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THE FORMATION OF STRUCTURE IN THE UNIVERSE*

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For almost two decades cosmologists have had a general picture of how the structure (galaxies, clusters of galaxies, etc.) in the Universe formed--small primordial density inhomogeneities grew up into the observed structure by the Jeans (or gravitational) instability. Thanks to 'hints' from the very early Universe, the details are now beginning to be filled in. We review some of the recent progress and developments, including the hot and cold dark matter scenarios, a statement of the Ω-problem, interpretation of microwave background anisotropies, biased galaxy formation, and the possible role of cosmic strings.

I. OVERVIEW

A. The Hot Big Bang Cosmology

The hot big bang cosmology seems to provide an accurate description of the evolution of the Universe from 10^{-2} sec after the bang until today, some 15 billion years later. The evidence which supports its validity include: the observed universal (Hubble) expansion, the 3K blackbody background radiation, and the concordance between the predictions of primordial nucleosynthesis and the observations of the primordial abundances of D, He^3, He^4, and Li^7 (ref. 1).

The hot big bang model is based upon the isotropic and homogeneous Friedmann-Robertson-Walker cosmological models.[2] In these models the evolution of the Universe is described by a single variable--the cosmic scale factor a(t). All physical distances, e.g., wavelength of a photon or the separation of two galaxies, scale

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up with $a(t)$. The redshift of a photon emitted at time t is related to $a(t)$: $1 + Z = a_{today}/a(t)$.

The evolution of $a(t)$ is given by the Friedmann equation

$$H^2 \equiv (\dot{a}/a)^2 = 8\pi G\rho/3 - k/a^2$$

where ρ is the total energy density (matter plus radiation), and $k = \pm 1$ or 0 is the curvature signature of the spatial hypersurfaces. Note that negatively-curved and flat models ($k \leq 0$) necessarily expand forever, while the positively-curved model must eventually recollapse.

The Hubble time, H^{-1}, is an important time-scale. It is the time required for the scale factor to roughly double, and as such, it sets the scale on which microphysics operates: coherent, causal microphysical processes operate only on scales less than H^{-1}. If, as is often the case, the scale factor grows as t^n (with $n<1$), then up to factors of order unity H^{-1} is also the age of the Universe and the distance a light signal could have propagated since 'the bang' (this distance is known as the particle horizon distance).

The curvature (and hence fate of the Universe) can be related to the energy density of the Universe by the Friedmann equation:

$$(k/a^2)/H^2 = \rho/(3H^2/8\pi G) - 1,$$
$$\equiv \Omega - 1,$$

where $\rho_{crit} = 3H^2/8\pi G$ is the critical density ($\simeq 1.88h^2 \times 10^{-29}$ g cm^{-3}), the present value of the Hubble parameter $H = 100$ h km sec^{-1} Mpc^{-1}, and $\Omega = \rho/\rho_{crit}$. The curvature signature is determined by Ω: $\Omega > 1 \Longleftrightarrow k > 0$; $\Omega < 1 \Longleftrightarrow k < 0$. For this reason it is useful to measure cosmological energy densities as their fraction (Ω_i) of the critical density.

The energy density in radiation (and relativistic particles) depends upon the temperature T and the effective number of relativistic degrees of freedom $g_* \; (\equiv 7/8 \sum_{Fermi} g_F + \sum_{Bose} g_B$, for species with mass $<< T$):

$$\rho_r = g_* \pi^2/30 \; T^4$$

(where thermal equilibrium has been assumed and $\hbar = k = c = 1$). As the Universe expands, $T \propto a(t)^{-1}$, so that $\rho_r \propto a^{-4}$.

On the other hand, the energy density in matter decreases simply due to the volume expansion of the Universe: $\rho_{matter} \propto a(t)^{-3}$. Today matter (baryons and other NR particles present) outweigh radiation (3K photons and 2K neutrinos) by a factor of $\simeq 3 \times 10^4 \; (\Omega h^2/\theta^4)$ (where 2.7θK is the temperature of the microwave background radiation).

Owing to their different scalings with $a(t)$ this has not always been the case. When $a(t)$ was less than $3 \times 10^{-5} \; (\theta^4/\Omega h^2) \; a_{today}$, the energy density in radiation was greater than that in matter. [Note, cosmologists often normalize $a(t)$ so that $a_{today} = 1$.] At the 'equal density' epoch: $t \simeq 3{\times}10^{10}$ sec $(\theta^3/\Omega h^2)^2$ and $T \simeq 6\text{eV}(\Omega h^2/\theta^3)$. Earlier than the equal density epoch the Universe was 'radiation-dominated' and

$$a(t) \propto t^{1/2},$$
$$T(\text{GeV}) \simeq (t/10^{-6}\text{ sec})^{-1/2}$$

During the radiation-dominated epoch, all kinds of interesting events occurred—primordial nucleosynthesis; production of various relics possibly including, monopoles, strings, weakly-interacting massive particles (or WIMP's), axions or coherent field oscillations; baryogenesis; and inflation, to mention a few (see Fig. 1). After the equal density epoch the Universe is matter-dominated and

$$T \propto a(t)^{-1},$$
$$a(t) \propto t^{2/3}.$$

Once the Universe becomes matter-dominated it is possible for structure to begin to form—earlier this is not possible because 'radiation' dominates the dynamics and relativistic particles are not unstable to gravitational collapse. In this mini-review we will focus on the current understanding of how structure in the Universe evolved, beginning our story at the equal density epoch.

B. Statement of the Problem

On small scales the Universe today is very lumpy. For example, the average density in a galaxy ($\simeq 10^{-24}$ g cm^{-3}) is about 10^5 times the average density of the Universe. The average density in a cluster of galaxies is about 100 times the average density in the Universe. Of course on large scales, say >> 100 Mpc, the Universe is smooth, as evidenced by the isotropy of the microwave background, number counts of radio sources, and the iostropy of the x-ray background.

The surface of last scattering for the 3K microwave background is the Universe at 300,000 years after 'the bang', when $T \simeq 1/3$ eV and $a \simeq 10^{-3}\, a_{today}$. Thus the μ-wave background is a fossil record of the Universe at that very early epoch. The isotropy of the μ-wave background, $\delta T/T \lesssim O(10^{-4})$ on angular scales ranging from 1' to 180° (see Fig. 2)[3], implies that the Universe was smooth at that early epoch: $\delta\rho/\rho << 1$. There is a calculable relationship between $\delta T/T$ and $\delta\rho/\rho$ (which depends upon the nature of density perturbations present—type and spectrum), but typically

$$(\delta\rho/\rho)_{DEC} = \# \, (\delta T/T) \simeq O(10^{-2}\text{-}10^{-3}),$$

where # is O(10-100); a detailed discussion of this relationship will be presented in Sec. III.

So, the Universe was very smooth, and today it is very lumpy--how did it get here from there? For the past decade, or so, cosmologists have a general picture of how this took place: starting at the equal density epoch the small primordial density inhomogeneities present grew via the Jeans or gravitational instability, into the large inhomogeneities we observe today, i.e., galaxies, clusters of galaxies, etc.[4] After decoupling, when the Universe is matter-dominated and baryons are free of the pressure support provided by photons, density inhomogeneities grow as

$$\begin{aligned} \delta\rho/\rho &\propto a(t) \qquad (\delta\rho/\rho \lesssim 1) \\ &> a(t)^3 \qquad (\delta\rho/\rho \gtrsim 1) \end{aligned}$$

The isotropy of the μ-wave background allows for perturbations as large as 10^{-2} - 10^{-3} at decoupling, and the cosmic scale factor $a(t)$ has grown by slightly more than a factor of 10^3 since decoupling, thus it is possible for the large perturbations we see today to have grown up from small perturbations present at decoupling. This is the basic picture which is generally accepted as part of the 'standard cosmology.'

One would now like to fill in the details, so that we understand the formation of structure in the same detail that we do primordial nucleosynthesis. The formation of structure (or galaxy formation as it is sometimes referred) began in earnest when the Universe became matter-dominated ($t \simeq 10^{10}$ sec, $T \simeq 10$ eV); that is the time when density perturbations in the matter component can begin to grow. In order to fill in the details of structure formation one needs to know the 'initial data' for that epoch; in this case, they include: the amount of stuff in the Universe, quantified by Ω; the composition, i.e., fraction Ω_i contributed by the various components— i = baryons, relic WIMP's (weakly-interacting massive particles), cosmological constant, relic WIRP's (weakly-interacting relativistic particles); spectrum and type (i.e., 'adiabatic' or 'isothermal') of density perturbations present. Given these initial data one can construct a detailed scenario, which can then be compared to the Universe we observe today.

I want to emphasize the importance of the initial data for this problem; without such, it is clear that a detailed picture of structure formation cannot be constructed. As I will discuss, it is in this regard that the Inner Space/Outer Space connection has been so important. Events which we believe took place during the earliest moments of the history of the Universe and which we are just now beginning to understand, have given us important hints as to the initial data for this problem. Because of these hints from the very early Universe, the problem has become much more focused, and at present two detailed scenarios exist--the 'cold dark matter' picture and the 'hot-dark matter' picture. As we shall discuss, neither picture, unfortunately, presents a totally satisfacory account at present. The yardstick by which any scenario must be measured is how well does that scenario reproduce the Universe we observe today?

C. The Universe We Observe Today

As far as the eye can tell, the basic building blocks of the Universe are galaxies--

typical mass 10^{12} $M_\odot$ (or 10^{69} baryons) and luminosity 10^{44} erg sec^{-1}. Galaxies to a first glance are uniformly distributed. However they do show a tendency to cluster, quantified by the galaxy-galaxy correlation function:

$$\xi_{gg}(r) \simeq (r/5h^{-1}Mpc)^{-1.8};$$

for reference, 1 Mpc $\simeq 3.26\times10^6$ light years $\simeq 3.1\times10^{24}$cm. Galaxies are found alone ('field galaxies'), in small groups of galaxies, and in rich clusters (bound systems containing hundreds of galaxies). Clusters themselves seem to show a tendency to cluster, quantified by the cluster-cluster correlation function,

$$\xi_{cc}(r) \simeq (r/30h^{-1}\ Mpc)^{-1.8}.$$

There is evidence for even larger-scale structure (i.e., on scales $\gtrsim$ 10 Mpc): superclusters, aggregates which are just becoming bound objects and contain several rich clusters; voids, regions of space as large as 10's of Mpc across which are deficient in bright galaxies; filaments, which are seen as long chains of galaxies. Thus far, these very large-scale structures have eluded useful quantification.

As mentioned earlier, there is the cosmic microwave background radiation--the fossil record of the Universe when $a(t) \simeq 10^{-3}\ a_{today}$. The interpretation of the anisotropy of the microwave background will be discussed in Sec. III. There are also the 'peculiar velocities' of galaxies (i.e., their motions relative to the Hubble flow). These peculiar velocities are presumably due to gravitational effects (and not rocket engines propelling galaxies) and so are related to the lumpiness of the Universe.

Finally, there is the composition of and the amount of stuff in the Universe, quantified as Ω_i (i = component in question). Luminous matter is easy to find, but only accounts for a tiny fraction of Ω: $\Omega_{LUM} \simeq 0.01$. The best measure of the baryon mass density comes from primordial nucleosynthesis; concordance of the predictions and observations of the light element abundances requires: $\Omega_B \simeq 0.014 - 0.15$--which is larger than Ω_{LUM} (thank God!), but far short of $\Omega \simeq 1$. Kepler's third law ($GM = v^2/r$) allows us to use the orbital motion of stars and gas clouds in galaxies to measure the masses of galaxies dynamically. The fact that orbital velocities remain constant as the distance r from the center of the galaxy exceeds the distance where the light 'craps out' (in virtually all spiral galaxies studied) indicates the presence of vast quantities of 'dark matter' in spiral galaxies: $\Omega_{HALO} \gtrsim 0.05 - 0.1$ (in the jargon--the so-called 'flat rotation curves'). A few comments are in order: (1) flat rotation curves are the best evidence for the existence of dark matter and from them it is clear that dark matter 'outweighs' luminous matter by a large factor (at least 3-10); (2) at present, only a lower bound can be placed on the amount of so called 'halo' dark matter, since there is no convincing evidence for a rotation curve which turns over (i.e., $v \sim r^{-1/2}$ as it would once most of the mass of the system was interior to r); (3) at present, primordial nucleosynthesis does not preclude the possibility that the dark matter is baryons. Other techniques (including weighing clusters by means of the virial theorem, weighing the Virgo cluster by means of our peculiar motion toward Virgo, etc.) have been employed to measure Ω on scales up to 30 Mpc (or so) and all yield values in the range of 0.2 ± 0.1. [With the exception of measuring the deceleration of the expansion, all techniques for determining Ω are only sensitive to the component of matter that clumps; the phrase 'to measure Ω on a given scale' then means to measure the amount of matter which clumps on that scale; for a more detailed discussion of determining Ω, see ref. 5.]

To summarize our knowledge of Ω: most of the matter in the Universe is dark, at present composition unknown; the observations can be summed up by: $\Omega_{OBS} \simeq 0.2 \pm$ '0.1' (see Fig. 3).

D. The Hints

As we have emphasized, the formation of structure problem can be viewed as an initial data problem: given the type (adiabatic or isothermal) and spectrum of density perturbations, specified by the power spectrum $k^{3/2}\ |\delta_k|$, the amount of matter, specified by $\Omega = \Sigma\Omega_i$, and the composition (i = baryons, WIMP's, etc.), the evolution of structure in the Universe can be numerically-simulated (see Sec. II). Adiabatic perturbations (known in the modern vernacular as curvature perturbations) are honest-to-God wrinkles in the spacetime manifold, characterized at early times by: $\delta n_i/n_i$ = same for all species i (i = photons, baryons, WIMP's). Isothermal perturbations (known in the modern vernacular as 'isocurvature' perturbations), on the other hand, are merely spatial variations in the equation-of-state, not honest-to-God wrinkles in the spacetime manifold, and at early times are characterized by: $\delta n_\gamma = 0$, (and hence $\delta\rho = 0$), $\delta(n_i/n_\gamma) \neq 0$ for some species i, i.e., some species is laid down non uniformly. The quantity δ_k is the kth Fourier component of $\delta\rho/\rho$, where $k = 2\pi/\lambda$ and λ are the comoving wavenumber and wavelength of the Fourier component δ_k. The physical wavenumber and wavelength are related to k and λ by: $k_{ph} = k/a$ and $\lambda_{ph} = a\lambda$. With $a_{today} = 1$, k and λ are the present physical wavenumber and wavelength of the perturbation. Physically, $k^{3/2}\,|\delta_k|$ is what most people call $\delta\rho/\rho$, more precisely it is the RMS mass function measured on the scale k (or λ).

Now for the hints. Baryogenesis scenarios for the origin of the baryon asymmetry predict that n_b/n_γ is only a function of microphysics and is therefore spatially constant, i.e., baryogensis strongly suggests that isothermal baryon perturbations should not be present.[6,7] [Isothermal baryon perturbations are one of the 'initial data' that in the past received much attention.[4]] The inflationary Universe scenarios[8] generically predict adiabatic perturbations, with the Zel'dovich spectrum (with the amplitude depending upon the details of the specific realization).[9] Inflationary models with axions also predict isothermal perturbations in the axions.[10] Cosmic strings can induce isothermal perturbations in the matter present in the Universe.[11] I should mention that until these hints came along, cosmologists had no clues as to the origin or the nature of the primordial density perturbations.

With regard to Ω. All theoretical prejudice argues for $\Omega = 1$, and the inflationary Universe scenarios provide an attractive means of implementing this prejudice. Concordance between the predictions of primordial nucleosynthesis and the inferred primordial abundances of D, He^3, He^4, and Li^7 requires: $0.014 \lesssim \Omega_B \lesssim 0.15$. Since the early Universe was hot, all kinds of particles, both familiar and unfamiliar to us, should have been present in great abundance ($n_x \simeq n_\gamma$, for $T \gtrsim m_x$). Of particular interest are those species which are stable (or at least very long-lived) and weakly-interacting, the so-called WIMP's. While stable, strongly-interacting particles will diminish in number by annihilation when the temperature of the Universe falls below their mass $[(n_x/n_\gamma)_{eq} \simeq (m_x/T)^{3/2}\exp(-m_x/T)]$ and become very scarce, WIMP's, owing to the feebleness of their interactions, will cease to annihilate when their abundance is still significant, and thus can be present today in significant numbers—perhaps enough to contribute the $\simeq 0.9$ of critical density needed to bring Ω to 1. Given the mass and interactions of a particle it is straightforward to compute its relic abundance and the mass density it contributes today. Particle physics has been very generous in supplying candidate WIMP's whose relic abundance could provide $\Omega_{WIMP} \simeq 0.9$ (see Table 1). Hopefully, laboratory experiments in the next 5-10 years will narrow the list of possibilities.

Given these hints as to the initial data for this problem, detailed scenarios can be played out (to be discussed in the next section). Two limiting scenarios follow—hot dark matter (neutrinos) and cold dark matter (essentially all the other candidates in Table 1). Unfortunately, neither picture is totally satisfactory. In the hot dark matter scenario, galaxies form too late (redshifts less than 1); in the cold dark matter case, the best fit to the observed Universe requires $\Omega h \simeq 0.2$.

This brings us to one of the current and pressing open questions—'the Ω-problem'.[12] Theoretical prejudice dictates $\Omega \simeq 1.0$, while observations indicate that $\Omega_{OBS} \simeq 0.2$ '± 0.1' —

far short of 1. Since the observations are only sensitive to the component of matter that clumps, the two views can be reconciled if there exists a smooth component which contributes $\Omega_{SMOOTH} \simeq 1 - \Omega_{OBS} \simeq 0.8 \pm$ '0.1'. A variety of possibilities have been suggested for the smooth component: a relic cosmological term[12,13]; WIRP's[12] (weakly-interacting relativistic particles), produced by the recent decay of WIMP's; fast-moving strings[14]; or 'failed galaxies', i.e., galaxies which for some reason failed to 'light up' (this very interesting possibility will be explored in IV). Another possibility is that cosmic strings play an important role in structure formation (this will be discussed in V).

II. Hot and Cold Dark Matter

Once the Universe is old enough for the cosmological horizon to encompass any given fluctuation, physical processes can alter the amplitude which it was given at the much earlier epoch when it was generated. WIMP's decouple from other matter in the Universe well before galaxy or cluster sized fluctuations become causally connected, and so the evolution of such perturbations is particularly simple. It depends only on whether the WIMP's are relativistic or not when the fluctuation enters the horizon, and, in the latter case, on whether the Universe is matter- or radiation-dominated at that time.[15] If the WIMP's are relativistic they stream out of the fluctuation in all directions and its amplitude is strongly suppressed. All fluctuations smaller than the horizon scale at the time the WIMP's go nonrelativistic are wiped out by this process. Larger fluctuations are scarcely affected by it, since the adiabatic decay of velocities prevents WIMP's from traversing them. However, if such fluctuations enter the horizon while the Universe is radiation-dominated, the dominant gravitational perturbation comes from the photon-baryon fluid which oscillates acoustically, thereby depriving the WIMP fluctuation of the driving force for its growth. This is known as the Meszaros effect; it does not affect the growth of large fluctuations which enter the horizon when the Universe is matter-dominated (by WIMP's). The linear transfer function describing the net effect of these processes can be calculated explicitly once the nature of the WIMP's is specified. In combination with a theory for the fluctuation generator (e.g. inflation) it specifies the linear fluctuation spectrum at the equal density

Table I - WIMP Candidates for the Dark Matter

Particle	Mass*	Place of origin
Invisible Axion	10^{-5}eV	10^{-30} sec, 10^{12} GeV
Neutrino	30eV	1 sec, 1 MeV
Photino/Gravitino/ Mirror Neutrino	keV	10^{-4} sec, 100 MeV
Photino/Sneutrino/Axino Gravitino/Shadow Matter/ Heavy Neutrino	GeV	10^{-3} sec, 10 MeV
Superheavy Magnetic Monopoles	10^{16} GeV	10^{-34} sec, 10^{14} GeV
Pyrgons/Maximons/Newtorites Perrypoles/Schwarz-schilds	$> 10^{19}$ GeV	10^{-43} sec, 10^{19} GeV
Quark Nuggets	$\simeq 10^{15}$ gram	10^{-5} sec, 300 MeV
Primordial Black Holes	$\gtrsim 10^{15}$ gram	$\gtrsim 10^{-12}$ sec, $\lesssim 10^{3}$ GeV

*Abundance required for closure density: $n_{WIMP} \simeq 1.05h^2 \times 10^{-5}$ cm$^{-3}/m_{WIMP}$ (GeV)

Fig. 1 - 'The Complete History of the Universe.'

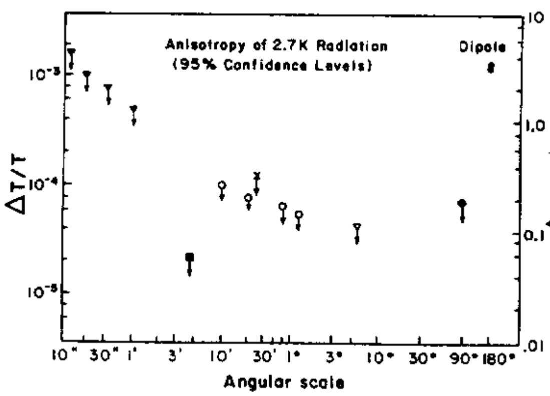

Fig. 2 - RMS anisotropy of the microwave background as a function of angular scale (from ref. 3).

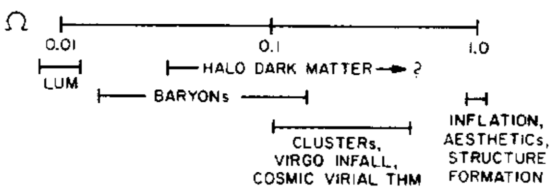

Fig. 3 - Summary of our present knowledge of Ω.

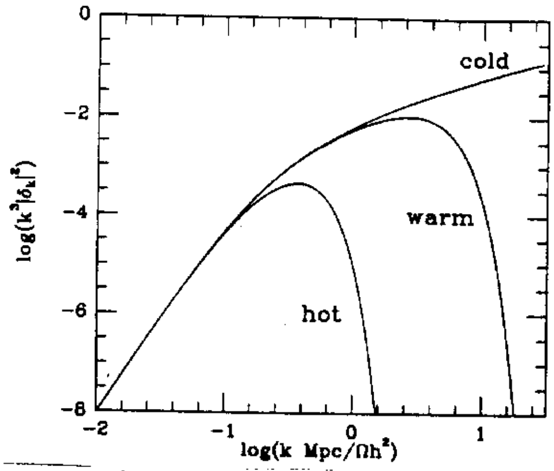

Fig. 5 - The power per octave as a function of spatial frequency for the linear density fluctuations expected at late times in a WIMP-dominated Universe which initially had the adiabatic, constant curvature fluctuations predicted by inflationary models. These curves were taken from calculations reported in ref. 19. For reference, wave number $k \approx 1\ \mathrm{Mpc}^{-1}$ corresponds to the mass scale of about $3\times10^{14} M_\odot$.

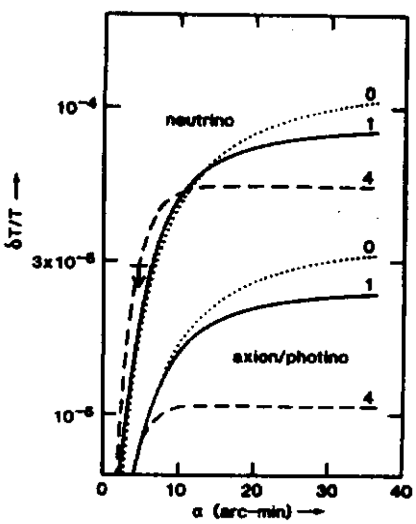

Fig. 6 - The predicted RMS temperature fluctuation as a function of angular scale for the CDM Universe (axion/photon) and the HDM Universe (neutrino), for $H = 50\ \mathrm{km\ sec^{-1}\ Mpc^{-1}}$ and spectral index of the primeval density fluctuations n = 0 (white noise), 1 (scale-invariant), and 4 (minimal fluctuations). Taken from Vittorio and Silk.[29]

epoch from which galaxies and the observed large-scale structure must grow.

The properties of the WIMP's are related to cosmological parameters by the equation

$$\frac{n_W}{n_\gamma} \; m_W = 28 \; \Omega_W h^2 \; \mathrm{eV}$$

where n_W/n_γ is the WIMP abundance relative to photons and m_W is their mass. The ratio n_W/n_γ emerging from the big bang depends on the WIMP mass and its coupling to other matter. For neutrino-type couplings values of Ω_W near 1 are obtained for masses of order 30 eV or of order 1 GeV[16]. In the former case WIMP velocities at early times are sufficient to wipe out galaxy and cluster sized fluctuations; such WIMP's are referred to as hot dark matter (HDM). In the latter case, streaming velocities are too small to influence galaxy or cluster fluctuations and the WIMP's are known as cold dark matter (CDM). For much more weakly interacting WIMP's such as SUSY particles n_W/n_γ is smaller and $\Omega_W \sim 1$ for $m_W \sim 1$ kev[17], galaxy-sized fluctuations are wiped out for such particles, but cluster-scale lumps survive. This intermediate case is known as warm dark matter (WDM). Other possible CDM candidtaes are heavy SUSY particles ($m_W \sim$ few GeV) or axions with $m_W \sim 10^{-5}$ eV which never had significant streaming velocities. Figure 5 shows the linear fluctuation spectra expected at late times[18] in WIMP-dominated Universes in which fluctuations were generated with the constant curvature spectrum predicted by inflationary models[9]. The sharp cut-off due to free-streaming is very evident in the HDM and WDM spectra; the bend due to the Meszaros effect can be seen for WDM and CDM.

The spectra of Fig. 5 are expected to grow maintaining their shape until the power per octave reaches unity on some scale. Objects of that scale will then separate from the expansion and recollapse into bounds lumps. For HDM the first objects are much more massive than galaxies, for WDM they are galaxy-sized, and for CDM they are much smaller than galaxies. In the last case, the first units are expected to aggregate rapidly into larger structures as fluctuations or larger scales go nonlinear; this process is known as hierarchical clustering. In the HDM case, the baryonic component of the very large first objects must fragment into galaxies during their initial (but recent) anisotropic collapse; this is an example of the 'pancake' picture for structure formation. In both cases the observed galaxies result from nonlinear dissipative processes affecting a baryonic component whose overall evolution is controlled by the gravitational field of the WIMP's.

The nonlinear stages of structure formation can be followed in full generality only by direct numerical simulation. Modern techniques are able to follow the evolution of the WIMP distribution accurately over a moderate range of scales, but the process of galaxy formation (which determines what we actually see) is too complex to be treated except in a schematic fashion, and this is the major uncertainty in using simulations to compare the structure of WIMP dominated Universes with the real world. The most effective simulation methods currently available represent the WIMP density distribution by a finite number of discrete 'particles' (typically $N = 3 \times 10^4 - 10^6$) moving under the influence of their combined gravitational field within the fundamental cube of a triply periodic Universe. Such a set-up can mimic the predicted linear fluctuation spectrum of a WIMP-dominated Universe over a wavenumber range of no more than $0.5\ N^{-1/3}$.[19] As a simulation evolves nonlinear structures separate out of the general expansion, recollapse, and rapidly become quite small in comparison with the expanding simulation volume. The dynamics of such clumps can be followed only if the simulation method gives accurate interparticle forces on scales much smaller than the mean particle separation. This requirement is usually a more stringent constraint on the dynamical range of a simulation than the initial constraint mentioned above. Thus substantial gains in overall performance are obtained by supplementing a standard grid Fourier Poisson-solver by a directly calculated short-range force correction, despite the fact that this considerably increases the amount of time needed to get the forces, and so reduces the number of particles that can be followed in a given amount of computer time.[19]

A neutrino-dominated Universe was the first WIMP model to be studied extensively by numerical simulation.[20] The large coherence length of the initial neutrino distribution (Fig. 5) manifests itself during the early nonlinear stages in large sheets and filaments of high density, in massive condensed lumps and in very large regions of below average density. While this bears some superficial resemblance to the observational picture of filamentary or sheet-like superclusters containing rich clusters of galaxies and surrounding large voids, it became clear that there is a large quantitative disagreement once it was realised that the observed galaxy distribution must be compared with the distribution of matter in regions which have undergone local collapse, rather than with the neutrino distribution as a whole.[21] The galaxy distribution in a ν-dominated Universe is expected to be much more clumpy than the distribution we actually see. It appears to be very difficult to circumvent this problem without giving up most of the attractive features of the theory[22]. In addition, it is not clear that gas could cool to form galaxies at all in such a Universe[23]. As a result of these conclusions the neutrino-dominated model does not, at present, appear viable.

Universes dominated by CDM are more difficult to simulate but appear much more promising. Hierarchical clustering produces a CDM distribution which is qualitatively very similar to the nearby galaxy distribution. However, a quantitative comparison requires specifying how the galaxy distribution is related to that of the CDM. In the absence of any real analysis, it has been traditional to assume that in a hierarchically clustering Universe galaxies and the dark matter must be well mixed on scales larger than that of an individual galaxy. An immediate consequence of this assumption is that our Universe is open, since the mass per bright galaxy measured by virial methods in rich clusters of galaxies falls short of the closure density by almost an order of magnitude when applied to the Universe as a whole. Simulations demonstrate that the predicted galaxy distribution in such an open CDM Universe matches observation quite well both in terms of quantitative measures such as low order position and velocity correlations and in more qualitative characteristics such as large-scale filamentary structure and large, low density regions[24]. The differences which remain, while significant, are small enough that it seems reasonable to attribute them to various important effects which the simulations do not model (e.g., the details of galaxy formation, or interactions and mergers between galaxies).

If, however, we accept $\Omega = 1$ as given, we must reject the hypothesis that the galaxy distribution is an unbiased representation of the CDM distribution on cluster scales. The number of bright galaxies per unit mass must exceed the average by almost a factor of ten in the high density regions to which virial analysis can be applied. Galaxy formation must therefore be biased to favor such regions. It has recently been realised that the standard picture (in which galaxies form as gas radiates its binding energy and sinks to the center of 'halos' of dark matter) will produce just such a bias if bright galaxies are able to form only in the densest halos (to be discussed in more detail in Sec. IV). When a 'galaxy' population is identified using a crude model of this process in simulations of an $\Omega = 1$ Universe, its properties are found to match the small-scale clustering properties of real galaxies very well[24]. Such a model may have difficulity in reproducing large-scale filaments and voids, but present simulations encompass too small a volume of space to address this question.

III. COSMIC MICROWAVE BACKGROUND

The cosmic microwave background (CMB) has proved to be one of the most important probes of galaxy formation theories. Density fluctuations, which later will evolve into galaxies and cluster of galaxies, necessarily induce angular anisotropies in the CMB at the epoch of the last scattering, redshift $z_* \simeq 1000$. Since then, the CMB photons have propagated freely, providing a unique window to this very early epoch. Moreover, once the 'chemistry' of the cosmological model is chosen (abundance of baryonic matter, abundance of non-baryonic dark matter, type of dark matter, i.e., hot or cold, etc.) a unique prediction can be made for the residual density fluctuation spectrum and, hence, for the CMB anisotropies. Comparison of the predictions with the observations constrain not only possible cosmological models but, also, the 'chemistry' of the Universe and the type of dark matter present.

CMB anisotropies are usually divided into three different categories: i) small-scale anisotropies (<1°); ii) large scale anisotropies (>1°); iii) dipole anisotropies. The only CMB anisotropy measured to date is a dipole anisotropy. The measurements of three different groups[25] are in excellent agreement and imply a motion of the Local Group relative to the CMB of 630 (±50) km sec^{-1}. Once the infall velocity toward the Virgo cluster is subtracted, the residual motion of the Local Group, pointing 45° away from Virgo, is:[26] $V_{LG} = 450$ km sec^{-1}. This motion may be purely due to local effects; so, V_{LG} must be considered as an upper limit on the velocity induced, if any, on the Local Group by the very large-scale matter distribution. The linear theory predicts that the contribution of a perturbation of wavelength l to the dipole anisotropy is: $\frac{\delta T}{T}\big|_{DIP} \simeq \frac{l}{ct_0}\,\delta(l,t_0)$, where t_0 is the present age of the Universe and $\delta \simeq \delta\rho/\rho$. Comparison with the observations places a severe contraint on the nature of the dark matter. If V_{LG} is taken as an upper limit, then all neutrino-dominated Universe scenarios are excluded.[27]

The large-scale anisotropies are associated with gravitational potential fluctuations. This effect is second order in l: $\frac{\delta T}{T} \simeq (\frac{l}{ct_0})^2\,\delta(l,t_0)$. The primary contribution to this kind of anisotropy comes from perturbations of wavelength comparable to the present horizon.[27,28] The evolution of very long wavelength perturbations is independent of the detailed recombination history of the Universe, because these perturbations were well outside the horizon at the epoch of recombination. For this reason, large-scale anisotropies are directly related to the primordial fluctuation spectrum.

The small-scale anisotropy has been calculated by several authors.[29,30] The situation here is more complicated because the scales of interest are well inside the horizon as matter and radiation decouple. As decoupling occurs the matter is beginning to fall into the potential wells formed by the WIMP's. The electron density determines the redshift of the last scattering surface and how far back we can look by observing the angular structure of the CMB. However, recombination is not an instantaneous process,[31] and is slower than predicted by the Saha equation because direct recombination to the ground state is strongly inhibited due to the presence of photons with a mean free time for photoionization which is very short compared to the expansion time. Thus the last scattering surface has a thickness l_*, and any imprint on the CMB due to perturbations of wavelength $l \ll l_*$ is smeared out.[32] If there has not been early reheating of the intergalactic medium, the final recombination of the primeval plasma occurred at $z_* \simeq 1000$ and the thickness of the last scattering surface is $\Delta z \simeq 100$, corresponding to a comoving length of $l_* \simeq 10\ (\Omega\ h^2)^{-1/2}$ Mpc. For this reason only a band of wavelengths ($10<l<100$ Mpc, say) is important in determining the angular structure in the CMB on scales up to a few tens of arcminutes. So, observations of the small scale CMB anisotropy constrain the amplitude of fluctuations in that region of the spectrum at the last scattering epoch.

On small angular scales, the most sensitive observational limit[33] on the anisotropy to date is at an angle of $\alpha = 4.5'$: at the 95% confidence level an upper limit of $\delta T/T < 3\times10^{-5}$. The technique used in this measurement involved beam switching between three positions in the sky, spaced an angle α apart. The rms temperature fluctuation is defined as

$$\delta T_{rms} = \langle\ |\ T_0 - 1/2\ (T_1 + T_2)\ |^2\ \rangle^{1/2}$$

where T_0 is the central beam temperature. By evaluating the fractional radiation brightness, it is possible to predict the CMB anisotropy expected in a particular cosmological model.[34] Small-scale anisotropy limits on angular scales in excess of several arc-minutes have unambiguously ruled out baryon dominated Universes for any power law adiabatic fluctuation power spectrum, including the scale invariant fluctuations predicted by inflation.[30] Attention has now been focused on non-baryonic, dark matter dominated Universes. Figure 6 shows the results for cold and hot dark matter, for flat models with a Hubble constant of 50 km s^{-1} Mpc^{-1}. The calculations were made assuming scale-free adiabatic perturbations. A detailed discussion of the normalization of the primeval fluctuation spectrum is given elsewhere.[27] If $\Omega = 1$ and $h = 0.5$, the cold matter scenario predicts a small-scale CMB anisotropy which is about a factor of 3 smaller than the 95% confidence level upper limit of $\delta T/T < 3\times10^{-5}$. In the massive neutrino scenario with $\Omega = 1$ and $h = 0.5$ the predictions for the small scale anisotropy are only marginally compatible with the observations.

On the other hand, observations of the large-scale galaxy distribution suggest values for the present cosmological density of the order of 20% of the critical density,[5] and recent studies of the ages of globular clusters suggest that the Universe is 17(±2) billion years old,[35] too old to be compatible with an high density Universe and zero cosmological constant (unless the Hubble constant is extremely small: < 40 km s^{-1} Mpc^{-1}). Reduction of the density parameter Ω cuts the growth period for density perturbations by a factor of $1/\Omega$, and so the required amplitude of density fluctuations at recombination must be larger by this factor (for a given normalization of the primordial spectrum at the present epoch). Thus the measured upper limit to the CMB fine scale anisotropy imposes a lower bound to Ω. Reducing Ω in a neutrino dominated universe has additional deterimental effects, because, the damping length $l_d \approx 13$ Mpc/(Ω h^2) increases thereby exacerbating all the problems associated

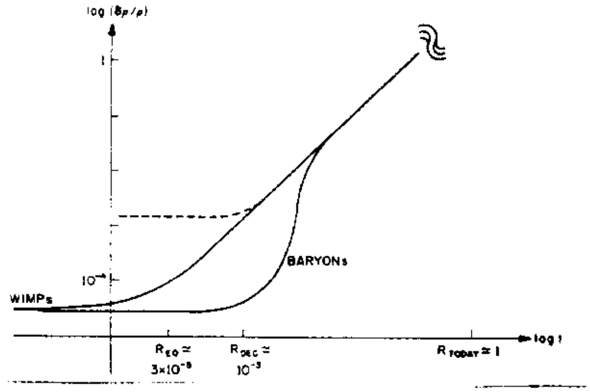

Fig. 4 - Schematic representation of the evolution of $\delta\rho/\rho$ in the WIMP's and baryons.

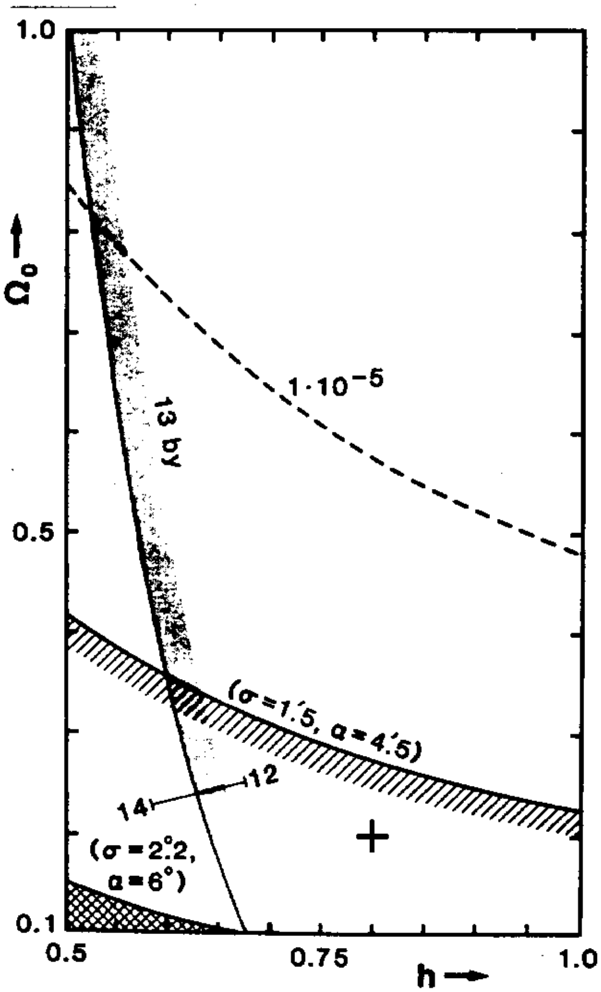

Fig. 7 - Regions of the Ω-h plane which are excluded (below and to the left) for CDM model Universes by the 4.5' measurement of $\delta T/T < 3\times10^{-5}$. The broken line labeled '1×10^{-5}' indicates the excluded region for a future measurement of $\delta T/T < 1\times10^{-5}$ on 4.5'.

with a neutrino dominated Universe. Figure 7 shows the excluded regions (at the 95% confidence level) in the Ω-h plane for cold dark-matter. Also shown are the constraints attainable by a future experiment with sensitivity $\delta T/T = 10^{-5}$ at the same angular scale of 4.5' and the inferred constraint if the age of the Universe is 13 billion years (and Λ=0). If mass and light are correlated on large scales, the CMB limit alone implies $\Omega > 0.4$ (if the Universe is at least 13 billion years old). The present upper limit to the small-scale anisotropy implies that a cold dark matter dominated Universe can only be reconciled with a low density ($\Omega \approx 0.2$) if the mass distribution is more weakly clustered than the observed galaxy distribution. We note that reionization of the Universe cannot affect this conclusion because fluctuations in the gaussian tail of the adopted fluctuation spectrum do not go non-linear sufficiently early to reionize the Universe and reduce the predicted anisotropy significantly.

IV. BIASED GALAXY FORMATION

In the preceeding sections, we have explored various aspects of the evolution of a Universe which is dominated by 'cold-dark-matter'. The nature and spectrum of the primordial density fluctuations were discussed in Sec. I and calculations of the microwave background anisotropy and of the non-linear evolution of the density field were presented in Secs. III and II. If the present density field could be observed directly, then, by comparing with the N-body results, one could determine both the parameter Ωh, which sets the slope of the spectrum, and the initial amplitude. If one makes the conventional assumption that bright galaxies give a fair representation of the present distribution of the matter, then it would appear that this picture can be ruled out since the N-body results require a low value for the density parameter while the microwave anisotropy limits require a high value.

This powerful and negative conclusion may be somewhat premature however, since there are several reasons to doubt the validity of the assumption that bright galaxies fairly trace the mass: It is well known, for instance, that different morphological types of galaxies have different clustering patterns[36] and there is some evidence that the clustering strength also depends on surface brightness.[37] A more extreme example of a set of objects which give a distorted picture of the matter distribution is provided by rich clusters of galaxies which have clustering strength which is at least an order of magnitude greater than that of galaxies.[38,39] Given this diversity of clustering strengths one must be suspicious of the idea that bright galaxies alone are fairly tracing the mass. Furthermore, it has also been argued that N_g/M, the number of galaxies per unit mass in bound systems, is a decreasing function of the mass of the system.[40] This tendency is also seen in the N-body studies[41] which can generate a correlation function with slope like that of the galaxies, but which are then unable to match the velocity field. Such behaviour is in conflict with the assumption that these bright galaxies fairly trace the mass since this would imply that N_g/M should be a universal constant.

In any hierarchical scenario, galaxies form before the large-scale structure. If galaxies form very early, then they will do so more or less uniformly throughout space and will then fairly sample the matter distribution on large scales. Under certain circumstances however, the galaxies can be born in a strongly clustered state even though the underlying large-scale density fluctuations were of very small amplitude at the time of galaxy formation. Sufficient conditions for this to occur are: i) that the initial density field be a Gaussian process; ii) that the spectrum of fluctuations be fairly flat; and iii) that galaxies form preferentially around those local maxima which lie above some global threshold.

Conditions i) and ii) are expected in the CDM dominated Universe. Since the process of galaxy formation is not very well understood, it is not known to what extent the 'biasing' of condition iii) takes place. It is not unreasonable though to imagine that galaxy formation may be a self-limiting process, so that once a few percent of the material has formed galaxies this may suppress galaxy formation elsewhere. While a detailed understanding of the feedback mechanism may not be immediately forthcoming, it is easy to enumerate some of the consequences of such a threshold.

One can show that, in order to significantly enhance the number of bright galaxies per unit mass in a rich cluster (as required to reconcile the results of virial analysis with a high density Universe), the threshold ν and galaxy formation redshift z_f should satisfy: $\nu^2 = 1 + z_f$ (ref. 42). Thus, if galaxies correspond to 3-sigma peaks forming at $z_f = 8$ for example, then the 'Ω-problem' is solved. In addition to enhancing the number of galaxies in proto-clusters, this mechanism will also strongly suppress galaxy formtion in proto-voids. One can also show that, on large scales, the galaxies will be born with a correlation function which is roughly equal to the present density correlation function. The present-day galaxy correlation function should then overestimate that of the matter by a factor ~ 4. This has the effect of reducing the predicted microwave anisotropy by a factor ~ 2 below that obtained using the conventional normalisation.

There are some further consequences of this hypothesis which can serve as tests of the 'biasing' hypothesis, and possibly to discriminate this segregation mechanism from other effects which may also be operating. Firstly, the 'light-to-mass' ratio N_g/M should be an increasing function of the density ρ of the system. One problem with applying this test is that most systems, such as groups and clusters of galaxies, for which N_g/M has been determined, have recently virialised and have similar densities. A more serious problem is that M and ρ are both 'derived parameters' and so, in the absence of any true correlation, would be expected to show a correlation in the opposite sense to that predicted. The absence of correlation between mass-to-light ratio and density in Dressler's[43] sample of very rich clusters is therefore consistent with an intrinsic correlation like that predicted. One would also expect N_g/M to be a decreasing function of radius within individual clusters. While there is no strong evidence for such a trend, the available data certainly do not rule out the possibility (Kent and Gunn's[44] claim to the contrary notwithstanding). Secondly, since galaxies are 'born' in a strongly clustered state, one can observe this, at least in principle, in very deep surveys. Since, at high enough redshift, the clustering pattern is 'frozen' in comoving coordinates, the velocity across a clustering length should actually increase with redshift, in marked contrast with the usual prediction of hierarchical clustering. The clustering pattern should also display a distinctive 3-point correlation function with a term varying as the cube of the 2-point function.[45] As a concrete example of the degree of clustering predicted, consider what is now a rich cluster: Prior to the collapse of this system it would appear as a density contrast of ~ 5 in a region of comoving radius ~ 5 - $10h^{-1}$ Mpc. Finally, in this picture, the great majority of the baryons in the Universe fail to make galaxies. The most convincing test of this idea would be to detect the 'failed galaxies' within the great voids and elsewhere. It may be that some of this gas is in a form like the 'Lyman-alpha clouds' seen in absorption against QSO's. These clouds should be observable at low redshift by the Hubble Space Telescope. Failure to detect such clouds though, would not be surprising, since the bulk of this gas would, at present, be clustered with the dark matter in potential wells of total mass ~ 10^{13} $M_\odot$. This gas would be at the virial temperature T ~ 1 - 3×10^6K and at the virial density contrast of about 200. If the total density of baryons is about one tenth of closure density then these group-mass clouds should generate a significant fraction of the soft X-ray background with anisotropy of a few percent on angular scale of a few arc-minutes.[46]

While the idea that galaxies are 'rare-events' is somewhat uncertain, it is fairly clear that rich clusters of galaxies, which contain only a few percent of the matter in the Universe, are the high-mass tail of the distribution of those fluctuations with sufficient amplitude to have collapsed by the present. If the initial fluctuations were Gaussian, then one can understand why these objects are so strongly clustered.[42] The volume of space which has been surveyed for these conspicuous objects is much larger than that contained in the galaxy surveys. Additionally, since these objects amplify the very clustering signal one would like to observe, they provide the best probe of the very-large-scale structure and therefore of the long-wavelength power spectrum of fluctuations. According to the prediction of the cold-dark-matter picture, the cluster-cluster correlation function ξ_{cc} should be negative for separations $r > 17(\Omega h^2)^{-1}$Mpc. The evidence for or against such a feature is rather weak at present. Bahcall and Soneira[39] claim, on the basis of the angular correlation studies, that the ξ_{cc} remains positive out to at least $100h^{-1}$Mpc. Taken at face value, this observation would severely constrain the parameters of the 'cold-dark-matter' picture, if not exclude the picture entirely. A careful search for this anticorrelation signature, using studies of this type, ideally with redshift information[47], provides the best hope of disproving the otherwise very promising 'cold-dark-matter' scenario.

V. COSMIC STRINGS AND GALAXY FORMATION

There are two 'natural' ways to produce density fluctuations in an inflationary universe. One is to introduce them at the inflationary epoch itself (i.e. at reheating), the other is the effect of cosmic strings. In the case of strings, a number of interesting astrophysical side-effects are produced which have nothing much to do with galaxy formation, but offers promising independent tests of the model. These effects include: (1) discontinuities in the microwave background temperature; (2 a stochastic background of gravitational radiation; (3) distinctive effects in gravitational lensing of quasars; and (4) various phenomena associated with local effects of the population of string loops on matter, such as formation of massive black holes in galactic nuclei and stochastic heating of stellar systems. Here we concentrate on an anecdotal summary of string astrophysics. An excellent guide to recent technical literature on string physics is given by Vilenkin.[48] A more detailed discussion of astrophysical effects can be found in ref. 49.

Strings are topologically stable defects or "knots" which have been frozen into the state of the physical ground-state vacuum. In the case of a very simple abelian Higgs model, strings are exact analogs of Landau-Ginzburg flux tubes. However, many varieties of string are possible, and occur generically in any theory where the set of degenerate vacua in the zero-temperature Universe contains closed loops which cannot be continuously deformed to a point without leaving the set. The simplest and most interesting strings are those which arise from local gauge theories and which are forbidden to to have ends, so that they must either be infinitely long or close into loops. The formation of strings and their relationship to other topological defects, such as monopoles, is beautifully discussed by Kibble.[50]

The original impetus for introducing strings into astrophysics at all came from grand unified theories, in which strings could have a mass per unit length ($\approx \langle\phi\rangle^2$) large enough to produce gravitational potential gradients comparable to those observed to occur in natural astronomical systems i.e., velocities $\approx \sqrt{\epsilon} \approx 300$ km sec^{-1} $(\epsilon/10^{-6})^{1/2}$, where ϵ is the dimensionless measure of the gravitational effects associated with strings:

$$\epsilon \equiv \frac{GM}{R} \simeq \left(\langle\phi\rangle/m_{pl}\right)^2$$

Zeldovich[11] wrote an important (if cryptic) paper suggesting that grand unified strings might be responsible for cosmological fluctuations. He recognized the main character of string evolution—that a cosmological string network evolves self-similarly in time, always maintaining about one open string passing through a horizon volume. This is basically a consequence of the gradient terms $D_\mu \phi$ which try to align ϕ everywhere in space to the same state. The added density due to strings leads to constant amplitude perturbations on each scale as they cross the horizon—the usual scale-invariant spectrum. In this case, the amplitude of the scale invariant potential perturbation is fixed by ε.

The next significant step was Vilenkin's[11] recognition that closed loops form when an open string crosses itself and intercommutes (i.e. changes partners), and that a loop smaller than the horizon is unlikely to intercommute again with the rest of the network. Thus a 'debris' of closed loops is left behind. Some of these loops bifurcate into two or more daughter loops, but a nonnegligible fraction undergo periodic oscillations in nonselfintersecting trajectories.[51] These loops lose mass (i.e., length) primarily by gravitational radiation, with a half-life of order ε^{-1} oscillations.

An oscillating loop looks like a point mass from far away. Upon closer examination, its time-averaged gravitational field is that of the surface swept out by the string, with a local surface density $\propto v^2$ where v is the local transverse velocity.[52] Note that although the 'apparent' length of string may appear to change during oscillations, this is always compensated by the transverse γ-factor—thus the Schwarschild mass does not fluctuate in time. However, there are both quadrupolar shear fluctuations and wake-type shear-free perturbations in the field (as discussed below) which vary on a light-crossing time.

Because of loops, perturbations created by strings differ significantly from 'inflationary' perturbations. Loops separate out and stabilize, eventually attaining very high relative densities (up to ε^{-1} by the time they decay) while the background matter expands away through them. Inflationary fluctuations from noninteracting quantum fields are Gaussian distributed noise, but here we find that very high density fluctuations are rare not by an exponential in $(\delta\rho/\rho)^2$, but only by a power law in $\delta\rho/\rho$. This may have profound effects, say, in galactic nuclei or other usually dense aggregations of matter. The linear power spectrum of perturbations is also modified by loops, because they are not subject or ordinary damping mechanisms processes.[53] As far as observable effects are concerned however, it seems likely that the alteration in the <u>character</u> of the noise is more significant than the alteration in the <u>spectrum</u> (that is, effects on scales comparable to the loop radius are likely to be more pronounced than effects on scales of order the loop mean separation). Some of these effects are discussed below.

Local strings obey an exact linear wave equation derived from an invariant action proportional to the surface area of the swath swept out in space-time.[54] Their inertia and tension are thus always exactly orthogonal, for arbitrary deformations. Thus, if it were not for gravitational radiation and other dissipative gravitational interactions, loop oscillations would always be <u>exactly</u> periodic. In general, strings move across their radius of curvature in a light-crossing time, so they are generally moving transversely close to the speed of light.

An infinite, straight, static string gives rise to a 'conical' spacetime, that is locally flat (no extrinsic curvature) but with an angle $4\pi\varepsilon$ missing. No tidal forces are detectable using particle trajectories which do not enclose the string; however, particles on parallel trajectories which pass on opposite sides of the string are deflected towards each other, each by a angle $4\pi\varepsilon$ independent of impact parameter.[55] Thus, if we consider a string moving through a medium of collisionless particles, and view impacts from the string's next frame, we find that a 'wake' is left behind the string with opening angle $8\pi\varepsilon$ in which two streams of particles are passing each other with velocity $\approx 8\pi\varepsilon c$.[56] This interaction is dissipative; the free energy of the particle motions is obtained from the string motion.

One side-effect of this wake phenomenon is observable in the microwave background radiation.[57] Suppose a string in the plane of the sky is moving transversely between us and the last scattering surface of the microwave background. Behind the string, we and the last scattering surface are moving towards each other with a relative velocity $\approx 8\pi\varepsilon$. Thus there is a sharp temperature discontinuity $(\delta T/T) \simeq 8\pi\varepsilon$ where the string is. For $\varepsilon \sim 10^{-6}$, $\delta T/T \approx 10^{-4.5}$, in a range which is observable using present technology.

The presence of loops has several interesting side-effects. Perhaps the most intriguing is that since loops release all of their mass into gravitational radiation, a substantial background is produced, with energy density per logarithmic frequency interval $\Omega_g \simeq \Omega_{rad}\ \varepsilon^{1/2} \simeq 10^{-7} (\varepsilon/10^{-6})^{1/2}$ at periods less than a few years (here Ω_{rad} is the microwave background energy density).[58] Recently, the discovery of the millisecond pulsar[59] PSR 1937 + 24 has brought detection of gravitational radiation backgrounds of this order of magnitude within the range of plausibility. An extremely clean, distant clock enables one to detect gravitational waves because the arrival times of pulses fluctuate with amplitude $\delta t \approx \Omega_g^{1/2}\ P^2/t_0$, where P is the wave period (here, roughly the period of observation) and t_0 is the age of the Universe. If pulses can be timed to an accuracy of about one microsecond, the string background should be detectable with a decade or so. The theory of this technique is discussed in ref. 60. It is also reasonably plausible that ground-based laser interferometers with 5 km baselines could attain the requisite sensitivity within a decade or so to detect the string background at $\simeq$ 1000 Hz. This is extremely interesting because such high frequencies probe much earlier epochs in the history of the Universe than the pulsar-detecting loops which formed at a temperature of order 10^{10} GeV. Gravitational waves must be taken increasingly seriously as probes of the early Universe now that instruments are capable of detecting $\Omega_g \lesssim \Omega_{rad}$—that is, gravitational-wave backgrounds comparable in flux to the microwave background. A more detailed discussion can be found in ref. 49. While observations of gravitational radiation provides a interesting window on loops at very high redshift, loops at lower redshift would also have observable consequences. Loops of $\simeq 10^{8-10}\ M_\odot$ (which are now extinct) would have provided high-density seed masses which would have led to the formation of massive black holes at recombination; these would have the right properties to power quasars and active galaxies.[61] Loops which are currently decaying, with mass $\sim 10^{12}\ (\varepsilon/10^{-6})^2\ M_\odot$, would act as gravitational lenses in front of distant quasars, and could lead to very distinctive time variability in the brightness and appearance of the lensed images.[62] Larger loops and open strings could also act as lenses, although in this case the most distinctive signature would have to be the existence of "chains" of lensed pairs of galaxies[63] or the coincidence of a microwave 'wake' discontinuity with a pair of images.[64] Currently decaying loops have a mean separation of order 30 $(\varepsilon/10^{-6})^{1/3}$ Mpc, and one might speculate that they are associated with rich clusters of galaxies. If that is the case, there is in principle a dissipative interaction between the oscillations of the loop and the mass in the cluster core, although a quantitative analysis[49] makes it appear unlikely that such effects would actually be observable.

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