Fermi National Accelerator Laboratory

FERMILAB-Pub-84/132-A
December 1984

QUANTITATIVE ANALYSIS OF THE THERMAL DAMPING
OF COHERENT AXION OSCILLATIONS

Michael S. Turner
Enrico Fermi Institute
The University of Chicago
Chicago, Illinois 60637

and

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Batavia, Illinois 60510

Abstract

Unruh and Wald[1] have recently discussed a new mechanism for damping coherent axion oscillations, 'thermal damping', which occurs due to the temperature-dependence of the axion mass and neutrino viscosity. We investigate the effect quantitatively and find that the present energy density in axions can be written as: $\rho_a = \rho_{ao}/(1 + J_{UW})$, where ρ_{ao} is what the axion energy density would be in the absence of the thermal damping effect and J_{UW} is an integral whose integrand depends upon $(dm_a/dT)^2$. As a function of f_{PQ} ($\equiv$ Peccei-Quinn symmetry breaking scale) J_{UW} achieves its maximum value for $f_{PQ} \simeq 3\times10^{12}$ GeV; unless the axion mass turn-on is very sudden, $|(T/m_a)(dm_a/dT)| >> 1$, J_{UW} is $<< 1$, implying that this damping mechanism is not significant.

I. Introduction

In models where a Peccei-Quinn[2-4] symmetry is broken at a very large energy scale,[5-6] $f_{PQ} >> 300$ GeV, coherent oscillations of the axion field (set in motion by the initial misalignment of the field with the minimum of its potential) behave like non-relativistic matter and contribute significantly to the energy density of the Universe today:[7-10]

$$\Omega_a h^2/\theta^3 \simeq 0.1\ \phi_i^2\ \Lambda_{200}^{-2/3}\ f_{12}^{11/9}, \tag{1}$$

where $\Omega_a = \rho_a/\rho_c$, $\rho_c \equiv 3H_o^2/8\pi G$ is the (critical) energy density of the flat, Einstein-de Sitter cosmological model, $H_o = 100h\ \mathrm{kms}^{-1}\ \mathrm{Mpc}^{-1}$ is the present value of the Hubble parameter, $\theta = (T_\gamma/2.7\ \mathrm{K})$, $\Lambda_{200} = \Lambda_{QCD}/200$ MeV, $f_{12} = f_{PQ}/10^{12}$ GeV, and ϕ_i is the initial misalignment angle of the axion field. The age of the Universe ($\gtrsim 10^{10}$ yr) and value of the Hubble parameter ($h \gtrsim 1/2$) restrict $\Omega_a h^2$ to be $\lesssim O(1)$, which, in the absence of significant entropy production for $T \lesssim \Lambda$ (if there is significant entropy production the r.h.s. is modified by a factor S_i/S_f, the ratio by which the entropy increases[10]), restricts f_{PQ} to be $\lesssim 3\phi_i^{-18/11} \times 10^{12}$ GeV $\simeq 3\times10^{12}$ GeV. [Note, it is usually presumed that the initial misalignment angle $\phi_i \simeq O(1)$ (however, see refs. 11, 12).] Of course the case of $f_{PQ} \simeq 10^{12}$ GeV is of particular interest--it corresponds to the axion-dominated Universe.

Recently, Unruh and Wald[1] have suggested an interesting new damping mechanism for axion oscillations, which is based upon the temperature dependence of the axion mass and neutrino viscosity, and which they claim could modify Eq. (1) significantly. It is the aim of this paper to investigate this effect quantitatively. To briefly summarize our results, we find that the importance of their effect depends critically upon the detailed

temperature dependence of the axion mass for $T \simeq O(\Lambda)$--which unfortunately is not well understood (see ref. 13); however, our results indicate that unless the axion mass is extremely temperature dependent, the thermal damping effect is not important and the usual formula for Ω_a is not modified.

II. Thermal Damping of Axion Oscillations

Let a be the complex scalar field with non-zero Peccei-Quinn (PQ) charge which breaks the PQ symmetry by developing a vacuum expectation value ($\sim f_{PQ}$) when the temperature of the Universe is of the order f_{PQ}. Write a as:

$$a = f_{PQ}\, e^{i\phi}; \tag{2}$$

then ϕ is the axion degree of freedom. The Langrangian density associated with the axion field is:

$$\mathcal{L} = R^3 f_{PQ}^2 \left[1/2\ \dot{\phi}^2 - V(\phi)\right], \tag{3}$$

where R is the Friedmann-Robertson-Walker cosmic scale factor, and we have assumed that ϕ is spatially homogeneous. In the absence of instanton effects $V(\phi)$ vanishes (as it must--the axion is the almost massless Nambu-Goldstone boson associated with the spontaneous breakdown of the global PQ symmetry); in any case $V(\phi)$ is periodic, with period $2\pi/N$ (N = integer which depends upon the details of the implementation of the PQ symmetry). For small ϕ, V can be written as

$$V \simeq 1/2\ m_a^2\ \phi^2, \tag{4}$$

where m_a is the axion mass. At low temperatures ($T << \Lambda$), the axion develops a small mass due to instanton effects: $m_a = m_\pi^2/f_{PQ}$, while at high temperatures ($T >> \Lambda$) $m_a = 0$. Thus when the PQ symmetry is broken ϕ is not determined by the minimization of $V(\phi)$ (since $V(\phi) = 0$), and the initial value of ϕ ($\equiv \phi_1$) will not necessarily coincide with the zero-temperature minimum of $V(\phi)$ ($\phi = 0$ by our conventions). In fact, one would expect ϕ_i to be of order unity (however, see refs. 11 and 12).

The equation of motion for ϕ is

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2\phi = 0, \tag{5}$$

where $H \equiv \dot{R}/R$ is the expansion rate of the Universe. The energy density associated with the axion field is

$$\rho_a = (1/2\ \dot{\phi}^2 + 1/2\ m_a^2\ \phi^2)\ f_{PQ}^2. \tag{6}$$

Qualitatively the behavior of ϕ is as follows: (1) when $m_a < 3H$ (true for $T >> \Lambda$), $\phi = \phi_i =$ constant; (2) when $m_a \gtrsim 3H$, ϕ oscillates with frequency m_a, and $\rho_a \propto m_a R^{-3}$. Eq. (5) can be cast into a more suggestive and useful form:

$$\dot{\rho}_a = (\dot{m}_a/m_a)\ m_a^2\ f_{PQ}^2\ \phi^2 - 3H\dot{\phi}^2\ f_{PQ}^2, \tag{7a}$$

$$= (m_a' T/m_a)\ (\dot{T}/T)\ m_a^2\ f_{PQ}^2\ \phi^2 - 3H\ \dot{\phi}^2\ f_{PQ}^2, \tag{7b}$$

where prime denotes d/dT and overdot d/dt. Once ϕ begins to oscillate, $\langle\dot{\phi}^2 f_{PQ}^2\rangle = \langle m^2 f_{PQ}^2\ \phi^2\rangle = \rho_a$, where $\langle\rangle$ indicates the average over an oscillation. Previously it has been assumed that the temperature variation is

just due to the expansion of the Universe, so that $\dot{T}/T = -H$ and T and m_a are changing slowly compared to ϕ. In this limit ($|\dot{m}_a/m_a| = |\dot{T}/T| = H << m_a$) Eq. (7a) can be averaged over an oscillation and becomes

$$\dot{\rho}_a = [(\dot{m}_a/m_a) - 3H]\ \rho_a; \tag{8}$$

this is the familiar adiabatic approximation. Eq. (8) is easily integrated, giving

$$(\rho_a\ R^3/m) = \text{constant}. \tag{9}$$

Taking the onset of oscillation to occur when $m_i \equiv m_a(T_i) = 3H(T_i)$, the usual result [7-9] follows

$$\rho_a = \rho_{ai}(m_a/m_i)(R_i/R)^3, \tag{10a}$$

$$= 1/2\ m_i\ m_a\ \phi_i^2\ f_{PQ}^2\ (R_i/R)^3; \tag{10b}$$

throughout we use the subscript i to refer to the value of a quantity when ϕ begins to oscillate (i.e., when $m_a = 3H$). While the Universe is radiation-dominated ($T \gtrsim 10$eV, $t < 10^{10}$ sec), $H = 1.7\ g_*^{1/2}T^2/m_{pl}$, where as usual g_* is the total effective number of relativistic degrees of freedom [$= \sum_{BOSE} g_B + (7/8) \sum_{FERMI} g_F$]; and as long as the expansion is isentropic $g_*^{1/3}RT =$ constant. Using these two facts, the usual estimate for ρ_a (which we designate by ρ_{ao}) follows directly from Eq. (10b),

$$\rho_{ao} = 2.5\ (g_*/g_{*i}{}^{1/2})\ (m_a\ f_{PQ}^2/m_{pl}\ T_i)\ \phi_i^2\ T^3. \tag{11}$$

[Note, when evaluated today ($T = 2.7$K) and compared to ρ_c, this reduced to Eq. (1) for Ω_a.]

Now let's consider the new, thermal-damping effect. Unruh and Wald[1] point out that in addition to the slow variation of T due to the expansion of the Universe, there is a rapid variation of T due to the oscillating axion field. They show that based on simple thermodynamic considerations the equilibrium temperature of the thermal plasma must oscillate as ϕ oscillates, specifically that:

$$\dot{T}_{EQ}/T_{EQ} = (m_a' T/m_a)\ (1/2\ m_a^2\ f_{PQ}^2\ \dot{\phi}\phi/\rho) \equiv \alpha_o, \tag{12}$$

where $\rho = g_*(\pi^2/30)T^4$ is the energy density of the thermal plasma.

In order to examine this effect quantitatively write $\phi = \phi_o \cos m_a t$ and neglect for the moment the effects of the expansion of the Universe (which will be negligible since we will be interested in time intervals of order the oscillation time). Then it follows that

$$\dot{\alpha}_o = -1/2\ m_a\ (m_a'\ T/m_a)(\rho_a/\rho)\ \sin 2m_a t, \tag{13a}$$

$$\alpha_o = \text{constant} + 1/4(\rho_a/\rho)(m_a' T/m_a)\ \cos 2m_a t, \tag{13b}$$

where in integrating Eq. (13a) we have ignored the temperature variation of all quantities. This is a justifiable approximation since $\Delta T/T \approx O(\dot{\alpha}_o m_a^{-1}) \approx (\rho_a/\rho)(m_a' T/m_a)$ which is $<< 1$ since ρ_a/ρ is $<< 1$ -- a fact which we will now demonstrate. Using $m_i = 3H(T_i)$, and taking $\phi_i \approx O(1)$ and $g_{*i} \approx 30$, it follows that

$$\begin{aligned}(\rho_a/\rho)_i &\approx 30(f_{PQ}/m_{pl})^2, \\ &\approx 2\times 10^{-13}\ f_{12}^{\ 2},\end{aligned} \tag{14}$$

which is indeed $<< 1$. Note that since α_o is proportional to (ρ_a/ρ), the variation in the equilibrium plasma temperature due to the oscillating axion field is very small (which is why the thermal damping effect turns out to be so small).

Let us assume for the moment that the plasma can adjust rapidly (on a timescale $<< m_a^{-1}$) to the oscillating equilibrium temperature, so that the plasma temperature $T = T_{EQ}$. Using $\dot{T}/T = \dot{T}_{EQ}/T_{EQ} \equiv \dot{\alpha}_o$ in Eq. (7b), consider the effect of the oscillating plasma temperature on the evolution of ρ_a:

$$\begin{aligned}\dot{\rho}_a &= \ldots . + \dot{\alpha}_0 (m_a' T/m_a)\ m_a^2\ f_{PQ}^2\ \phi^2 \\ &\approx (1 + \cos 2\ m_a t)\ \sin 2m_a t.\end{aligned}$$

Because of the relative phasing of $\dot{T}/T$ and ϕ^2 the effect of the oscillating plasma temperature on ρ_a integrates to zero over a full oscillation period.

Can the plasma respond rapidly-enough to the oscillating equilibrium temperature to maintain $T = T_{EQ}$ for all species? [At temperatures $\gtrsim$ few 100 MeV the relativistic species present are: $e^\pm$, $\mu^\pm$, γ, $\nu\bar{\nu}$, gluons, $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$.] Unruh and Wald argue (and we believe correctly so) that the strongly-and electromagnetically-interacting degrees of freedom (quarks, gluons, photons, and charged leptons) interact rapidly-enough (interaction rate $\Gamma >> m_a$) so that their temperature can closely track T_{EQ}, while the weakly-interacting neutrinos cannot since Γ_ν is only comparable to m_a (we'll be more quantitative later). Let us assume then that the temperatures of all species except the neutrinos closely track T_{EQ}. Because of neutrino interactions with the photon-quark-gluon-charged lepton plasma (here after referred to as 'photons' for brevity), the neutrino temperature will attempt to track T_{EQ}. This then results in a back reaction on 'the photons', which spoils the phase relationship of the photon temperature with T_{EQ}, and leads to a non-zero cycle average for $(m_a'T/m_a)\,(\dot{T}/T)\; m_a{}^2\, f_{PQ}{}^2\, \phi^2$. This is the thermal damping effect of Unruh and Wald.

We will use the Boltzmann equation to estimate the effect of the oscillating photon temperature on T_ν. Let n_ν and $n_{\nu EQ}$ respectively denote the actual neutrino number density and the equilibrium neutrino number density; note $n_i \simeq (g_i/\pi^2)T_i^3$. The Boltzmann equation for the evolution of n_ν is

$$\dot{n}_\nu = -\langle\sigma v\rangle\,(n_\nu{}^2 - n_{\nu EQ}{}^2), \tag{15}$$

where $\langle\sigma v\rangle$ is the thermally-averaged cross section times velocity ($= c$) for neutrino-'photon' interactions. [For simplicity only processes like $\nu + \bar{\nu} \to$ '$\gamma + \gamma$' and '$\gamma + \gamma$' $\to \nu + \bar{\nu}$ have been considered; the $n_\nu{}^2$ term on the r.h.s. accounts for $\nu\bar{\nu}$ annihilations, and the $n_{\nu EQ}{}^2$ term accounts for $\nu\bar{\nu}$ pair creations by 'the photons'.] Again, for the time intervals of interest we can ignore the effects of the expansion of the Universe.

Now write $T_\nu = T(1+\delta)$ and $T_{\nu EQ} \simeq T_\gamma = T(1 + \alpha)$. Since $g_\nu \simeq 6$ (for 3 ν species) $\ll g_\gamma \gtrsim 30$ (for $T \gtrsim \Lambda$) we can neglect the effect of the neutrinos on the photon temperature when calculating the evolution of T_ν and assume that the photon temperature tracks T_{EQ} (i.e., take $\alpha = \alpha_0$). Eq. (15) then reduces to

$$\dot{\delta} = -\Gamma(\delta - \alpha_0), \tag{16}$$

where since α_0, $\delta \ll 1$ we have retained only terms to linear order, and $\Gamma = 2n_{\nu EQ}\langle\sigma v\rangle$. The solution to Eq. (16) is

$$\delta(t) = e^{-\Gamma t}\,[c + \Gamma\textstyle\int_0^t \alpha_0\,(t')e^{\Gamma t'}dt')]; \tag{17}$$

taking $\alpha_0 = 1/4\,(m'T/m_a)(\rho_a/\rho_\gamma)\cos(2m_a t)$, we find that

$$\delta(t) = \Gamma/4(m_a'T/m_a)(\rho_a/\rho_\gamma)\;(\Gamma^2 + 4m_a^2)^{-1}\,(\Gamma\cos 2m_a t + 2m_a \sin 2m_a t), \tag{18}$$

where the term $ce^{-\Gamma t}$ has been dropped since it $\to 0$ as $t \to \infty$.

The oscillating energy density in the neutrinos must be supplied by the 'photon bath'; thus the neutrinos will have a back reaction on the 'photon bath':

$$(\dot{T}_\gamma/T_\gamma)_{\text{due to }\nu} \simeq -\dot{\delta}\,(g_\nu/g_\gamma),$$

where the g_ν/g_γ factor reflects the ratio of their heat capacities. Taking this effect into account we have for the oscillating component of $\dot{T}_\gamma/T_\gamma$:

$$\dot{T}_\gamma/T_\gamma = \dot{\alpha}_0 - \dot{\delta}(g_\nu/g_\gamma)$$

$$=(m_a'T/m_a)\;(\rho_a/\rho_\gamma)\;[-m_a/2\,\sin 2m_a t$$

$$-(g_\nu/g_\gamma)\;m_a\Gamma/2\;(\Gamma^2 + 4m_a{}^2)^{-1}\;(-\Gamma\sin 2m_a t + 2m_a\cos 2m_a t)]. \tag{19}$$

Because of the neutrinos, a phase lag between T_{EQ} and T_γ has been introduced; now $(m_a'T/m_a)(\dot{T}/T)\; m_a^2\, f_{PQ}^2\, \phi^2$ does not average to zero over a cycle; instead we find that

$$\langle (m_a'T/m_a)(\dot{T}/T)\;\; m_a^2\, f_{PQ}^2\phi^2\rangle = -\frac{m_a}{4}\;\frac{g_\nu}{g_\gamma}\,\Big(\frac{m_a'\,T}{m_a}\Big)^2\;\frac{\rho_a^2}{\rho_\gamma}\;\Big(\frac{2m_a}{\Gamma} + \frac{\Gamma}{2m_a}\Big)^{-1}.$$

This is the thermal damping effect discussed by Unruh and Wald [and is, in the limit of $\Gamma \gg m_a$, equivalent (up to factors of 2) to their Eq. (3.9)].

Taking thermal damping into account and again using an adiabatic approximation, Eq. (7b) can be written in a form analogous to Eq. (8):

$$\dot{\rho}_a = (\dot{m}_a/m_a - 3H)\ \rho_a - m_a/4\ (g_\nu/g_\gamma)(m_a' T/m_a)^2 (2m_a/\Gamma + \Gamma/2m_a)^{-1}\ \rho_a^2/\rho_\gamma, \tag{20}$$

which when integrated gives

$$\rho_a(t) = \rho_{ao}(t)/[1 + J_{UW}(t)], \tag{21}$$

where

$$J_{UW} = \frac{1}{4}\ \frac{g_\nu}{g_\gamma} \left(\frac{\rho_a}{\rho_\gamma}\right)_{t_1} \int_{t_1}^{t} \left(\frac{g_*(t)}{g_*(t_1)}\right)^{1/3} \times \tag{22}$$

$$\times \left(\frac{m'_a T}{m_a}\right)^2\ \frac{m_a^2}{m_1}\ R/R_1 \left(\frac{2m_a}{\Gamma} + \frac{\Gamma}{2m_a}\right)^{-1} dt,$$

$\rho_{ao}(t)$ is the result in the absence of thermal damping [given by Eq. (11)], t_1 is the epoch when the axion field begins to oscillate ($m_a(t_1) \equiv m_1 = 3H(t_1)$), and the constancy of the entropy per comoving volume, i.e., $g_*^{1/3}RT$ = constant, has been assumed. Significant thermal damping is signaled by $J_{UW} \gtrsim 1$. Note that in the limits $\Gamma \gg m_a$ (neutrinos well coupled to the 'photons' on the oscillation timescale) and $\Gamma \ll m_a$ (neutrinos decoupled from the 'photons' on the oscillation timescale) the thermal damping effect vanishes (i.e., $J_{UW} \to 0$). As is usually the case with viscous effects, the effect is maximized when $\Gamma \approx m_a$.

III. Numerical Results

During the epoch of 'mass turn-on' (100 MeV $\lesssim T \lesssim$ few GeV) the Universe is radiation-dominated, so that

$$R/R_1 = (t/t_1)^{1/2},$$

$$\dot{R}/R = H \simeq 1.7\ g_*^{1/2}\ T^2/m_{pl};$$

the total number of relativistic degrees of freedom g_* is of order 30, implying that

$$H \simeq 10\ T^2/m_{pl}.$$

Here and throughout we have used units in which $\hbar = k_B = c = 1$, and $G^{-1/2} = m_{pl} = 1.22\times10^{19}$ GeV.

Now consider the neutrino-interaction rate Γ. The thermally-averaged rate for an electron or muon neutrino to interact with the charged leptons present ($e^\pm$, $\mu^\pm$) via reactions like $\nu\bar{\nu} \rightleftarrows e^+e^-$, $\mu^+\mu^-$; $\nu e^\pm \rightleftarrows \nu e^\pm$; etc. is $\simeq 0.7\ G_F^2T^5$ (summed over all reactions). For the τ neutrino the rate is $\simeq 0.1\ G_F^2T^5$.

The interaction rates with quarks ($u\bar{u}$, $d\bar{d}$, and $s\bar{s}$) are similar. As an estimate for the neutrino interaction rate (per species) we use

$$\Gamma \simeq (1\text{-}2)\ G_F^2\ T^5. \tag{23}$$

Note that because of the $(\Gamma/2m_a + 2m_a/\Gamma)^{-1}$ factor in the integrand of J_{UW}, changing the magnitude of Γ merely shifts the axion mass (i.e., value of f_{PQ}) for which the effect is maximized.

To simplify the expression for J_{UW} we define the following dimensionless qualitities:

$$x = T/m_\pi,$$

$$g(x) = m_a(T)/m_o,$$

where m_π is the pion mass ($\simeq$ 140 MeV).

Taking $m_o \simeq m_\pi^2/f_{PQ}$ and $1/4\ (g_\nu/g_\gamma) \simeq 1/10$, expression (22) for J_{UW} becomes:

$$J_{UW} = 6\times10^{-3}\ x_i^{-1} \int_0^{x_i} g'(x)^2 x^{-2} \left(\frac{2g(x)}{f_{12}x^5} + \frac{f_{12}x^5}{2g(x)}\right)^{-1} (g_{*i}/g_*)^{1/6}\, dx, \tag{24}$$

where x_i is specified by

$$x_i^2/g(x_i) \simeq 3\times10^5\ f_{12}^{-1} \tag{25}$$

(just the condition that $m_i \simeq 3H_i$). Note that x_i is typically $\simeq$ 1-30 (corresponding to $T_i \simeq$ 100 MeV - few GeV). The small coefficient in front of the integral $(6\times10^{-3}/x_i)$ is traceable to the smallness of $(\rho_a/\rho_\gamma)_i$, cf. Eqs. (14 and 22).

In order to evaluate J_{UW} a model for $g(x) \equiv m(T)/m_o$ is needed. At low temperatures ($T \ll \Lambda$), $m(T) \simeq m_o$ is temperature independent ($g(x) \to 1$ for $x \to 0$). At very high temperatures ($T \gg \Lambda$) Gross, Pisarski, and Yaffe[13] (GPY) have calculated $m_a(T)$ using the dilute instanton gas approximation, and find that

$$m_a(T) \simeq 15\,\frac{\Lambda^2}{f_{PQ}} \left(\frac{m_u m_d m_s}{\Lambda^3}\right)^{1/2} (\Lambda/\pi T)^4$$

$$\times\ [\ln(\pi T/\Lambda)]^3. \tag{26}$$

where m_u, m_d, m_s are the u, d, s current algebra quark masses. [Of course, the region of greatest interest is the intermediate region, $T \simeq \Lambda$.] For simplicity take $\pi T/\Lambda \simeq T/m_\pi = x$; then for $x \gg 1$ the GPY result implies that

$$g(x) \simeq (\ln x)^3/x^4$$

which is shown in Fig. 1. For $30 \gtrsim x \gtrsim 3$, $g(x) \propto x^{-3}$, suggesting the following form for $g(x)$:

$$g(x) = \begin{cases} 1 & x < 1, \\ x^{-3} & x > 1. \end{cases}$$

Using this form for $g(x)$ we have numerically evaluated J_{UW}, and the results are shown in Fig. 2. J_{UW} peaks for $f_{PQ} \simeq 3\times10^{12}$ GeV achieving a value of $\simeq 2\times10^{-4}$.

Consider the following form for $g(x)$ (suggested to us by Wald):

$$g(\alpha,x) = \begin{cases} 1 & x < 1 \\ 1 - \exp[-\alpha/(x-1)] & x > 1 \end{cases}$$

where α is an adjustable parameter which permits one to control how rapidly the axion mass turns on. The function $g(\alpha,x)$ and all of its derivatives are continuous. At $x = 1 + \alpha/2$, $|g'(x)|$ achieves its maximum value, $4/(e^2\alpha)$; for $x < 1$, $g'(x) = 0$ and $g(x) = 1$. The function $g(\alpha,x)$ is plotted in Fig. 1 for $\alpha = 10^{-1}$, 10^{-2}, 10^{-3}. The functional form $x^{-1}\ g(\alpha = 10^{-1}, x)$ closely approximates the GPY calculation for $x \gtrsim 3$ and is also shown in Fig. 1.

We have numerically evaluated J_{UW} using $g(\alpha = 10^{-3}, x)$, $g(\alpha = 10^{-2}, x)$, and $x^{-1}\, g(\alpha = 10^{-1}, x)$, and our results are shown in Fig. 2. Only for $\alpha = 10^{-3}$ (which corresponds to a very rapid 'mass turn-on') does J_{UW} start to approach order unity. In the small α limit most of the integral comes from the interval near $x \simeq 1 + \alpha/2$ where $|g'(\alpha, x)| \approx \alpha^{-1}$ is very large. In this limit J_{UW} can be evaluated approximately analytically:

$$J_{UW} \simeq 10^{-3}\, \alpha^{-1}\, x_1^{-1}\, (1/2\, \epsilon_{12} + 2/\epsilon_{12})^{-1}. \tag{26}$$

The implications of the numerical results are clear. Unless the axion mass turn-on is extremely rapid, $|T\, m_a'/m_a| >> 1$, the thermal damping effect is not important. [Again we emphasize that the smallness of the effect traces directly to the smallness of ρ_a/ρ_γ, cf. Eq. (14).] Lest the reader be misled, we must point out that there comes a point of diminishing returns for the thermal damping effect as the rapidness of the axion mass turn-on is increased. As Preskill, Wise, and Wilczek[7] pointed out, if the mass turn-on is extremely rapid $|\dot{m}_a/m_a| \gtrsim m_a$, the adiabatic approximation used here (and in the earlier papers)[7-10] is not valid. In the limit of $|\dot{m}_a/m_a| >> m_a$ the impulse approximation is the relevant approximation, in which case there is no adiabatic damping, or thermal damping, and so ρ_a will exceed ρ_{ao}. The adiabatic approximation used here become invalid when

$$|\dot{m}_a/m_a| = |m_a'T/m_a|\; |\dot{T}/T| \gtrsim m_a,$$

Recall that $\dot{T}/T = -H + \dot{\alpha}_0$ and $\dot{\alpha}_0 = O[(m_a'T/m_a)(\rho_a/\rho_\gamma)\, m_a]$. Thus the adiabatic approximation is suspect if

$$|m_a'T/m_a|(H/m_a) \gtrsim 1$$

or

$$(m_a'T/m_a)^2\, (\rho_a/\rho_\gamma) \gtrsim 1.$$

Acknowledgements

I wish to thank Bob Wald for many useful and stimulating conversations; in addition I also thank Rob Pisarski, John Preskill and Bill Bardeen for helpful conversations. This work was supported by the DOE (at the University of Chicago and at Fermilab), by NASA (at Fermilab), and by an A. P. Sloan Fellowship.

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Figure Captions

Figure 1 - Various models for the 'mass turn-on' function $g(x) = m_a(T)/m_o$, where $x = T/m_s$. The curve labeled GPY is the high temperature calculation of Gross, Pisarski, and Yaffe[13] (valid in the limit $x >> 1$; see Eq. (26)). The broken curves labeled 10^{-3}, 10^{-2}, and 10^{-1} show the functional form $g(\alpha,x) = 1 - \exp[-\alpha/(x - 1)]$ for $\alpha = 10^{-3}$, 10^{-2}, and 10^{-1} respectively. The unlabeled solid curve is given by $x^{-1}\, g(\alpha = 10^{-1}, x)$. The solid line labeled '$3H/m_o$' is $3H/m_o$ for $f_{PQ} = 10^{14}$ GeV; the axion field starts to oscillate when $m_a(T)/m_o = 3H/m_o$.

Figure 2 - Results of numerically evaluating J_{UW} {see Eq. (24)} as a function of f_{PQ}. The curves labeled 10^{-3}, 10^{-2}, and 10^{-1} were obtained using the functional forms $g(\alpha = 10^{-3}, x)$, $g(\alpha = 10^{-2}, x)$, and $x^{-1}\, g(\alpha = 10^{-1}, x)$ respectively. The unlabeled curve was obtained using $g(x) = \{x^{-3}, x > 1;\ 1, x < 1\}$.

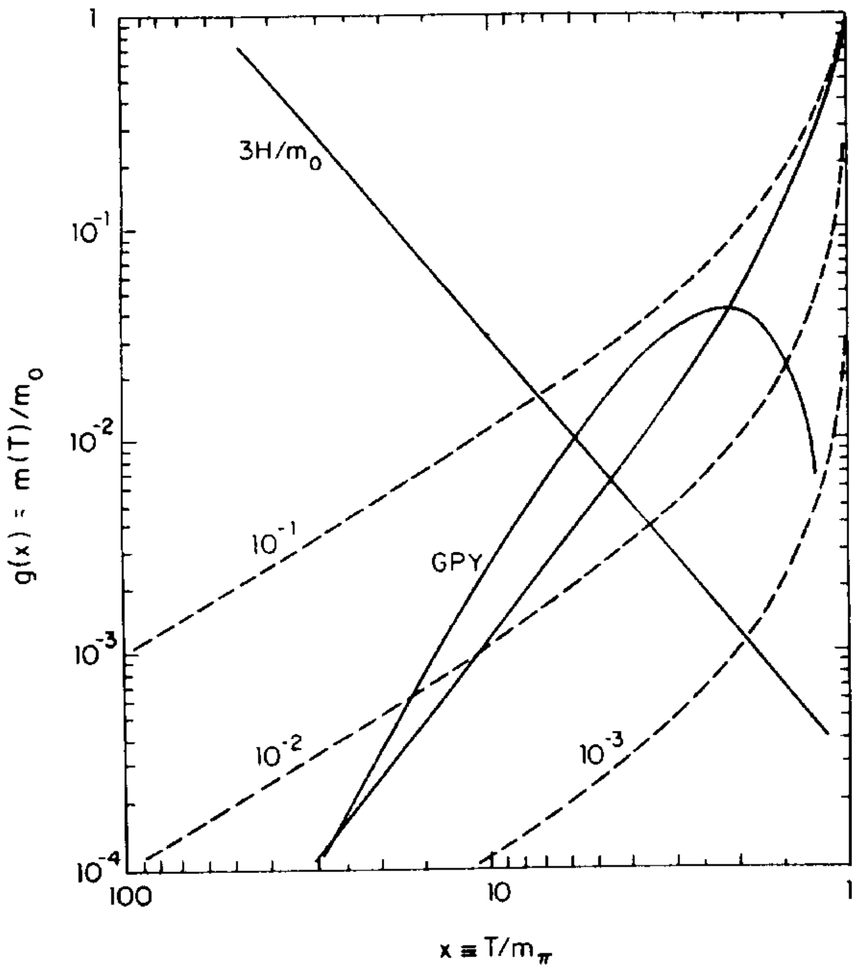

FIGURE 1

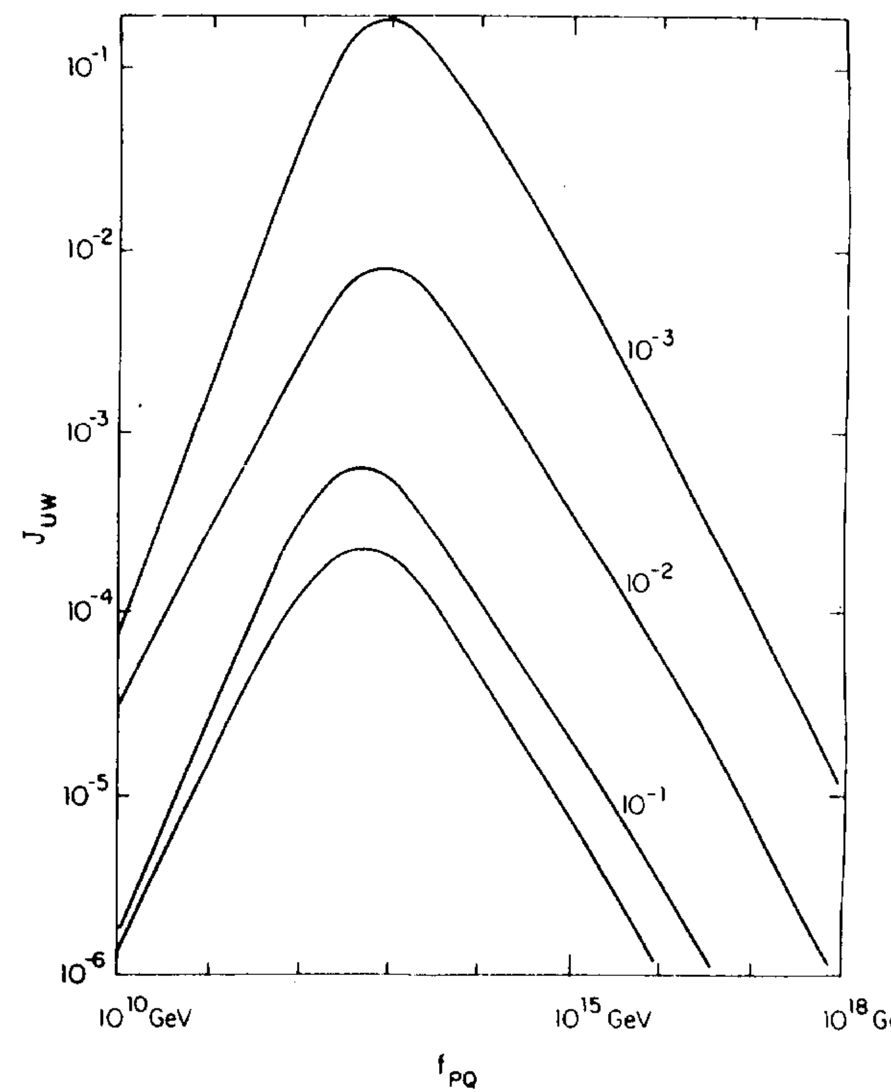

FIGURE 2