Prescription for successful new inflation

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The conditions on the effective (de Sitter space) scalar potential that are required to obtain a successful new-inflationary-Universe scenario are codified into a general prescription. These conditions ensure that sufficient inflation, density fluctuations of an acceptable magnitude, and reheating to a high enough temperature to produce the astrophysically observed baryon asymmetry result. We exemplify our prescription for a quartic potential and show that if the scalar field is a gauge singlet, then it is possible to tune the parameters of the potential to satisfy the conditions we have prescribed.

I. INTRODUCTION

The new-inflationary-Universe scenario[1–3] based on grand unified theories (GUT's) with Coleman-Weinberg[4] (CW)-type spontaneous symmetry breaking (SSB) has proven to be an interesting failure as an attempt to resolve the cosmological puzzles of large-scale homogeneity, isotropy, near-critical expansion rate, small-scale inhomogeneity, the baryon asymmetry, and the glut of topological defects (monopoles, domain walls, etc.) that plague the standard hot big-bang model. In this scenario the Universe in its very early history ($t \simeq 10^{-34}$ sec, $T \simeq 10^{14}$ GeV) undergoes a strongly first-order phase transition in which it supercools in the metastable, symmetric phase ($\phi = 0$ where $\phi =$ the Higgs field responsible for the SSB of the GUT). When the Universe has supercooled to a temperature of the order of the Hawking temperature[5] ($\equiv H/2\pi$, where $H = \dot{R}/R =$ expansion rate), the metastable phase becomes unstable, and ϕ very slowly begins to evolve ("roll") toward the SSB vacuum ($\phi = \sigma$).[6] Owing to the flatness of the CW potential near $\phi = 0$ it takes many Hubble times (H^{-1}) for ϕ to increase significantly from its initial value. During this "slow-rollover" period the energy density of the Universe is dominated by the nearly constant energy density of the Higgs field potential and the cosmic-scale factor R grows exponentially (this is the key element of the inflationary scenario). As ϕ gets further away from the origin, the potential steepens, and the evolution of ϕ quickens. The rapid time variation of ϕ ($\Delta t \ll H^{-1}$) near the SSB minimum leads to the conversion of the coherent Higgs field energy into radiation by quantum particle creation,[7] thereby reheating the Universe.

Although the notion of slow evolution from the symmetric to the SSB phase is an attractive approach for the successful implementation of inflation, several serious problems associated with using CW potentials have been discovered. For the usual range of coupling constants in a CW GUT model, de Sitter-space-produced quantum fluctuations in the scalar field ϕ drive it across the flat region of the potential too quickly, thereby preventing the slow rollover necessary to achieve sufficient inflation.[8] There is also the problem of competing phases. In the CW SU(5) model it is possible that the direction of steepest descent near $\phi = 0$ is not in the direction of the global, SSB minimum [$SU(3) \times SU(2) \times U(1)$], and so the transition may proceed through other metastable phases [e.g., $SU(4) \times U(1)$], resulting in disastrous consequences.[9] Finally, the most severe problem is that the SU(5) CW GUT leads to density perturbations that are far too large ($\delta\rho/\rho \simeq 10$–$100$) to be consistent with the observed isotropy ($\Delta T/T \lesssim 10^{-4}$) of the cosmic microwave background.[10–13]

Other models have been investigated, but thus far they too have had their difficulties. For example, the supersymmetric geometric-hierarchy model of Dimopoulos and Raby[14] which employs the Witten reverse-hierarchy scheme[15] with an O'Raifeartaigh potential[16] avoids the problem of excessively large density perturbations and difficulties associated with de Sitter-space quantum fluctuations, but thus far it appears that these models cannot reheat to a sufficiently high temperature to account for the baryon asymmetry of the Universe.[17]

Because the inflationary scenario is so cosmologically attractive, it is important to search for a field-theoretic model that can successfully implement new inflation. To this end, we have codified in this article a precise set of rules which a completely general (e.g., nonpolynomial) effective potential must satisfy in order to effect an untroubled inflationary-Universe scenario. In the remainder of this section we continue with preliminaries, then in Sec. II we present our prescription. In Sec. III we apply our prescription to a quartic polynomial potential. We finish with some concluding remarks in Sec. IV.

A. Preliminaries

We will assume that the effective potential $V(\phi)$ can be expressed in terms of a single-parameter scalar field ϕ, whose value at the global, SSB minimum[18] is $\phi=\sigma$. [In accord with this assumption, the Higgs field of interest could either be a gauge-singlet field, or could have nontrivial transformation properties under the gauge group and be described by a fixed direction (in group space) and magnitude ϕ.] We also assume that at some point in time ϕ is beyond any barrier which may exist between the metastable and true vacua and that within a region whose physical size is of the order of the Hubble radius ($\simeq H^{-1}$), the average value of ϕ is ϕ_0 (see Fig. 1). (Without loss of generality we have taken $0\leq\phi_0\leq\sigma$.) We will not be concerned with the dynamics of how the scalar field made its way to $\phi=\phi_0$ (e.g., by loss of metastability of the false vacuum, or by bubble nucleation), or whether the region we are focusing on is the interior of a bubble or is a "fluctuation region" produced in the course of spinodal decomposition.[3,6]

The key equation for describing the semiclassical evolution of the scalar field from $\phi=\phi_0$ to $\phi=\sigma$ is[7]

$$\ddot{\phi}+3H\dot{\phi}+\Gamma\dot{\phi}+V'(\phi)=0\ , \tag{1}$$

where the field ϕ is normalized so that its kinetic term in the Lagrangian is $\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi)$, and the expansion rate (Hubble parameter) $H\equiv\dot{R}/R$ is determined by the usual Friedmann equation

$$H^2=(8\pi/3m_{\rm Pl}{}^2)[V(\phi)+\tfrac{1}{2}\dot{\phi}^2+\rho_r]\ . \tag{2}$$

Here

$$m_{\rm Pl}=G^{-1/2}=1.22\times10^{19}\ \text{GeV}$$

is the Planck mass, the overdot denotes d/dt, the prime denotes $\partial/\partial\phi$, and ρ_r is the energy density in radiation. The $\Gamma\dot{\phi}$ term accounts for particle creation[7] due to the time variation of ϕ and is only important when the time variation of ϕ is rapid compared to the expansion rate.[19] It is, of course, this particle creation which is responsible for converting the coherent Higgs field energy $(\frac{1}{2}\dot{\phi}^2+V)$ into radiation, thereby reheating the Universe. The quantity $\Gamma\simeq$ (lifetime of the scalar Higgs particle)$^{-1}$ is determined by the fields which couple to ϕ and the strength with which they couple.[20] The evolution of ρ_r is given by[7,21]

$$\dot{\rho}_r+H\rho_r-\Gamma\dot{\phi}^2=0\ . \tag{3}$$

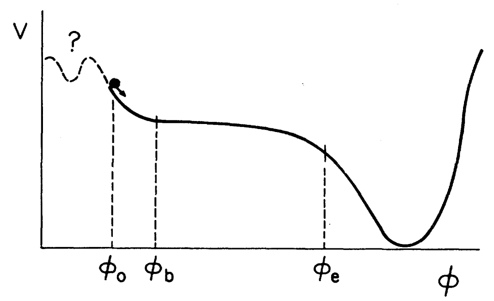

FIG. 1. The de Sitter-space effective scalar potential discussed throughout this article. We are interested in the evolution of ϕ from $\phi=\phi_0$ to $\phi=\sigma$, the interval during which its evolution is described by a semiclassical equation of motion, Eq. (1). The details of how ϕ evolved from $\phi=0$ to $\phi=\phi_0$ (e.g., by quantum tunneling, thermal fluctuations, etc.) are not considered here. In the interval $[\phi_b,\phi_e]$ the motion of ϕ is friction dominated [i.e., the $\ddot{\phi}$ term in Eq. (1) can be neglected], and ϕ evolves very slowly (time scale $\gtrsim H^{-1}$); see Sec. II for the precise definitions of ϕ_b and ϕ_e.

II. THE PRESCRIPTION

During the slow-rollover phase the $\ddot{\phi}$ term is negligible compared to the friction term in Eq. (1), and the motion of ϕ is "friction dominated." When the $\ddot{\phi}$ term is neglected Eq. (1) becomes

$$\dot{\phi}=-V'(\phi)/3H \tag{4}$$

(recall the $\Gamma\dot{\phi}$ term is not important during the slow-rollover phase). Using expression (4) for $\dot{\phi}$ to compute $\ddot{\phi}$ it follows that

$$\ddot{\phi}/3H\dot{\phi}=V''/9H^2+(V'm_{\rm Pl}/V)^2/48\pi\ . \tag{5a}$$

Therefore, neglecting the $\ddot{\phi}$ term is consistent when

$$|V''(\phi)|\lesssim9H^2\ , \tag{5b}$$

$$|V'm_{\rm Pl}/V|\lesssim(48\pi)^{1/2}\ . \tag{5c}$$

For most potentials $V''\simeq V'/\phi$; in this case inequality (5c) follows directly from inequality (5b). Thus in most instances, condition (5c) is redundant and condition (5b) alone suffices. Conditions (5b) and (5c) also guarantee that $\dot{\phi}^2/2<V(\phi)$, so that during the slow-rollover phase $H^2\simeq8\pi V(\phi)/3m_{\rm Pl}{}^2$. During this phase the equation of motion for ϕ can be written as

$$\frac{d}{dt}(\tfrac{1}{2}\dot{\phi}^2+V)=-3H\dot{\phi}^2$$

which implies that $H^2[\simeq8\pi(\frac{1}{2}\dot{\phi}^2+V)/3m_{\rm Pl}{}^2]$ must be strictly decreasing. If conditions (5b) and (5c) were satisfied all the way to $\phi=\sigma$, then both $\dot{\phi}^2/2$ $(<V)$ and $V(\phi)$ would vanish at $\phi=\sigma$ [the vanishing of the cosmological constant requires $V(\sigma)=0$], and so $H(\sigma)$ would also vanish. On the other hand, $V''(\sigma)$ must be >0, so at some value of $\phi<\sigma$, say, $\phi=\phi_e$, inequalities (5b) and (5c) must cease to be valid.

Denote the "flat interval of $V(\phi)$" in which constraint (5) is satisfied by $\phi_b<\phi<\phi_e$ (if there is more than one such interval each must be analyzed in turn). Of course, we cannot consider $\phi<\phi_0$; so, if ϕ_b is $<\phi_0$, we must take the interval to begin at $\phi=\phi_0$. Neglecting the possibility of more than one such interval, we can precisely define ϕ_b and ϕ_e by

$$\phi_b=\max\,[\phi_0,\ \text{minimum value of } \phi \text{ for which } |V''(\phi)|\lesssim9H^2 \text{ and } |V'm_{\rm Pl}/V|\lesssim(48\pi)^{1/2}]\ ,$$

$$\phi_e=[\ \text{maximum value of } \phi(<\sigma) \text{ for which } |V''(\phi)|\lesssim9H^2 \text{ and } |V'm_{\rm Pl}/V|\lesssim(48\pi)^{1/2}]$$

(see Fig. 1). Slow rollover, then, occurs when $\phi\in[\phi_b,\phi_e]$ and its motion is adequately described by Eq. (4). With the preliminaries and definitions taken care of, we will proceed to present our prescription for successful new inflation.

(a) $\phi_e \lesssim \sigma \lesssim f m_{Pl}$, $f \simeq 1$.

The inflationary-Universe scenario does not necessarily require an understanding of quantum gravity, and by demanding that the minimum of the potential occur at a value less than the order of the Planck scale, we hope to avoid having to consider its effects. The "fudge factor" f has been included because even if $\sigma > m_{\rm Pl}$, masses and vacuum energies need not necessarily exceed the Planck scale if ϕ is only weakly coupled to itself and other fields. We expect for reasonable models that f will be at most of order unity. The condition that $\sigma > \phi_e$ follows from our earlier discussion and the definition of ϕ_e.

(b) $\Delta\phi \equiv \phi_e - \phi_b \gtrsim$ *many* H.

During the de Sitter phase (which the Universe enters after many e-folds of supercooling) the Hawking temperature ($\equiv H/2\pi$) sets the scale for quantum fluctuations in the scalar field ϕ. If the interval over which the potential is flat ($\Delta\phi$) is not large compared to H, then quantum fluctuations will drive ϕ quickly across the flat part of the potential, rendering the semiclassical description of $\phi(t)$ [cf. Eqs. (1) and (4)] invalid.[8] Roughly speaking, since H sets the scale of "quantum fuzziness" in ϕ, we do not want any of the details of our inflationary scenario (which is semiclassical in nature) to involve the behavior of ϕ over a range of values as small as H.

More precisely, it has been shown that quantum zero-point fluctuations in ϕ grow as[8] $\langle \phi^2 \rangle = H^3 t/4\pi^2$, where t is the time elapsed since the beginning of the de Sitter phase. Consistency of our semiclassical treatment demands that these quantum fluctuations not drive ϕ across the interval $[\phi_b, \phi_e]$ faster than ϕ would have rolled [according to the semiclassical equation of motion, Eq. (1)]. To ensure that quantum fluctuations do not quickly drive ϕ across the interval $[\phi_b, \phi_e]$ it suffices that $\Delta\phi$ must be $\gtrsim N^{1/2}(H/2\pi)$, where NH^{-1} is the time required for ϕ to roll from ϕ_b to ϕ_e. In practice this constraint is usually satisfied whenever the density perturbation constraint is, and so for convenience we shall take "many H" to be $O(10)\,H$ rather than $(N^{1/2}/2\pi)H$.

(c) $\int_{\phi_b}^{\phi_e} H\,dt \simeq -\int_{\phi_b}^{\phi_e} 3H^2 d\phi / V'(\phi) \equiv N \gtrsim 60$.

The growth of the cosmic scale factor R during inflation is determined by

$$\exp\left[\int H\,dt\right],$$

and most of the time required for ϕ to go from ϕ_0 to σ elapses during the friction-dominated epoch. For a GUT phase transition, Guth[1] argued that about 60 e-folds of expansion were required to resolve the large-scale homogeneity and flatness puzzles. Constraint (c) is just that requirement; however, let us examine the requirement in a little more generality.

The basic idea is that a small, smooth patch is inflated to encompass what will become the present observable Universe (size $\simeq 10^{28}$ cm). The original physical size of the smooth patch must be $\lesssim O(H^{-1})$—the distance over which a light signal can travel in an expansion time. During inflation it grows in size by a factor of

$$\exp\left[\int H\,dt\right] \simeq e^N .$$

From the end of inflation (i.e., when $\phi \simeq \sigma$) until reheating, it may undergo additional growth (if reheating does not occur rapidly). Assume that the energy density of the coherent Higgs field $[=\frac{1}{2}\dot{\phi}^2 + V(\phi)]$ is $O(\mu^4)$ when $\phi \simeq \sigma$. As ϕ oscillates about the SSB minimum, the Higgs field stress energy behaves like nonrelativistic matter ($p=0$).[21] Until reheating ρ_ϕ red-shifts $\propto R^{-3}$; therefore, from the end of inflation to reheating, R grows by a factor of $\simeq (\mu/T_{\rm RH})^{4/3}$ ($T_{\rm RH}$ = temperature to which the Universe is eventually reheated). Thus at the epoch of reheating, the smooth patch had a physical radius $\lesssim H^{-1} e^N (\mu/T_{\rm RH})^{4/3}$. If the evolution of the Universe was approximately adiabatic from this epoch on, then the size of our present observable Universe when $T \simeq T_{\rm RH}$ was just $\simeq 10^{28}$ cm $(3K/T_{\rm RH})$ (or smaller by a factor of $\gamma^{1/3}$ if between then and now the entropy increased by a factor γ). Taking $H^{-1} \simeq m_{\rm Pl}/\mu^2$, the requirement that the smooth patch encompass what will become our observable Universe is

$$N \gtrsim 55 + \ln[(\mu/10^{14}\ {\rm GeV})^{2/3} (T_{\rm RH}/10^{14}\ {\rm GeV})^{1/3} \gamma^{-1/3}] , \tag{6}$$

which reduces to $N \simeq 60$ for $\mu \simeq T_{\rm RH} \simeq 10^{15}$ GeV (a GUT phase transition which reheats with 100% efficiency). For 10^{19} GeV $\gtrsim \mu \gtrsim 10^2$ GeV and 10^{19} GeV $\gtrsim T_{\rm RH} \gtrsim 1$ MeV, N only varies between 24 and 67. That is, the amount of inflation necessary is very insensitive to the details ($\mu, T_{\rm RH}$, etc.) of the inflationary model.

Returning now to our constraint (c), for most potentials this expression can be simplified considerably. If ϕ_m is the value of ϕ in the interval $[\phi_b, \phi_e]$ at which V' attains its minimum, then by expanding V' around ϕ_m, $V' = V'(\phi_m) + V''(\phi_m)(\phi - \phi_m)$, constraint (c) becomes

$$-\int_{\phi_b}^{\phi_e} 3H^2 d\phi/V' \gtrsim 3H_e^2 \ln[1 + \Delta\phi\, |V''(\phi_m)/V'(\phi_m)|\,] / |V''(\phi_m)| \gtrsim 60 , \tag{7}$$

where $H_e = H(\phi_e)$. If the potential is a polynomial with no linear term, then the log factor is ~ 1, and the constraint is further simplified:

$$N \gtrsim 3H_e^2/|V''(\phi_m)| \gtrsim 60 . \tag{8}$$

The more complicated expression, Eq. (7), contains some fail-safe protections that eliminate more general potentials which might satisfy Eq. (8), but which on closer inspection do not actually inflate enough. For example, criterion (8) only involves V'' and is insensitive to the presence of a large linear term in $V(\phi)$. Such a term would, of course, speed up the evolution of ϕ, and could destroy the slow rollover. The more complicated version of constraint (c), Eq. (7), is sensitive to such a term since a large value of V' would cause the argument of the logarithm to approach unity [in which case Eq. (7) would be difficult to satisfy]. Finally, we mention one possible difficulty with constraint (7)—the vanishing of $V'(\phi_m)$ and/or $V''(\phi_m)$. If this occurs one can return to the integral form of the constraint, or expand V' around a nearby value of the field, say, $\phi'=\phi_m+H$, so that the expression in Eq. (7) is well defined. Since $\Delta\phi >$ many H, the shift from expanding around ϕ_m to expanding around ϕ' should be insignificant.

(d) $\dot{\phi}_{60}=\delta^{-1}10^4H^2$.

This constraint is imposed to ensure that the density fluctuations that result from the de Sitter-space quantum fluctuations in ϕ have an amplitude $(\delta\rho/\rho)_H\simeq\delta\times10^{-4}$ when the mass scales relevant for galaxy formation, say, $10^{12}M_\odot$ (galaxy) to $10^{15}M_\odot$ (supercluster), reenter the horizon. A scale-invariant spectrum of density perturbations (the so-called Zel'dovich spectrum)[22] is predicted in the new-inflationary-Universe scenario[10–13] and is consistent with present models of galaxy formation and the measured fluctuations in the microwave background on angular scales $\gg 1^0$ if $\delta\simeq1$.[23] Constraint (d) follows from the fact that the density fluctuation on a comoving scale l which results from the quantum fluctuations in ϕ has an amplitude

$$(\delta\rho/\rho)_H\simeq(H^2/\dot{\phi})\,|_{N_l} \tag{9}$$

when the scale reenters the horizon during the post-inflationary Friedmann-Robertson-Walker (FRW) phase.[10–13] Here $\dot{\phi}$ and H are to be evaluated when the scale in question, here specified by N_l, crossed outside the horizon [i.e., $\lambda_{\text{phys}}=R(t)\times l=$physical size of the perturbation $\simeq H^{-1}$] during the inflationary epoch. The quantity N_l is the number of e-folds in $R(t)$ between horizon crossing and the end of inflation ($\phi=\sigma$) for the comoving scale l. In Appendix A we show that

$$N_l\simeq\ln[\mu^2T_{\text{RH}}M/(\text{MeV}^3-M_\odot)]/3\ ,$$

where T_{RH} is the temperature to which the Universe is reheated, and M is the mass within the comoving scale l. The subscript 60 which appears in the expression for constraint (d) refers to the fact that $\dot{\phi}$ and H are to be evaluated when the mass scales of interest today ($\simeq10^{12}-10^{22}\ M_\odot$) crossed the Hubble radius during inflation. For these scales $N_l\simeq60$. Roughly speaking, during inflation $\dot{\phi}\simeq$constant, so that we expect $\dot{\phi}_{60}\simeq\dot{\phi}_e$; however, we have not found this to be a very accurate approximation in practice. As ϕ increases, the potential steepens, and so $\dot{\phi}$ also increases. Thus we expect $\dot{\phi}_{60}$ to be less than $\dot{\phi}_e$. To investigate this quantitatively, consider the case of a potential $V(\phi)$ where for the friction-dominated, slow-rollover phase $V(\phi)\simeq V_0-a\phi^n$ $(n>2)$. Integrating Eq. (4), one finds that

$$\begin{aligned}\phi_{N_l}&\equiv\phi(t_l)\\&\simeq[3(n-2)N_l/(n-1)]^{-1/(n-2)}\phi_e\ ,\end{aligned} \tag{10a}$$

$$\begin{aligned}\dot{\phi}_{N_l}&\equiv\dot{\phi}(t_l)\\&\simeq[3(n-2)N_l/(n-1)]^{-(n-1)/(n-2)}\dot{\phi}_e\ ,\end{aligned} \tag{10b}$$

where t_l corresponds to the time when the scale l crosses the horizon during inflation, t_e corresponds to the end of the slow-rollover phase ($\phi\simeq\phi_e$), and $N_l\equiv H(t_e-t_l)$. [Equation (10) is only valid for $N_l\gg1$, i.e., $\phi\ll\phi_e$.] For the scales of interest ($10^{12}-10^{22}\ M_\odot$) $N_l\simeq60$, and so

$$\dot{\phi}_{60}\simeq[180(n-2)/(n-1)]^{-(n-1)/(n-2)}\dot{\phi}_e\ ; \tag{11}$$

for $n=3$, 4, 5, and 6, $\dot{\phi}_e$ is a factor of 8×10^3, 10^3, 700, and 500 larger than $\dot{\phi}_{60}$. That is, $\dot{\phi}_{60}\simeq\dot{\phi}_e$ is not a very good approximation. For a potential of this form ($V\simeq V_0-a\phi^n$), H does not vary significantly during the slow-rollover phase, and so $(\delta\rho/\rho)_H\propto\dot{\phi}$. Since $\dot{\phi}$ varies with N_l the spectrum of perturbations is *not* exactly scale invariant—see Appendix A for further discussion of this point. In terms of $\dot{\phi}_e$, the density-perturbation constraint (d) is

$$\dot{\phi}_e\simeq[180(n-2)/(n-1)]^{(n-1)/(n-2)}\delta^{-1}10^4H^2\ . \tag{12a}$$

Or equivalently, this can be expressed as a constraint on $V'(\phi_e)$:

$$\begin{aligned}V'(\phi_e)\ &\simeq-3\times10^4[180(n-2)/(n-1)]^{(n-1)/(n-2)}\\&\quad\times\delta^{-1}H^3\ .\end{aligned} \tag{12b}$$

Finally, let us finish our discussion of constraint (d) by remarking that the choice of δ can be rather important. As we will see in our "worked" example in Sec. III, the coefficient of the quartic term $\sim\delta^2$ (this seems to be a rather general result; also, see Ref. 26), and so the prescribed potential depends sensitively on the choice of δ. As we have discussed in Ref. 23, $\delta\simeq1$ seems to be what is required for galaxy formation.

(e) Reheating. $T_{RH}\simeq min\{(m_{Pl}H_e)^{1/2},\ (\Gamma m_{Pl})^{1/2}\}$.

When the scalar field ϕ reaches $\phi\simeq\sigma$ it begins to oscillate about the SSB minimum. Due to quantum-particle creation [accounted for in Eq. (1) by the $\Gamma\dot{\phi}$ term], the coherent Higgs-field energy $(=\frac{1}{2}\dot{\phi}^2+V)$ is eventually converted into radiation.[7,19–21] It is straightforward to show that this occurs in a time $\simeq\Gamma^{-1}$ (=lifetime of the Higgs boson associated with ϕ).[19–21] If $\Gamma>H_e$, then the reheating process takes less than an expansion time ($\simeq H_e{}^{-1}$), and the coherent field energy is efficiently converted to radiation, reheating the Universe to a temperature

$$\begin{aligned}T_{\text{RH}}&\simeq(30\rho_\phi/\pi^2g_*)^{1/4}\\&\simeq(45/4\pi^3g_*)^{1/4}(H_em_{\text{Pl}})^{1/2}\ ,\end{aligned} \tag{13}$$

where, as usual, g_* counts the effective number of relativistic degrees of freedom (1 for each bosonic degree of freedom, $\frac{7}{8}$ for each fermionic degree of freedom). For $T_{\text{RH}}\gg10^3$ GeV, one expects $g_*(T_{\text{RH}})\gtrsim100$. The Higgs field energy density at reheating $(=\rho_\phi)$ is given by its value at $\phi=\phi_e$ $(\rho_\phi\simeq3m_{\text{Pl}}{}^2H_e{}^2/8\pi)$, since as ϕ evolves from $\phi=\phi_e$ to $\phi=\sigma$ the change in ρ_ϕ, $-d\rho_\phi=3H\dot{\phi}^2dt$, is $\ll\rho_\phi$, as dt is $\ll H^{-1}$. Taking $V(\phi_e)\simeq0(\mu^4)$, it follows that $T_{\text{RH}}\simeq(30/\pi^2g_*)^{1/4}\mu$.

On the other hand, if $\Gamma\lesssim H_e$, then the reheating process takes longer than an expansion time. Until $t\simeq\Gamma^{-1}$, the field ϕ oscillates about $\phi=\sigma$. The energy density associated with these coherent field oscillations $(=\frac{1}{2}\dot{\phi}^2+V)$ dominates the energy density of the Universe and behaves like nonrelativistic matter,[21] with $\rho_\phi\propto R^{-3}$. [The coherent field oscillations of ϕ are essentially equivalent to a very cold ($T\ll m_\phi$) gas of Higgs bosons—a point to which we shall return.] During this epoch the scale factor

$R(t)$ grows $\propto t^{2/3}$. When $t \simeq \Gamma^{-1}$ these oscillations are damped (in a few expansion times), reheating the Universe to a temperature[21]

$$T_{\rm RH} \simeq (45/4\pi^3 g_*)^{1/4} (\Gamma m_{\rm Pl})^{1/2}$$
$$\simeq (30/\pi^2 g_*)^{1/4} (\Gamma/H_e)^{1/2} \mu \ , \tag{14}$$

which is a factor of $(\Gamma/H_e)^{1/2}$ smaller than in the rapid (or "good") reheating case ($\Gamma > H_e$). If $\Gamma \ll H_e$, $T_{\rm RH}$ can be very low; for the Dimopoulos-Raby geometric-hierarchy model: $\mu \simeq 10^{12}$ GeV and $\Gamma/H_e \simeq 10^{-28}$, resulting in $T_{\rm RH} \simeq 10$ MeV (see Ref. 17).

The crucial constraints on the reheating temperature are (1) big-bang nucleosynthesis—to achieve the usual concordance of the predictions of primordial nucleosynthesis with the observed abundances of the light elements[24] (D, ^{3}He, ^{4}He, and ^{7}Li) $T_{\rm RH}$ must be $\gtrsim$ few MeV and (2) baryogenesis—since any preinflationary baryon asymmetry has been exponentially diluted, a new baryon asymmetry, of magnitude $n_B/s \simeq 10^{-10}$ (see Ref. 24), must be generated. The first consideration ($T_{\rm RH} \gtrsim$ few MeV) implies that Γ must be $\gtrsim 10^{-23}$ GeV. The second consideration results in a potentially much more stringent constraint which we shall now discuss in detail. We remind the reader that solving the isotropy and homogeneity problems does not stringently constrain $T_{\rm RH}$—in fact, fewer e-folds of inflation are required when $T_{\rm RH}$ is smaller, cf. Eq. (6).

(f) Baryogenesis ($n_B/s \simeq 10^{-10}$).

If the reheat temperature is sufficiently high, then baryogenesis can proceed as it does in the standard cosmology, through the out-of-equilibrium decays of superheavy bosons whose interactions violate B, C, and CP conservation. Of the superheavy bosons whose decays can lead to baryogenesis, the color triplet, isosinglet Higgs boson [triplet component of the $\underset{\sim}{5}_H$ in SU(5)] seems to be the candidate superheavy boson which can be the lightest—possibly as light as $M \simeq 10^{10}$ GeV. If baryogenesis is to proceed in the standard manner, then $T_{\rm RH}$ must be $\gtrsim M/10 \gtrsim 10^9$ GeV (for further discussion and a recent review of baryogenesis, see Ref. 25).

It has also been suggested that the baryon asymmetry could be produced directly by the decay of the coherent Higgs-field oscillations.[7,19,20] If collective effects are not important, then the Higgs-field oscillations can be treated as a very cold (i.e., nonrelativistic) gas of Higgs particles of mass $m_\phi{}^2 = V''(\sigma)$, with a particle-number density of $n_\phi \simeq \rho_\phi / m_\phi$. Viewed this way, the reheating process is then just the decay of these Higgs particles at $t \simeq \Gamma^{-1} \simeq$ lifetime of the Higgs boson. Consider the baryon asymmetry produced directly in this process. If the decay of each Higgs boson produces on average a net baryon number ϵ, then the net baryon number density produced by Higgs decays is $n_B \simeq \epsilon n_\phi \simeq \epsilon \rho_\phi / m_\phi$. (So long as $T_{\rm RH}$ is $\ll m_\phi$, this process is automatically out of equilibrium, and reverse reactions can be ignored.) As usual, ϵ is related to the branching ratio of ϕ into channels which have net baryon number, and the C,CP violation in the B-nonconserving decay modes. For simplicity suppose that only two decay channels have net baryon number, say, equal to B_1 and B_2, then $\epsilon \simeq (B_1 - B_2)(r_1 - \bar{r}_1)(r_1 + r_2)$, where r_i ($\bar{r}_i$) is the branching ratio into channel i ($\bar{i}$). Since the C-, CP-violating effects involve higher-order loop corrections $(r_1 - \bar{r}_1) \lesssim O(\alpha) \lesssim 10^{-2}$; α = coupling strength of the particle exchanged in the loop. Note that if the decay of ϕ proceeds primarily through channels which do not carry net baryon number, then $(r_1 + r_2) \ll 1$ and so $\epsilon \propto (r_1 + r_2) \ll 1$.

After reheating the entropy density $s \simeq 2\pi^2 g_* T_{\rm RH}{}^3/45$. Assuming that the ϕ decays occur in a time $\lesssim$ few expansion times, the Higgs-field energy density $\rho_\phi \simeq n_\phi m_\phi$ and the energy density in radiation after reheating $\rho_r \simeq \pi^2 g_* T_{\rm RH}{}^4/30$ are approximately equal. Using this fact it follows that the baryon asymmetry generated in this way is

$$n_B/s \simeq 0.75 \epsilon (T_{\rm RH}/m_\phi) \ . \tag{15}$$

If $T_{\rm RH}$ is sufficiently low ($\ll 10^{10}$ GeV), B-nonconserving processes will be ineffective (rates $\ll H$), and this asymmetry will not be "washed-out ," or even significantly diluted by subsequent scattering processes.

The crucial point regarding this scenario is that the baryon asymmetry which can be produced depends upon the *ratio* $T_{\rm RH}/m_\phi$ and not $T_{\rm RH}$ *itself.* Given ϵ, the reheat temperature required for baryogenesis ($n_B/s \simeq 10^{-10}$) is *only*

$$T_{\rm RH} \simeq 10^{-10} \epsilon^{-1} V''(\sigma)^{1/2} \ , \tag{16a}$$

or, equivalently,

$$\Gamma m_{\rm Pl} / V''(\sigma) \simeq 10^{-20} \epsilon^{-2} \ , \tag{16b}$$

where $m_\phi{}^2 = V''(\sigma)$. For $\epsilon \simeq 10^{-3}$ and $m_\phi \simeq 10^5$ GeV, a reheating temperature of only 10 MeV is acceptable. For the Dimopoulos-Raby model[14,17] $m_\phi \simeq 10^5$ GeV and $T_{\rm RH} \simeq 10$ MeV; however in the present version of the model, $\epsilon \ll 10^{-3}$.

To summarize, baryogenesis is potentially the most restrictive constraint on reheating. If the baryon asymmetry is to be produced in the "standard way" then $T_{\rm RH}$ must be $\gtrsim 10^9$ GeV, implying that $\mu \gtrsim 10^9$ GeV and $\Gamma \gtrsim 1$ GeV. If it can be produced directly in Higgs decays, then the constraint depends only upon $T_{\rm RH}/m_\phi$, cf. Eq. (15), and $T_{\rm RH}$ as low as a few MeV can be tolerated.

(g) Sensible Particle Physics.

Lest we get carried away with cosmological considerations, we should insist that the potential $V(\phi)$ be part of a model which leads to sensible particle physics phenomenology.

III. A "WORKED" EXAMPLE

To illustrate the use of "our prescription" we will now apply constraints (a)—(f) to an effective potential of the form[26]

$$V(\phi) = V_0 - \alpha\phi^2 - \beta\phi^3 + \lambda\phi^4 \ , \tag{17}$$

where α, β, λ, are assumed to be approximately constant on the interval $[\phi_0, \sigma]$. Without loss of generality β can be taken to be > 0, and $\lambda > 0$ is required so that the potential is bounded. We will also assume that $\phi_0 = 0$, e.g., due to high-temperature symmetry restoration, followed by supercooling in a metastable symmetric phase. The SSB minimum of the potential is determined by $V'(\sigma)=0$ and $V''(\sigma)>0$, and V_0 is determined by the vanishing of the cosmological constant of the true vacuum, i.e., $V(\sigma)=0$. Rather than considering this quartic potential in its most generic form, we will consider two limiting cases: (i) the case where the quadratic term is not important—the SSB minimum is determined by β and λ, and ϕ "rolls off" the cubic term; (ii) the case where the cubic term is not important—the SSB minimum is determined by α and λ, and ϕ rolls off the quadratic term. The intermediate case, where ϕ rolls off a combination of the quadratic and cubic terms should not be qualitatively different and only occupies a tiny fraction of the "α-β phase space."

Case (i): $V\simeq V_0-\beta\phi^3+\lambda\phi^4$.

In this limit the SSB minimum and V_0 are given by

$$\sigma\simeq 3\beta/4\lambda \ , \tag{18}$$

$$V_0=27\beta^4/256\lambda^3 \ , \tag{19}$$

respectively. It then follows that the Hubble parameter for $\phi=\phi_0$ ($\equiv H_0$) is just

$$H_0{}^2=8\pi V_0/3m_{\rm Pl}{}^2\simeq\beta^4/\lambda^3 m_{\rm Pl}{}^2 \ . \tag{20}$$

For $\phi \ll \sigma$ the $\lambda\phi^4$ term can be neglected and $V''\simeq -6\beta\phi$. During the slow-rolling period when the motion of ϕ is friction dominated $|V''|\lesssim 9H^2$; this interval is evidently $\phi\in[0,\phi_e]$, where $V''(\phi_e)\simeq 9H_e{}^2$, so that

$$\phi_e\simeq 4\beta^3/3\lambda^3 m_{\rm Pl}{}^2 \ . \tag{21}$$

Our constraints (a)—(f) then imply

(a) $\phi_e<\sigma<fm_{\rm Pl} \Longrightarrow \beta\lesssim 3\lambda m_{\rm Pl}/4$.

(b) $\Delta\phi\simeq\phi_e\gtrsim$ many, say, $10H_e \Longrightarrow \beta\gtrsim 7\lambda^{3/2}m_{\rm Pl}$.

(c) For this potential $\phi_b=0$; due to quantum fuzziness, however, it does not make any sense to consider $\phi\lesssim O(H)$. The condition for sufficient inflation ($\int H\,dt\gtrsim 60$) implies

$$\beta\gtrsim 130\lambda^{3/2}m_{\rm Pl} \ .$$

Note that this constraint can be obtained either by actually integrating the equation of motion for ϕ, or by the approximation

$$\int H\,dt\simeq 3H^2/|V''(\phi_b\simeq H)|$$

cf. Eq. (8).

(d) $\dot\phi_{60}\simeq\delta^{-1}10^4H^2$, or equivalently,

$$\dot\phi_e\simeq 8\delta^{-1}10^7H_e{}^2 \Longrightarrow$$

$$\beta\simeq 4\delta^{-1}10^7\lambda^{3/2}m_{\rm Pl} \ .$$

By combining (a) and (d) it follows that $\lambda\lesssim 4\times 10^{-16}\delta^2$. For $\delta<10^6$ (i.e., $\delta\rho/\rho\lesssim 100$—definitely our range of interest) condition (d) guarantees that condition (b) is automatically satisfied. Bringing everything together we find that conditions (a)—(d) are satisfied for the following range of parameters:

$$\lambda\lesssim 4\times 10^{-16}\delta^2 \ , \tag{22a}$$

$$\begin{aligned}\beta&\simeq 4\delta^{-1}\times 10^7\lambda^{3/2}m_{\rm Pl}\\ &\simeq 3\delta\times 10^{-8}H_0 \ ,\end{aligned} \tag{22b}$$

$$\sigma\simeq 3\times 10^7\delta^{-1}\lambda^{1/2}m_{\rm Pl} \ , \tag{22c}$$

where $H_0\simeq\delta^{-2}\times 10^{15}\lambda^{3/2}m_{\rm Pl}$. [Or equivalently: $\lambda\simeq(\sigma/m_{\rm Pl})^2\delta^2 10^{-15}$, $\beta\simeq\lambda\sigma$, and $\sigma\lesssim m_{\rm Pl}$.] For $\delta\simeq 1$ and $\sigma\simeq m_{\rm Pl}\simeq 10^{19}$ GeV we have $\lambda\simeq 4\times 10^{-16}$, $\beta\simeq 3\times 10^3$ GeV, and $H_0\simeq 10^{11}$ GeV. Although we have neglected the quadratic term in this case, it is straightforward to show that it will not upset the successful implementation of new inflation obtained for these parameters so long as

$$\alpha\lesssim H_0{}^2/40 \ . \tag{22d}$$

We have verified these analytic results by numerically integrating the equations of motion [Eqs. (1) and (2)] and find good agreement.

(e), (f) The condition for good reheating ($\Gamma\gtrsim H_e$) implies

$$\Gamma\gtrsim 2\delta^{-2}\times 10^{15}\lambda^{3/2}m_{\rm Pl} \ ,$$

with a resulting reheat temperature of

$$\begin{aligned}T_{\rm RH}&\simeq 4\delta^{-1}\times 10^7\lambda^{3/4}m_{\rm Pl}\\ &\simeq\delta^{-1}\times 10^{15}(\lambda/4\times 10^{-16})^{3/4}\ {\rm GeV} \ .\end{aligned}$$

For the case of good reheating $T_{\rm RH}$ can easily be high enough for primordial nucleosynthesis to proceed as usual (λ need only be $\gtrsim 10^{-38}$), and possibly high enough for baryogenesis to proceed in the usual way ($T_{\rm RH}\gtrsim 10^{10}$ GeV for $\lambda\gtrsim\delta^{4/3}\times 10^{-22}$).

For the case of "poor reheating" ($\Gamma\lesssim H_e$) the reheat temperature is $T_{\rm RH}\simeq(\Gamma m_{\rm Pl})^{1/2}$ [cf. Eq. (14)]. As before, primordial nucleosynthesis requires that $\Gamma\gtrsim 10^{-23}$ GeV. If the baryon asymmetry can be produced directly in the decay of the coherent field oscillations, then

$$\begin{aligned}\Gamma&\simeq 10^{-20}\epsilon^{-2}[V''(\sigma)/m_{\rm Pl}]\\ &\simeq 4\times 10^{-5}\epsilon^{-2}\delta^{-2}\lambda^2 m_{\rm Pl}\\ &\simeq\epsilon^{-2}\delta^{-2}(\lambda/4\times 10^{-16})^2 8\times 10^{-17}\ {\rm GeV}\end{aligned}$$

is required to produce the observed baryon asymmetry ($n_B/s\simeq 10^{-10}$) [cf. Eq. (16b)]. Note that $V''(\sigma)^{1/2}\simeq 6\delta^{-1}\times 10^7\lambda m_{\rm Pl}$ is the mass of the "rolling Higgs field."

Case (ii): $V\simeq V_0-\alpha\phi^2+\lambda\phi^4$.

In this limit the SSB minimum and V_0 are given by

$$\sigma^2=\alpha/2\lambda \ , \tag{23}$$

$$V_0=\alpha^2/4\lambda \ , \tag{24}$$

respectively. For the following analysis we will use σ and λ in favor of α and λ; note $V=\lambda(\sigma^2-\phi^2)^2$. The Hubble parameter is then given by

$$\begin{aligned}H^2(\phi)&=8\pi V(\phi)/3m_{\rm Pl}{}^2\\ &\simeq 8\lambda(\phi^2-\sigma^2)^2/m_{\rm Pl}{}^2\\ &\simeq 8\lambda\sigma^4/m_{\rm Pl}{}^2 \quad (\phi\ll\sigma) \ .\end{aligned} \tag{25}$$

We have taken into account the dependence of H upon ϕ because for this potential the variation of H with ϕ is important.

(a), (b) The condition for a "slow-roll" is $|V''|\lesssim 9H^2$, which implies

$$\sigma^2 \gtrsim m_{\rm Pl}^2/18 \; . \tag{26}$$

The end of the slow-rolling interval is determined by $|V''(\phi_e)| \simeq 9H_e^2$, which implies that

$$\phi_e^2 = \sigma^2 + m_{\rm Pl}^2/12 - \sigma m_{\rm Pl}(1 + m_{\rm Pl}^2/16\sigma^2)^{1/2}/3 \; .$$

For $\sigma \gtrsim m_{\rm Pl}$ [which is the range of interest, cf. Eq. (26)],

$$\phi_e \simeq \sigma - m_{\rm Pl}/6 \; ; \tag{27}$$

i.e., ϕ rolls very slowly from ϕ_b ($\gtrsim H_0$) to ϕ_e, which is essentially equal to σ. [When $\sigma > m_{\rm Pl}$, we have one of the rare cases where condition (5c), as well as (5b), is important in the determination of ϕ_e; taking both conditions into account we find that $\phi_e = \sigma - m_{\rm Pl}/4.3$. The numerical difference between this and Eq. (27) is never very important; however, in constructing Table I we have used this more accurate estimate for ϕ_e.]

The requirement that $\phi_e - \phi_b \gtrsim$ many H is just

$$\sigma \gtrsim 30\lambda^{1/2}\sigma^2/m_{\rm Pl} \Longrightarrow \sigma/m_{\rm Pl} \lesssim \lambda^{-1/2}/30 \; .$$

(c) During the slow-rollover $3H\dot{\phi} \simeq -V'$ and

$$N(\phi) \equiv \int_0^{t(\phi)} H(\phi)dt$$
$$\simeq 3\sigma^2/m_{\rm Pl}^2[\ln(\phi/\phi_b)^2 - (\phi/\sigma)^2 + (\phi_b/\sigma)^2] \; . \tag{28}$$

In integrating the equation of motion we have taken into account the variation of H with ϕ. For $\phi \gg \phi_b \gtrsim H_0$, Eq. (28) can be simplified to

$$\phi(t) \simeq \phi_b \exp[(m_{\rm Pl}/\sigma)^2 N(\phi)/6] \; . \tag{29}$$

Physically, $N(\phi)$ is the number of e-folds the cosmic scale factor $R(t)$ undergoes during the time it takes ϕ to go from ϕ_b to ϕ; for $\phi \ll \sigma$, $H(\phi) \simeq H_0 \simeq$ constant and $N(\phi) \simeq H_0 t$.

During the journey of ϕ from $\phi = \phi_b$ to $\phi = \phi_e$ $R(t)$ undergoes $N \equiv N(\phi_e)$ e-folds of growth. The values of ϕ and $\dot{\phi}$ 60 e-folds before $\phi \simeq \phi_e$ are needed to compute $(\delta\rho/\rho)_H$ on the scales which are of interest today. From Eq. (29) it follows that

$$\phi_{N_l} \simeq \phi_e \exp[-(m_{\rm Pl}/\sigma)^2 N_l/6] \; , \tag{30}$$

and $\dot{\phi}_{N_l}$ is related to ϕ_{N_l} by the equation of motion for ϕ ($\dot{\phi} = -V'/3H$). The density perturbation on the scale which crossed outside the horizon N_l e-folds before $\phi \simeq \phi_e$ (when it reenters the horizon) is

$$(\delta\rho/\rho)_H \simeq H^2(\phi_{N_l})/\dot{\phi}_{N_l} \tag{31a}$$
$$\simeq -3H^3(\phi_{N_l})/V'(\phi_{N_l}) \tag{31b}$$
$$\simeq 12\sqrt{2}\lambda^{1/2}(\sigma/m_{\rm Pl})^3(\sigma/\phi)[1-(\phi/\sigma)^2]^2 \; . \tag{31c}$$

For the scale of interest ($N_l \simeq 60$), $\phi_{60} \simeq \phi_e \times \exp[-10(m_{\rm Pl}/\sigma)^2]$, and if $\sigma \lesssim 3m_{\rm Pl}$, ϕ_{60} is substantially less than ϕ_e (and σ) so that Eq. (31c) reduces to

$$(\delta\rho/\rho)_H \simeq 12\sqrt{2}\lambda^{1/2}(\sigma/m_{\rm Pl})^3 \exp[N_l(m_{\rm Pl}/\sigma)^2/6] \quad (\sigma \lesssim 3m_{\rm Pl}) \; . \tag{32}$$

[In this regime $H(\phi_{N_l}) \simeq H_0$ and $V' \simeq -2\alpha\phi_{N_l}$—the variation of H with ϕ can be ignored, and the quartic term can be ignored when computing V'. Equation (32) could also have been derived by direct substitution of H_0 and $2\alpha\phi_{N_l}$ into Eq. (31b).]

Computing $(\delta\rho/\rho)_H$ when $\sigma \gg 3m_{\rm Pl}$ is not as straightforward. In this regime Eq. (30) is a poor approximation to the solution for ϕ_{60} in Eq. (28), and so we have directly solved Eq. (28) for ϕ_{60}. For $\sigma = 3m_{\rm Pl}$ the error made in computing $(\delta\rho/\rho)_H$ by using Eq. (30) for ϕ_{60} is a factor of 1.7, increasing to an error of a factor of 30 for $\sigma = 30m_{\rm Pl}$.

Bringing together conditions (26), (28) and (32), we find that successful inflation can be achieved for $\sigma \gtrsim m_{\rm Pl}/3$. The prescribed values of σ, λ, α, and H_0 are tabulated in Table I. In order to ensure that the cubic term does not interfere with the success of this potential we must also have $\beta \lesssim \lambda\sigma$. We have also checked these analytic results against numerical integrations of the equations of motion and find good agreement.

As discussed in Sec. II [see discussion of constraint (e)], the energy available for reheating the Universe is determined by the value of H at $\phi = \phi_e$. For this potential H varies significantly between $\phi \simeq 0$ and $\phi \simeq \phi_e$, and so H_e is not accurately approximated by H_0. Substituting $\phi_e \simeq \sigma - m_{\rm Pl}/6$ into Eq. (25), we find that $H_e^2 \simeq \lambda\sigma^2$. [Note that $H_e^2/H_0^2 \simeq 1/(8\sigma^2/m_{\rm Pl}^2)$.] Thus the maximum reheat temperature is

$$T_{\rm RH} \simeq (H_e m_{\rm Pl})^{1/2}$$
$$\simeq (\lambda\sigma^2/m_{\rm Pl}^2)^{1/4} m_{\rm Pl}$$
$$\simeq (\lambda/10^{-14})^{1/4}(\sigma/m_{\rm Pl})^{1/2} 10^{12}\ \text{GeV} \; . \tag{33}$$

TABLE I. Prescribed parameters for the potential $V = V_0 - \alpha\phi^2 + \lambda\phi^4 \equiv \lambda(\phi^2 - \sigma^2)^2$. Note that $\alpha = 2\lambda\sigma^2$ and $\alpha/H_0^2 \simeq (m_{\rm Pl}/2\sigma)^2$. To compute these parameters we have used $\phi_b = 10H_0$, $\phi_e = \sigma - m_{\rm Pl}/4.3$, and solved Eq. (28) for ϕ_{60}.

$\sigma/m_{\rm Pl}$	λ/δ^2	$N = \int Hdt$	H_0/δ	$T_{\rm RH}(\max) = (H_e m_{\rm Pl})^{1/2}/\delta^{1/2}$
0.5	2×10^{-44}	70	10^{-3} GeV	1×10^{8} GeV
1	5×10^{-20}	120	5×10^{9} GeV	2×10^{14} GeV
2	1×10^{-15}	330	5×10^{12} GeV	4×10^{15} GeV
3	2×10^{-15}	740	2×10^{13} GeV	6×10^{15} GeV
10	3×10^{-16}	8100	6×10^{13} GeV	10^{16} GeV
30	3×10^{-17}	73000	2×10^{14} GeV	3×10^{16} GeV

[a]To calculate N, we have taken $\phi_b \simeq H_0$ [cf. Eq. (28)].

For the case of poor reheating ($\Gamma \lesssim H_e \simeq \lambda^{1/2}\sigma$) $T_{\rm RH}$ is, as before, $\simeq(\Gamma m_{\rm Pl})^{1/2}$, and nucleosynthesis constrains Γ to be $>10^{-23}$ GeV. If the baryon asymmetry can be produced directly in the decay of the coherent field oscillations, then we need

$$\Gamma \simeq 10^{-20}\epsilon^{-2}[V''(\sigma)/m_{\rm Pl}]$$
$$\simeq \epsilon^{-2}(\lambda/10^{-14})(\sigma/m_{\rm Pl})^2 10^{-14}\ \text{GeV}\ ,$$

where as usual $m_\phi = V''(\sigma)^{1/2} \simeq (8\lambda)^{1/2}\sigma$ is the mass of the rolling Higgs field.

The potential we have just analyzed ($V = V_0 - \alpha\phi^2 - \beta\phi^3 + \lambda\phi^4$) is not necessarily an arbitrary potential; it would, for example, represent the renormalizable effective potential for a scalar field which is weakly coupled to ordinary matter. (Some consideration must be given to the high-temperature effects which must push the Universe towards $\phi = 0$ in such a model so that a slow-rollover transition indeed takes place. This issue is considered in Ref. 29.) Fine tuning of the quartic self-coupling and of all the dimensional parameters (with respect to the SSB scale σ) is obviously required, but this fine tuning may be "natural" in certain contexts (e.g., supergravity or supersymmetry theories).[28,29] Because of the weak coupling of the ϕ field to ordinary matter, sufficient reheating may be a difficulty (as in the Dimopoulos-Raby model[14,17]). Finally, we warn the reader that it would not be correct to assume that fine tuning a GUT Higgs potential according to this prescription [Eqs. (22a)—(22d) or Table I], would result in a successful inflationary model. In such a model radiative corrections due to the gauge fields would have to be considered (if the scalar field is not a gauge singlet, or otherwise couples to the gauge fields). Since those corrections involve terms quartic in the scalar field, with coefficients that depend logarithmically on ϕ, the analysis is quite different. (Recall, for our example we assumed constant coefficients α, β, λ.) We have found that the radiative corrections make it impossible to satisfy our prescription for a successful inflationary scenario.

IV. CONCLUDING REMARKS

New inflation is an extremely attractive scenario for resolving the handful of very fundamental cosmological conundrums which plague the standard cosmology. Thus far, the idea of a slow-rollover transition,[2,3] which solves in a very simple way the fundamental difficulty with Guth's original inflationary picture: the "graceful return" to the standard cosmology,[27] has not been implemented with complete success.[28,29] In Sec. II we have discussed in detail the set of constraints a scalar potential of a single Higgs field must satisfy in order to give rise to a successful inflationary model. We believe that the prescription outlined in Sec. II will serve as a useful guide for developing successful inflationary-Universe models; in fact, this set of rules has already proven useful in studying the efficacy of certain supergravity potentials.[29]

From our "worked" example[26] it is clear that the constraints seem to require extreme fine tuning of parameters (compared to the "typical choice" of parameters in a GUT potential). If this fine tuning is indeed necessary, there is the hope that it can eventually be understood as being due to "new physics." In fact, the constraints are likely to be even tighter when the dynamics of the initial phase ($\phi = 0 \to \phi = \phi_0$) of the transition are taken into consideration. For example, high-temperature effects may push the Universe into the stable phase before the transition even takes place.[29] Also, if the transition is to be initiated by a Hawking-Moss-type nucleation event,[30] then the Universe must begin on the metastable-phase side of a barrier that is small enough for the Hawking-Moss limit to be applicable.[30,31] There is also the question, which we mentioned earlier, of competing phases[9]—although the SU(3) $\times$SU(2)$\times$U(1) SSB minimum may be the true vacuum, that does not guarantee that the Higgs field goes directly to that minimum, and not through some other metastable phase first.

There is a possible "loophole" in our set of constraints—the assumption that the potential is a function of a single scalar field. Since the constraints seem to be very restrictive (almost mutually exclusive), it may be somewhat easier to achieve successful inflation with a potential that involves several fields, with one being responsible for slow rollover and another responsible for reheating. We are presently investigating this possibility.

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APPENDIX A: HOW SCALE INVARIANT IS THE DENSITY PERTURBATION SPECTRUM IN THE NEW-INFLATIONARY-UNIVERSE SCENARIO?

Roughly speaking, quantum fluctuations in the scalar field ϕ give rise to a scale-invariant spectrum of density perturbations after reheating.[10–13] There is, however, a logarithmic dependence of the spectrum on scale. In this appendix we study the quantitative effect of this deviation from exact scale invariance.

The density perturbation on a comoving scale l at the time of horizon crossing during the FRW epoch after inflation is given by [cf. Eq. (9)]

$$(\delta\rho/\rho)_H \simeq (H^2/\dot\phi)\,|_{t_l}\ , \tag{A1}$$

where $H^2/\dot\phi$ is evaluated at the time $t = t_l$, when the scale crossed outside the horizon during the inflationary epoch. For a potential V which is approximately $V \simeq V_0 - a\phi^n$ during the slow-rollover phase we have [cf. Eq. (10)]

$$\dot\phi(t_l) \propto N_l^{-(n-1)/(n-2)}\ , \tag{A2}$$

where $n > 2$, and N_l is the number of e-foldings from

$t = t_l$ to the "end of inflation" (when $\phi \simeq \sigma$). (*Note*: if $\Gamma \ll H$, the "end of inflation" is not reheating, but the beginning of the coherent-field-oscillation phase.) The comoving mass contained within the comoving scale l is

$$M(N_l) = \exp(3N_l)M_0 \ , \tag{A3}$$

where M_0 is the mass within the horizon at the end of inflation. Using the fact that during the radiation-dominated FRW epoch the horizon mass is $M_H \simeq (t/\mathrm{sec})^{3/2} M_\odot$ and that the age of the Universe is $t \simeq (T/\mathrm{MeV})^{-2}$ sec, it follows that $M_0 \simeq (\mu^2 T_{\mathrm{RH}}/\mathrm{MeV}^3)^{-1} M_\odot$, so that

$$N_l \simeq \tfrac{1}{3} \ln(\mu^2 T_{\mathrm{RH}} M) \ . \tag{A4}$$

Here $V(\phi=0) \simeq \mu^4$, μ and T_{RH} are measured in MeV, and M is measured in solar masses.

Combining Eqs. (A2) and (A4) we have

$$(\delta\rho/\rho)_{H,M_1}/(\delta\rho/\rho)_{H,M_2} = [\ln(M_1\mu^2 T_{\mathrm{RH}})/\ln(M_2\mu^2 T_{\mathrm{RH}})]^{(n-1)/(n-2)} \ ; \tag{A5}$$

the deviation from scale invariance *increases* with decreasing values of n, μ, and T_{RH}. For $n=3$, $\mu = 10^{10}$ GeV, and $T_{\mathrm{RH}} \simeq 1$ MeV, the variation from $M_1 = 10^{22}$ $M_\odot$ (=present horizon mass) to $M_2 = 10^{12}$ $M_\odot$ (=galactic mass) is a factor of 1.60; while for $n=3$, $\mu = T_{\mathrm{RH}} = 10^{10}$ GeV, the variation is only a factor of 1.43.

For $n=2$ and $\sigma \lesssim 3m_{\mathrm{Pl}}$ we have [cf. Eq. (32)]

$$(\delta\rho/\rho)_H \propto \exp[(m_{\mathrm{Pl}}/\sigma)^2 N_l/6] \ , \tag{A6}$$

and so

$$(\delta\rho/\rho)_{H,M_1}/(\delta\rho/\rho)_{H,M_2} \simeq (M_1/M_2)^{m_{\mathrm{Pl}}^2/18\sigma^2} \ . \tag{A7}$$

For $\sigma \simeq m_{\mathrm{Pl}}$, $M_1 = 10^{22}$ $M_\odot$, and $M_2 = 10^{12}$ $M_\odot$, the deviation from scale invariance is a factor of 3.60. A deviation of less than a factor of $O(3)$ from exact scale invariance is probably not significant. For $n=2$ and $\sigma \gg 3m_{\mathrm{Pl}}$, $(\delta\rho/\rho)_H$ is essentially independent of N_l [cf. Eq. (31)].

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