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COSMOLOGICAL CONSTRAINTS ON THE LIFETIME OF MASSIVE PARTICLES

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ABSTRACT

Particles with masses more than a few MeV, decaying into photons or electrons, can cause destruction by photofission of cosmologically produced light elements. A previous calculation of this effect is corrected and extended, and used to derive maximum lifetimes for massive neutrinos; these range from a few thousand seconds upwards, depending on the particle mass. Some approximate expressions are given enabling lifetime limits to be obtained for other particles, with different masses and abundances, such as gravitinos. These limits are generally stronger than previously determined constraints, such as the distortion of the microwave background by energetic photons.

I. INTRODUCTION

In recent years standard big bang cosmology has become a routine proving ground for innovations in high energy particle physics. New types of particle or interaction which may be difficult or impossible to investigate by direct experiment often have significant consequences for the evolution of the Universe. An example of this procedure is in its application to the case of hypothetical massive neutrinos. Simply by calculating the abundance of such particles relative to photons, it was concluded that neutrinos between about 100eV and 1GeV in mass must be unstable, otherwise they would dominate the present mass density of the Universe by an intolerable amount (Dicus, et al. 1978). But if neutrinos are unstable, they will produce other particles (photons, electrons, neutrinos etc.) on decaying, and consideration of the effects of the decay products leads to other constraints on the lifetime of the neutrinos. In a previous paper (Lindley 1979) it was shown that, if neutrinos produce photons of high enough energy as they decay, a strong limit on the lifetime comes from requiring that these photons should not destroy deuterium by photofission reactions. The same idea has been used to limit the abundance of evaporating primordial black holes (Lindley 1980), and has been mentioned as a constraint on the properties of supersymmetric particles (Kim, et al. 1984). Unfortunately, the original calculations given by Lindley (1979) were both inaccurate and partly in error. It is the purpose of this paper to correct those errors and omissions, and to derive more accurate constraints on the lifetimes of massive neutrinos, or particles such as gravitinos. In addition, we consider decays which produce electron-positron pairs, as well as direct radiative decays.

The essence of the following calculations can be explained concisely. A photon with energy above threshold for photofission of a particular nucleus has a small chance of destroying that nucleus; the probability is just the ratio of the rate for photoreaction to the total scattering rate. A single energetic photon from particle decay produces a cascade of lower energy photons as thermalisation proceeds, and by adding all the above probabilities for the photons in the cascade, we find the mean number of nuclei destroyed by a single initial photon. If we then know the abundance of the decaying particle, and therefore the abundance of energetic photons, we can estimate the total number of nuclei destroyed. A point to note, which will be important in what follows, is that the destruction probability per photon is inversely proportional to the total photon scattering rate; therefore, the faster photon thermalisation, the slower element destruction. In the next section we discuss the details of photon and electron thermalisation, then estimate the 'destruction efficiency' of photons as a function of energy. Finally, these estimates are used to put constraints on particle lifetimes.

This paper deals only with the destruction of light elements by photoreactions. There is also the interesting possibility that for some combination of particle mass and lifetime a small fraction of helium 4 can be destroyed to produce helium 3 and deuterium, without the latter being themselves destroyed. This question is dealt with elsewhere (Audouze, et al. 1984).

II. THRESHOLDS

Photons of modest energy (around 100MeV) can lose energy by three principal means; Compton scattering off thermal electrons, pair-production off nuclei, and pair-production off thermal photons. It was the omission of the third process which was the major error made by Lindley (1979). For higher photon energies, more exotic channels open (such as muon pair-production or hadronic showers) which we will not attempt to discuss. Double photon pair-production has a cross-section which differs from the Klein-Nishina cross-section only by phase-space factors (Jauch and Rohrlich 1976), and a threshold given by $(Ew)^{1/2} \approx m_e c^2$ (where E and w denote the energies of the decay and thermal photon respectively). Its significance is that when this process is above threshold, energetic photons thermalise extremely rapidly because thermal photons are much more numerous than electrons and nuclei. Whether or not double photon scattering is important for photons of energy $E \gg kT$ depends on the temperature; as the Universe cools, fewer and fewer photons in the thermal Planck distribution are sufficiently energetic to exceed the threshold requirement. The critical temperature when double photon scattering becomes unimportant compared to Compton scattering can be estimated by finding the point when the number of thermal photons above threshold is equal to the total number of thermal electrons, the cross-sections being comparable. The double photon cross-section has a peak value, of order the Thomson cross-section, when $(Ew)^{1/2} \simeq 1.4\ m_e c^2$ (Jauch and Rohrlich 1976). The fraction of photons above energy w in a black body distribution falls to 10^{-9} when $w/kT \simeq 25$. Therefore, at the point when

$$Ekt = \frac{(1.4m_ec^2)^2}{25} \qquad (1')$$

double photon and Compton scattering are of roughly equal importance for an electron to photon ratio of 10^{-9}; we could have chosen 10^{-8} or 10^{-10} without altering the factor 25 very much. The critical temperature and photon energy are then related by

$$E_*kT_* = 1/50 \ \text{MeV}^2 \qquad (2)$$

Photons of energy E_* are thermalised by electronic or nuclear scattering when the temperature is T_* or below; equally, at this temperature, photons of energy E_* or below are unaffected by double photon pair-production.

The threshold for photodestruction of deuterium is $Q_D = 2.225$MeV, and we are obviously interested only in photon energies higher than this. From the foregoing, the critical temperature for photon energy 2.225MeV is $9\text{x}10^{-3}$MeV. We will adopt a simple temperature-time relation for the Universe of

$$(kt/\text{MeV})^2 \ (t/\text{sec}) = 1 \qquad (3)$$

in which case the critical temperature occurs at time $t_D = 1.24\text{x}10^4$s. (We could obviously use a more accurate temperature-time relation than this by taking care to get the value of the right-hand side correct. However, we will show below that the lifetime limits obtained can be scaled easily to cosmologies in which the combination T^2t has an arbitrary value. This means that the results of this paper can be adapted to certain kinds of non-standard cosmologies, for example with different numbers of particle species.) Before t_D, all photons which are sufficiently energetic to cause photofission are very rapidly thermalised by double photon pair-production, which means that element destruction is effectively suppressed. Because helium 3 and helium 4 have higher thresholds (5.5MeV and 19.8MeV respectively), they become vulnerable to photodestruction at later times $t_3 = 7.6\text{x}10^4$s and $t_4 = 9.9\text{x}10^5$s. We will assume for the time being that photofission is completely negligible as long as double photon pair-production dominates thermalisation; this will be justified later.

The thermalisation history of photons has now become more complicated than it was in Lindley (1979). To illustrate what happens as the Universe cools, let us consider the fate of 100MeV photons. Until t_D, nothing happens. Just after t_D, the decay photons are still well into the double photon regime, and therefore produce electron-positron pairs which lose energy by inverse Compton scattering. From a photon of energy w_o, inverse Compton photons have a spectrum ranging up to a maximum energy (Jauch and Rohrlich 1976)

$$\frac{w_m}{w_o} = \frac{1 + \beta}{(1-\beta)+2w_o/\varepsilon} \qquad (4)$$

where β is the velocity of the electron, and ε its energy. Some fraction of these inverse Compton secondary photons will be above Q_D but below E_*, the double photon threshold, and will be capable of photodestroying deuterium. Later, helium 3 and then helium 4 will become vulnerable to destruction in the same way. Not until $kT = 2\times10^{-4}$MeV, $t = 2.5\times10^{7}$s, will the decay photon itself fall below the double photon threshold, and begin to lose energy by Compton scattering or nuclear pair-production. At this point, the spectrum of secondary photons, and therefore the element destruction rates, will change. However, since the energetic photons come from decaying particles whose abundance is decreasing exponentially with time, the earlier effects will be the more important. Consequently, the largest contribution to element destruction is from the inverse Compton photons resulting from double photon pairs.

Since the destructive effect of photons is almost entirely due to the secondary photons from electron-positron pairs, it is convenient to estimate the destructive power of an electron of given energy. (Positrons do not annihilate significantly until they have lost energy and become non-relativistic, so we need not distinguish them from electrons.) As we are dealing with very energetic electrons, with $\beta \simeq 1$, equation (4) for the maximum scattered photon energy can be written

$$w_m = (1+\beta)^2 w_o/[(1-\beta^2) + 2(1+\beta)w_o/\varepsilon]$$

$$\simeq 12\gamma^2 kT/[1+12\gamma kT/m_e c^2] \quad (5)$$

where the electron energy is now $\varepsilon = \gamma m_e c^2$. We have also substituted $w_o = 3kT$ for the typical energy of a thermal photon. In the low temperature limit, the maximum scattered energy is $12\gamma^2 kT$, and at high temperature, the maximum is ε. Figure 1, which illustrates the double photon threshold and the maximum scattered photon energy as functions of temperature, will help to explain what happens as an electron of given energy thermalises at different cosmological epochs. As an example, consider an electron of 50MeV, and its effect on deuterium. After time t_D, secondary photons between Q_D and E_* can destroy deuterium. At a later time (t_1 on fig. 1), the maximum secondary photon energy is equal to the double photon threshold energy, after which all scattered photons above Q_D can destroy deuterium. Finally, at time t_2, the maximum scattered photon energy falls below Q_D, and no element destruction can occur. For any electron energy, there is consequently a limited range of times when photodestruction by inverse Compton scattered photons is effective. Moreover, ^{4}He is destroyed for less time than ^{3}He, and ^{3}He for less time than D. It is also seen from fig. 1 that for each nucleus the electron energy must exceed some value (marked on the diagram), or else the maximum scattered energy will fall below the photodestruction threshold before the double photon threshold has risen above it, in which case no destruction occurs.

In order to calculate (in the next section) the element destruction rates per electron, we would like to know the complete distribution in energy of all photons resulting from the thermalisation of a single electron. This is obviously a complicated thing to do; the first set of inverse Compton photons will themselves either pair-produce or Compton scatter, leading to more photons, and so on. Instead of this, we will simply estimate the distribution of the first set of inverse Compton photons, and use that to get destruction efficiencies. This should be a reasonably accurate approach for the electrons whose energy is not much higher than the photodestruction thresholds, because photons scattered more than once will mostly be below these thresholds. Even calculating exactly the energy distribution of scattered photons for fixed electron and initial photon energy is algebraically intricate, so we will use an approximation. For most of the area in fig. 1 below the double gamma threshold, the maximum scattered energy is approximately $w_m = 12\gamma^2 kT \ll \epsilon$, and for this asymptotic case the distribution of inverse Compton scattered photons is given by Rybicki and Lightman (1979) as

$$n(w,w_m)dw = 2\,\frac{dw}{w_m}\,\left(1 - \frac{w}{w_m}\right) \tag{6}$$

This has been normalised to give an integral over $0 < w < w_m$ of unity, so that it can be interpreted as the probability density function for the distribution of scattered photon energy. We can use expression (6) for the photon energy distribution when double photon scattering is unimportant. However, we are also interested in the photon spectrum when the maximum photon energy is above the double gamma threshold in fig. 1. In this case, our assumption of a sharp transition in photon scattering behaviour at the threshold is obviously inaccurate, so there is little point in trying to make a better estimate of the inverse Compton photon spectrum. Therefore we will use equation (6) for the photon distribution in all cases, but for consistency in the energetics w_m will be given for all electron energies and temperatures by equation (5). This is what we will now use to calculate the element destruction rates by electrons.

III. DESTRUCTION RATES

The probability that a single photon causes photofission of a nucleus is the inverse ratio of the mean free path for photofission to the total mean free path. Let us first consider the case when double photon pair-production is unimportant, so that a photon either undergoes Compton scattering or pair-produces in the presence of a nucleus. (The mean free path for photoreaction is so large compared to these two that it can be ignored in calculating the total scattering rate.) The pair-production cross-section is proportional to the square of the nuclear charge, so we can write the total mean free path as

$$\lambda_T^{-1} = n_e \sigma_{KN} + (n_H + 4n_{He})\sigma_{pp} \tag{7}$$

where σ_{KN} is the Klein-Nishina cross-section, and σ_{pp} is the pair-production cross-section on a proton. This can be rewritten

$$\lambda_T^{-1} = n_e\left(\sigma_{KN} + \frac{(1+4n_{He}/n_H)}{(1+2n_{He}/n_H)}\,\sigma_{pp}\right) \tag{8}$$

The factor multiplying σ_{pp} is 8/7 for a helium abundance of 25% by mass, 1/13 by number, so we will set it to unity for simplicity; this makes the scattering rate independent of He abundance. With this approximation, the probability $p_i(w)$ that a photon of energy w destroys a nucleus of element i is therefore

$$p_i(w) = \frac{\lambda_T}{\lambda_i} = \frac{n_i \sigma_i}{n_e(\sigma_{KN}+\sigma_{pp})} \tag{9}$$

where σ_i is the appropriate photoreaction cross-section. An electron produces the inverse Compton photon spectrum $n(w,w_m)dw$ derived above so defining the quantity

$$\Sigma_i(\varepsilon,T) = \int_{Q_i}^{w_1} \frac{\sigma_i \, dw}{(\sigma_{KN}+\sigma_{pp})} \cdot n(w,w_m) \tag{10}$$

the expectation value of the number of nuclei destroyed per electron is

$$\frac{\Delta N_i}{N_i} = - \frac{n_i}{n_e} \Sigma_i(\varepsilon,T) \tag{11}$$

The dependence of Σ_i on ε and T comes from the w_m in the photon spectrum, and from the upper limit in the integral, which is $w_1 = \min(w_m,E_*)$. As explained in the previous section, only the inverse Compton photons below the double photon threshold are included in the calculation of destruction rates. The quantity N_i can be taken as the number of nuclei in a constant comoving volume, and this is clearly unchanging in the absence of photofission. If dN_E is the number of high energy electrons released into the same volume over a certain time interval, we can write the destruction of nucleus i as

$$\begin{aligned} dN_i &= -n_i\Sigma_i(\varepsilon,T)\, dN_E/n_e \\ &= -N_i\Sigma_i(\varepsilon,T)\, dn_E/n_e \end{aligned} \tag{12}$$

where we have first replaced the ratio of densities n_i/n_e by the corresponding ratio of numbers in a fixed volume, and then replaced N_E/N_e by the corresponding ratio of densities. This gives us an expression for the rate of change of N_i which does not explicitly depend on cosmological epoch or expansion. We can notionally divide N_i by the total number of baryons in the comoving volume, which means it can be directly interpreted as the fractional number density of nucleus i; this is the most convenient physical quantity to deal with. The independent variable may be regarded as n_E/n_e, the ratio of energetic electron density to thermal electron density. The equation for N_i is implicitly dependent on cosmological epoch through the temperature dependence of Σ_i. For a given electron energy, the Σ_i are easily found numerically by integrating the inverse Compton photon spectra over the cross-section ratios. The photoreaction cross-sections are experimentally determined to good accuracy at these energies, and Table 1 gives a list of references for them. All reactions have been included, except $^4He(\gamma,D)D$; the cross-section for this is not so well-known, but it is less than 1% of the total cross-section for helium 4 (Arkatov et al 1974). Figure 2 illustrates the values of the Σ_i for helium 4, helium 3, and deuterium, as a function of temperature for electron energy 100 MeV. Also illustrated are the quantities f_{ij}, which are the branching fractions for production of nucleus j from nucleus i; for instance, helium 3 produces deuterium in a fraction f_{3D} of photoreactions, otherwise producing free nucleons only. (The f_{ij} are not used in this paper, but are important in the calculations of Audouze, et al. (1984)). The shapes of the curves for the Σ_i, and the typical magnitudes, are much the same for different electron energies.

When double photon pair-production dominates thermalisation, a similar calculation can be done. The difference is that the total photon scattering rate is controlled by the thermal photon density rather than the electron density. Consequently, we can rewrite equation (12) as

$$dN_i = -N_i \Sigma_i' \, dn_E/n_\gamma \tag{13}$$

where Σ_i' is now calculated using the photon-photon pair-production cross-section instead of the $\sigma_{KN}+\sigma_{pp}$ of equation (10), and the independent variable is the ratio of energetic electron density to photon density. We expect that Σ_i' will have the same order of magnitude as Σ_i. However, if the decay electrons come from gravitinos their total abundance is perhaps 1/50 of the photon density, and in the case of massive neutrinos even less. Therefore we expect the maximum possible change in N_i due to particle decay to be no more than $\Delta N_i/N_i = 10^{-3}$ if double photon scattering dominates thermalisation. This justifies our neglect of this regime in deriving constraints on the particle lifetimes, but it is interesting to note that one might be able to change a fraction 10^{-3} or 10^{-4} of helium 4 into lighter elements; this would be in the right range for present day abundances of helium 3 and deuterium. This point is discussed in greater detail by Audouze, et al. (1984).

The same argument can be used to show that electrofission reactions (i.e. destructions of nuclei through the impact of electrons or positrons) are also negligible. The cross-sections for such reactions are similar in magnitude to the corresponding photoreaction cross-sections, because an electron passing by a nucleus can be regarded as producing a field of virtual photons which interact with the nucleus (Bishop 1964). Electrofission reactions occur in competition with inverse Compton scattering of energetic electrons by thermal photons, and therefore the destruction rate by electrons is of similar magnitude to the destruction rate by photons when double photon scattering is important. By the reasoning of the previous paragraph, we can neglect the destructive effect of electrons.

IV. LIFETIME LIMITS

Consider a particle X with primordial abundance x_0 relative to thermal electron density, with mass m_X and lifetime τ. There are two decay paths which we can discuss using the quantities calculated above. The simpler possibility is $X \to e^+e^-Y$, where Y is a particle which we assume to be non-interacting cosmologically. In this case we can use directly the destruction rates Σ_i and write the change in abundance of element i as

$$\Delta N_i/N_i = -2x_o \int_{t_i}^{t_2} \Sigma_i(\varepsilon,T)e^{-t/\tau}\, dt/\tau \qquad (14)$$

where the factor 2 is because there are two electrons per X decay, and we assume that the decay electrons all have the same energy, $\varepsilon = m_X/3$. An alternative decay is $X \to \gamma Y$; at early times (before t_1, when the photon energy is greater than the double photon threshold), an electron-positron pair is produced, and photodestruction proceeds according to equation (14). After t_1, thermalisation changes, the decay photon either Compton scattering or pair-producing. If the latter dominates (which is the case for photon energies greater than about 120MeV), then equation (14) is exactly right, but if the photon Compton scatters then we should calculate Σ_i differently. However, because we are dealing with an exponentially decaying source of photons or electrons, it makes essentially no difference what happens at late times; one could truncate the integral in equation (14) at t_1 instead of t_2 and get the same answer. We will use equation (14) regardless of whether decay occurs to electrons or to photons. The only difference is that in the first case, we shall take the electron energy to be $m_X/3$, while in the second case it will be $m_X/4$. This is significant if the abundance of X depends on its mass. It should be noted that this analysis is applicable only up to recombination, after which the Universe is essentially transparent to photons. In calculating the integral of equation (14), we put t_2 equal to 10^{12}s if it is calculated to be greater than that. This really makes no difference, however.

To get an estimate for the critical lifetime, we demand that the right hand side of equation (14) should be one; in other words, we take the maximum allowed lifetime to be that which gives one e-fold in element abundance. We can now calculate permitted lifetimes for massive neutrinos and for gravitinos.

a) Massive Neutrinos

Assuming standard weak interactions, Dicus, et al. (1978) calculated the freeze-out temperature of massive neutrinos, and hence their abundance relative to photons. For masses greater than a few MeV (which we need to cause photodestruction) we obtain from their results

$$x_o = 5 \times 10^5 \, (m_\nu/100\text{MeV})^{-2.75} \eta^{-1} \tag{15}$$

The electron to photon ratio enters into this abundance ratio as the parameter η, defined by $n_e/n_\gamma = 5\eta \times 10^{-10}$. For a standard cosmology, η has a value close to one (Yang et al. 1984). The more likely decay is to give e^+e^- directly once the neutrino is more massive than a few MeV, and figure 3 shows the maximum allowed lifetime, according to equation (14), for this decay, and for $\eta = 1$ or 10. A limit is obtained by considering destruction of each of helium 4, helium 3 and deuterium, but clearly the last gives the strongest limit. Also shown in fig. 3 is the η=1 limit from deuterium, assuming a decay giving photons rather than pairs, so that $m_\nu = 4\varepsilon$. This curve is barely distinguishable from the pair decay limit.

The dependence of the maximum lifetime on neutrino mass turns out to come mostly from the variation of x_0 with m_ν, not from the variation of Σ_i with electron energy. A simple argument illustrates this. If we take the integral of equation (14) and integrate twice by parts we obtain (note that $\Sigma_i(t_i) = \Sigma_i(t_2) = 0$)

$$\int_{t_i}^{t_2} e^{-t/\tau}\Sigma_i(dt/\tau)$$

$$= [-\tau e^{-t/\tau}(d\Sigma_i/dt)]_{t_i}^{t_2} + \tau\int_{t_i}^{t_2} e^{-t/\tau}(d^2\Sigma_i/dt^2)dt \tag{16}$$

From fig. 2, we see that Σ_i rises rather abruptly from zero at t_i, so let us assume that the largest contribution to the above comes from the integrated part, evaluated at t_i (the endpoint contribution is exponentially suppressed). Further, if Σ_i is roughly independent of electron energy, then we expect the shape of the lifetime curves of fig. 3 to be given roughly by

$$\ln \tau/t_i - t_i/\tau = \text{const} + \ln x_o(m_\nu,\eta) \tag{17}$$

In fig. 3, such a curve has been fitted to the deuterium limit at the point η=1 and m_ν = 100MeV, and gives a reasonable approximation to the exact results. The fit is worse at lower abundances (higher lifetimes) because the approximation of the integral by the first term integrated by parts is worse as the lifetime increases. However, the calculation of the destruction efficiencies Σ_i using only the primary scattered photon spectrum is itself unreliable for higher energies. The results of this section are trustworthy up to electron energies of perhaps 100MeV, but doubtful above that. When the lifetime limits of fig. 3 begin to rise sharply, the calculation is very sensitive to small errors. In the next subsection we give an approximate method of dealing with very high energy particles.

(b) Gravitinos

Supersymmetric theories introduce a variety of new particles. Two which may be of cosmological importance are the fermionic partners of the photon and graviton, the photino and gravitino. (For a review of some aspects of cosmology with supersymmetry, see Olive 1983). The mass of these particles depends on the particular supersymmetric theory being offered, but in general is expected to be of order 100GeV. In standard supersymmetry, one of these two particles is lighter than the other and absolutely stable; if the gravitino is heavier, it can decay into a photon and a photino. There are more exotic schemes, such as the proposal of Kim et al (1984) to incorporate Peccei-Quinn symmetry breaking into supersymmetry. This has the axino (the partner of the axion) as the lightest stable superpartner, and there is a decay in which the photino turns into a photon and an axino. In addition, photino and gravitino masses are strongly model dependent, and in specific theories, may be in the MeV or GeV range (Dawson et al. 1983). Rather than get involved in the intricacies of supercosmology, we shall call a gravitino any particle which decays into a photon and a cosmologically uninteresting (from our point of view) particle. The major distinction between gravitinos and massive neutrinos is that they decouple much earlier, while still relativistic, and are consequently about as abundant as photons, regardless of mass. This immediately introduces a problem, since one cannot have a Universe containing particles of 100GeV with the same abundance as photons at times of around one second without running into difficulties with the total mass density. This gravitino problem may be solved by inflation (Nanopoulos, et al. 1984), which can put the gravitino abundance at essentially zero. However, reheating after the inflationary phase may produce gravitinos again. The upshot of all this is that supercosmology can provide new unstable particles of almost arbitrary abundance and mass. We will therefore give some general consideration of the limits that photodestruction can place on such a population.

For gravitinos of a particular mass and abundance, there is a maximum lifetime given by requiring that they should not dominate the mass density of the Universe. Using the temperature-time relation above, and taking the sum of photon and neutrino densities for the total cosmic density, one easily finds

$$\tau < 2.1 \times 10^{-3}\mathrm{s}/(n_{\tilde{g}}/n_\gamma \cdot m_{\tilde{g}}/100\mathrm{MeV})^2 \qquad (18)$$

If gravitinos come to dominate the Universe, and decay later, they will upset the standard relations between nucleosynthesis and baryon to photon ratio, so the above lifetime can be taken as an upper limit. (It may be possible to construct a non-standard cosmology in which gravitinos temporarily dominate the Universe, but we will not consider this here.) We want now to consider the photodestructive effects of decays to see if a stronger limit can be obtained. This could be done by using equation (14), putting in a mass and abundance, and finding numerically the maximum lifetime. However, this would be both time consuming and, at electron energies of more than a few hundred MeV, inaccurate because of the approximations made in deriving the Σ_i.

Instead, we shall try to give an approximate analytic expression for maximum lifetime in terms of particle mass and abundance. The destruction efficiencies Σ_i, for electron energies not far above the photonuclear thresholds, vary in a way which is determined by the complicated form of the inverse Compton photon spectrum and the various photoreaction cross-sections. However, for higher energies, one might expect an approximate scaling behaviour of the Σ_i with energy. For photon energies greater than a hundred MeV or so, pair-production is always the dominant scattering process, whether off thermal photons or baryons; in addition, for electron energies somewhat higher than this, more than a few GeV, inverse Compton scattering is in the regime where the maximum photon energy is equal to the electron energy. Under these conditions it is possible to show that the spectrum of scattered photons, including mutiple scattering, should be of constant shape, and scale in magnitude proportionately to the energy of the initial particle (Lindley 1980, Appendix). Because all the photoreaction cross-sections peak at some tens of MeV, and fall off as power-laws beyond that, the existence of a cut-off in the scattered photon spectrum at some high energy is unimportant in determining the destruction rates Σ_i. For initial decay particles, either photons or electrons, of sufficiently high energy, we therefore expect approximate behaviour

$$\Sigma(\varepsilon,T) \approx \varepsilon/\varepsilon_0 \; \Sigma\,(\varepsilon_0,T) \tag{19}$$

since the low energy part of the scattered photon spectrum is of constant shape. This scaling in Σ_i, when applied to the integral of equation (14), can be treated in the same way as the scaling with x_0; a tenfold increase in particle mass is equivalent to a tenfold increase in abundance. We can therefore approximate the lifetime limit for very massive particles (greater than about 1GeV) as the solution of

$$2x_o \cdot \varepsilon/\varepsilon_o \int_{t_i}^{t_2} \Sigma\,(\varepsilon_o,T)e^{-t/\tau}\,dt/\tau = 1 \tag{20}$$

It should be noted that this scaling does not apply very well to the calculated Σ_i above, partly for the reasons given, that the variation of the photon spectrum and the photonuclear cross-section is more complicated at lower energies, and partly because the approximation of using only the once-scattered inverse Compton photons to find Σ_i is worse at higher energies. In fact, the calculated values of the Σ_i tend to decrease roughly in proportion to the electron energy, because the spectrum (6) goes inversely with the maximum scattered photon energy, and near t_D, w_m is approximately equal to initial electron energy. Use of equation (4.7) means that the maximum lifetime can be found in terms of a single parameter, the product of mass and abundance; the only other variation comes from choosing different electron energies from which to do the scaling. This new parameter is of course nothing more than the mass density of gravitinos, appropriately normalised; it equivalently measures the energy density of decay electrons. Figure 4 illustrates the lifetime limit in terms of this parameter, with the scaling point chosen to be 100MeV in the hope that this is high enough for the scaling to be reasonably good, but low enough for the numerical calculation of Σ_i to be reasonably accurate. We define the lifetime limit as a function of

$$\rho_{100} = x_o(\varepsilon/100\text{MeV}) \tag{21}$$

which can be thought of as the equivalent density of 100MeV decay particles. The 'error bars' in fig. 4 are intended to convey the difference in maximum lifetime obtained using different scaling points. They were obtained by scaling from 50MeV and 200MeV. The straight line on fig. 4 illustrates the lifetime limit (18), using $m_{\tilde{g}} \approx 3\varepsilon$, which requires that gravitinos should never dominate the density of the Universe. Except for particles which are near the bottom right-hand corner of the figure, the photodestruction limit is stronger than the simple mass density limit. (The exact position of the density limit line depends on the baryon to photon ratio, since the number density of gravitinos is taken relative to the thermal electron density; we chose $\eta = 1$ for this line.) Also shown on this diagram is the lifetime limit of the previous section on massive neutrinos, for the $\eta = 1$, $m_\nu = 3\varepsilon$ case, translated into the new parameter. For low neutrinos masses (short lifetimes), the directly calculated limit is presumably better than the scaling law limit, while for high neutrino masses (long lifetimes), the direct calculation of Σ_i is inaccurate, and the scaling law gives the better limit. A point to mention is that the form of the scaling law can be justified better than the rather arbitrary normalisation to 100MeV electrons that was chosen. To get a more reliable line on fig. 4, one could in principle do a complete numerical calculation, including all multiply scattered photons and electrons, which would allow the proportionality of the Σ_i with energy to be fixed absolutely in magnitude. However, doing such a calculation involves either solving a pair of coupled integro-differential equations for the energy distribution of scattered photons and electrons, or else performing a Monte Carlo simulation of all the scattering processes. Even so, only energies of order some hundreds of MeV could be reliably analysed because of the increasing number of scattering processes which open up at higher energies. Such a calculation has not been attempted. An appropriate conclusion to this section is the rule-of-thumb that for decay particles of less than about 100MeV, the numerical calculations of the previous sections should be employed, while for higher energies, fig. 4 is useful, with the caveat that the normalisation is somewhat uncertain.

(c) Scaling to Different Cosmologies

It was mentioned in section II that the lifetime limits can be adapted to a cosmological model in which the temperature-time relation has T^2t equal to an arbitrary constant. The only place that time enters explicitly into the calculation is in the integral (14). All other times, such as t_D, are derived from temperatures. In this integral, any scaling of cosmic time t can be accompanied by the same scaling in lifetime τ to leave the expression unchanged in value. The adopted relation (3) implies that a temperature of 0.1MeV occurs at a time of 100s. In standard cosmology, with a density during the radiation era made up of photons and three species of massless neutrinos, the correct temperature-time relation has 0.1MeV occur at 132s (Weinberg 1972). In this case, one should therefore increase the upper limits on lifetime in all the above by 32%. However, in a cosmology with a variety of supersymmetric relic particles, the expansion rate will be faster than standard; by calculating the temperature-time relation in one's favourite cosmology, the lifetime limits of this paper can be scaled appropriately.

V. CONCLUSIONS

The calculations in this paper are an improvement on the estimates given earlier (Lindley 1979), and somewhat more general, but contain a number of faults. The two major sources of inaccuracy are the neglect of multiple scatterings (we used only the primary inverse Compton spectrum), and the rather brusque treatment of the double photon pair-production threshold. In principle, better estimates could be made in both these areas, but it would demand some tedious calculation. However, for a considerable range in particle mass and abundance, (the flatter part of the curve in fig. 4), the values of the maximum lifetime that we obtained are only moderately sensitive to the values of the destruction coefficients Σ_i, because of the exponential dependence on the lifetime of the number of decay particles causing photofission. Uncertainties are much more significant on the steeper part of fig. 4, because as the abundance of decaying particles falls there is a rather rapid transition from considerable photodestruction to none. The numerically calculated results given above should be quite accurate, especially for electron energies not much greater then about 100MeV. The approximate results embodied in figure 4 allow lifetime limits to be estimated for particles with higher masses than the direct calculations can accommodate. This also allows limits to be deduced for other massive decaying particles, such as the generic 'gravitino' discussed in section IV.

The lifetime limits obtained above are of order some thousands of seconds or more. This is in many cases stricter than other upper limits; the next most severe constraint probably comes from consideration of the distortion of the microwave background by high energy photons and electrons (Silk and Stebbins 1984; Dicus, et al. 1978). The latter is a hard thing to calculate, but one usually gets maximum lifetimes of order 10^4 to 10^5s, depending on the photon to baryon ratio and the present division of cosmological density into baryons, photons and possible missing mass candidates. We therefore give the main conclusion that photodestruction of light elements gives, for particles of sufficient mass, the strongest limit on lifetime.

Finally, we refer again to Audouze, et al. (1984), who discuss the possible creation of interesting quantities of deuterium and helium 3 by the photodestruction of small amounts of helium 4. This does not affect the lifetime limits of this paper; what one finds is that for particle lifetimes a little less than the permitted maximum for helium 4 destruction, creation of the lighter elements can indeed occur. Once the maximum is reached, however, all elements are destroyed to an unacceptable degree.

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TABLE I

Reaction	Threshold (MeV)	Reference
$^{4}He(\gamma,p)^{3}H$	19.8	Arkatov et al. 1971
$^{4}He(\gamma,n)^{3}He$	20.6	Arkatov et al. 1976
		Ferrero et al. 1966
$^{4}He(\gamma,pn)^{2}H$	26.1	Arkatov et al. 1976
$^{4}He(\gamma,2p)2n$	28.3	"
$^{3}He(\gamma,p)^{2}H$	5.5	Gorbunov & Varfolomeev 1964
$^{3}He(\gamma,n)2p$	7.7	"
		Berman et al. 1974
$^{2}H(\gamma,n)p$	2.225	Evans 1955
		Partovi 1964

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FIGURE CAPTIONS

Figure 1: Threshold energies as a function of temperature. Above the straight solid line, photon-photon scattering dominates and no photodestruction occurs. The curved lines are the maximum photon energy from inverse Compton scattering of an electron of the energy indicated on each curve. The horizontal broken lines are the photo-nuclear thresholds for (from top to bottom) ^{4}He, ^{3}He, and D. The three broken curved lines show the minimum electron energy necessary to cause any photodestruction of the three nuclei.

Figure 2: The solid lines are the destruction efficiencies as a function of temperature (defined in the text) of 100 MeV electrons for ^{4}He, ^{3}He, and D, as marked. The broken lines are the branching functions for splitting ^{4}He into ^{3}He, or D, and for ^{3}He into D. (Photodestruction of ^{4}He also yields ^{3}H, but this decays into ^{3}He and is included in f_{43}.)

Figure 3: Lifetime limits for massive neutrinos. The curves show the maximum lifetime against destruction of ^{4}He, ^{3}He, and D as a function of neutrino mass, with electron energy taken to be one-third of the mass. The upper solid line for D has fewer neutrinos per thermal electron (η=10 instead of η=1), and the broken line has electron energy one-quarter of the neutrino mass. The crosses show an analytic fit to the curves.

Figure 4: Estimated maximum lifetime for particles of arbitrary mass and abundance, as a function of the parameter ρ_{100} = $(n_x/n_e)(\varepsilon/100\mathrm{MeV})$, and with electron energy one-third of the particle mass. The straight line is the limit from requiring that the universe is never dominated by these particles. The broken line

is the η=1 limit from deuterium, of the previous figure, translated into the parameter ρ_{100}.

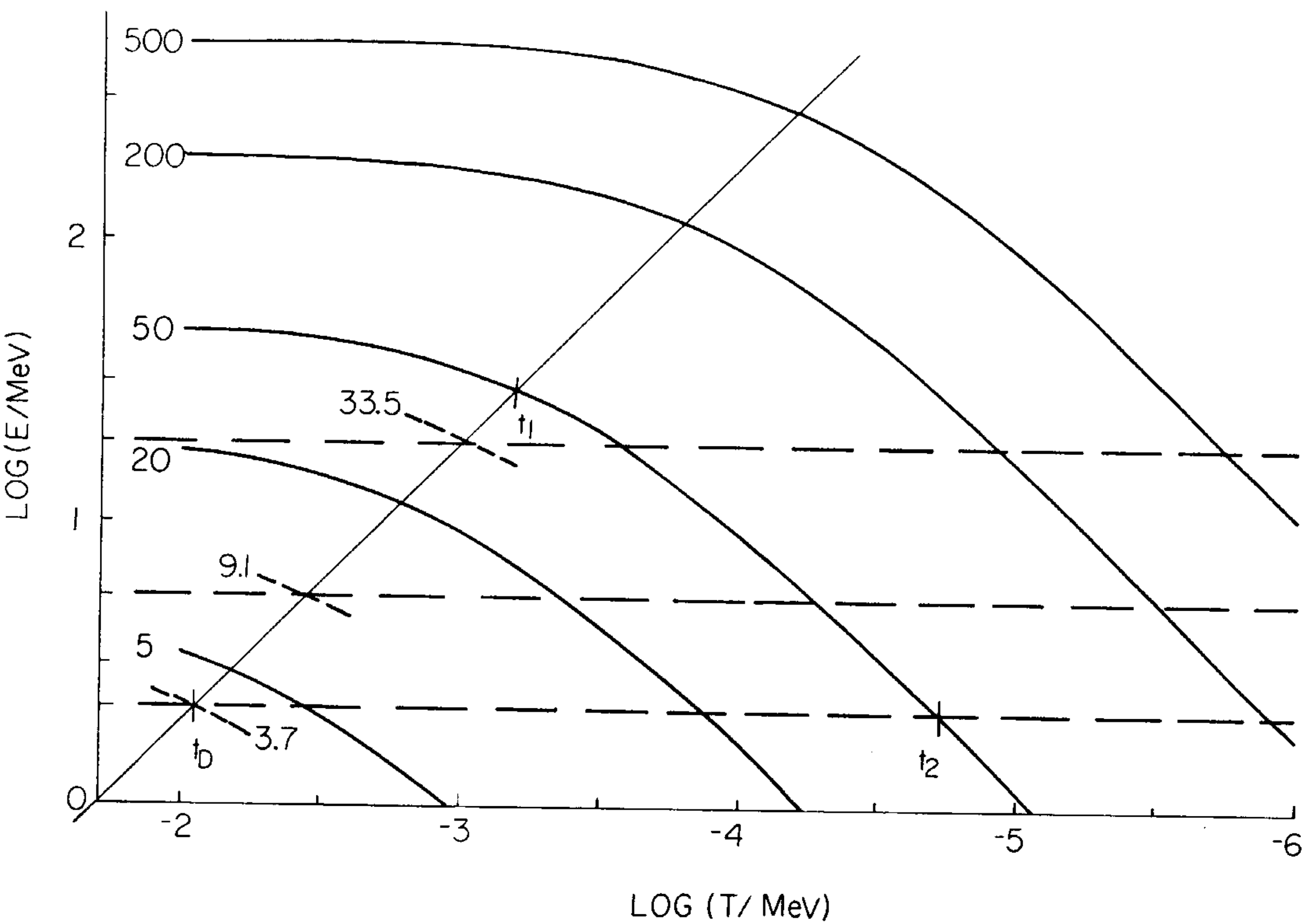
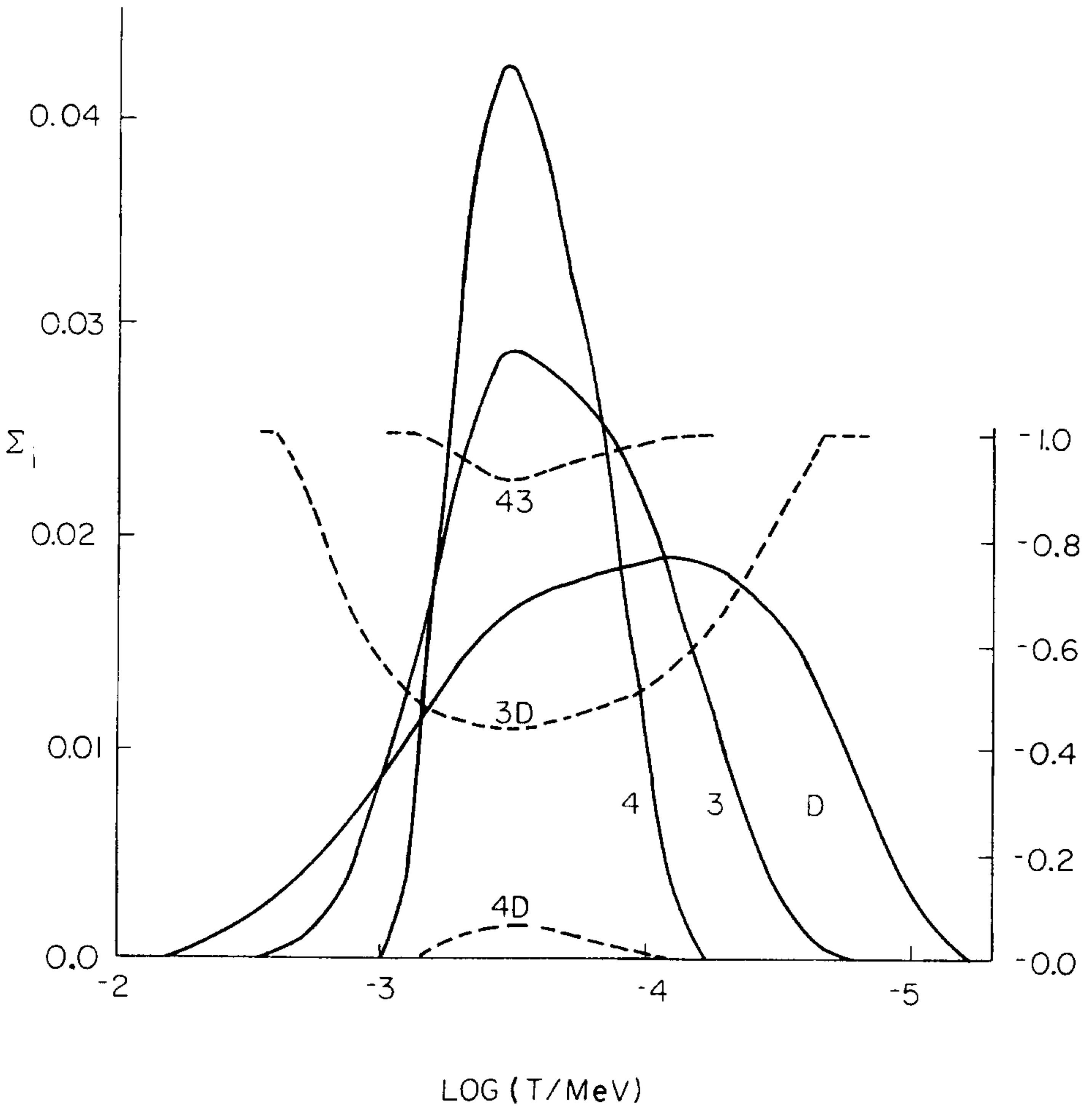


FIGURE 1



0.04
0.03
0.02
0.01
0.0
Σ_i
43
3D
4D
4
3
D
-1.0
-0.8
-0.6
-0.4
-0.2
-0.0
-2
-3
-4
-5
LOG (T/MeV)

FIGURE 2

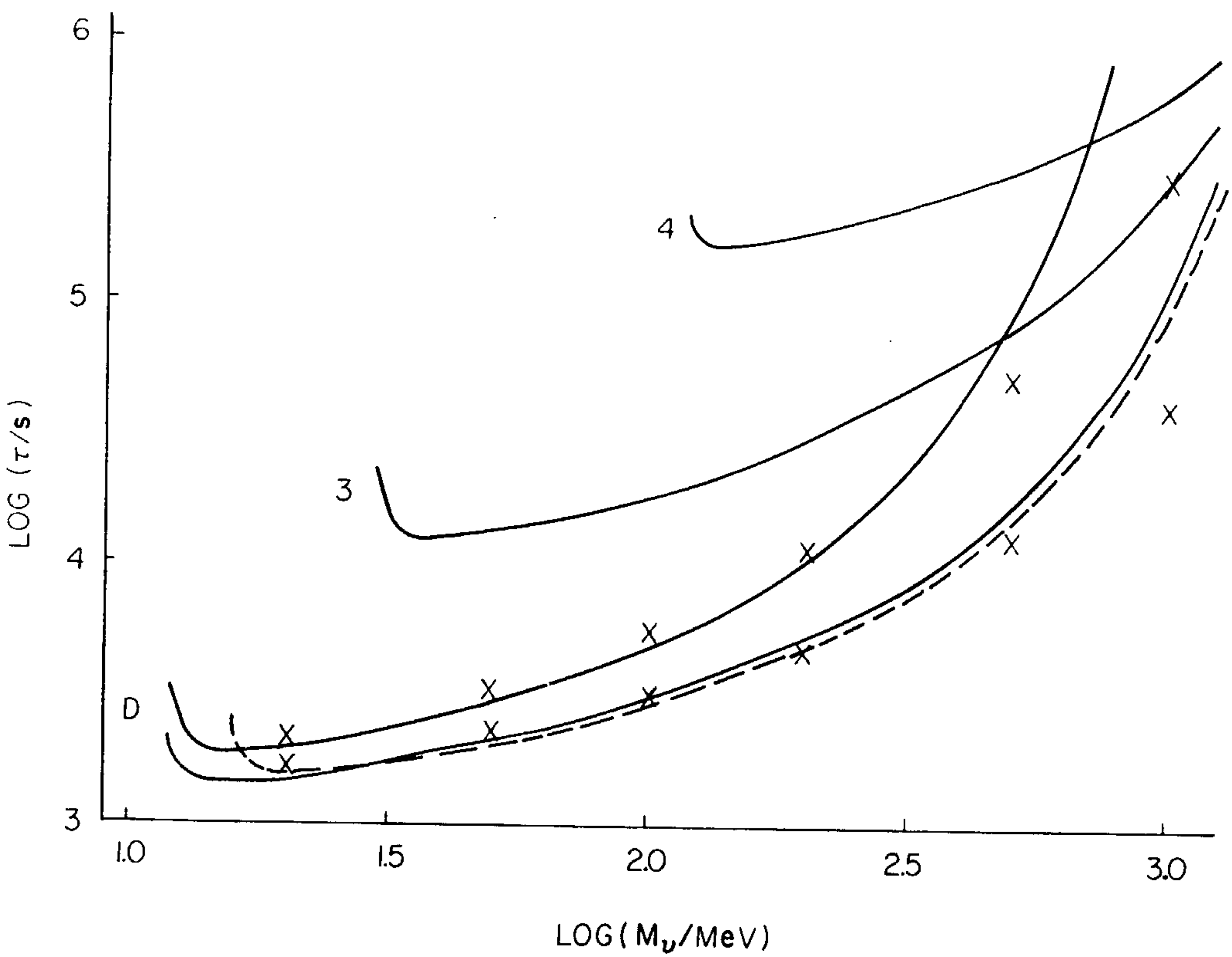


FIGURE 3

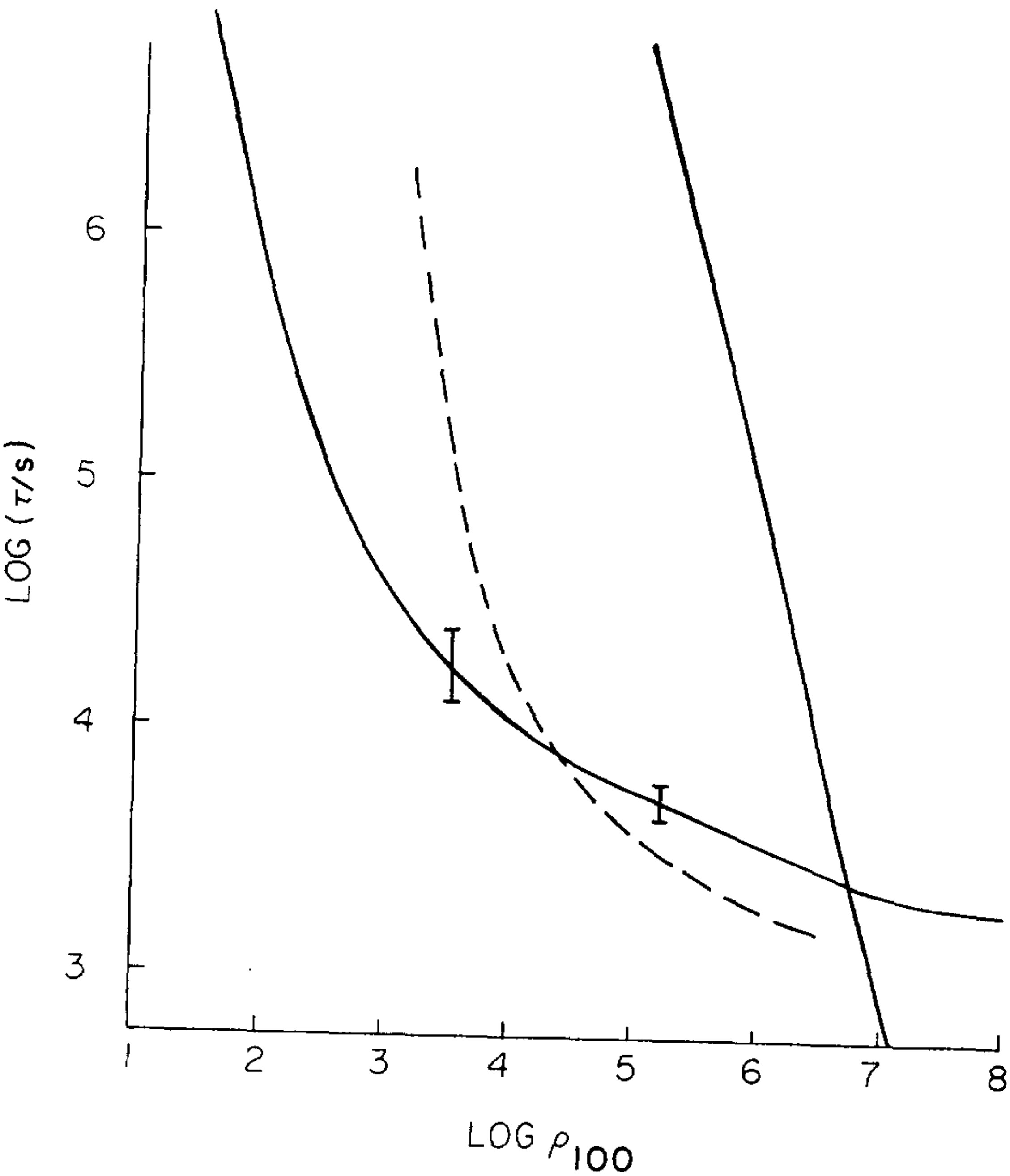


FIGURE 4