the spectrum predicted by eqn (25) should only be adopted with caution.

3.5 The Evaporation of PBHs

The crucial feature of sufficiently small PBHs is that they can shrink through quantum effects[11,137]. In general a black hole emits particles with energy in the range (E, E+dE) at a rate given by

$$\dot{dN} = \frac{\Gamma dE}{2\pi h}\left[\exp\left(\frac{E-nh\Omega-e\Phi}{h\kappa/2\pi c}\right) \pm 1\right]^{-1}. \qquad (32)$$

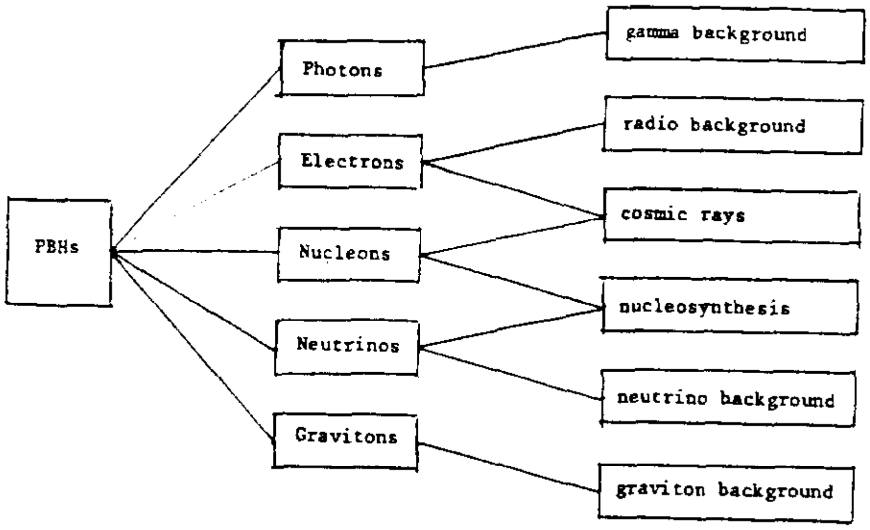
Here Ω, Φ and κ are the angular velocity, electric potential and surface gravity of the hole; Γ is the absorption probability for the species of particle involved (in general a function of its spin); and the + and - signs apply for fermions and bosons respectively. One would expect Φ = 0 since eqn (32) implies that a black hole discharges on a much shorter timescale that it evaporates, at least for $M < 10^5 M_\odot$[138]. Although it is not so clear why angular momentum should be lost on a shorter timescale, unless there exists a massless scalar particle[139], we will assume $\Omega = 0$ also. In this case eqn (32) implies that the hole emits approximately as a black body with temperature

$$\theta = \frac{h\kappa}{2\pi c} = \frac{hc^3}{8\pi GkM} \approx 10^{26}\ M^{-1} K \qquad (33)$$

where here and throughout this section M is in grams. The radiation is not exactly black-body because the Γ factor in eqn (32) is frequency-dependent. At high frequencies the effective cross-section is $27\pi G^2M^2/c^4$ for all particle species, but at low frequencies the cross-section is reduced in a way which depends on the spin of the particle. The overall emission rate tends to decrease with increasing spin.

A black hole should emit any particle species whose rest mass is less than its emission temperature. Thus all holes should emit zero-rest-mass particles like photons, neutrinos, and gravitons, and

Table (3): Cosmological consequences of PBH evaporations



sufficiently small ones should also emit electrons and nucleons. Table (3) shows that interesting cosmological effects may be associated with each of these particle species. However it must be stressed that there is no necessity to invoke PBHs to explain any of the cosmological features alluded to in Table (3), since each of them has other possible explanations. The table should merely be regarded as exemplifying the wide range of cosmological effects which may be associated with PBH evaporations. As we shall see, it also indicates the various ways in which one can infer upper limits on the number of PBHs which may have formed in various mass ranges.

Page [139] finds that, for holes with $M > 10^{17}$ g (which can emit only zero-rest-mass particles), the fractions of the initial mass emitted in gravitons, photons, and neutrinos are $\varepsilon_g = 0.02$, $\varepsilon_\phi = 0.17$, and $\varepsilon_\nu =$

0.81, respectively. This assumes there are three neutrino species. For holes with 10^{15} g < M < 10^{17} g (which are hot enough to emit electrons and positrons), the values are $\varepsilon_g = 0.01$, $\varepsilon_\phi = 0.09$, $\varepsilon_\nu = 0.45$, and $\varepsilon_e = 0.45$. For holes with 10^{14} g < M < 10^{15} g (which are hot enough to emit muons, these subsequently decaying into electrons and neutrinos), $\varepsilon_g = 0.01$, $\varepsilon_\phi = 0.07$, $\varepsilon_\nu = 0.51$, and $\varepsilon_e = 0.41$. For $M < 10^{14}$ g, the hole can also emit hadrons and the emission fractions depend on highly uncertain details of particle physics. These considerations suggest that one may write the mass loss rate as

$$\frac{dM}{dt} = -\,3 \times 10^{25}\ M^{-2}\ f(M)\ \mathrm{g\ s^{-1}} \tag{34}$$

where f(M) depends on the number of particle species which can be emitted and is normalized to be 1 for holes which emit only massless particles ($M > 10^{17}$g). The associated lifetime is

$$t_{evap} = 9 \times 10^{-27}\ f(M)^{-1}\ M^3\ \mathrm{s} \tag{35}$$

and this implies that a PBH evaporates within the age of the Universe (10^{10}y) if its mass is less than

$$M_* = 5 \times 10^{14}\mathrm{g}. \tag{36}$$

Page has made more refined calculations of M_*, accounting for the effects of the black hole having spin[140] or charge[141]; these modify eqn (34) but only by about 10%.

The nature of the final explosive phase of a black hole's evaporation depends crucially on the form of f(M) for $M < 10^{14}$g. In the "Elementary Particle" picture, all hadrons are supposed to be made up of a finite number of fundamental particles like quarks and gluons. In this case only these fundamental particles are emitted directly and f(M) never exceeds ~100. The most natural picture would be one in which the black hole emits quark and gluon jets, these subsequently fragmenting into hadrons and leptons. If we regard the explosive phase as beginning when $t_{evap} \approx 10$ s, it occurs when M reaches about 10^{10}g. In the "Composite Particle" picture, the hadrons can be regarded as being made up of each other[142]. In this case all hadrons are equally fundamental and all can be emitted directly. This means that f increases exponentially when T reaches the "ultimate" temperature of 160 MeV or, equivalently, when M falls to 6×10^{13}g. However, this picture is not now regarded as being very plausible.

In discussing the cosmological effects of PBH evaporations, it is useful to note that the total number of particles of a given species emitted by a PBH of initial mass M is of order $10^{11}\varepsilon_i M^2$, where ε_i is the fraction of mass that goes into that species. Most of these particles will have an energy of order $10^{22}\ M^{-1}$eV but there will also be an E^{-3} tail of higher energy particles emitted in the later phases of evaporation. Note that the fraction of the Universe which is in PBHs of initial mass M at their evaporation epoch, $\alpha(M)$, is related to the corresponding fraction at the formation epoch, $\beta(M)$, by[143]

$$\alpha(M) = \beta(M)\left(\frac{t_{evap}}{t_{form}}\right)^{\frac{2\gamma}{1+\gamma}} = \beta(M)\left(\frac{M}{M_p}\right)^{\frac{4\gamma}{1+\gamma}}$$

where γ specifies the equation of state between t_{form} and t_{evap}. Thus, if $\gamma = 1/3$, the conversion factor is just M/M_p. The effect of evaporations on the PBH mass spectrum is to change eqn (25) to

$$\Pi(M) \propto M^{-\alpha}\left[1+\frac{t_{now}}{t_{evap(M)}}\right]^{-\left(\frac{\alpha+2}{3}\right)} \propto \begin{cases} M^{-\alpha} & (M>M_*) \\ M^{2} & (M<M_*) \end{cases} \quad . \qquad (37)$$

Thus the spectrum is unchanged above M_*; its form below M_* is associated with the E^{-3} tail effect.

3.6 The Contribution to the Photon Background

Particles emitted by PBHs after some redshift z_{free} will not have interacted with the background Universe; they will therefore preserve their original spectrum apart from being redshifted. Their present background spectrum should thus have the form indicated in Fig.5, α being the exponent of the PBH mass spectrum[144]. The E^{-3} part for $E>$ 100 MeV comes from the tail of the PBHs of mass M_* which explode today and the contribution at lower values of E comes from PBHs with $M < M_*$ which evaporated earlier. There are changes of slope at two points; these derive from the different relationships between redshift and time during the free-expansion, matter-dominated, and radiation-dominated eras. Page and Hawking show that the photon spectrum drops off below

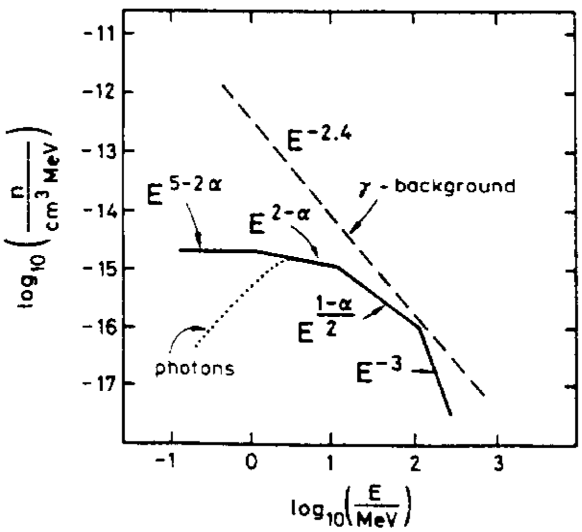


Figure (5): Spectrum of particles from evaporating PBHs

about 1 MeV (corresponding to a value for z_{free} of order 100) primarily due to pair-production off background nuclei[145]. Comparison with the observed γ-ray spectrum[146], which goes like $E^{-2.4}$, shows that the best limit on the number of PBHs derives from those of mass M_* providing α < 5.8 (as expected). Since the observed background γ-ray density at 10^2MeV is about $10^{-14}\mathrm{cm}^{-3}$, or $\Omega_\gamma \sim 10^{-9}$ in units of the critical density, and since the fraction of mass coming out in γ-rays is of order 0.1, we infer an upper limit[147] on $\Omega_{PBH}(M_*)$ of 10^{-8}. The corresponding limit on $\beta(M*)$ is

$$\beta(M_*) = \frac{\Omega_{PBH}(M_*)}{\Omega_R}\left[1+z_{form}(M_*)\right]^{-1} < 10^{-26} \tag{38}$$

where Ω_R denotes the background radiation density and we assume γ = 1/3 before t_{eq} . This shows that the fraction of the Universe going into black holes at 10^{-23}s must have been even tinier than indicated by eqn (29) and the upper limit on ε drops from 0.05 to 0.03. Since eqn (25) implies that the mass density in holes larger than M_* should be smaller than $\Omega_{PBH}(M_*)$ by a factor $(M/M_*)^{-1/2}$, one might conclude that the total PBH density is also less than 10^{-8}. However, this conclusion would fail if PBH formation were inhibited on scales smaller than M_* (e.g. by the equation of state being stiff [73] or by the Universe being highly anisotropic [128] before 10^{-23} s) or if the PBHs formed at a phase transition after 10^{-23}s.

Photons which are emitted sufficiently early, before a redshift $z_{therm} \approx 10^6 \Omega_i^{-4}$ where Ω_i specifies the ionized gas density in units of the critical density[148], will be completely thermalized, and so PBHs smaller than

$$M_{therm} \approx 10^{11} \Omega_i^{8/3} \text{ g}$$

will merely boost the primordial photon-to-baryon ratio. As shown by Zeldovich and Starobinskii[149], an initial ratio of S_o would by today have been increased to

$$S = (1 + S_o) \beta(M) \left(\frac{M}{M_P} \right) \quad . \tag{39}$$

If the PBHs exist over an extended mass range, we expect $\beta(M)$ to be constant, so the largest contribution to S should come from the holes with mass M_{therm}. These PBHs can generate <u>all</u> of the 3K background ($S \approx 10^8 \Omega^{-1}$, $S_o = 0$) providing $\beta \approx 10^{-8} \Omega_i^{-11/3}$. This is compatible with the γ-ray limit only if the PBH spectrum is cut off before M_* or if β falls off faster than M^{-2} rather than being constant. In any case, eqn (39) yields an upper limit

$$\beta(M) < 10^3 M^{-1} \Omega^{-1} \qquad (M < M_{therm}) . \tag{40}$$

For initial fluctuations which fall off faster than $M^{-2/3}$, the PBH mass spectrum declines exponentially above 10^{-5}g, so one cannot generate an appreciable value of S.

Photons emitted at epochs intermediate between z_{therm} and z_{free}, rather than being thermalized or propagating freely, will merely distort the 3K background spectrum. Calculations of Naselskii[150] indicate that PBHs in the range 10^{11}g < M < 10^{13}g must have

$$\beta(M) < 10^{-18} \left(\frac{M}{10^{11} g} \right)^{-1} \tag{41}$$

if the distortion is not to exceed the observational limits.

3.7 PBH Explosions Today

We have seen that the γ-ray background observations require $\Omega_{PBH}(M_*) < 10^{-8}$. This implies that the mean number density of such holes can be at most $10^4 pc^{-3}$, although, if the holes are clustered inside galactic halos, the local density could be as high as $10^{10} pc^{-3}$. The corresponding explosion rate is at most $10^{-6} pc^{-3} y^{-1}$ for unclustered holes or $1\ pc^{-3} y^{-1}$ for clustered holes. We now discuss the prospect of detecting these explosions.

In the Composite Particle picture , we have seen that the evaporation becomes catastrophic at a mass $M_{crit} \simeq 6 \times 10^{13}$g. This should generate a "hadron fireball"[151], releasing around 10^{34} ergs in 250 MeV γ-rays over a period of about 10^{-7}s[145]. To detect one explosion per month, one would need a detecting area of at least $4 \times 10^5 cm^2$ for unclustered holes or $40\ cm^2$ for clustered holes. Observations of COSMOS 561[152], with an area of $300\ cm^2$, give a very weak upper limit of 75 explosions $pc^{-3} y^{-1}$; this is nearly two orders of magnitude larger than that permitted by the γ-background observations. On the other hand, Porter and Weekes[153] show that atmospheric Cerenkov techniques, with an effective area of $10^9 cm^2$, give a limit of 0.04 explosions $pc^{-3} y^{-1}$, which is better than the background limit providing the holes are clustered inside halos.

In the Elementary Particle picture, evaporation becomes explosive at a mass $M_{crit} \simeq 10^{10}$g, some 10^{30} ergs being released as 5×10^6 MeV γ-rays in around 10 s[144]. The number flux of photons is now so small that only atmospheric Cerenkov techniques can be used. However, all the limits presently available are weaker than the γ-background limit: Porter and Weekes[154] get an upper limit of 3×10^4 explosions $pc^{-3} y^{-1}$ and Fegan et al.[155] get a limit of 6×10^3 explosions $pc^{-3} y^{-1}$. This suggests that there is little prospect of detecting the photons emitted from PBH explosions in the Elementary Particle picture. However, this conclusion may be overly pessimistic. Recent calculations by MacGibbon suggest that most of the explosive energy may be emitted as quark jets in the Elementary Particle picture and each of these may generate a low energy tail of photons with energy down to 100 MeV.[173] The prospect of detecting these soft photons may be much better.

Rees[156] has pointed out that the situation could also be improved if the PBHs explode in a region where there is an appreciable magnetic field. In the interstellar medium, for example, $B = 5 \times 10^{-6}$G. In this case, the interactions with this field of the shell of electrons and positrons emitted will generate a burst of radiation. The wavelength at which the burst appears is $r_{max}\gamma^{-2}$ where $\gamma \simeq (M_{crit}/10^{16}g)^{-1}$ is the Lorentz factor of the electrons and r_{max} is either the radius where the $e^{\pm}$ shell is braked,

$$r \approx 10^{16}\gamma^{-1} \left(\frac{B}{5\times10^{-6}G}\right)^{-2/3} \text{cm} \tag{42}$$

or the radius at which its conductivity breaks down,

$$r \approx 4 \times 10^{19}\gamma^{-3/2} \left(\frac{B}{5\times10^{-6}G}\right)^{1/2} \text{cm} \tag{43}$$

whichever is smaller. In the interstellar medium, the braking radius is smaller (so that most of the $e^{\pm}$ energy goes into electromagnetic waves) providing $\gamma < 10^7$ or $M_{crit} > 10^9$g. Thus, if $M_{crit} \approx 10^9$ g, one gets about 10^{30} ergs released at a wavelength of 10^4Å; such an optical burst would be detectable out to 1 kpc. If $M_{crit} \approx 10^{11}$g, one gets about 10^{32} ergs being released at a wavelength of 10 cm. Arecibo could detect such a radio burst as far away as Andromeda. More detailed calculations of the radio burst characteristics have been presented by Blandford[157].

In fact, Rees' mechanism does not work in all circumstances. A pulse is produced only if the explosion timescale is less than the characteristic period of the generated radiation and this applies only if there exist many extra particle species which can be emitted above 10^2 GeV. On the other hand, one needs $\gamma > 10^3$ ($M_{crit} < 10^{13}$g) to avoid most of the energy going into swept-up plasma and $\gamma > 10^5$ ($M_{crit} < 10^{11}$g) to avoid electrons and positrons annihilating too quickly. One therefore requires a picture for the black hole explosion intermediate between the Composite Particle picture and usual Elementary Particle picture. If radio bursts <u>are</u> produced, one can already infer very strong limits on the number of PBH explosions. Observations at 400 Hz[158] give an upper limit $2 \times 10^{-9}\ pc^{-3} y^{-1}$; and observations at 10^2 MHz and 10^3 MHz [159] give limits of $5 \times 10^{-7}\ pc^{-3}\ y^{-1}$ and $4 \times 10^{-5}\ pc^{-3}\ y^{-1}$, respectively. These are much better than the γ-ray background limits. However, the optical burst limit[160-162] is $0.03\ pc^{-3}\ y^{-1}$, which is considerably weaker.

3.8 The Contribution to the Positron Background

The electrons and positrons emitted by black hole evaporations could be of interest in their own right, even if the Rees mechanism is not operative. For PBHs in the mass range $10^{14} g < M < 10^{17} g$, both ε_{e^-} and ε_{e^+} should be about 0.2. Since observations of the 100 MeV positron background[163] show that $n_{e^+}(100\ \text{MeV}) \sim 10^{-11} cm^{-3}$, this implies a limit $\Omega_{PBH}(M_*) < 10^{-6}$ if the PBHs are unclustered or $\Omega_{PBH}(M_*) < 10^{-10}(t_{leak}/10^8 y)^{-1}$ if they are clustered inside galactic halos with the positrons escaping in a time t_{leak}. Since t_{leak} would probably be about $10^8 y$,[166] the positron limit on $\Omega_{PBH}(M_*)$, and hence $\beta(M_*)$, may be better than the γ-ray background limit by two orders of magnitude.[144] The background electron density at 100 MeV is larger than the positron density by a factor of 10 and is therefore less interesting. If one considers the spectrum of cosmic ray electrons expected from PBHs evaporating at previous epochs, as well as today, one gets a form similar to that shown in Fig.5 except that it is scaled by a factor $\varepsilon_e/\varepsilon_\phi$ and falls off below 10 MeV (i.e. for $t_{evap} < 10^{15}$ s) on account of the electrons being degraded by inverse Compton scattering off the 3K background photons. The predicted spectrum is conceivably compatible with that observed if $\Omega_{PBH}(M_*) \sim 10^{-10}$, although solar modulation effects make a direct comparison difficult.

More refined calculations of the background of positrons expected from PBHs evaporating in the <u>present</u> epoch, allowing for their diffusion and degradation within the galactic halo, have been presented by Nazel'skii and Pelikhov[167]. These authors also calculate an indirect limit for PBHs exploding in the interstellar medium associated with the fact that both electrons and positrons there will generate synchotron radio emission via interaction with the interstellar magnetic field. From observations of the 300 MHz background, they infer $\Omega_{PBH} < 10^{-9}$.

Another interesting effect could derive from the positrons generated by PBHs exploding near the galactic centre, since the PBH density should be higher there. One would expect some of these positrons to annihilate, producing a 0.511 MeV line, so it is relevant that such a line has indeed been detected from the galactic centre [164]. The intensity of the line corresponds to about 8×10^{42} annihilations s^{-1}. Okeke and Rees[165] have shown that any positrons from PBHs will be slowed by ionization losses, thus permitting their annihilation, providing their energy is less than $E_{slow} \simeq (50 - 100)$ MeV, i.e. providing they come from PBHs larger than $M_e \simeq 10^{14}$ g. Given the form of the PBH mass spectrum [viz.eqn (37)], the associated annihilation rate goes like $M^{-\alpha-1}$ for $10^{17}g>M>M_*$, like M for $M_*>M>M_e$ and like M^4 for $M<M_e$. The biggest contribution therefore comes from PBHs with $M \sim M_*$ and one would need about 10^{20} of them (i.e. about $10^2 M_\odot$ worth) within the central kpc of the galaxy to produce the observed 0.511 MeV line. If one asumes that the number density of PBHs in the halo falls as R^{-2} with galactocentric distance, like the rest of the halo material[51], one infers a limit $\Omega_{PBH}(M_*) < 10^{-9}$ which is about one order of magnitude stronger than the γ-ray background limit, though possibly weaker than the positron limit itself.

3.9 The Contribution to Cosmic Ray Antiprotons

PBHs smaller than about 10^{14} g would be hot enough to emit protons and antiprotons; those emitted in the tail of the PBH explosions occurring today would contribute to the cosmic ray background. However, the cosmic ray proton flux falls off as $E^{-2.6}$ in the energy range 1GeV to 10^{11} GeV (compared to the E^{-3} expected from PBHs) and the integrated energy density is $\Omega_{CR} \sim 10^{-4}$ in units of the critical density [168]. Both features would seem to preclude cosmic ray protons deriving from evaporating black holes; indeed, in view of the γ-ray limit [$\Omega_{PBH}(M_*) < 10^{-8}$], one would infer that at most 10^{-4} of the energy in cosmic ray protons could so derive. The situation with antiprotons is much more interesting since observations [169] suggest that, in the energy range 130-320 MeV, $n_{\bar{p}} / n_p = (2.2 \pm 0.6) \times 10^{-4}$. The $\bar{p}$ flux is therefore comparable to that which could have been generated by PBH evaporations. This possibility is accentuated by the fact that more conventional explanations for the antiproton flux seem to be unsatisfactory. It is usually assumed that antiproton cosmic rays are secondary particles, produced by the spallation of the interstellar medium by primary cosmic rays. However, the observed $\bar{p}$ flux at 130-320 MeV exceeds the predicted secondary flux by a factor of 10^2 ; even at energies around 10 GeV, the observed $\bar{p}$ flux still exceeds the predicted value by a factor of 3[170].

These considerations have prompted Kiraly et al.[171] to examine whether PBH evaporations could produce the antiprotons. If one normalizes the expected flux to that observed at 10 GeV, one expects a spectrum of the form

$$\frac{dN}{dE} \simeq 10^{-2} \left(\frac{E}{\text{GeV}} \right)^{-3} \text{cm}^{-2}\ \text{s}^{-1}\ \text{GeV}^{-1}. \tag{44}$$

This accords with observation providing $\Omega_{PBH}(M_*)$ is of order 10^{-9}, which is a factor of 10 smaller than the maximum permitted by the γ-ray background limit. Turner[172] has suggested a similar scheme, allowing for an extended spectrum of PBHs. He finds that the dominant contribution derives from those PBHs with $M \sim 10^{13}$g which evaporate at about 10^{15} s, antiprotons produced before then annihilating with protons in the background Universe. Both the Kiraly et al. and Turner models are rather simplistic since they assume that the fraction of mass emitted as antiprotons is $\epsilon_{\bar{p}} \simeq 0.1$ (independent of M). More detailed calculations[173], based on the jet picture allow one to predict the value of $\epsilon_{\bar{p}}$ and its energy dependence more precisely. These calculations also allow one to relate the production of antiprotons by PBHs with that of positrons and gamma rays. Thus there is a close connection between the considerations of this subsection and the last. If cosmic ray positrons and antiprotons really do derive from PBHs, their observed spectra could give vital information about particle physics.

3.10 The Effect on Cosmological Nucleosynthesis

Several limits can be placed on the fraction of the early Universe which goes into evaporating PBHs by considering ways in which the evaporations would mar the standard picture of cosmological nucleosynthesis (discussed in Section 2.4.1). Firstly, if the number of photons generated by the PBHs in the period after nucleosynthesis is large enough to change the primordial photon-to-baryon ratio [cf. eqn (39)], the value of S at nucleosynthesis will be less than its present value of $10^{8}\Omega^{-1}$. This will increase the helium abundance and decrease the deuterium abundance. Detailed calculations by Miyama and Sato [174] show that this imposes a limit

$$\beta(M) < 10^{-15}\,\Omega\left(\frac{M}{10^{9}g}\right)^{-1} \qquad (10^{9}g < M < 10^{13}g). \tag{45}$$

This limit has also been obtained by Vainer and Nasel'skii [175]. Such limits do not apply for PBHs smaller than 10^{9}g (which evaporate before $t_F \sim 1$ s) but, from eqn (40), one also has a limit in this mass range from the requirement that one does not generate too many thermalized photons.

A more subtle effect of the photons emitted from PBHs after cosmological nucleosynthesis is that they could photodissociate the small amount of deuterium produced previously. Lindley shows that the survival of deuterium requires[176]

$$\beta(M) < 10^{-21}\,\Omega\left(\frac{M}{10^{10}g}\right)^{1/2} \qquad (M > 10^{10}g) \tag{46}$$

which is stronger than the Miyama and Sato limit. It should be noted, however, that Lindley does not consider the photodissociation of helium, which could conceivably <u>increase</u> the final deuterium abundance.

Vainer and Nasel'skii [177] point out that neutrinos from PBHs with $10^{9}g < M < 10^{11}g$, which evaporate in the period $1\ s < t < 10^{5}s$, could also effect nucleosynthesis by modifying the n_N/n_P freeze-out ratio. However, Zeldovich *et al.*[178] show that the effect of the neutrinos on nucleosynthesis is much less important than that of the nucleons emitted. While protons or antiprotons may be confined near the holes[179], the neutrons and antineutrons will not be so confined and they will modify nucleosynthesis both by altering the n_N/n_P freeze-out value for $10^{9}g < M < 10^{10}g$ and by spallation of already formed helium for $10^{10}g < M < 10^{13}g$. The limit associated with the first effect, which is also discussed by Rothman and Matzner[180], is

$$\beta(M) < 10^{-16}\,\Omega\left(\frac{M}{10^{9}g}\right)^{-1/2} \qquad (10^{9}g < M < 10^{10}g) \tag{47}$$

The helium spallation limit derives from the requirement that the deuterium abundance produced by the spallation not exceed that observed. Since any deuterium produced after around 10^3s will survive, whereas the spallation will continue to operate until arund 10^5s, one requires

$$\beta(M) < 10^{-25}\,\Omega^{1/3}\left(\frac{M}{10^{9}g}\right)^{5/2} \qquad (10^{11}g < M < 10^{13}g) \tag{48}$$

The most interesting aspect of this result is that it means only a tiny fraction of the Universe would need to go into PBHs in the mass range $10^{10}g \lesssim M < 10^{13}g$ in order to generate the observed deuterium abundance. This means that the usual conclusion that Ω must be less than 0.1 in order for the cosmological production of deuterium to be correct[109] can be circumvented. However, the value of β required is consistent with the γ-ray limit, $\beta(M_*) < 10^{-26}$, only if the spectrum falls off faster than $M^{-3.5}$ or has an upper cut-off below M_*.

3.11 CONCLUSION

We have seen that there are a wide variety of ways in which PBH evaporations could have affected the history of the Universe and that there are several cosmological problems which they could resolve. However, most of these problems have other possible resolutions, so it would be premature to infer that evaporating PBHs <u>must</u> have existed. Indeed most of the preceding discussion only serves to indicate that the fraction of the Universe going into PBHs must have been tiny on all scales above 10^5g, at least if the early Universe had a hard equation of state. The limits on $\beta(M)$ in this context are summarized in Fig. 6, which is reproduced from Novikov *et al.*[143]. The limits in the case of a universe which is dust-like at early times are somewhat weaker but they still require that $\beta(M)$ be tiny for $M > 10^{10}$g. We have seen that one would *expect* β to be tiny unless the early Universe was

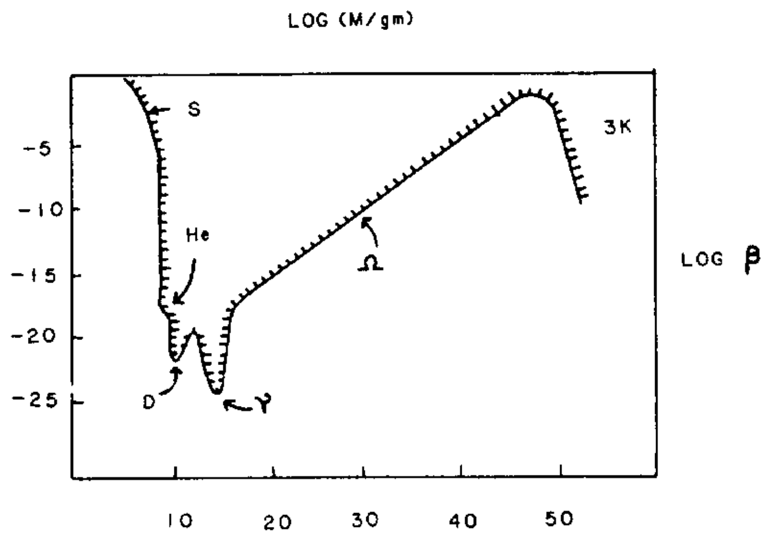


Figure (6): Constraints on fraction of Universe contained in PBHs

chaotic. Indeed the main point of the limits discussed above is that they reinforce the conclusion that the Universe was quiescent[217], containing horizon-epoch fluctuations no larger than 1% in amplitude, at all times after 10^{-33}s.

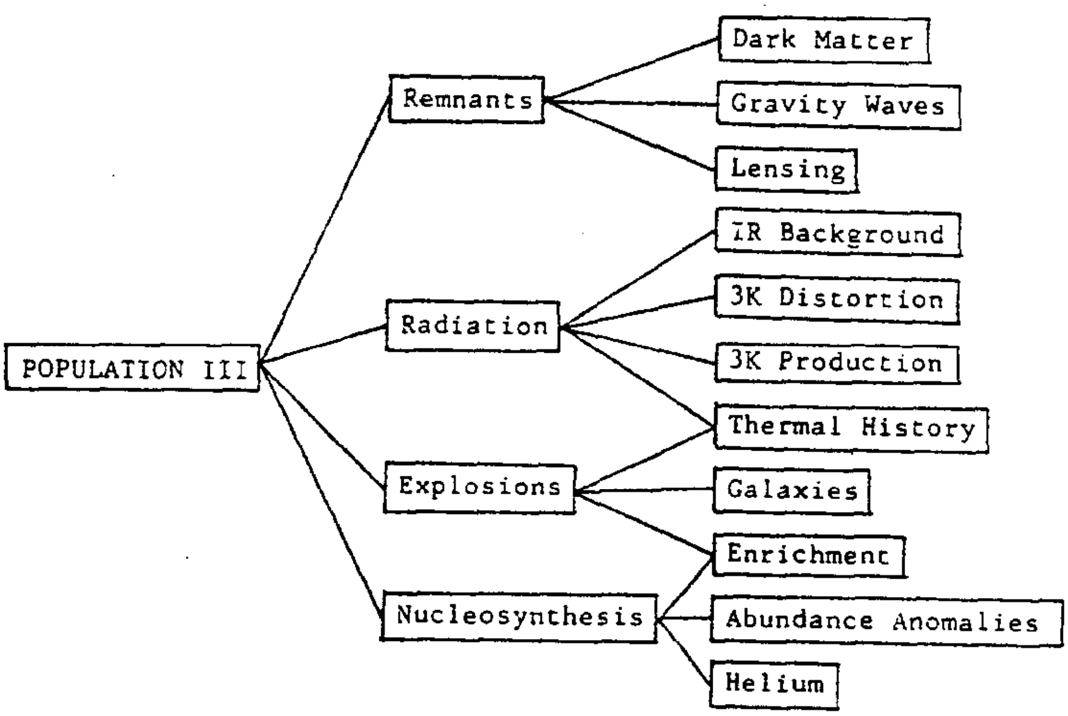
Of course the striking point is that PBH evaporations could have interesting cosmological effects even if β is tiny. However, we have seen that it is a priori unlikely that β would have the value required for Ω_{PBH} to be significant without being negligible. If PBHs did form abundantly, it is more likely that they did so at a phase transition. In this case, the holes would not be of the evaporating kind if the phase transition occurred after 10^{-23}s, but even large PBHs could have interesting cosmological consequences. In particular, PBHs in the mass range 10^{16}g - 0.1 $M_{\odot}$ could still be viable candidates for the dark matter in galactic halos.

4. POPULATION III STARS

4.1 Introduction

Population III stars may be defined as the stars that formed before the Universe had any appreciable heavy element content. The existence of a few such stars is inevitable since heavy elements can only be generated through stellar nucleosynthesis. In this section we will focus on the much more exciting and controversial possibility that most of the Universe may have been processed through them, perhaps before galaxies formed. We have seen that the prime motive for this suggestion is the possibility that the dark matter in galactic halos may consist of Population III remnants. We will examine why such a large number of stars might form in Section 4.2. However, the cosmological interest in Population III stars is not confined to the dark matter issue. They would also be expected to produce radiation,

Table (4): Cosmological consequences of Population III stars



explosions, and nucleosynthesis products. As illustrated in Table (4), each of these could have important cosmological consequences. It must be stressed at the outset, however, that there are no observations which definitely require their existence. The most conservative approach therefore is to use some of the effects indicated in Table (4) to constrain the Population III hypothesis; this we do in Section 4.3. The more positive aspect of what Population III stars could explain will be explored in Section 4.4.

4.2 Scenarios For Population III Formation

The existence of galaxies and clusters of galaxies implies that density fluctuations must have been present in the early Universe. The likelihood that these fluctuations could give rise to a population of pregalactic stars depends on what model one adopts for the nature of the fluctuations.

4.2.1 Isothermal fluctuations. In this case, the fluctuations are entirely in the baryons and one expects the smallest surviving scale after decoupling to be

$$M_{Jb} \simeq 10^6 \; \Omega_b^{-1/2} \; M_\odot \tag{49}$$

corresponding to the baryon Jeans mass then.[42] (Here Ω_b is the baryon density in units of the critical density.) Larger structure would then build up through a process of hierarchical clustering [81]. The form of the galaxy correlation function[181,182] suggests that the fluctuations at decoupling must have had the form

$$\delta_{dec} = \left(\frac{M}{M_1} \right)^{-\beta} , \quad M_1 = 10^6\text{-}10^8 M_\odot , \; \beta = \frac{1}{3} - \frac{1}{2} \; . \tag{50}$$

The precise values of M_1 and β required depend on the value of Ω_b. This implies that one would expect the first pregalactic clouds (of mass M_{Jb}) to bind shortly after decoupling ($z \simeq 10^3$). These clouds would presumably fragment into stars, though the typical fragment mass and the efficiency of star formation is very uncertain. The lack of heavy elements[183,184] and substructure,[185] as well as the effects of the microwave background,[186] would tend to make the final fragments larger than at the present epoch and perhaps in the VMO range. Possibly there would be no fragmentation at all, in which case one could end up with a single SMO. On the other hand, enhanced molecular hyudrogen formation[187] and associated thermal instabilities[188] could reduce the fragment mass to 0.01 $M_\odot$. The mass of the first stars is thus uncertain by 10^8 orders of magnitude. A hybrid scheme has been suggested in which each cloud forms a disc due to rotational effects; one then gets stars of mass 0.01 $M_\odot$ in the disc,together with a VMO which forms through relaxation at the center.[186]

4.2.2 <u>Adiabatic fluctuations in a "cold" universe.</u> If the fluctuations are contained in the total density and the Universe's mass is presently dominated by collisionless "cold" particles, like axions or photinos or primordial black holes, one expects bound clumps of these particles to form down to very small scales. Baryons would then fall into the potential wells, forming bound clouds, on baryon scales above

$$M_{Ja} = 10^6 \, \Omega_b \, \Omega_a^{-3/2} \, M_\odot \tag{51}$$

where Ω_a is the cold particle density.[42] These clouds could then form pregalactic stars just as in the previous case. In fact, the formation of the pregalactic clouds is even easier in this case because the cold particle fluctations grow by an exta factor of $10\Omega_a$ between the time when the cold particles dominate the density and decoupling.

4.2.3 <u>Adiabatic fluctuations in a "hot" universe.</u> If the Universe's mass is dominated by baryons or collisionless "hot" particles, like neutrinos with non-zero rest mass, then adiabatic fluctuations are erased on subgalactic scales by photon diffusion for[189]

$$M < M_s \simeq 10^{13} \, \Omega_b^{-5/4} \, M_\odot \tag{52}$$

or neutrino free-streaming for[90]

$$M < M_\nu \simeq 10^{15} \Omega_\nu^{-2} \, M_\odot \tag{53}$$

Thus, the first objects to form are "pancakes" of cluster scale.[190] However, one still expects these pancakes to fragment into clumps of mass $10^{8} M_{\odot}$ and these clumps might in turn fragment into stars before clustering into galaxies.[191] Even in this case, therefore, one might expect pregalactic stars to form, albeit at a relatively low redshift ($z < 10$). The mass of the resulting stars is still very uncertain but at least two of the previous arguments for high mass fragments (lack of metallicity and substructure) would still pertain.

This discussion emphasizes that pregalactic stars could form in all of the most likely scenarios for the density fluctuations. However, it is not clear how much of the Universe would be expected to go into the stars. If they formed very efficiently, cosmological nucleosynthesis constraints on the baryon density ($\Omega_b < 0.1$) would permit their remnants to provide the dark matter in halos but not a critical density.[109] On the other hand, in the present epoch (e.g. in giant molecular clouds) star formation seems to be rather inefficient, so - if one wants to argue that most of the Universe has been processsed through Population III stars - one needs to suppose that the conditions for star formation are very different at early times. One possibility is that the stars may be massive enough to generate a lot of explosive energy; in this case, the explosions may have amplified the fraction of the Universe going into stars,[192] as discussed in Section 4.4.10.

4.3 Cosmological Constraints

4.3.1 <u>Remnants</u>. Only stars larger than $M_c \approx 200\ M_{\odot}$ can generate black holes with high efficiency (i.e., with the fraction of the initial star mass which collapses ϕ_B being close to 1). Providing the Population III spectrum is such that most of the mass is in stars this large, the fraction of the Universe ending up in black hole remnants should be $f_B = f_* \phi_B$, where f_* is the fraction of the Universe going into the stars. Observationally, we require $0.9 \lesssim f_B \lesssim 0.99$ (the lower limit being required to explain the dark mass in halos[51] and the upper limit to ensure that enough visible material is left over). It might seem unlikely that f_B would be this high since we could not expect both ϕ_B and f_* to be so close to 1. In particular, ϕ_B must be less than 0.5 if envelope-ejection occurs at hydrogen-shell burning.[27] However, one could in principle boost f_B by invoking black hole accretion or many generations of stars. We have seen that dynamical constraints, associated with the puffing up of the disc in our own galaxy by traversing halo holes,[117,192] require that the mass of the hole which dominates the halo not exceed $10^6 M_\odot$. More specifically, we require

$$\Omega_B(M) < 0.1 \min \left[1, \left(\frac{M}{10^6 M_\odot} \right)^{-1} \right] . \tag{54}$$

as indicated in Fig. 7. We therefore conclude that the mass of the halo holes must lie between $10^2 M_\odot$ and $10^6 M_\odot$ (i.e., they must derive from VMOs or low mass SMOs). The other possibility for explaining the dark matter is to suppose that the Population III stars are so small[112] that their mass-to-light ratio exceeds 100; this requires $M < 0.1\ M_\odot$. As discussed in Section 2.3.3, infrared observations of other spiral galaxies may exclude any hydrogen-burning stars.

4.3.2 Background Light. Since stars turn $\epsilon_R \approx 0.01$ of their rest mass into radiation over their nuclear-burning time, the background light density they generate should be[192-194]

$$\Omega_R \approx 10^{-2} \Omega_* f_R (1+z_*)^{-1} \left(\frac{\epsilon_R}{0.01} \right) \tag{55}$$

where the densities are in units of the critical density, z_* is the redshift at which they burn most of their fuel, and f_R - the fraction of the radiation which goes into background light rather than heating effects - should be close to 1. Since the observed background density over all wavebands cannot exceed about 10^{-4} (with the possible exception of the far IR band, which is presently unobserved), this implies

$$\Omega_* < 0.03 \max \left[1, \left(\frac{M}{10^2 M_\odot} \right)^{-1/2} \right] (1+z_*) . \tag{56}$$

Thus the stars must form before a redshift of 10 if $\Omega_* > 0.1$. This also requires that the stars be larger than about 10 $M_\odot$ in order to burn out by then; the precise value depends on rather uncertain cosmological parameters. The value of Ω_* permitted for lower values of z_* is indicated in Fig. 7. Of course, these limits do not apply for stars with $M < 1\ M_\odot$ since these are still burning. For these, the background light constraint requires [192]

$$\Omega_* < 0.07 \left(\frac{M}{M_\odot} \right)^{-3} \qquad (57)$$

which is somewhat weaker than the constraint associated with infrared observations of individual galaxies. This limit is also shown in Fig. 7. The background light limits would be more interesting if one knew that the starlight should presently be in a waveband where the density is less than 10^{-4}. However, as discussed in Section 4.4.2, the effects of dust make this assumption questionable.

4.3.3 Enrichment. One of the strongest constraints on the spectrum of Population III stars comes from the fact that stars in the mass range $4\ M_\odot - M_c$ should eventually explode, producing an appreciable heavy element yield. The dependence of the yield on M can be expressed as

$$Z_{ej} \approx \begin{cases} 0.5 - (M/6M_\odot)^{-1} & (8M_\odot < M < 100M_\odot) \\ 0.1 & (4M_\odot < M < 8M_\odot) \\ 0.5 & (100\ M_\odot < M < M_c) \end{cases} \qquad (58)$$

Since Population II stars are observed with metallicity as low as 10^{-5}, this implies that the pregalactic enrichment cannot exceed this amount unless Population II stars are themselves pregalactic.[197] In any case, it cannot exceed the lowest metallicity observed in Population I stars $Z = 10^{-3}$). The associated constraint on the Population III mass spectrum, assuming $\Omega_g = 0.1$, is shown in Fig. 7. If one wants to produce the dark matter without contravening this constraint, the most straightforward solution is to assume that the spectrum either begins above M_c or ends below $4\ M_\odot$. As mentioned in Section 4.2, it is possible that the first stars were either very large or very small, so this solution is not unreasonable.

The effects discussed above place interesting constraints on the density of stars with mass M which could have formed at a redshift z; these constraints are summarized in Fig.7 . The black hole constraint is most important for high M , the nucleosynthetic constaint for intermediate M , and the background light constraint for low M. Only the last constraint is dependent on z and it is interesting in the intermediate mass range only for z < 10. Fig. 7 indicates that the only mass ranges in which a large fraction of the Universe can be processed through Population III stars are $M < 0.1\ M_\odot$ and $M_c < M < 10^6 M_\odot$. It also restricts the form of the mass spectrum: for example, if the spectrum encompasses the exploding mass range, then the nucleosynthetic constraints require a stellar mass spectrum which is either very steep if the dark matter is in low mass stars or very shallow if it is in high mass stars.

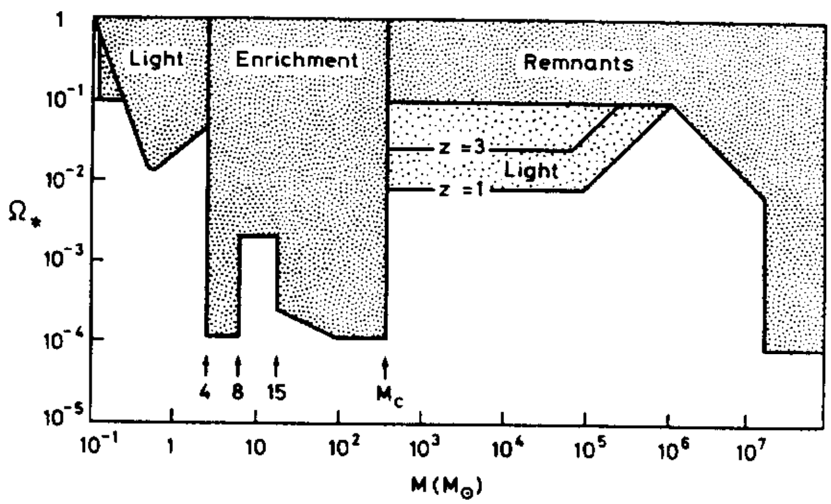


Figure (7): Constraints on Population III mass spectrum

4.4 What Population III Stars Can Achieve

In this section we will discuss various cosmological observations (some of them rather tentative) which Population III stars may be able to explain. It should be stressed, however, that Population III stars are not the only explanation of these observations, so the considerations below do not provide unequivocal evidence for their existence.

4.4.1 Near infrared background. We have seen that an important constraint on the number of Population III stars is associated with the observational upper limit on the background radiation density. On the other hand, the existence of large background radiation density would be one of the chief hallmarks of the Population III scenario. Recently the detection of an IR background in the waveband 2 - 5 μ with a density $\Omega_{IR} \simeq 10^{-4}$ and a black-body spectrum with temperature 1500 K has been reported.[198] Although this claim remains to be confirmed, it is obviously interesting to inquire what sort of stars would be required to explain the data. If we assume that the stars are VMOs with $M > M_c$ (in order to avoid overenrichment) and that their radiation is not modified by grain absorption, then the expected spectrum can be shown to be[67]

$$\Omega_R(\nu) \equiv \frac{4\pi\nu i(\nu)}{\rho_{crit}c^3} \approx 6 \times 10^{-4} \left(\frac{f_b x_o}{0.6}\right) \left(\frac{\Omega_*}{1+z_*}\right) \left(\frac{x^4}{e^x - 1}\right) \qquad (59)$$

where $i(\nu)$ is the intensity, $x = h\nu(1 + z_*)/kT_s$, and $f_b X_o$ is the fraction of the star's mass burnt to helium (normalized to the value appropriate for a VMO with $X_o = 0.75$). Since VMOs have a surface temperature of $T_s \approx 10^5$K (independent of M), the quantity $\Omega_R(\nu)$ peaks at a present wavelength

$$\lambda_{max} \approx 0.04\ (1+z_*)\mu \qquad (60)$$

which lies in the observed waveband for $40 < z_* < 140$. A comparison with the data shows that no single curve passes through all the data points. If one excludes the 5 μ point (which is the most dubious since it could be contaminated by interplanetary dust emission), the best fit curve[67] corresponds to $z_* = 75$ and $\Omega_* = 1.6h^{-2}$ where $h \equiv (H_o/50)$. The value of Ω_* is rather high if one believes the cosmological nucleosynthesis constraint.

4.4.2 Microwave distortions. The above discussion fails if absorption by dust occurs. For some range of values of z and the dust fraction χ, the starlight would be absorbed and re-radiated in the far IR. One can show[199] that the dust temperature should have a value

$$T_d = T_c \left[1 + 10 \left(\frac{\Omega_R}{\Omega_c}\right) \left(\frac{r}{0.1}\right)^{-1} \left(\frac{1+z_*}{10^3}\right)^{-1} \right]^{1/5} \tag{61}$$

where T_c and Ω_c are the temperature and density of the 3 K background, Ω_R is the density of the stellar radiation, and r is the grain radius. This means that the re-radiated light should presently reside in the waveband 300 - 900μ, with only a weak dependence on z_* and χ. This is presently unobservable, but could become so with future space experiments (e.g. SIRTF and LDR). For sufficiently high values of z_* and χ, the dust temperature would be held so close to the microwave background temperature thatthe re-radiated starlight would distort its black-body spectrum.[200-202] A few years ago, such a distortion was, in fact, reported: the best-fit black-body temperature around the peak was found to be 2.9 K (significantly higher than the 2.7 K found in the Rayleigh-Jeans region) and there also appeared to be a trough just shortward of the peak.[203] This could be explained if the 2.9 K component was thermalized pregalactic starlight, with the trough reflecting the reduced absorption efficiency of grains in the 1 - 10μ region; the trough would be redshifted to the observed wavelength for $z_* \simeq 100$. This proposal would require that 25% of the 3K background energy come from stars, which in turn would require $\Omega_* \simeq 0.1$. However, more recent data suggest that the distortion may be going away.[204]

4.4.3 Generation of the 3K background. Even if the 3K background has an undistorted spectrum, it could still have been generated entirely by stars providing one can invoke a suitable thermalizing agent[205]; this may be feasible even at low redshifts if one adopt special types of elongated grains.[206-209] However, these models seem rather contrived: it is very difficult to ensure thermalization at both long and short wavelengths. As discussed in Section 1.3, an alternative scheme is to assume that the black hole remnants of the stars generate the radiation through accretion [48]. Of course, any scheme which envisages the 3K background deriving from Population III stars or their remnants must also require that the early Universe was cold or tepid [45] (with the primordial photon-to-baryon ratio being much less than its present value of 10^9). The advantages originally claimed for the cold and tepid models were: (i) that the photon-to-baryon ratio could be explained naturally rather than being assigned arbitrarily as an initial condition of the Universe; and (ii) that the existence of a prolonged matter-dominated phase before decoupling would permit galactic-scale density fluctuations a longer period of growth. However, both these arguments have become less compelling in recent years. The grand unified theories [211] permit alternative explanations for the value of S by invoking baryon-non-conserving processes at 10^{-35} s; and the currently favoured picture of galaxy formation requires that the early Universe was cold anyway, but in the sense of being dominated by non-relativistic particles for some period before decoupling rather than in the sense of having no radiation.

4.4.4 Thermal history. The light generated by Population III stars could have an important effect on the thermal history of the Universe even in the conventional hot Big Bang scenario. During its main-sequence phase, each star (or cluster of stars) would be surrounded by an HII region. The fraction of the Universe in such regions would progressively grow on account of both the increasing number of stars and the increasing size of the individual HII regions. For $z < 60\,\delta^{-1/3}$, where δ is the gas clumpiness, the recombination time within each HII region exceeds the lifetime of the stars, so the whole Universe is ionized once the number of ionizing photons generated exceeds the number of atoms [212,213]; the fraction of the gas in stars is then just

$$f_* \simeq 3 \times 10^{-5}\,\delta . \tag{62}$$

For $z > 60\,\delta^{-1/3}$, one gets a fully developed HII region (whose structure is determined by the balance of ionizations and recombinations) and the situation is more complicated; one now needs a larger value of f_* to reionize the Universe but it is still small.[192] Such reionization could have important cosmological implications (e.g., in reducing anisotropies in the 3K background). After the HII regions have merged, the Universe would maintain a high degree of ionization,

$$1-x \approx 10^{-10}\, f_*^{-1} \left(\frac{T}{10^4 K} \right)^{-0.8} \tag{63}$$

and the gas temperature T would tend to a value of $10^4 - 10^5$K until the stars cease burning.[192] If the stars finally explode, or if they leave black holes which heat the Universe through accretion,[56,64] the temperature may be boosted even higher.

4.4.5. Helium production by VMOs. Because the pulsational instability leads to mass-shedding of material convected from its core, a VMO is expected to return helium to the background medium during core-hydrogen burning.[214] The net yield depends sensitively on the mass loss fraction ϕ_L. If this is very high, the yield will be low because most of the mass will be lost before significant core burning occurs. However, for $\phi_L < (1- Y_i)/(2 - Y_i)$, the mass loss is always slower than the shrinkage of the convective core and one can show that the fraction of mass returned as new helium is[215]

$$\Delta Y = (1-\tfrac{1}{2}Y_i)\phi_L^{\;2} \leq 0.25 \frac{(1-Y_i)^2}{(1-Y_i/2)} \quad . \tag{64}$$

Here Y_i is the initial (primordial) helium abundance and the equality sign on the right applies only if ϕ_L has the critical value. This does not necessarily impose a constraint on the number of VMOs since ΔY is very small if ϕ_L is well below the critical value. However, there is some indication from numerical calculations that hydrogen-shell burning may produce a super-Eddington luminosity which completely ejects the stellar envelope.[26,27] If true, this means that the maximal helium production permitted by eqn (64) is guaranteed. This would have profound cosmological implications. If $Y_i = 0.22$, corresponding to the conventional primordial value, $\Delta Y = 0.17$, so one would substantially overproduce helium if much of the Universe went into VMOs. In this case, the only remaining candidate for the dark matter would be SMOs in the mass range 10^5 - $10^6 M_\odot$. On the other hand, if $Y_i = 0$, corresponding to no primordial production, $\Delta Y = 0.25$, which is tantalizingly close to the standard primordial value. One can show that, if a fraction F of the Universe is processed through VMOs, then the resulting helium abundance is given by [215]

$$F = \int_0^Y \frac{2(2-Y)}{(1-Y)^2} \exp\left(\frac{2Y}{Y-1}\right) dY . \tag{65}$$

For small Y, this just gives $F \simeq 4Y$, so one naturally gets the sort of value required if F is close to 1. More precisely, Y lies between 0.20 and 0.25 for $0.8 < F < 0.9$. This raises the question of whether the Population III VMOs invoked to produce the dark matter might not also generate the helium which is usually attributed to cosmological nucleosynthesis. Unlike stars smaller than M_c, which could not do so without overproducing heavy elements, stars larger than M_c may produce no appreciable yield since the heavy elements they generate collapse with the core to form a black hole. One still has to find a way to suppress primordial helium production. One way to do this would be to suppose that the early Universe was cold. In this case, the amount of helium production is determined entirely by the neutrino degeneracy factor [216,217] and one could avoid any helium production at all for a lepton-to-baryon ratio exceeding 1.5.

4.4.6 <u>Heavy element production</u>. We saw in Section 4.3.3 that one of the strongest constraints on the Population III hypothesis derives from the requirement that the stars not produce too many metals. Nevertheless, for some purposes it would be advantageous to have a slight pregalactic enrichment (e.g., to explain the G-dwarf problem[218] or the small grain abundance required to produce alleged distortions in the 3K spectrum[200] or the possible lack of a metallicity gradient in globular clusters[219]), so the question arises of whether it is likely that Population III stars could generate just a small metallicity (e.g., $Z \simeq 10^{-5}$). One natural way in which this would occur might be to suppose that the first stars do indeed explode but that their formation is self-limited due to reionization suspending star formation once f_* reaches the value given by eqn (62). This would naturally generate an enrichment of about 10^{-5}. Another possibility is that the heating effect of the first stars might shift the typical stellar mass into the non-exploding range once f_* gets sufficiently large. In this case, most of the Population III density could end up in dark remnants, even though[220] some enrichment is produced. If the first stars are unusual, one might expect their metal yield to dispay unusual abundance ratios. It is therefore of interest that various chemical anomalies have been observed for metal-poor stars. For example, the high oxygen-to-iron ratio at low Z[221] suggests that the first stars were either MOs or VMOs (the latter producing a lot of oxygen since they explode in their oxygen burning phase)[222]; and the possible evidence for primary nitrogen[223,224] might also be explained by VMOs if the carbon and oxygen produced during helium-core burning could be convected through the hydrogen-burning shell and there CNO-processed to nitrogen.[215,225]

4.4.7 <u>Dynamical effects of halo holes</u>. It was mentioned in Section 4.3.1 that the most interesting constraint on the mass of any holes in our own halo is associated with their tendency to puff up the disc. A more detailed calculation of this effect suggests that halo holes could actually be responsible for the amount of disc-puffing which is observed.[117] The velocity dispersion of the disc stars in the radial, transverse, and vertical directions should evolve according to

$$\sigma_u(t) = (\sigma_{uo}^2 + D_e t)^{1/2}, \quad \sigma_v(t) = \beta^{-1}\sigma_u(t), \quad \sigma_w(t) = (\sigma_{wo}^2 + D_z t)^{1/2} \tag{66}$$

respectively, where

$$D_{e(z)} = 2\pi G^2 n M^2 \ln \left(\frac{V^3 T_{orb}}{GM}\right) F_{e(z)}(V/\sigma,\ \beta) \ . \tag{67}$$

Here σ_o is the initial velocity dispersion of the stars, σ is the velocity dispersion of the holes, V is the relative velocity of the holes and the stars (close to the circular velocity in the disc if the halo is assumed to have little rotation), n and M are the mass and number density of the holes, β is the ratio of the circular to epicyclic frequency, and F_e and F_z are determined in terms of the error function. The important features are: (i) that all three components of σ grow asymptotically as $t^{1/2}$, thus explaining the empirical relationship between the observed scale heights of stars and their age; and (ii) that the predicted ratio of the vertical and radial components σ_w/σ_u agrees with observation.[226] Attempts to explain the puffing up of the disc through heating by giant molecular clouds, for example, would seem to be less successful on both accounts: one then expects $\sigma \propto t^{1/4}$ and σ_w/σ_u is too small.[230] In order to normalize the $\sigma(t)$ relationship correctly, one needs

$$n M^2 \simeq 3 \times 10^4 M_{\odot}^{\ 2}\ pc^{-3}. \tag{68}$$

Combining this with the inferred halo density at our own galactocentric distance, implies a hole mass $M \simeq 2 \times 10^6 M_{\odot}$. Note that this argument does not strictly require that the halo object be a single hole; even a cluster of smaller holes - or indeed a cluster of any other sort of object - would suffice. In this context, it may be relevant that M is close to the size of the first clouds one would expect to form in the "isothermal" or "cold" scenarios discussed in Section 4.2.

4.4.8 Gravity waves from single holes. The formation of black holes would be expected to generate bursts of gravitational radiation. The characteristic period and density of the waves would be

$$P_o \simeq 10\,\frac{GM}{c^3}\,(1+z_B) \simeq 10^{-2}\left(\frac{M}{10^2 M_{\odot}}\right)(1+z_B)\ \mathrm{s},\ \ \Omega_g = \varepsilon_g \Omega_B (1+z_B)^{-1} \tag{69}$$

where z_B is the redshift at which the holes form and ε_g is the efficiency with which the collapsing matter generates gravity waves. One can show that the expected time between bursts (as seen today) would be less than their characteristic duration providing $\Omega_B > 10^{-2}\Omega^{-2}$. If the holes make up galactic halos, one would therefore expect the bursts to form a background of waves.[227] This background can be detectable by laser interferometry if M is below $10^2 M_{\odot}$ or by the Doppler tracking of interplanetary spacecraft if M is in the range $10^5 - 10^{10} M_{\odot}$.

4.4.9 Gravity waves from binary holes. The prospects of detecting the gravitational radiation would be even better if the holes formed in binaries.[228] This is because two sorts of radiation would then be generated: (i) continuous waves as the binaries gradually spiral inwards due to quadrupole emission; and (ii) a final burst of waves when the components finally merge. The burst would have the same characteristics as that associated with isolated hole formation except it would be postponed to a lower redshift and ϵ_g would be larger ($\epsilon_g \approx 0.08$) because of the larger asymmetry; from eqn (69) both factors would increase Ω_g. The continuous waves would also be interesting since they would extend the spectrum to considerably longer periods, thus making the waves detectable by a wider variety of techniques. Over most wavebands, the spectrum of the waves would be dominated by the holes whose initial separation was such that they are coalescing at the present epoch. This corresponds to a separation

$$a_{crit} \approx 10^2 \left(\frac{M}{10^2 M_\odot}\right)^{3/4} R_\odot \tag{70}$$

If the fraction of holes which form in the binaries with this separation is f_{crit}, the spectrum should have the form: [228]

$$P \frac{d\Omega_g}{dP} \approx 0.08\, \Omega_B f_{crit} \left(\frac{P}{P_o}\right)^{-2/3} \left[10^{-2} \left(\frac{M}{10^2 M_\odot}\right) s < M < 10^5 \left(\frac{M}{10^2 M_\odot}\right)^{5/8} s\right] \tag{71}$$

Providing $\Omega_B f_{crit}$ is not too small, this background should be detectable by laser interferometry for $M < 400\, M_\odot$ and by Doppler tracking of interplanetary spacecraft for $4 \times 10^4 M_\odot < M < 6 \times 10^{10} M_\odot$. One could also hope to observe the coalescences which are occurring in our own galactic halo at the present epoch. The average time between halo bursts and their expected amplitude would be

$$t_{burst} \approx 10 \left(\frac{M}{10^2 M_\odot}\right) f_{crit}^{-1}\, h^{-1} y, \quad h_{burst} \approx 7 \times 10^{-17} \left(\frac{M}{10^2 M_\odot}\right), \tag{72}$$

respectively. The crucial issue, of course, is whether one can expect black holes to form in binaries. Since at least 50% of O stars (the largest stars forming in the present epoch) appear to be in binaries,[229] it does not seem implausible that VMO binaries should also be abundant. While O stars appear to have a separation spectrum[231] which peaks at $a \approx 30 R_\odot$, this could be raised by about a factor of 4 for VMOs due to mass loss effects. Therefore, a reasonably large fraction of the resulting black holes binaries could have the separation a_{crit}.

4.4.10 <u>Pregalactic explosions</u> Stars in the mass range 4 - 200$M_\odot$ should produce explosive energy with an efficiency $\varepsilon = 10^{-5} - 10^{-4}$; those in the range 200 - $10^5 M_\odot$ may explode with comparable efficiency providing the shell ejection mechanism discussed in § 4.4.5 works.[232] This explosive release could have an important effect on the large-scale structure of the Universe.[233-237] One would expect the shock-wave generated by each exploding star (or cluster of stars) to sweep up a shell of gas. Under suitable circumstances, this shell could eventually fragment into more stars. If the new stars themselves explode, one could then initiate a bootstrap process in which the shells grow successively larger until they overlap. This mechanism has been proposed in three contexts: (i) as a means to boost the fraction of the Universe being processed through pregalactic stars[192]; (ii) as a means to generate many galaxies from a few seed galaxies[234]; and (iii) as a way of producing the giant voids and filaments, whose existence is indicated by observational data[238,239], from much smaller scale initial structure. During the Compton cooling era ($z > 10$), one can show that the maximum amplification which can be achieved in a single step is

$$\xi_1 \equiv \frac{M_{shell}}{M_1} \approx 4\times10^5 (1+z)^{-1.7} \left(\frac{\varepsilon}{10^{-4}}\right)^{0.6} \left(\frac{M_1}{10^6 M_\odot}\right)^{-0.4} \quad (73)$$

where M_1 is the seed mass and z is the redshift at which the shell fragments. The shell mass at the nth step is[237]

$$M_n \simeq \xi_1^{\,2.5\,\{1-(0.6)^{n-1}\}}\, M_1 \quad (74)$$

and this tends asymptotically to

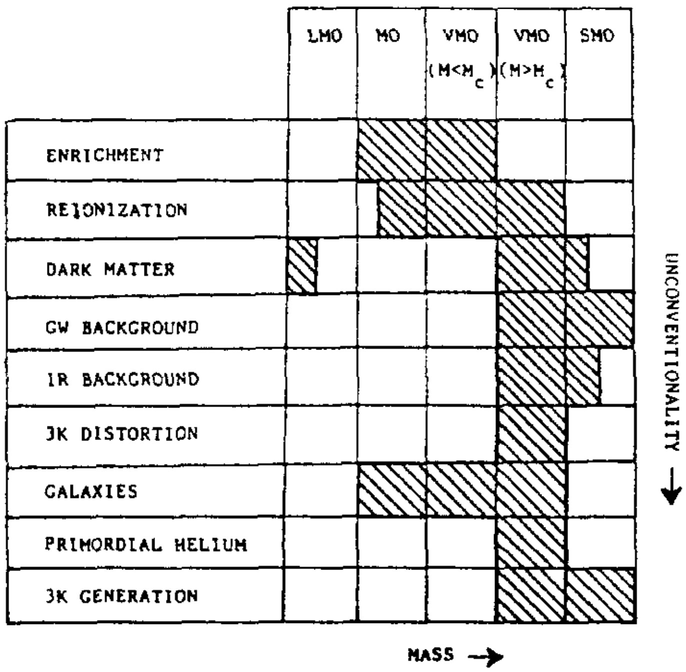
$$M_\infty \simeq \xi_1^{\,2.5} M_1 \simeq 1\times10^{20} (1+z)^{-4.3} \left(\frac{\varepsilon}{10^{-4}}\right)^{1.5} M_\odot \quad . \quad (75)$$

providing the shells do not overlap first. Equation (75) specifies the maximum scale of structure that can be attained at a redshift z. By the end of the Compton era, M_∞ is already of order $10^{15} M_\odot$, independent of the original seed mass. For $z < 10$, radiative cooling dominates and the situation becomes more complicated. An upper limit to the final shell size in all circumstances is $\sqrt{\varepsilon}\,ct$. This is 30 Mpc for $\varepsilon \simeq 10^{-4}$, which is just about large enough to explain the largest voids.

4.5 Conclusion

The shaded regions in Table (5) indicate what sorts of Population III stars are required to explain the various cosmological effects alluded to in this section. We have ordered these effects according to the extent to which they require a deviation from the standard Big Bang picture; only the last two require that one gives up the conventional hot scenario. The "IR background" refers specifically to the Japanese measurement and the "GW background" is required to be detectable. The stars which solve most of the problems would seem to be the VMOs: indeed, the combination of exploding and collapsing VMOs would appear to explain everything. Therefore, if one is prepare to forego some of the attractions of the standard picture, VMOs would seem to be the Population III candidate with most explanatory power. On the other hand, if one wants to preserve the standard picture as much as possible, the most attractive Population III candidate would be SMOs of $10^6 M_{\odot}$: such objects could avoid making helium, light, enrichment, and explosions, while at the same time producing dark halos, disc heating, and detectable gravitational waves.

Table (5): Explanatory power of different Population III candidates



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