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PRIMORDIAL INFLATION AND SUPERGRAVITY

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ABSTRACT

The present status of supersymmetric inflationary models is reviewed. In particular, problems in finding a successful model in minimal N=1 supergravity theories are resolved in more general non-minimal theories.

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It is well known by now, that problems associated with initial conditions in the standard big bang model can be neatly resolved by an early period of exponential expansion or inflation.[1] The inflationary epoch is triggered during a first order phase transition in which the Universe is trapped in a false vacuum state. In models of new inflation,[2] the Universe begins its exponential expansion as a scalar field rolls into a potential well, thus picking up a vacuum expectation value resulting in symmetry breaking. Early attempts to find a model for new inflation associated the phase transition with SU(5) symmetry breaking. It had quickly become apparent, however that problems in these SU(5) inflationary models began to outnumber their benefits.[3-5]

It was noticed[5,7] that if the scalar field responsible for inflation, the inflaton, were a gauge singlet, many of the difficulties could disappear. As we will see, models in which the inflaton picks up a very large vacuum expectation value $\langle\phi\rangle=v\gg M_X$ (where M_X is the GUT scale) allowed for a longer period of inflation, and are called primordial inflation.[7] As we will also see, supersymmetry removes other difficulties in ways similar to those in which supersymmetry relaxes the gauge hierarchy problem in GUTs. Primordial inflation combined with supersymmetric inflation naturally leads one to look for models derived from N=1 supergravity.[8] (For a review of the general interplay between cosmology and supersymmetry see the contribution[9] by J. Ellis in these proceedings.) Here, I will examine the present state of inflationary models in supergravity focusing on the resolution[10-12] of some general problems[13] regarding initial conditions.

To begin, I will very briefly review the conditions necessary for inflation. The expansion rate of the Universe is determined by the Friedmann equation

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi\rho}{3M_p^2} - \frac{k}{R^2} + \frac{\Lambda}{3} \qquad (1)$$

where H is the Hubble parameter, R is the Robertson-Walker scale factor, ρ is the total mass-energy density of the Universe, $M_p = 1.2\times10^{19}$ GeV is the Planck mass, $k = \pm1,0$ is the curvature constant determining whether the Universe is open (-1), closed (+1) or flat (0) and finally Λ represents the cosmological constant. If during a phase transition, the Universe supercools into a state with a large vacuum energy density which acts as a cosmological constant, the expansion rate will soon be dominated by this contribution. Recall that the energy density in radiation falls off as T^4 as the Universe expands and cools. Thus if the phase transition is described by a scalar potential $V(\phi)$, such that $V(0)$ is large, the expansion rate will be

$$H^2 \simeq 8\pi V(0)/3M_p^2 \qquad (2)$$

and

$$R \sim \exp(Ht) \qquad (3)$$

where the vacuum energy density plays the roll of the cosmological constant. An expansion of the form of eq. (3) characterizes what we call inflation.

The search for an inflationary model becomes the search for a scalar potential $V(\phi)$ with several key properties.[14] First of all, we must require that the timescale for the field to pick up its vacuum expectation value be large, so that $\exp(H\tau) > 10^{28}$ or $H\tau \gtrsim 65$ where τ is the rollover timescale. The timescale is determined by the classical equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \partial V/\partial\phi = 0 \tag{4}$$

to be

$$\tau^{-1} \approx \dot{\phi}/\phi \sim (\partial^2 V/\partial\phi^2)/3H \tag{5}$$

where $\ddot{\phi} \approx 0$ initially and the effects of interactions are neglected until after the rollover is completed. Our first major requirement is that near the origin, $\phi < H << v$

$$\partial^2 V/\partial\phi^2|_{\phi\approx 0} < 3H^2/65 \tag{6}$$

Clearly, flat potentials of this type are more easily attainable when v is made as large as possible, i.e. in models of primordial inflation.

A second key constraint concerns the production of density fluctuations during the phase transition. In general there will be a time spread over which in certain regions of space, ϕ rolls down faster or slower than others. Density perturbations have been calculated[4] in terms of this time spread

$$\delta\rho/\rho \propto H\delta\tau \tag{7}$$

where $\delta\rho/\rho$ is the magnitude of the perturbation as it enters the horizon and $\delta\tau$ is calculated in terms of H and $\dot{\phi}$. Limits coming from the isotropy of the microwave background radiation[15] imply that

$$\delta\rho/\rho \leq 10^{-4} \tag{8}$$

This limit generally feeds into a constraint on the coupling constants in V.[4] Supersymmetry is helpful in keeping these couplings small enough to satisfy this constraint.[5,7]

Our goal therefore is to find a potential V of the form

$$V = \delta + \gamma\phi^2 + \beta\phi^3 + \alpha\phi^4 \;.... \tag{9}$$

where the coupling $\alpha, \beta, \gamma, \delta ...$ satisfy the inflationary constraints (... refers to the possible inclusion of non-renormalizable terms in V). In the context of supergravity, the search for V translates into a search for a superpotential or more generally a Kähler potential. In N=1 supergravity, the scalar potential is expressed in terms of the Kähler potential as[16]

$$V = e^G[G_i(G^{-1})^i_j\, G^j - 3] \tag{10}$$

where $G_i = \partial G/\partial\phi^i$, $G^j_i \equiv \partial^2 G/\partial\phi^i\partial\phi^*_j$ etc. and G is a real function of the chiral superfield ϕ^i. The scalar kinetic terms of the theory are expressed as

$$\mathcal{L}_{K.T.} = -G^j_i\,(\partial_\mu\phi^i)(\partial^\mu\phi^*_j) \tag{11}$$

so the theory is only well defined for $G^j_i > 0$. Minimal N=1 supergravity refers to those theories in which $G^j_i = \delta^j_i$ and G is expressed as

$$G = \phi^i\phi^*_i + \ln|F|^2 \tag{12}$$

where $F(\phi)$ is the superpotential. (I will be using throughout, units such that $M_p = \sqrt{8\pi}$.

For G of the form in eq. (12), the scalar potential is given by

$$V(\phi^i,\phi_i^*) = \exp(\phi^i\phi_i^*)\ [|F_i+\phi_i^*F|^2-3|F|^2] \tag{13}$$

where $F_i = \partial F/\partial\phi^i$. One must now apply the inflationary constraints to find a superpotential F. Let us write F in the following general form[8)]

$$F(\phi) = m^2(\lambda + \sum_n \frac{\lambda n}{n+1}\ \phi^{n+1}) \tag{14}$$

where m is an overall scale for F and we will assume that the λ_i are all of the same order of magnitude for naturalness. We will from now on concentrate on a single field, the inflaton ϕ, which is a gauge singlet. One constraint on F comes from the fact that we will not want to break supersymmetry with ϕ. The reason being that, if broken by ϕ, the supersymmetry breaking scale would be $M_s \sim m^2/M_p$. As we will see, $m \simeq 10^{-4}M_p$ implying that $M_s \sim 10^{10}$GeV >> $M_W \sim 10^2$ GeV, thus destroying the gauge hierarchy we set out to solve initially with supersymmetry. The condition to preserve supersymmetry is that

$$e^{G/2}G_\phi = F_\phi + \phi^*F = 0 \tag{15}$$

at the global minimum (we will take the global minimum to be at $\langle\phi\rangle = v = 1$ for convenience). Secondly we will also want the cosmological constant to vanish at the minimum implying that[8)] $F(1) = 0$ as well.

The couplings $\alpha, \beta, \gamma, \delta$ etc. can be expressed in terms of the couplings λ_i. For example, for V along the real ϕ axis

$$\alpha = \left(\frac{1}{2}\lambda_o^2 + \frac{9}{8}\lambda_1^2 + \lambda_2^2 + \frac{5}{3}\lambda_o\lambda_2 + \frac{9}{4}\lambda_1\lambda_3 + 2\lambda_o\lambda_4\right)m^4 \tag{16a}$$

$$\beta = -(2\lambda_o\lambda_3 + 2\lambda_1\lambda_2 + \lambda_o\lambda_1)m^4 \tag{16b}$$

$$\gamma = 2\lambda_o\lambda_2 \; m^4 \tag{16c}$$

$$\delta = (\lambda_o^2 - 3\lambda^2)m^4 \tag{16d}$$

where $\lambda_{1=2\lambda}$ was in order to cancel the linear term. The first inflationary constraint (eq. 6) is satisfied if γ or $\lambda_2 = 0$. The constraint coming from density perturbations becomes[17)]

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{(2\pi 3)^{\frac{1}{2}}} \left(\frac{\beta}{H}\right) \ln^2(Hk^{-1}) \lesssim 10^{-4} \tag{17}$$

where k is the wave number of the perturbation. This then becomes a constraint on the scale m, $m \lesssim 10^{-4}$. The simplest superpotential which satisfies all of this is[18)]

$$F = (1 - \phi)^2 m^2 \tag{18}$$

As we will see however there is one additional constraint to be considered.

The normal picture for a phase transition in the early Universe is that at very high temperatures, symmetries are restored and $\langle\phi\rangle = 0$. As the Universe cools the high temperature minimum may become metastable triggering the phase transition. Since we are dealing here with a gauge singlet this picture is not guaranteed. For example, if V is asymmetric about $\phi = 0$, due to a cubic term we would expect the high temperature minimum to be shifted. In supergravity, the picture becomes less intuitive. Finite temperature minima may be shifted from $\phi = 0$ even if V is symmetric. This is true however only when the superpotential F is asymmetric about the origin. For very high temperatures, one loop corrections to V in N=1 supergravity are[19)]

$$V_1 = \frac{T^2}{24} \, Tr(M_B^2 + \frac{1}{2} M_F^2) \tag{19}$$

where

$$TrM_B^2 = 2V_k^k \tag{20a}$$

$$TrM_F^2 = 2e^G|T_{ij}|^2 - 4e^G \tag{20b}$$

$$T_{ij} = G_{ij} + G_iG_j - G_k(G^{-1})_p^k G_{ij}^p \tag{20c}$$

The importance of the finite temperature potential, is that it enables the theory to predict the initial value for $\langle\phi\rangle$, i.e. the starting place for the inflationary rollover. Our final requirement[20)] is therefore that there exist a high temperature minimum at $\phi \approx 0$ (or wherever the potential is suitably flat. Unfortunately, for the simple example of eq. (18), the finite temperature minimum is far from the origin so that it is rather unnatural to expect $\langle\phi\rangle = 0$ initially. If one chooses $\langle\phi\rangle=0$ as an initial condition (i.e. ignoring V_1) then the model will work. The benefit of using V_1 is that it is the theory which chooses the initial conditions rather than doing so by hand.

It turns out however, that without choosing couplings λ_i which differ by many orders of magnitude, it is impossible[13] within the context of minimal N=1 supergravity. This is most easily seen when one considers the form for V_1 when a large number n, of chiral supermultiplets are also present in the theory,[19]

$$V_1 = \frac{nT^2}{24}[G_\phi G^\phi - 2]e^G \tag{21}$$

In a minimal theory, the zero temperature potential V takes the form

$$V = e^G[G_\phi G^\phi - 3] \tag{22}$$

It is easy to show that $V_\phi(0) = V_1(0) = 0$ is only satisfied if $G_\phi(0) = 0$. But then $V(0) < 0$ making a transition to a state with V=0 impossible.

One can however, remedy this situation by considering non-minimal supergravity theories, i.e. those in which $G^j_i \neq \delta^j_i$. One can show, for example that the inflationary constraints along with the thermal constraint can be satisfied by a more general choice of the Kähler potential[12]

$$G = g_\phi(\phi+\phi^*) + g^\phi_\phi(\phi\phi^*) + g_{\phi\phi}(\phi^2+\phi^{*2}) + g^\phi_{\phi\phi}(\phi\phi^*)(\phi+\phi^*) + \ldots$$
$$+ \ln|F|^2 \tag{23}$$

where g_ϕ, g_ϕ^ϕ etc. are coupling constants. The inclusion of the superpotential is simply for the convenience of ensuring unbroken supersymmetry and a vanishing cosmological constant at the global minimum. For example

$$F = m^2(1 - \phi^5)^2 \tag{24}$$

with $m \sim 10^{-4}$ ensures that $F(1)=F_\phi(1)=0$ and gives the correct scale for $\delta\rho/\rho$ without altering the structure of the potential V near $\phi \approx 0$.

In order to satisfy the necessary constraints, one finds a relationship between the couplings.[12] For example for $g_\phi^\phi=1$ and $g_{\phi\phi}^\phi = 0$

$$g_\phi = a > \sqrt{3} \tag{25a}$$

$$g_{\phi\phi} = 1 - a^2/2 \tag{25b}$$

$$g_{\phi\phi\phi} = (ab + 2a^3 - 5a - a^2c/3)/6 \tag{25c}$$

$$g_{\phi\phi\phi}^\phi = b/6 \tag{25d}$$

$$g_{\phi\phi}^{\phi\phi} = (1 - ac)/4 \tag{25e}$$

$$g_{\phi\phi\phi}^{\phi\phi} = c/12 \tag{25f}$$

$$3a^2 < 6 + a^3c \tag{25g}$$

ensures a flat potential and a high temperature minimum at $\phi=0$. It is straightforward to check that the choice $a=-2$, $b=4$, $c=-3$ produces a satisfactory potential. When one takes into account the effects of n chiral supermultiplets eq. (25a)-(25g) are slightly altered by letting $c \to c - n/a$ except in eq. (25f).

Another possibility for finding a model for inflation within the context of non-minimal supergravity theories is SU(n,1) supergravity.[21,22,11] At the tree level, these theories produce perfectly flat potentials.[21] Minima are determined therefore by radiative corrections which fix the scale of supersymmetry breaking as well as the scale for weak interactions. Hence, these theories are called no-scale theories as only the Planck scale is put in by hand. The form for the Kähler potential in SU(n,1) supergravity (with n chiral supermultiplets) is

$$G = -3 \ln(f(z,z^*) - \phi_i^* \phi^i/3) + \ln |F|^2 \tag{26}$$

where z is the singlet responsible for supersymmetry breaking. Recall that in our discussion of minimal supergravity, we required that the inflaton preserve supersymmetry. That rule is left to a second field z which is not directly coupled to other matter fields and is commonly referred to as the hidden sector. In a minimal theory, we could have chosen[23]

$$G(z,z^*) = zz^* + \ln|F_1(z)|^2 \tag{27a}$$

$$F_1 = \mu^2(z+\Delta) \tag{27b}$$

where $\mu \sim 10^{-8}$ and Δ adjusts the cosmological constant. In eq. (26), the z field serves essentially the same role as in minimal theories although the scale μ is no longer put in by hand.

We will assume that in eq. (26), the second, third, and fourth derivatives of $f(z,z^*)$ vanish at the minimum $z=z_o$ and that $|f_z|$=const. The scalar potential then takes the very simple form[21]

$$V = e^{2\bar{G}/3}|F_i|^2 \tag{28}$$

where $\bar{G} = G-\ln|F|^2$. It is interesting to note that the potential (28) resembles very closely the scalar potential in global supersymmetry. The value of $\bar{G}$ is determined by radiative corrections. For simplicity we can take[22] $e^{\bar{G}}=1$.

Concentrating again, only on the inflaton, we can find[11] a superpotential similar to the simple form of eq. (18)

$$F = m^2(\phi - \phi^4/4) \tag{29}$$

leading to the very simple potential

$$V = m^4|1 - \phi 3|^2 \tag{30}$$

where again $m \sim 10^{-4}$ from the constraint coming from density perturbations. Although more difficult to check, one can show that the above model also satisfies the thermal constraint, namely $V_1(\phi,\phi^*)$ has a minimum at $\langle\phi\rangle = 0$. Thus we see that previous problems[13] associated with inflation and minimal supergravity are not general and can be alleviated in non-minimal models.

Finally, I would like to point out another class of inflationary models which avoid the thermal problems, they are Linde's chaotic inflationary scenarios.[24] In chaotic inflation, a potential as simple as $V = \lambda\phi^4$ can be made to inflate and produce density perturbations with $(\delta\rho/\rho) \sim 10^{-4}$. The idea is that at very early times there is a reasonable probability that ϕ will be very far from the origin say $\phi \gtrsim M_p$. For $\lambda \sim 10^{-12}$ (as is required by $(\delta\rho/\rho)$) $V \leq M_p^4$ even for $\phi \sim 10^3 M_p$. As ϕ rolls back to $\phi = 0$, the Universe can go through an inflationary epoch. Thus it is irrelevant whether or not $V_1(\phi)$ has a minimum at $\phi = 0$. The drawback on this type of model is that one must assume that ϕ is uniform on scales much larger than the horizon $(\Delta L \sim 10^3 H^{-1})$ in order to ensure that the kinetic terms do not dominate the Lagrangian. This is perhaps an unreasonable assumption, but again, once made, the model works.

It is interesting however, that chaotic models also become[25] very simple in SU(n,1) supergravity theories. In minimal supergravity the superpotential was necessarily very complicated,[26] whereas in SU(n,1) models $F = \lambda^{1/2}\phi^3$ will produce[25] the desired scalar potential.

Finally, what remains is a physical explanation (rather than a cosmological one) for the inflaton. Our general philosophy has been that N=1 supergravity is only an effective theory which is a remnant of some extended theory, e.g. N=8 supergravity. We hope that perhaps a more natural explanation for inflation would occur there. Indeed, the SU(n,1) theories[21] do represent left over symmetries for extended (N > 4) supergravity indicating that perhaps we are searching in the right direction.

REFERENCES

1) E.B. Gliner and I.G. Dymnikova, Sov. Astron. Lett. 1 (1975) 93; A.H. Guth, Phys. Rev. D23 (1981) 347.

2) A.D. Linde, Phys. Lett. 108 (1982) 389; A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.

3) A.D. Linde, Phys. Lett. 116B (1982) 335.

4) S.W. Hawking, Phys. Lett. 115B (1982) 295; A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49 (1982) 1110; A.A. Starobinski, Phys. Lett. 117B (1982) 175; J.M. Bardeen, P.J. Steinhardt and M.S. Turner, Phys. Rev. D28 (1983) 679.

5) J. Ellis, D.V. Nanopoulos, K.A. Olive, and K. Tamvakis, Phys. Lett. 118B (1982) 335.

6) J. Breit, S. Gupta and A. Zaks, Phys. Rev. Lett. 51 (1983) 1007.

7) J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Nucl. Phys. B221 (1983) 224.

8) D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. 123B (1983) 41.

9) J. Ellis, these proceedings.

10) G. Gelmini, C. Kounnas and D.V. Nanopoulos, CERN preprint Th. 3777 (1984); C. Kounnas and M. Quiros, Paris preprint LPTENS 84/21 (1984).

11) J. Ellis, K. Enqvist, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Fermilab preprint 84-126 and CERN preprint Th. 4039.

12) L. Jensen and K.A. Olive, submitted to Nucl. Phys. B (1985).

13) B.A. Ovrut and P.J. Steinhardt, Phys. Lett. 133B (1983) 161.

14) P. Steinhardt and M.S. Turner, Phys. Rev. Lett. D29 (1984) 2105.

15) See various contributions, these proceedings.

16) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P.

Van Nieuwenhuizen, Phys. Lett. 79B (1978) 231 and Nucl. Phys. B147 (1979) 105; E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 231 and Nucl. Phys. B212 (1983) 413.

17) J. Ellis, D. V. Nanopoulos, K.A. Olive, and K. Tamvakis, Phys. Lett. 120B (1983) 331.

18) R. Holman, P. Ramond, G.G. Ross, Phys. Lett. 137B (1884) 343.

19) K.A. Olive and M. Srednicki, Phys. Lett. B (in press) 1984; P. Binetruy and M.K. Gaillard, Berkeley preprint UCB-PTH-84/19, LBL 18072 (1984).

20) G. Gelmini, D.V. Nanopoulos, and K.A. Olive, Phys. Lett. 131B (1983) 53.

21) E. Cremmer, S. Ferrara, C. Kounnas, and D.V. Nanopoulos, Phys. Lett. 133B (1983) 287; J. Ellis, A.B. Lahanas, D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. 134B (1984) 29; J. Ellis, C. Kounnas, and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406; J. Ellis, C. Kounnas and D.V. Nanopoulos, CERN preprint Th. 3824; J. Ellis, K. Enqvist and D.V. Nanopoulos, CERN preprint Th. 3890.

22) J. Ellis, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 143B (1984) 410.

23) J. Polonyi, Budapest preprint KFKI-1977-93 (1977).

24) A.D. Linde, Phys. Lett. 129B (1983) 177.

25) A.S. Goncharov and A.D. Linde, Lebedev Physical Institute preprint 1984.

26) A.S. Goncharov and A.D. Linde, Phys. Lett. 139B (1984) 27.