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Dynamical Resonances in Strongly Coupled Higgs Models

Pham Quang Hung†
Department of Physics
University of Virginia
Charlottesville, VA 22901

and

H. B. Thacker
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, IL 60510

Abstract

The scattering of Goldstone and longitudinal gauge bosons in weak interaction theories with a strongly coupled Higgs sector is studied. By combining low energy theorems with analyticity, unitarity, and crossing symmetry, it is argued that, quite generally in such theories, one would expect a prominent resonance in the I=J=1 channel (where I=weak isospin) analogous to the ρ resonance in π-π scattering. The possibility that this resonance is at 170 GeV and is responsible for some of the recent unusual CERN collider events is explored. The minimum requirement for such an interpretation is a model with two Higgs doublets whose vacuum expectation values differ by a factor of five to ten. This is the same model and about the same ratio of vacuum expectation values needed to interpret the recently discovered $\zeta(8.3)$ as a Higgs particle.

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I. Introduction

In spite of the overwhelming success of the standard Glashow-Weinberg-Salam model of electroweak interactions, the details of the Higgs sector of the theory are still poorly understood. An intriguing possibility which was discussed some time ago by Veltman[1] and by Lee et. al.[2], is that the Higgs sector of the theory is strongly interacting, with a self-coupling of order unity.[3-4] The low energy weak interactions are effectively screened from the dynamics of the Higgs field, being sensitive only to its vacuum expectation value, and because of this there are presently no experimental limits on the strength of the Higgs self-coupling.

Recent unusual events from the CERN UA1 and UA2 experiments[5] have prompted us to reconsider the dynamics of a strongly interacting Higgs sector. Although the interpretation of these events is far from clear,[6] some of them might be regarded as decays of an object with mass around 160-180 GeV. Indeed Veltman[7] has suggested that this object may be a bound state of gauge bosons. Such bound states might occur quite naturally in a theory with strongly interacting Higgs fields, since longitudinal gauge bosons also exhibit strong self-interactions in such a theory. In addition to this possible experimental motivation, theoretical considerations lead us to take the possibility of a strongly interacting Higgs sector seriously. If the Higgs fields which are responsible for the breakdown of SU(2)xU(1) are composite, such as in technicolor models, it might be expected that they would have strong self-interactions due to residual technicolor forces, just as pions have strong interactions due to residual color forces.

In this paper we shall investigate the properties of a strongly interacting Higgs sector, focusing particularly on the question of whether such a theory might give rise to a bound state or resonance in the 160-180 GeV range which could account for some of the CERN events. The approach we take to this problem is similar to that of Ref. 2. We consider the implications of partial wave unitarity in various two-body amplitudes which describe the scattering of longitudinal gauge and Higgs bosons. Since the gauge couplings are small, we will neglect them and retain only the interactions of the Higgs sector. Longitudinal gauge bosons are represented by the corresponding Goldstone modes of the Higgs field. In Ref. 2, the analysis of the full weak interaction theory was reduced to a study of the Higgs Lagrangian in the high energy limit ($s \gg m_W^2$). Here we are considering the possibility of a resonance with a mass around 170 GeV, and so do not wish to restrict our discussion to the case $s \gg m_W^2$. A sensible approximation at all energies is to neglect terms in the Lagrangian which represent real gauge interactions of order g, but retain the gauge boson mass terms (of order g^2v^2) which are generated by spontaneous symmetry breakdown. Thus, the theory we will investigate is described by the Higgs Lagrangian plus explicit mass terms for the Goldstone (longitudinal gauge) bosons. This theory is just a linear sigma model, and the study of longitudinal gauge boson scattering is quite similar to that of π-π scattering in hadron physics. The gauge boson mass terms are analogous to a pion mass term which explicitly breaks chiral symmetry. To the extent that these mass terms are small (e.g. compared to the mass of the physical Higgs scalar in a minimal one doublet model) one may derive a low energy theorem for the scattering of gauge bosons analogous to Weinberg's result for π-π

scattering.[8] In the latter case, the behavior of π-π scattering below 1 GeV is to a large extent determined by the low energy current algebra constraints combined with analyticity, unitarity, and crossing symmetry. Our treatment of gauge boson scattering parallels the discussion of π-π scattering by Brown and Goble.[9] In that treatment, unitary amplitudes are constructed via an effective range expansion for s- and p-wave phase shifts. Similar conclusions are reached if one follows the approach of Basdevant and Lee,[10] who construct unitarized π-π scattering amplitudes by Padé approximation of σ-model perturbation theory.[11]

Some of our main results concerning the behavior of strongly interacting Higgs theories can be understood by pursuing the π-π analogy. Let us first consider the minimal Weinberg-Salam model with a single complex doublet. In Ref. 2, the implications of partial wave unitarity for the s-wave scattering of longitudinal gauge bosons was considered. In that work, a unitarized s-wave amplitude was constructed via the N/D method (with the neglect of left-hand cuts, this is essentially equivalent to the effective range procedure employed here). The elementary Higgs scalar appears as a pole in this amplitude. It was found that for weak Higgs self-coupling λ ($\lambda \ll 1$) the s-wave pole is near the real axis and the Higgs scalar is a narrow, well-defined state with a mass near the perturbative value of $m_H^2=2\lambda v^2$. As λ approaches unity, the Higgs particle becomes heavier and more unstable. For a strongly coupled theory ($\lambda \gtrsim 8$) the Higgs pole has migrated far away from the real axis on the second sheet of the complex s plane and is no longer a well-defined resonance. As we will see in the following Section, the nature of the elementary Higgs scalar in the strongly coupled one-doublet model is expected to be similar to that of the I=J=0 partial wave in π-π scattering. In the latter case the unitarized current algebra result is in reasonable agreement with experimental data. (Here the Padé result is in somewhat better quantitative agreement than the effective range formula.) Both theory and experiment suggest that the I=J=0 π-π phase shift is positive, increases to near 90° at around 600 MeV, and remains close to 90° over a broad range of energies (from 600 to at least 900 MeV). Since this behavior seems to be essentially dictated by current algebra, analyticity, unitarity, and crossing symmetry, we expect that these results can be carried directly over to the strongly coupled one-doublet model of weak interactions with an appropriate change of scale for f_π. Thus, the s-wave scattering of longitudinal gauge bosons should exhibit strong interactions (large phase shift) but no clearly defined scalar resonance. (There is of course a whole range of possibilities between weak and strong coupling for which the Higgs scalar would be more or less well-defined.) Next we apply the same considerations to the I=J=1 partial wave. This is in many ways the most interesting channel because of the prominent ρ resonance that appears in the p-wave π-π amplitude. As in the s-wave case, the behavior of the p-wave phase shift is dictated largely by general principles, which would suggest a similar behavior in the weak interaction model. In Section II we consider p-wave scattering of longitudinal gauge (Goldstone) bosons and obtain one of the main results of our analysis: the expectation that a new I=J=1 (where I=weak isospin) resonance will appear in the Weinberg-Salam model with strongly coupled Higgs fields. In technicolor models, this resonance would be identified with the technirho. But our analysis suggests that its existence is a general property of strongly interacting Higgs models, independent of any underlying dynamical model for the Higgs fields. We might also expect higher spin resonances analogous to the f^0, g, etc., though with somewhat less certainty, because current algebra makes no statement about partial wave amplitudes for J>1. Our main focus in this paper will be on the I=J=1 resonance, which we will refer to as $\mathscr{V}$, since, as we will see, such a resonance (in a two-doublet model) is a possible candidate for some of the CERN collider events. The I=J=1 resonance is singled out by the fact that it can mix with the ordinary W and Z bosons. This provides a production mechanism which, at present energies, is much more efficient than other mechanisms such as virtual W-W scattering.

Having suggested this new vector boson resonance, we then address the question of whether it could possibly have a mass in the 170 GeV region. For a one-doublet model, the answer is probably negative. The effective range parametrization of the p-wave π-π phase shift has two parameters, the scattering length and effective range, which determine the mass and width of the ρ. Of these, only the scattering length is determined by current algebra. This yields a successful relation between m_ρ and Γ_ρ or $g_{\rho\pi\pi}$ but does not determine m_ρ. However, it seems reasonable to expect the analogy between π-π scattering and longitudinal gauge boson scattering to go beyond current algebra, since they are both presumed to be well-described by a linear σ-model with $m_\sigma^2 \gg m_\pi^2$ $(m_H^2 \gg m_W^2)$. From this more general point of view we would expect the mass of the vector resonance to be determined from the basic parameters m_π^2 and f_π (m_W^2 and v=Higgs vacuum expectation value), and to be fairly insensitive to m_σ^2. Such arguments suggest that in the one-doublet model, the new vector resonance would be in the 2 TeV mass range, far too heavy to be relevant to the CERN events.

The same arguments which suggest that $m_{\mathcal{V}} \approx 2$ TeV in a one-doublet model also show that what is needed to produce a resonance of mass $m_{\mathcal{V}} \approx 170$ GeV is a Higgs field which is strongly coupled but has a much smaller vacuum expectation value than the one-doublet value of $v=250$ GeV. This is easily arranged by enlarging the Higgs sector to include more than one field which acquire vacuum expectation values. Since the observed Fermi constant only requires $(\Sigma v_i^2)^{1/2}=250$ GeV, the individual VEV's may be much smaller than in the one-doublet model. The easiest way to enlarge the Higgs sector without destroying the successful prediction of the ρ-parameter (i.e. $m_W^2/m_Z^2\cos^2\theta$) from the minimal model is to introduce additional Higgs doublets.[12-14] It is easy to see that additional doublets do not alter the result $\rho=1$. Thus, we will consider a model with two Higgs doublets. Interpreting the CERN events as an $I=J=1$ resonance, we are led to choose the two vacuum expectation values to be $v\approx 50$ GeV and $V\approx 245$ GeV. The current algebra-effective range analysis applied to this model yields a value for the $\zeta\zeta$ coupling constant (where $\zeta^{\pm,0}$ are the charged and neutral physical Goldstone bosons of the two-doublet model). Using this result we can study W-$\mathcal{V}$ and Z-$\mathcal{V}$ mixing and estimate the production cross-section via this mechanism. This leads to a prediction of around 5 $\mathcal{V}$'s produced for the integrated luminosity of the CERN collider experiments. Since this must be multiplied by the branching ratio into a particular exotic signature (e.g. a monojet), a total of 5 $\mathcal{V}$'s is not large enough to explain the CERN data. However, the production rate is only loosely constrained by our arguments and may in fact be larger.

In the investigation described here we were led by consideration of the CERN events to favor a model with two Higgs doublets with vacuum expectation values which differ by about a factor of 5. Shortly after the completion of this work, a remarkable new resonance with a mass of 8.3 GeV was announced by the Crystal Ball collaboration.[15] This resonance, called ζ, was seen in the decay $\Upsilon \to \gamma + \zeta$. It is tempting to interpret the ζ as some type of Higgs particle, but the observed branching ratio in Υ decay is about a factor of 100 too large to be the Higgs scalar of the minimal model. Several authors[16] have pointed out that, in a two-doublet model, the ζ has a natural interpretation. In addition to the gauge bosons, this model contains three neutral and two charged particles which are physical. Two of the neutral scalars are radial Higgs modes analogous to the Higgs scalar of the minimal theory. In our strongly interacting scenario, we would expect at least one of these scalars to be very broad and massive [See note below]. The third neutral particle is a Goldstone mode. It's mass results from a term in the Lagrangian of the form $\left(\lambda_4(\phi_1^\dagger\phi_2)^2 + \mathrm{c.c}\right)$ which produces a mixing between the two Higgs doublets after spontaneous symmetry breakdown. If this term is small, then the physical neutral Goldstone particle can be quite light even in a theory with strong self interactions. It is this particle which will be identified with the $\zeta(8.3)$.

[In this paper we have adopted the pseudoscalar interpretation of the $\zeta(8.3)$. Another possibility is that the observed resonance is the lighter of the two Higgs scalars (referred to as H_1 in the Appendix). Much of our analysis might still apply in this latter case. The smallness of the 8.3 GeV mass constrains λ_1 to be small, but λ_3 may still be large. In this case, the H_1H_1 scattering amplitude satisfies a low energy theorem of exactly the same form as Eq.(2.9) for $\zeta\zeta$ scattering. (The H_1-particle here plays the role of an approximate Goldstone boson of the higher O(8) symmetry associated with the λ_3 term in (A.2).) The effective range analysis of H_1H_1 scattering leads to phenomenological results similar to the case discussed in the text.]

In order to understand the observed branching ratio for $\Upsilon\to\gamma+\zeta(8.3)$, one again needs a ratio of VEV's on the order of $v/V\approx0.1$-0.2.[16] The fermions are prevented from coupling to ϕ_2, the field with the larger VEV, by a discrete symmetry. The Yukawa coupling of the b quark to ϕ_1 is thus increased by a factor of V/v over the minimal model, and the expected branching ratio is correspondingly increased.[12-13] We find it intriguing that our consideration of the CERN events has led us to the same model which may also explain the properties of the $\zeta(8.3)$. We look forward to more information from both the CERN collider and from e^+e^- storage rings to provide definitive tests of these ideas.

II. New Resonances in the Strongly Coupled Higgs Sector

In this section we will examine the implications of current algebra combined with analyticity, unitarity, and crossing symmetry for the scattering of Goldstone particles (either longitudinal gauge bosons or physical Goldstone bosons) in a strongly coupled Higgs theory. Our procedure will be to construct unitary partial wave amplitudes by an effective range parametrization of the s- and p-wave phase shifts, using the low energy current algebra constraints to fix the scattering length parameter. The analogous treatment of π-π scattering by Brown and Goble[9] yields a reasonable description of the s- and p-wave phase shifts below 1 GeV. It may be worth remarking that the implications of crossing symmetry for these unitarized amplitudes may also be investigated. In particular, the s- and p-wave phase shifts must satisfy certain integral constraints derived by Roskies.[17] In the π-π case, these constraints have been discussed by Basdevant and Lee.[10] From their results we see that the Brown-Goble effective range amplitudes violate the Roskies crossing relations by as much as 30%, whereas the Padé amplitudes of Basdevant and Lee satisfy them to within 5%.[10] By this criterion, the Padé amplitudes are somewhat more acceptable than the effective range amplitudes. [See, however, the second paper in Ref. 9.] On the other hand the general behavior of the phase shifts is quite similar in the two approaches, and the effective range calculation is considerably simpler. Based on the results of Basdevant and Lee, we believe that a full Padé analysis of Higgs model perturbation theory would merely strengthen and not alter our main conclusions.

To begin, we will study longitudinal gauge boson scattering in the minimal one-doublet model. Ignoring fermions, the Lagrangian for the Higgs sector of the theory is

$$L_H = (D_\mu\phi)^\dagger(D_\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \tag{2.1}$$

where ϕ is a complex scalar doublet,

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \tag{2.2}$$

and D_μ is the SU(2)×U(1) gauge covariant derivative. After symmetry breaking, (2.1) describes a theory with three Goldstone bosons w^+, w^-, and z (which represent longitudinal gauge bosons) and a neutral Higgs scalar h, which interact according to a tree-level effective potential of the form

$$\Gamma_H = \lambda v^2h^2 + \lambda vh(2w^+w^-+z^2+h^2) + \frac{\lambda}{4}(2w^+w^-+z^2+h^2)^2 \tag{2.3}$$

To this we add an explicit longitudinal gauge boson mass term generated by the gauge covariant derivatives of the scalar field. For simplicity, we will neglect the difference between m_W and m_Z and approximate the gauge boson mass term by the isospin symmetric form

$$\Gamma' = \frac{1}{2}m_W^2(2w^+w^-+z^2). \tag{2.4}$$

This makes it somewhat easier to derive the low energy theorem for gauge boson scattering, which depends on the transformation properties of this term.

The Higgs Lagrangian (2.1), without the gauge terms, is invariant under a global $O(4) \approx SU(2) \times SU(2)$ group consisting of orthogonal transformations among the four real components of ϕ. This is spontaneously broken down to $O(3) \approx SU(2)$ = "weak isospin" by the vacuum expectation value of the Higgs field. The $w^{\pm}$ and z form an $I=1$ multiplet and the h has $I=0$. The model is isomorphic to the meson sector of the chirally invariant $SU(2) \times SU(2)$ linear σ model with $w^{\pm}, z$ corresponding to $\pi^{\pm,0}$, and h corresponding to σ. The current algebra arguments first discussed by Weinberg lead to a low energy theorem for the scattering of Goldstone bosons. Let w_i, $i=1,2,3$, be the isospin components of $w^{\pm}, z$, and let $T_{ij,kl}(s,t)$ be the scattering amplitude for the process

$$w_i + w_j \to w_k + w_l \tag{2.5}$$

Then, the low energy behavior of this amplitude required by current algebra is $T \to \hat{T}$, where

$$\hat{T}_{ij,kl}(s,t) = \frac{1}{v}\{\delta_{ij}\delta_{kl}(s-\gamma m_W^2) + \delta_{ik}\delta_{jl}(t-\gamma m_W^2) + \delta_{il}\delta_{jk}(u-\gamma m_W^2)\} \tag{2.6}$$

where s, t, and u are the usual Mandelstam variables. In this formula γ is a constant which depends on the transformation properties of the term in the Lagrangian which explicitly violates chiral symmetry. The original Weinberg result for π-π scattering, $\gamma=1$, assumed that this symmetry breaking term transformed like the isoscalar component of the

$[\frac{1}{2},\frac{1}{2}]$ representation of SU(2)×SU(2). In the σ-model this corresponds to a term proportional to the σ field,

$$\Gamma' = \text{const.}\times\ \sigma \tag{2.7}$$

More generally, the theorem (2.6) can be derived for a symmetry breaking term which transforms like the isoscalar component of the representation $[\frac{N}{2},\frac{N}{2}]$, which are the only irreducible representations containing the I=0 representation of the isospin group. For this case, the constant γ in (2.6) is given by[11]

$$\gamma = \frac{8-N(N+2)}{5} \tag{2.8}$$

For the case we are considering, it is easy to see that the isospin symmetric W mass term (2.4) belongs to the [1,1] representation, for which γ=0. Thus the low energy theorem (2.6) reads

$$\hat{T} = \frac{1}{v}\{\delta_{ij}\delta_{kl}s + \delta_{ik}\delta_{jl}t + \delta_{il}\delta_{jk}u\} \tag{2.9}$$

which may be verified by an explicit tree-level Feynman graph calculation. It should be remarked that, even though (2.9) might also have been obtained if we had ignored m_W^2 from the start, we have here derived this result in the presence of a finite W mass. Aside from the approximation $m_W=m_Z$, it should be valid at low energies up to corrections of $O(g^2)$.

Now let us consider how large s, t, or u can be before the low energy theorem (2.9) will start to fail. For a weakly coupled theory, the scale at which (2.9) fails is set by the perturbative Higgs mass $m_h^2=2\lambda v^2$, i.e. the corrections are of order s/m_h^2. However, in a strongly coupled theory m_h is very large ($\gtrsim$1TeV), and the range of validity of (2.9) is set instead by unitarity. In this case, corrections are of order s/v^2, and we should be able to extend the range of validity of the current algebra result by unitarization. Experience with the π-π system suggests that a unitarized version of the partial wave amplitudes which follow from (2.9) should provide a reasonable description of the s- and p-wave w-w phase shifts from threshold up to about 2 TeV. (Neglecting Goldstone boson masses, the scale conversion factor is $v/f_\pi \approx 5\times10^3$.)

The scattering amplitude $T_{ij,kl}(s,t)$ may be decomposed into s-channel partial wave amplitudes t_{IJ} with definite isospin I and angular momentum J by writing

$$T_{ij,kl}(s,t) = 16\pi \sum_{I,J} t_{IJ}(s)\ (2J+1)\ P_J(\cos\theta)\ P^{(I)}_{ij,kl} \tag{2.10}$$

where $P^{(I)}$ are isospin projection matrices. From the current algebra result (2.9) we obtain three nonvanishing partial wave amplitudes given by

$$\hat{t}_{00} = (4\pi v^2)^{-1}[2q^2+3m_W^2] \tag{2.11a}$$
$$\hat{t}_{20} = -(4\pi v^2)^{-1}q^2 \tag{2.11b}$$
$$\hat{t}_{11} = (12\pi v^2)^{-1}q^2 \tag{2.11c}$$

Following Brown and Goble, we can now use the effective range procedure to construct unitary amplitudes with low energy behavior given by (2.11). Unitarity requires that, in the elastic region $4m_W^2<s<16m_W^2$, the partial wave amplitudes must satisfy

$$\mathrm{Im}\ t_{IJ} = \frac{k}{\sqrt{s}}|t_{IJ}|^2 \tag{2.12}$$

The phase shift δ_{IJ} is defined by

$$t_{IJ} = (\sqrt{s}/k)e^{i\delta_{IJ}}\sin\delta_{IJ}, \tag{2.13}$$

and the unitarity condition (2.12) implies that the phase shifts are real in the elastic region. For our discussion, we will assume that these results are approximately valid well beyond the elastic region, an assumption which works quite well in the case of π-π scattering. The unitarity condition implies that the inverse amplitude has a discontinuity across the real axis for $s>4m_W^2$ given by

$$\mathrm{Im}\ t_{IJ}^{-1} = -k/\sqrt{s}\ . \tag{2.14}$$

Assuming no other singularities, the function t_{IJ}^{-1} is easily reconstructed from its discontinuity by evaluating the integral

$$-\frac{s}{\pi}\int_{4m_W^2}^{\infty}\frac{\sqrt{1-(4m_W^2/s')}}{s'(s'-s)}\ ds' \quad . \tag{2.15}$$

This gives the result

$$[t_{IJ}(s)]^{-1} = h(s) + g_{IJ}(s) \tag{2.16}$$

where

$$h(s) = -i(k/\sqrt{s}) + h_1(s) \tag{2.17a}$$

and

$$h_1(s) = \frac{k}{\pi\sqrt{s}}\ln\left[\frac{\sqrt{s}+2k}{\sqrt{s}-2k}\right] \quad . \tag{2.17b}$$

and, in our approximation of neglecting inelastic channels, $g_{IJ}(s)$ is a meromorphic function. The formula (2.16) is a generalized effective range expansion. We can now use the current algebra amplitudes (2.11) to constrain the form of $g_{IJ}(s)$. Consider first the s-wave isoscalar amplitude $t_{00}(s)$. A standard effective range expansion would approximate $g_{00}(s)$ by a finite order polynomial in k^2. But it can easily be seen from (2.11a) that such an expansion would have a small radius of convergence ($k^2<3m_W^2/2$) even if the amplitude were given exactly by current algebra. Brown and Goble construct a unitarized s-wave amplitude by simply approximating $g_{00}(s)$ by its current algebra value, and writing

$$t_{00}(s) = \frac{\hat{t}_{00}(s)}{1+h(s)\hat{t}_{00}(s)} \tag{2.18}$$

The phase shift corresponding to the amplitude (2.18) is positive and rises to around 30°-40° in the 1-2 TeV region.[9] As we mentioned in the

Introduction, a more elaborate Padé analysis[10] gives a phase shift closer to 90°, but in either case, the Higgs scalar does not appear as a well-defined narrow resonance in the strongly coupled theory.

The I=2 s-wave current algebra phase shift (2.11b) is negative, and we therefore do not expect any resonance in this channel. Finally, we turn to the most interesting channel, I=J=1. Taking into account the correct threshold behavior of the p-wave phase shift, the effective range approximation consists of writing

$$g_{11}(s) = \frac{a_{11}}{k^2} + \frac{1}{2}r_{11} \tag{2.19}$$

where a_{11} and r_{11} are the scattering length and the effective range. From the current algebra result (2.11c), the scattering length is determined to be

$$a_{11} = 12\pi v^2 \tag{2.20}$$

The amplitude is given by

$$t_{11}(s) = \frac{k^2}{a_{11}+\frac{1}{2}k^2 r_{11}+k^2 h_1(s)-i(k/\sqrt{s})} \tag{2.21}$$

For a sufficiently negative effective range parameter, this amplitude will have a resonance (i.e. the real part of its denominator will vanish) somewhere above threshold at $s=m_\nu^2$. Although current algebra does not in itself determine r_{11}, as discussed in the Introduction we will rely on the π-π analogy and on the Padé analysis of σ-model perturbation theory to argue that the I=J=1 amplitude does in fact exhibit a resonance. Neglecting the variation of $h_1(s)$ in the resonance region, the amplitude (2.21) may be written

$$t_{11} = \frac{m_{\mathcal{V}}\Gamma_{\mathcal{V}}(k/k_{\mathcal{V}})^2(m_{\mathcal{V}}/k_{\mathcal{V}})}{m_{\mathcal{V}}^2-s-im_{\mathcal{V}}\Gamma_{\mathcal{V}}(k/k_{\mathcal{V}})^3(m_{\mathcal{V}}/\sqrt{s})} \tag{2.22}$$

where

$$k_{\mathcal{V}}^2 = (m_{\mathcal{V}}^2-4m_W^2)/4 \tag{2.23}$$

The current algebra result for the scattering length (2.20) gives an expression for the resonance width in terms of its mass,

$$\Gamma_{\mathcal{V}} = (3\pi v^2 m_{\mathcal{V}}^2)^{-1}k_{\mathcal{V}}^5 \tag{2.24}$$

The $\mathcal{V}$WW coupling constant may be obtained from the standard formula

$$\Gamma_{\mathcal{V}} = \frac{f^2_{\mathcal{V}WW}k_{\mathcal{V}}^3}{6\pi m_{\mathcal{V}}^2} \tag{2.25}$$

giving the result

$$f_{\mathcal{V}WW} = \sqrt{2}(k_{\mathcal{V}}/v) \tag{2.26}$$

Note that, for the π-π system, this gives a coupling $f_{\rho\pi\pi}=5.3$ which is in reasonable agreement with the experimental value of about 6.0. (If the pion mass is neglected, the formula (2.26) is equivalent to the KSRF relation.) In the case of a one-doublet weak interaction theory, we have $v \approx 250$ GeV. It seems quite unlikely that the vector resonance in this system could be in the 170 GeV region of the CERN events, since this is very close to the w-w threshold ($k_{\mathcal{V}} \approx 0$). Eq.(2.26) would give a very small coupling constant, in contradiction with our experience with the π-π system. If we instead assume that $f_{\mathcal{V}ww} = f_{\rho\pi\pi}$, then (2.26) implies that $m_{\mathcal{V}} \approx 2$ TeV for the one-doublet model.

It is clear from these arguments that what is needed to produce an I=J=1 resonance at 170 GeV is: (1) A Higgs doublet with a much smaller vacuum expectation value, and (2) A pair of Goldstone bosons whose mass is substantially less than m_w so that the 170 GeV resonance is well above threshold. Both of these conditions can be satisfied by introducing a two-doublet model of weak interactions. This model has been studied by a number of people,[13-14] and has recently received considerable attention due to a possible interpretation[16] of the $\zeta(8.3)$ resonance discovered at Crystal Ball.[15] The details of this model which are relevant to the present discussion are presented in the Appendix. From Eq.(A.2) we see that, assuming the discrete symmetry (A.3), there are five independent coupling constants, λ_1-λ_5, and two vacuum expectation values, $v=\langle\phi_1\rangle$ and $V=\langle\phi_2\rangle$. Of the five coupling constants, λ_1 and λ_2 describe the self-coupling of each doublet, and λ_3-λ_5 describe the interaction between the two doublets. The vacuum expectation values are constrained by the observed Fermi decay constant to satisfy $(v^2+V^2)^{1/2}=250$ GeV. In addition to the three longitudinal gauge bosons, the perturbative spectrum of the theory consists of five particles. In the case of weak interdoublet coupling, two of these, H_1 and H_2, correspond to radial oscillations of the Higgs fields while the other three (hereafter referred to as $\zeta^{\pm}$ and ζ^{0}) are Goldstone modes. The masses of the ζ's arise from the cross couplings λ_4 and λ_5 which induce a mixing between the two doublets upon spontaneous symmetry breaking. In fact if we accept the two-doublet pseudoscalar interpretation of the $\zeta(8.3)$, the coupling constant λ_5 is determined from the ζ mass (c.f. Eq.(A.14)) to be quite small,

$$\lambda_5 \approx 2.2\times10^{-3} \tag{2.27}$$

We will assume that the charged Goldstone particles $\zeta^{\pm}$, like ζ^{0}, have a fairly small mass (say $\lesssim$30 GeV),, which implies, by (A.12), that the coupling constant λ_4 is also very small. This leaves only λ_1, λ_2, and λ_3 which are potentially large. In Appendix A we have listed the Feynman vertices for the two-Higgs model, neglecting λ_4 and λ_5 terms.

Before discussing the general case where λ_1, λ_2, and λ_3 are all large, it is useful to consider the particularly simple case where λ_3 is also small. In this case the two doublets are strongly interacting with themselves but weakly interacting with each other. In this situation, our previous analysis of the one-doublet model can be applied to each doublet separately. In each of the two sectors, ϕ_1 and ϕ_2, we would expect an I=J=1 resonance with a mass reduced compared to the one-doublet model by approximately a factor V/(250 GeV)$\approx$1 and v/(250 GeV)$\approx$0.1-0.2 respectively. The latter resonance may have a mass small enough to be relevant to the CERN events. The important point here is that the scale factor set by the ratio of the CERN event energy (170 GeV) to the natural weak scale of the Fermi constant (1-2 TeV) is roughly the same factor that is needed to explain the enhanced branching ratio for $\Upsilon\to\gamma+\zeta(8.3)$.

The analysis is somewhat more complicated for the case when λ_1, λ_2, and λ_3 are all large. For general values of (v/V) one must consider the coupled channel unitarity equations connecting ww, wζ, and $\zeta\zeta$ states. However, in the limit (v/V)<<1, the problem again simplifies and effectively reduces to a set of single-channel problems. To see this consider the three-point Feynman vertices (A.20) in the limit (v/V)<<1,

$$\Gamma(\zeta^+\zeta^- H_1) = 2v\left(\frac{\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3}{\lambda_2+\lambda_3}\right) \tag{2.28a}$$

$$\Gamma(\zeta^+ w^- H_1) = -\frac{2v^2}{V}\left(\frac{\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3}{\lambda_2+\lambda_3}\right) \tag{2.28b}$$

$$\Gamma(w^+ w^- H_1) = O(v^3/V^2) \tag{2.28c}$$

We will limit our discussion to the I=J=1 channel and therefore do not need the four-point vertices, which only contribute to the J=0 amplitudes in tree approximation. The low energy theorem for p-wave scattering is obtained from the t- and u-channel Higgs exchange graphs. To extract this low energy behavior the Higgs propagators are replaced by

$$(m_H^2-t)^{-1} \rightarrow \frac{t}{m_H^4} \tag{2.29}$$

(discarding a constant term which only contributes to the s-wave). From Eqs.(A.18) and (A.19) we see that m_{H_1} is of order v, while m_{H_2} is of order V. Thus, for (v/V)<<1 we need only keep the H_1 exchange graphs. Moreover, it is seen from (2.28) that only the $\zeta\zeta H_1$ coupling is non-vanishing to leading order in (v/V), and we only need to consider the $\zeta\zeta$ channel by itself. The λ-dependence of the factor $(1/m_{H_1}^4)$ in (2.29) is exactly canceled by the square of the vertex (2.28a), and the low energy theorem for $\zeta\zeta$ scattering reduces to the same result that was obtained for the decoupled case $\lambda_3=0$. The unitarity analysis again implies the existence of a vector resonance in the $\zeta\zeta$ channel with a mass in the 170 GeV range. Since the value of (v/V) is presumed to be about 0.2, the (v/V)<<1 approximation should be fairly reliable. Note, however that in this approximation, the 170 GeV vector resonance $\mathscr{V}$ decays exclusively into $\zeta\zeta$ pairs. In the discussion of decay modes in Section IV, we will find it desirable to relax this restriction and consider the possible decay $\mathscr{V}\to\zeta$+gauge boson, which may be necessary for interpreting some of the CERN events. Naively we would expect the rate for this process to be suppressed by a factor $(v/V)^2$ which would imply about a 4% branching ratio, but it is possible that the branching ratio for $\mathscr{V}\to W+\zeta$ may be larger than the naive estimate. The decay $\mathscr{V}\to WW$ is probably unimportant even if it is allowed by phase space, since it is supressed by a factor $(v/V)^4 \approx 10^{-3}$.

III. Production and Decay of $\mathscr{V}$ Resonances

In this section, we will discuss the production of I=J=1 resonances and estimate the number of such events that would be expected in the CERN collider experiments. The most efficient mechanism for the production of $\mathscr{V}$'s is via their mixing with the gauge bosons.

The quantum numbers of the $\mathscr{V}^{\pm,0}$ are the same as those of $W^{\pm,0}$ (where we use W^0 to represent the neutral SU(2) gauge boson which mixes with the U(1) boson to form Z^0 and γ). This allows a mixing between the $\mathscr{V}^{\pm,0}$ and the gauge bosons $W^{\pm,0}$ analogous to the ρ-γ mixing familiar from vector dominance arguments in hadron physics. In fact the vector dominance argument can be carried directly over to the $\mathscr{V}$-W mixing problem. Since the sine of the relevant mixing angle, $(g/2f_{\mathscr{V}})\approx 0.1$ is quite small, a lowest order Feynman diagram treatment is sufficient. In the pure Higgs theory without gauge interactions we define the global weak isospin current in terms of the Higgs fields. For the two-doublet model this is given by

$$j_\mu^a = \epsilon_{abc}[\phi_1^b \overleftrightarrow{\partial} \phi_1^c + \phi_2^b \overleftrightarrow{\partial} \phi_2^c] \tag{3.1}$$

where ϕ_1^i and ϕ_2^i, i=1 to 3 are the isospin components of the two Higgs fields. Recall that, in the ρ-γ mixing problem, the ρ-γ coupling constant is related to the $\rho\pi\pi$ coupling constant by assuming ρ-dominance of the isovector current form factor of the pion at $q^2=0$. If the ρ-γ vertex is denoted by (em_ρ^2/f_ρ), the vector dominance assumption leads to the result $f_\rho=f_{\rho\pi\pi}$. This agrees well with the ρ-γ vertex measured in the decay $\rho \to e^+e^-$. For the case of interest here, we apply a $\mathscr{V}$-dominance assumption to the form factor

$$\langle 0|j_\mu^a|\zeta^b\zeta^c\rangle \quad , \tag{3.2}$$

where j_μ^a is the weak isospin current of the Higgs sector, defined in (3.1). Denoting the $\mathcal{V}$-W mixing vertex by

$$\gamma_{W-\mathcal{V}} = \frac{gm_{\mathcal{V}}^2}{2f_{\mathcal{V}}} , \tag{3.3}$$

vector dominance gives

$$f_{\mathcal{V}} = f_{\mathcal{V}\zeta\zeta} \quad . \tag{3.4}$$

As we argued in the last section, to the extent that (v/V)<<1, we may unitarize the single channel $\zeta\zeta$ scattering amplitude by itself. The $\mathcal{V}$-ζ-ζ coupling in terms of v and $m_{\mathcal{V}}$ is then given by (c.f.(2.23))

$$f_{\mathcal{V}\zeta\zeta} = \frac{m_{\mathcal{V}}}{\sqrt{2}v} \quad , \tag{3.5}$$

Here and in the following, we will neglect m_ζ compared to m . The W-$\mathcal{V}$ vertex is then

$$\gamma_{W-\mathcal{V}} = gvm_{\mathcal{V}}/\sqrt{2} \tag{3.6}$$

The W-$\mathcal{V}$ mixing problem and its effect on the W propagator may be treated by methods which have previously been used to study Υ-W^0 mixing.[18] Applying this formalism to the $\mathcal{Y}$-W mixing problem, the W propagator is found to be proportional to

$$\left\{q^2 - \left[m_{W'}^2 + \left(\frac{\tilde{g} m_{\mathcal{Y}}}{2f_{\mathcal{Y}}}\right)^2 \frac{q^2}{q^2 - m_{\mathcal{Y}}^2}\right]\right\}^{-1} \tag{3.7}$$

where

$$m_{W'} = m_W/\sqrt{1-(g/2f_{\mathcal{Y}})^2} \quad . \tag{3.8}$$

$$\tilde{g} = g/\sqrt{1-(g/2f_{\mathcal{Y}})^2} \tag{3.9}$$

From these formulae, it is easily seen that the behavior of the W propagator in the low energy region $0 \lesssim q^2 \lesssim m_W^2$ is only very slightly affected by $\mathcal{Y}$-W mixing. For example, if the position of the W pole is taken to be 84 GeV, the effective mass measured at $q^2 \approx 0$ is 84.2 GeV.

The production cross-section for $\mathcal{Y}$'s via $\mathcal{Y}$-W mixing can now be estimated from the measured cross-section for W production. This gives

$$\sigma(p\bar{p} \to \mathcal{Y} + X) = R_L \times \frac{\gamma_{W-\mathcal{Y}}^2}{(m_{\mathcal{Y}}^2 - m_W^2)^2} \times \sigma(p\bar{p} \to W + X) \tag{3.10}$$

Here R_L is the ratio of the Drell-Yan $q\bar{q}$ luminosity factor at $\tau = m_{\mathcal{Y}}^2/s \approx 0.1$ to that at $\tau = m_W^2/s \approx 0.025$,

$$R_L = \frac{\left(\tau\frac{dL}{d\tau}\right)_{\tau=m_{\mathcal{Y}}^2/s}}{\left(\tau\frac{dL}{d\tau}\right)_{\tau=m_W^2/s}} \tag{3.11}$$

A rough estimate gives about $R_L \approx 0.1$. The second factor in (3.7) may be evaluated from (3.6) and gives

$$\frac{\gamma_{W-\mathcal{Y}}^2}{(m_{\mathcal{Y}}^2 - m_W^2)} \approx 0.03 \tag{3.12}$$

If we assume that the UA1 and UA2 experiments each produced a total of about 650 W's in their last run (assuming an 8% branching ratio for $W \to e\nu$) then the total number of charged $\mathcal{Y}$'s produced in the combined experiments is predicted from (3.10) to be about 4 (2 for each experiment). A similar estimate, using the observed Z^0 production cross-section and a 3% branching ratio for $Z^0 \to e^+e^-$, predicts about 1 neutral $\mathcal{Y}^0$ produced.

Since the cross sections for events with particular anomalous signatures will be reduced by branching ratio factors, this is certainly a smaller number of $\mathcal{Y}$'s than one would need for a unified explanation of all the anomalous CERN events. Nevertheless, the predicted number of $\mathcal{Y}$'s is not insignificant, and we are encouraged enough to go on and consider some of the individual event configurations and see how well they conform to our expectations for the decay of $\mathcal{Y}$ resonances.

There appear to be at least three types of anomalous collider events: (1) The "monojet" events (A through F) of UA1, consisting of a single hard jet and large missing transverse momentum; (2) The electromagnetic shower events (G and H) of UA1 consisting of a hard "photon" (an isolated electromagnetic shower with no tracks in the central detector) and large missing transverse momentum; and (3) The UA2 events consisting of a hard electron, one or more jets, and missing transverse momentum. (We will not discuss the $Z^0 \rightarrow e^+e^-\gamma$ and $\mu^+\mu^-\gamma$ events.)

Of course, some of these events may be due to conventional background sources, e.g. the production of a gauge boson with a hard gluon bremsstrahlung. But events of the latter conventional type have been observed and appear to be in good agreement with QCD predictions. The jet energies in at least some of the anomalous events are beyond the range where a significant QCD background is expected.

As we discussed in the last Section, the dominant decay modes of the $\mathscr{Y}^{\pm,0}$ are expected to be into $\zeta\zeta$ and ζ + gauge boson. For the present discussion we will make no assumption about the relative importance of these two modes, although, in the two-doublet model we have considered, the ζ + gauge boson mode is expected to be supressed by a factor of $(v/V)^2$. In fact this decay mode appears to provide a more acceptable interpretation of several of the most interesting events than does the theoretically favored $\zeta\zeta$ decay mode.

The easiest way that a $\mathscr{Y}$ decay can produce a monojet event is via the decay $\mathscr{Y}\rightarrow\zeta Z^0$ followed by $Z^0\rightarrow\nu\bar{\nu}$. The latter decay is expected to have a large ($\approx$18%) branching ratio. The single jet in event B of UA1 has a total E_T of 48 GeV with only three charged tracks and a low invariant mass of 0.79±0.12 GeV/c^2. We interpret this event as a $\mathscr{Y}^+ \to \zeta^+ Z^0$ decay followed by $Z^0 \to \nu\bar{\nu}$ and $\zeta^+ \to \tau^+\nu_\tau$ and finally $\tau^+ \to A_1\bar{\nu}_\tau$. (Here we use A_1 to denote the fact that the 3π decay mode of the τ has a broad peak in the 1 GeV region.) Event A of UA1 also admits a natural interpretation in our scheme. The monojet in this event has a striking character, consisting of a very hard prompt muon and a hadronic and electromagnetic shower at a finite angle (≈0.1 radian) with respect to the muon direction. The total E_T is 71 GeV and the invariant mass of the muon + shower system is around 5 GeV. We interpret this jet as arising from the decay $\zeta^0 \to \tau\bar{\tau}$ followed by $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ and $\bar{\tau} \to$ hadrons + $\bar{\nu}_t$. The observed transverse momentum of the muon is surprisingly large in this interpretation, but is not impossible for sufficiently asymmetric decays. In this event, there is some jet-like activity opposite the monojet, and the recoiling object may be a $\zeta^\pm \to \tau\nu_\tau$ with the τ decaying into hadrons which are soft enough to not be counted as a jet. Thus, event A would be interpreted as a $\mathscr{Y}^\pm \to \zeta^0\zeta^\pm$ decay. The "monoshower" event H of UA1 consists of a single isolated electromagnetic shower of 54 GeV opposite a large missing p_T of about 40 GeV. We note that, in the decay $\mathscr{Y} \to \zeta$ + neutral gauge boson, the neutral gauge boson is the linear combination of Z^0 and γ that forms the weak isovector W^0. Thus, we would expect the decay $\mathscr{Y} \to \zeta + \gamma$ to occur at a rate supressed by a factor $\tan^2\theta_W$ compared to the Z^0 mode. Noting also that the lego plot for event H does show some jet-like structure opposite the shower, we suggest the following interpretation: $\mathscr{Y}^\pm \to \zeta^\pm + \gamma$ followed by $\zeta^\pm \to \tau\nu_\tau$ with the τ decaying into soft enough hadrons to not qualify as a jet. Finally, the UA2 events consist of a hard electron, missing p_T, and one or more jets. One of these (event D) appears likely to be a heavy quark decay, while the other three (A-C) are consistent with being W + jet(s). The invariant mass of the W + jet(s) system in all three events is in the 170 GeV range (though it must be cautioned that this region is kinematically favored by the event selection criteria). Events of this general type could arise from the decay $\mathscr{Y}\to\zeta+W$ with the ζ decaying nonleptonically. However, event C does not fit this interpretation, because the invariant mass of the two jets in this event (which should equal the mass of the charged or neutral ζ) is 63 GeV. Event A is likely to be QCD background. Event B may be consistent with a $\mathscr{Y}\to\zeta+W$ interpretation if the invariant mass of the single jet is large enough. Needless to say, all of the interpretations we have discussed here are quite tentative, even if the basic ideas are correct. Although the general argument that a strongly coupled Higgs sector will give rise to dynamical vector resonances seems fairly compelling, the quantitative details of the production and decay of these resonances, as well as the relation between the resonance mass and the VEV of the Higgs field, are not very tightly constrained by our arguments. A larger sample of anomalous collider events together with a study of the decay modes of the $\zeta(8.3)$ should provide a more detailed confrontation with the ideas we are proposing.

IV Conclusions

In this paper we have studied properties of weak interaction models with strongly coupled Higgs sectors. By imposing unitarity constraints on the low energy current algebra amplitudes for Goldstone and longitudinal gauge boson scattering, it was concluded that, in addition to the spinless bosons associated with the Higgs field degrees of freedom, there should be dynamical resonances. Our main focus was on the I=J=1 resonance, referred to as $\mathscr{Y}$, which is analogous to the ρ resonance in $\pi\pi$ scattering. Although in the minimal one-doublet model this resonance should have a mass in the 2 TeV range, the mass of the lightest I=J=1 resonance in a two doublet model can be much lower. If one takes the ratio of VEV's in this model to be that required for the interpretation of the $\zeta(8.3)$ as a Higgs particle, the mass of the $\mathscr{Y}$ should be roughly in the 170 GeV range, and it is thus a possible candidate for the anomalous CERN collider events. There are some apparent problems with this scheme, particularly in the rather small number (≈ 5) of $\mathscr{Y}$'s predicted for the last CERN collider run. This would have to be enhanced by as much as an order of magnitude or more to agree with the observed rate of anomalous events. There is also a problem with the decay branching ratios in that the decay $\mathscr{Y}\to\zeta$+gauge boson seems to be less suppressed relative to $\mathscr{Y}\to\zeta\zeta$ than naive unitarized tree-level calculations would suggest. On the other hand, we find the idea that the anomalous CERN events are the manifestation of a dynamical resonance in the Higgs sector to be quite appealing. The results we have discussed in this paper encourage us to take this possibility seriously as further data become available. It would be a very interesting development indeed if both the collider events and the $\zeta(8.3)$ turned out to be evidence of an emerging Higgs sector.

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Appendix A

In this Appendix we discuss some of the properties of the two-doublet Higgs sector which are used in our analysis of Goldstone boson scattering amplitudes. With some changes of notation, we will follow the general conventions of Ref. 13. The model consists of two complex scalar doublet fields,

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \tag{A.1}$$

which interact according to an effective potential

$$\Gamma = \lambda_1(\phi_1^\dagger\phi_1 - v^2/2)^2 + \lambda_2(\phi_2^\dagger\phi_2 - V^2/2)^2 + \lambda_3[(\phi_1^\dagger\phi_1 - v^2/2) + (\phi_2^\dagger\phi_2 - V^2/2)]^2$$
$$+ \lambda_4[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)] + \lambda_5(\mathrm{Im}\phi_1^\dagger\phi_2)^2 \tag{A.2}$$

In choosing this form for the potential we have imposed a discrete symmetry on Γ under the transformation

$$\begin{aligned} \phi_1 &\to \phi_1 \\ \phi_2 &\to -\phi_2 \end{aligned} \tag{A.3}$$

The vacuum expectation values of the fields are given by

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ V \end{pmatrix} \tag{A.4}$$

There are eight scalar degrees of freedom in the two-doublet system, and the mass matrix is easily obtained by shifting the field origin,

$$\begin{aligned} \phi_1^0 &= v + \hat{\phi}_1^0 \\ \phi_2^0 &= V + \hat{\phi}_2^0 \end{aligned} \tag{A.5}$$

and calculating all quadratic terms in (A.2). This matrix is 2×2 block diagonal with the charged and neutral Goldstone modes and the Higgs mode of each doublet mixing separately. Let us denote the real and imaginary components of ϕ_1^0 and ϕ_2^0 by

$$\phi_1^0 = (h+v) + i\phi_1^3 \tag{A.6}$$

$$\phi_2^0 = (H+V) + i\phi_2^3 \tag{A.7}$$

where h and ϕ_1^3 (H and ϕ_2^3) are the Higgs and neutral Goldstone modes of ϕ_1 (ϕ_2) respectively. The 2×2 mass matrix for each of the three Goldstone modes is proportional to the same matrix,

$$\begin{pmatrix} V^2 & -vV \\ -vV & v^2 \end{pmatrix} \tag{A.8}$$

and thus the mixing angle for the Goldstone sector is given by

$$\tan\beta = \frac{v}{V} \tag{A.9}$$

The mass eigenstates are then

$$w^{\pm} = \cos\beta\ \Phi^{\pm} + \sin\beta\ \phi^{\pm} \tag{A.10a}$$

$$\zeta^{\pm} = -\sin\beta\ \Phi^{\pm} + \cos\beta\ \phi^{\pm} \tag{A.10b}$$

and

$$z^{0} = \cos\beta\ \Phi_3 + \sin\beta\ \phi_3 \qquad \text{(A.11b)}$$

$$\zeta^{0} = -\sin\beta\ \Phi_3 + \cos\beta\ \phi_3 \qquad \text{(A.11b)}$$

The $w^{\pm}$ and z^{0} are the massless modes which are eaten by the gauge fields, while the masses of the physical Goldstone bosons are

$$m^2_{\zeta^\pm} = \lambda_4(v^2+V^2)/2 \tag{A.12}$$

$$m^2_{\zeta^0} = \lambda_5(v^2+V^2)/2 \tag{A.13}$$

The mass matrix for the neutral Higgs modes h and H is

$$2\begin{pmatrix} (\lambda_1+\lambda_3)v^2 & -\lambda_3 vV \\ -\lambda_3 vV & (\lambda_2+\lambda_3)V^2 \end{pmatrix} \tag{A.14}$$

The masses and mixing angle α in this sector may be easily worked out. Define the eigenmodes H_1 and H_2 by

$$H_1 = \cos\alpha\ h + \sin\alpha\ H \tag{A.15}$$

$$H_2 = -\sin\alpha\ h + \cos\alpha\ H \tag{A.16}$$

The mixing angle is given by a fairly complicated expression, but for $(v/V)<<1$ it reduces to

$$\alpha \approx \frac{\lambda_3}{\lambda_2+\lambda_3}\frac{v}{V} \tag{A.17}$$

In this same approximation, the masses are

$$m_{H_1}^2 \approx 2v^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)/(\lambda_2+\lambda_3) \tag{A.18}$$

$$m_{H_2}^2 \approx 2V^2(\lambda_2+\lambda_3)(1+\alpha^2) \tag{A.19}$$

Next let us consider the three-point Feynman vertices corresponding to the potential (A.2). As pointed out in Section II, we do not need the four-point vertices for consideration of p-wave scattering at tree level. In view of the discussion in Section I, we will assume that λ_4 and λ_5 are both <<1 and only include those pieces of the interaction which are proportional to λ_1, λ_2, or λ_3. With these restrictions, we obtain the following vertices:

$$\Gamma(\zeta^+\zeta^-H_1) = 2\lambda_1 v \cos^2\beta \cos\alpha + 2\lambda_2 V \sin^2\beta \sin\alpha + 2\lambda_3 \cos 2\beta (v \cos\alpha - V \sin\alpha) \tag{A.20a}$$

$$\Gamma(\zeta^+ w^- H_1) = -2\lambda_1 v \sin\beta \cos\beta \cos\alpha - 2\lambda_2 V \sin\beta \cos\beta \sin\alpha \tag{A.20b}$$

$$\Gamma(w^+ w^- H_1) = 2\lambda_1 v \sin\beta \cos\alpha + 2\lambda_2 V \cos\beta \sin\alpha + 2\lambda_3 \cos 2\beta (V \sin\alpha - v \cos\alpha) \tag{A.20c}$$

In the limit $(v/V) \ll 1$, these vertices reduce to the expressions given in Eq. (2.28).

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