Fermi National Accelerator Laboratory

FERMILAB-Pub.-84/92-A
September, 1984

WALL DOMINATED INFLATION

David Seckel
Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia IL 60510

Abstract

The possibility that a wall dominated phase, during which $S \sim t^2$, could solve the horizon and flatness problems is examined. It is difficult to do this and still have a homogenous universe at the time of nucleosynthesis. The issues of baryogenesis, the monopole problem, and density fluctuations are also discussed. In order to gracefully exit the wall dominated phase it is proposed that the walls decay via hole formation. The details of that process are discussed in an Appendix.

I. Introduction

In this paper I will address the question of whether or not a wall dominated era can account for the current state of the universe. In a 1974 paper Zel'dovich etal.[1] considered the possibility that a wall dominated era could solve the horizon problem. The essential point of their paper was that if domain walls dominate the stress-energy of the universe the scale factor of the universe grows quadratically in time, i.e. $S \sim t^2$. In any model where S grows faster than t one has the potential to solve the horizon problem since causal regions get stretched out.[2] This situation is reminiscent of inflation[3] as introduced by Guth, hence the name -- wall dominated inflation. To avoid confusion I will refer to any model where S grows faster than t as inflation, the Guth scenario as "normal" inflation and wall dominated inflation as "wall dominated" inflation.

The original motivation for this paper was to notice that a wall dominated era would improve not only the horizon problem, but the flatness and monopole problems as well. Unfortunately, in the original paper by Zel'dovich etal. there was no clear proposal for ending the wall dominated era. Then, in the past few years, Vilenkin and Everett[4] and Sikivie[5] have discussed models where domain walls form that are classically stable but topologically unstable. As a result the walls decay via hole formation. The process is a tunneling phenomenon quite similar to the decay of the false vacuum.[6] With this in mind it seemed that wall dominated inflation might solve all the cosmological problems solved by normal inflation. It is the main issue of this paper to determine if this might indeed be possible.

Before proceeding, I would like to suggest two possible advantages that wall dominated inflation might have over normal inflation. First, wall dominated inflation does not appear to have a graceful exit problem.[3,7] The decay of the walls is exponential in time while the expansion of the universe is only a power of t. Eventually the tunneling process wins out; the universe always escapes from the wall dominated era. Admittedly, the graceful exit problem has been solved quite nicely in "new"[8] versions of inflation; however, the solutions always entail some unnatural setting of parameters.[8,9] Second, normal inflation requires that the effective cosmological constant be zero (Λ = 0) today but non-zero at some time in the past. If one assumes, as is usually done, that the current value of the cosmological constant is zero due to a fine tuning of parameters in the Lagrangian then one has an extreme problem of naturalness. On the other hand, if one assumes a dynamical explanation for $\Lambda = 0$ today, then it is difficult to understand why the same dynamical mechanism was not effective during the inflationary epoch. Wall dominated inflation does not require a non-zero cosmological constant and so a dynamical mechanism explaining $\Lambda = 0$ might be acceptable.

The rest of this paper consists of parts II-V and an appendix. The second part outlines a simple scenario for studying wall dominated inflation, defines all the relevant terms, and shows how the horizon and flatness problems are improved by wall dominated inflation. The third part examines the horizon and flatness problems in detail with, alas, a negative result. The fourth part is a discussion of other cosmological issues in the context of wall dominated inflation: the graceful exit, baryon production, galaxy formation and the monopole problem. Section V presents a summary. Throughout the body of the paper it is assumed that the walls will decay after some time τ_d. There is an appendix which discusses at some length how to build a model where classically stable but topologically unstable walls may be created and subsequently decay.

II. Wall Dominated Inflation

In this section I present the basic scenario which will form the framework for the rest of this paper. I will assume that the stress energy of the universe can be described by a Friedman-Robertson-Walker[10] metric. Although one can argue with the validity of these assumptions, they do form a basis for studying the effects of different microphysics on the history of the universe.

At the Planck time the universe is assumed to be radiation dominated. It is also assumed that the curvature term in the FRW equations is initially small enough that the universe can evolve to the point where inflation begins. At some time, t_w, the universe cools to the point where it passes through a phase transition[11] in which walls are formed (see the Appendix for details of the wall structure). For some time after that the universe continues to be radiation dominated. During this time the walls are becoming a more important component of the stress energy tensor, until at time t_* the walls dominate the radiation. At this point inflation begins and continues from then until the walls decay at time t_d.

When the walls decay the energy of the walls goes into reheating the universe. Directly after the walls decay the matter of the universe is mostly contained in hot sheets of relativistic particles, i.e. the fossil walls. It is assumed that the sheets thicken at roughly the speed of light until t_1, when the sheets will have thickened to the point that neighboring sheets have overlapped. During the time when the fossil walls are thickening I have used a radiation ($\rho = 3p$) equation of state. This is an assumption, based on the observation that matter within the fossil walls is relativistic. It neglects the energy associated with the bulk fluid motion within the thickening walls. Also, during this era the temperature is not homogeneous. Rather, it is a function not only of time but of position within the sheets. When I refer to a temperature during this epoch it should be thought of as an effective temperature $T(t_d < t < t_1) \sim \langle\rho\rangle^{1/4}$, where $\langle\rho\rangle$ is an average over a volume that includes many sheets. After the sheets merge the universe will continue to expand as per the standard big bang picture.

For future reference, I use the following definitions; t_N -- time of nucleosynthesis, t_m -- time when matter dominates, t_o -- today. Other variables are used with appropriate subscripts, as defined in the previous paragraphs. For example, the density when the walls first dominate the stress-energy is $\rho_* \equiv \rho(t_*)$.

I can now give a description of wall dominated inflation. The Einstein equations for a FRW universe reduce to

$$(\dot{S}/S)^2 = 8\pi G\rho/3 - K/S^2 \tag{1}$$

$$\ddot{S}/S = -4\pi G/3\ (\rho + 3p) \tag{2}$$

where ρ is the density, Newton's constant is $G = 1/m_p^2$, $m_p = 1.2 \times 10^{19}$ GeV, p is the pressure, $\cdot$ indicates d/dt, S is the cosmic scale factor, and K = 1, 0, -1 is the curvature. To solve eqns. (1,2) one must have an equation of state or, equivalently, some knowledge of how the density scales with S. For comparison with what follows recall that during a radiation dominated era $p = 1/3\rho$, $\rho \sim 1/S^4$, $S \sim t^{1/2}$ and that during normal inflation $p = -\rho$, $\rho = \Lambda^4$ = constant, $S \sim e^{Ht}$ where $H = (8\pi/3)^{1/2} \Lambda^2/m_p$ is the Hubble constant during inflation.

The energy density in walls is

$$\rho = \sigma/\lambda \tag{3}$$

where σ is the energy density per unit area and λ is the typical spacing between walls. For most purposes it is sufficient to set $\sigma = \mu^3$, where μ is the mass scale associated with the underlying theory responsible for the walls. It is expected that the walls form when the temperature drops below a critical value $T_c \approx \mu$.

For most of this paper I will assume that $\lambda \gtrsim t$. If the scale of walls is less than the horizon then it is assumed that short range physics can act over a horizon length to dissipate the energy in walls. On the other hand, if $\lambda > t$ no microphysics can act due to causality. As a result, $\lambda \sim S$ as long as the walls are outside each other's horizon. During a radiation or matter dominated era t grows faster than S. Eventually walls come within each other's horizon and, as a result, $\lambda \approx t$. During an inflationary epoch S grows faster than t and $\lambda \sim S$.

Zel'dovich, etal.[1] have discussed what happens when walls dominate the stress-energy tensor and $\lambda \sim S$. From eqn. (3) one finds $\rho \sim 1/S$. Since the curvature term in the Einstein equations scales as $1/S^2$, one may conclude that curvature becomes less important during a wall dominated era. The flatness problem is improved. Further, since I assumed curvature was unimportant at the start of inflation it can be neglected throughout the wall dominated era. This allows one to find simple solutions to the Einstein equations. The pressure is defined by $p = -dE/dV$. Since $E = \rho V^3 \sim V^{2/3}$, the pressure during wall dominance is $p = -2/3\ \rho$. With this pressure and $\rho \sim 1/S$, the solution to the Einstein equations is $S \sim t^2$. A wall dominated era is inflationary.

It may appear that the above argument is circular. I started by assuming that $\lambda \sim S$, found that $S \sim t^2$, and concluded that $\lambda \sim S$. What if λ develops in a different manner? To study this question it is convenient to use comoving coordinates; $\lambda \sim xS$, where x is the typical interwall spacing in comoving coordinates. Then

$$\dot{\lambda} = \dot{x}S + x\dot{S} = \dot{x}S + \lambda\dot{S}/S \tag{4}$$

The first term in eqn. (4) is a dynamical term arising from local processes such as walls annihilating with each other. In general, causality restricts this term to $\dot{x}S \lesssim 1$. The second term is due to cosmological expansion. If $\lambda\dot{S}/S > \dot{x}S$, then expansion dominates the behavior of λ and one finds $\lambda \sim S$. In our case, $\lambda\dot{S}/S \sim \lambda(G\rho)^{1/2} \sim \lambda(\mu^3/m_p^2\lambda)^{1/2}$. At the beginning of wall dominance I have supposed that λ_* $= t_* = m_p/g^{1/2}T_*^2$, where g is the effective number of degrees of freedom in thermal equilibrium. Further, at that time $\mu^3/\lambda_* = gT_*^4$, and as a result $(\lambda\dot{S}/S)_* \approx 1$. Cosmological expansion dominates the behavior of λ immediately upon wall dominance.

It is possible that the annihilation of walls is not particularly efficient so that $\dot{x}S = \varepsilon << 1$. As long as the dynamic term dominates eqn. (4) one finds $\dot{\lambda} = \varepsilon$ or $\lambda \approx \varepsilon t$. At the time of wall dominance $\lambda_* =$ εt_*, with the consequence that $(\lambda\dot{S}/S)_* \approx \varepsilon$. Again one finds inflation, λ

$\approx S \sim t^2$, since expansion dominates dynamics as soon as wall dominance starts.

III. The Horizon and Flatness Problems

Up till now I have shown that wall dominated inflation has the potential to solve the horizon and flatness problems, but does it really work? How much inflation is necessary?

Begin by considering the horizon problem. An observer looking at the night sky will notice that the universe appears to be isotropic and homogeneous. This fact could be nicely explained if different parts of the observable universe were to have been in thermal contact at some time in the past, but otherwise it is somewhat of a mystery. Suppose our observer sees photons, originating from different regions, that were emitted at some time, t_e. The separation of the emitting regions is currently $s_o = S_o \int_{t_e}^{t_o} 1/S\, dt \approx t_o$, where I have omitted factors of order 1 involving details of integration and the angular separation of the sources. At the time the photons were emitted the separation was

$$s_e = s_o\, S_e/S_o = t_o\, S_e/S_o \tag{5}$$

In order for these two regions to have been in thermal contact by the time of emission it is necessary that the particle horizon at t_e be greater than the separation of the sources; i.e. $d_e > s_e$; where $d_e = S_e \int^{t_e} 1/S\, dt$. As long as S grows slower than t, such as in a radiation ($S \sim t^{1/2}$) or matter dominated universe ($S \sim t^{2/3}$), d_e is determined by the upper limit of the integral; $d_e \approx t_e$. This fact leads to the horizon problem; $t_e < t_o\, S_e/S_o$ for matter or radiation dominance and therefore

$d_e < s_e$. The two regions under observation could not have been in causal contact during the past.

The horizon problem can be solved if at some time in the past the universe was inflationary. In the case at hand the universe is supposed to have been wall dominated from t_* until t_d, with $S \sim t^2$ during that time. In that case

$$d_e = S_e \left[\int^{t_d} 1/S \, dt + \int_{t_d}^{t_e} 1/S \, dt\right] \tag{6}$$

There are two contributions to the horizon. The second, after inflation, is given by t_e -- the upper limit of integration. The first contribution comes during the inflationary epoch when the particle horizon is dominated by the lower limit of the integral. During this time a region of past causal contact is increasing in size due to the cosmic expansion. Photon propagation is relatively unimportant during this phase. It follows that the first contribution to d_e is roughly $t_*(S_e/S_*)$, where t_* is the particle horizon at the onset of inflation. Together the two terms give

$$d_e \simeq t_*(S_e/S_*) + t_e. \tag{7}$$

It is possible to have the first term be much greater than the second. To solve the horizon problem it is required that $d_e > s_e$ or using eqn. (5)

$$S_o/S_* \geq t_o/t_*. \tag{8}$$

If one uses the scaling laws for the various epochs: wall $(S/S_*) = (t/t_*)^2$, radiation $(S/S_d) = (t/t_d)^{1/2}$ and matter dominated $(S/S_m) = (t/t_m)^{2/3}$, then eqn. (8) gives a result for the scale factor and temperature at the time of wall decay.

$$S_d \gtrsim (S_* \, S_m \, S_o)^{1/3} \tag{9}$$

or $$T_d \lesssim (T_*^4 \, T_m \, T_o)^{1/6} \tag{10}$$

In the last expression T_d is the effective temperature after reheating, $T_d \equiv (\rho_d)^{1/4}$. Eqns. (9,10) give a condition on the amount of wall dominated inflation that must occur if the horizon problem is to be solved.

Now consider the flatness problem. A measure of the importance of curvature in the universe is given by the ratio of the two terms on the right hand side of the Einstein equation (1),

$$\alpha \equiv 3K/8\pi G\rho S^2. \tag{11}$$

(In more common terminology one defines the Hubble constant $H \equiv \dot{S}/S$, the critical density $\rho_c \equiv 3H^2/8\pi G$, and the density parameter $\Omega = \rho/\rho_c$. Then, $\alpha = \Omega(1 - \Omega)$.) During a radiation era $\alpha \sim S^2$, during matter dominance $\alpha \sim S$. The flatness problem arises when one realizes that the universe today is near critical density so that $\alpha_o \approx 1$. At early times, $\alpha \approx (S_m/S_o)(S/S_m)^2 << 1$; for example, in the standard big bang model, at the grand unification scale ($T \sim 10^{15}$ GeV) one must impose the condition $\alpha \lesssim 10^{-52}$.

A wall dominated era can solve the flatness problem. During the inflationary epoch $\rho \sim 1/S$ and therefore $\alpha \sim 1/S$. For demonstrative purposes assume that $\alpha_* = 1$ and ask how much inflation is required to have $\alpha_o \leq 1$. (This does not really solve the flatness problem as one must still explain why α_* should be equal to one; however, it is a minimum condition that must be met.) The simplest procedure is to note that $\rho \sim 1/t^2$ during wall, radiation, and matter dominance. Then, $\alpha_o \leq 1$ implies

$$\alpha_o = (\rho_* S_*^2/\rho_o S_o^2)\alpha_* = (S_*^2/t_*^2)(t_o^2/S_o^2) \lesssim 1 \tag{12}$$

which is identical to eqn. (8) derived from the analysis of the horizon problem. It follows that eqns. (9,10) describe the minimum amount of inflation necessary to solve the flatness problem. The result that the same condition applies for both the horizon and flatness problems should not be surprising. If one thinks of α as the amplitude of a curvature fluctuation then α should have roughly the same value at both horizon crossings,[11,12] which is the result shown in eqn. (12).

It would appear that the previous analysis tells us how much inflation is needed and we can proceed to other concerns, such as baryogenesis. Unfortunately, there is a major problem: ensuring that the universe is homogeneous and isotropic. Consider the state of the universe just after the walls decay. The matter is not distributed homogeneously. Rather it is in sheets that mimic the location of the walls just prior to their decay. As the universe ages these sheets thicken at roughly the speed of light. Once the sheets have thickened enough that the voids between the sheets are filled in the universe can achieve homogeneity. This happens when the thickness of the sheets, ℓ, is equal to the typical spacing of the walls, λ. In order to achieve homogeneity one must have $\ell \gtrsim \lambda$. Since the sheets thicken at the speed of light $\ell = S \int_{t_d}^{t} 1/S \, dt \simeq t$. From the beginning of the inflationary epoch on, the walls scale with the expansion of the universe, $\lambda = \lambda_* S/S_*$. In the model it was supposed that before inflation started microphysics could act and thus λ was roughly equal to the particle horizon, $\lambda_* \simeq t_*$. The condition for homogeneity today is then $\ell_o = t_o > t_* S_o/S_* = \lambda_o$ or $S_o/S_* < t_o/t_*$. Comparing this result with eqn. (8) one arrives at the disappointing conclusion that it is not possible to solve the horizon and flatness problems via wall dominated inflation and still have a homogeneous universe today.

Actually, the contradiction is much worse. There is good evidence from big bang nucleosynthesis[14] that the universe was homogeneous at the time of nucleosynthesis, t_N. One might barely hope to solve the horizon and flatness problems at t_N by setting $S_N/S_* = t_N/t_*$, but then one must account for the additional growth of these problems between nucleosynthesis and today. Is it possible that Helium production might have occurred before the sheets merged? I think that the answer is most certainly no. The problem is that the temperature might be expected to fall faster than in the standard Helium production scenarios due to the free expansion perpendicular to the walls. Recall that the fraction of Helium depends on how long neutrons can freely beta decay before they are frozen into Helium. If that time is too short then too many neutrons survive and the Helium fraction is too high. The temperature probably cools too fast in the remnant walls; however, a detailed calculation for an expanding wall is needed to confirm this suspicion.

A possible way out of this predicament would be to start the inflationary period with more than one wall per horizon. One can estimate how many walls would be necessary by demanding that the horizon and flatness problems be solved today and that the sheets merge by t_N. Let the scale of the walls at the beginning of inflation be $\lambda_* = \varepsilon\ t_*$. In order to solve our problem we need $\lambda_N = t_N$, where $\lambda_N = (\varepsilon t_*)(S_d/S_*)(S_N/S_d)$ and $t_N = t_*\ (S_d/S_*)^{1/2}(S_N/S_d)^2$. The value of S_d is determined from eqn. (9) so as to solve the horizon problem. Solving for ε, one finds that

$$\varepsilon = [S_N^2/S_m S_o]^{1/2} = [T_m T_o/T_N^2]^{1/2} \approx 10^{-8}. \tag{13}$$

It is difficult to understand how such a small value of ε could occur. This is especially true if the phase transition during which the walls were formed occurred at a mass scale, μ, associated with grand unification. The smallest reasonable value for λ would be a coherence length of order $1/\mu$. In that case, $\varepsilon \approx 1/\mu t \approx \mu/m_p \approx 10^{-4}$, which is, unfortunately, much greater than 10^{-8}.

IV. Other Cosmological Issues

Despite the bad news about the flatness and horizon problems it is still worth asking what happens to other cosmological issues in the context of a wall dominated era.

Since it was one of the motivations for considering wall dominated inflation, the first problem I will address is that of exitting the inflationary phase. Guth realized that his original inflationary scenario had a fatal flaw.[3,7] He hoped that during a first order phase transition bubbles of true vacuum would grow and coalesce. The time for the phase transition to complete would be the tunneling time for bubble formation. Unfortunately the phase transition is never completed. Once the universe gets into an inflationary phase the scale of the universe expands exponentially. There is then a competition between two exponential processes. If bubble formation wins the competition it will win it very quickly, before any inflation occurs. If inflation wins the competition then the phase transition is never completed. In short (although subsequent work[8,9] has seemingly solved the "graceful exit" problem) the original inflationary scenario did not work.

It is easy to show that the decay of walls does not have a graceful exit problem. This is done by showing that the proper area of walls goes to zero at large time. By comparison, in normal inflation the comoving volume of false vacuum goes to zero, but the proper volume in false vacuum still grows exponentially. Furthermore, the bubbles of true vacuum do not percolate, i.e. they do not form clusters of infinite extent.

Consider the proper area of wall, A, within some comoving volume. As the universe expands, $A \sim S^2 \sim t^4$. Then, $dA/dt = 4A/t$, due to expansion. Let the rate of hole formation per proper area be Γ. The rate in the comoving volume is then ΓA. One can show that holes which form at time t reach a limiting comoving radius, $x = t/S$, where S is the scale factor at the time the hole forms. Therefore, a hole that forms at time t effectively reduces the area of wall by $\pi S^2 x^2 = \pi t^2$. The total effective loss of area is $dA/dt = -\pi\Gamma t^2 A$. Combining this with the expansion effect

$$dA/dt = (4A/t) - \pi\Gamma t^2 A \tag{14}$$

or $$A = t^4 e^{-(\pi\Gamma/3)t^3}$$

Eventually there will be no area left in walls and, therefore, no graceful exit problem. The era of wall decay, t_d, is roughly defined by $dA/dt < 0$ or

$$t_d = (\pi\Gamma/4)^{1/3} \tag{15}$$

The next problem to consider is that of baryogenesis.[15] The first step is to calculate the reheating temperature. This can be done using eqn. (10) if one knows the value of T_*. Suppose that the scale of walls at the time of wall dominance is $\lambda_* = \varepsilon t_*$. Then the condition that $\rho_{rad} = \rho_{wall}$ gives $g_* T_*^4 = \mu^3/\lambda_* = \mu^3 g_*^{1/2} T_*^2/\varepsilon m_\rho$. Solving for T_* one finds

$$T_* = \mu\,[\mu/\varepsilon g_*^{1/2} m_p]^{1/2} \tag{16}$$

Plug into eqn. (10) and find

$$T_d = \mu\,[T_m T_o/\varepsilon^2 g_* m_p^2]^{1/6} \approx \mu\, 10^{-10}\ \varepsilon^{-1/3} \tag{17}$$

For the case where $\varepsilon = 1$ and the mass scale of the walls is in the range of $10^{15} < \mu < 10^{19}$ GeV, as per grand unification ideas, then eqn. (17) gives a reheating temperature of $T_d \sim 10^5 - 10^9$ GeV. This is uncomfortably low for the usual picture of baryon production. Fortunately, the physics is different in our case as well. When the wall is just going through its decay the material right around the wall is very far out of thermal equilibrium. One can consider the wall as a collection of scalars, each of mass μ which are decaying freely. Let the probability that any one scalar produce a baryon be p. Then the number density of baryons is $n_b = n_s p$, where $n_s \sim \rho_d/\mu$ is the number density of scalars. After thermalization the number density of photons is $n_\gamma \sim T_d^3$. The baryon to photon ratio is then given as

$$n_b/n_\gamma \cong pn_s/T_d^3 \cong pT_d/\mu \tag{18}$$

or, using eqn. (16) with $\varepsilon = 1$

$$n_b/n_\gamma \cong 10^{-10}p. \tag{19}$$

The observed baryon to photon ratio is about 10^{-10}. To get this result it is necessary to postulate a net production of one baryon per scalar decay. This is probably not acceptable within the current knowledge of grand unified theories.[15] If one relaxes the assumption of one wall per horizon at t_* then

$$n_b/n_\gamma = \varepsilon^{-1/3}10^{-10}p. \tag{20}$$

The situation gets better. With maximal CP violation in scalar decay one can only get $p \sim 10^{-2}$. If one chooses $\varepsilon \sim 10^{-8}$, so as to solve the flatness and horizon problems, an acceptable baryon number is produced.

Next, I will address myself to the problem of density perturbations in the context of wall dominated inflation. Recall that the prediction of the so called Harrison-Zel'dovich[16], spectrum was first a great failing and then a great success for new models of inflation. There are two major points to be studied: the initial perturbation spectrum at t_* and the evolution of that spectrum until today.

First, examine the evolution problem. In order to solve the horizon problem, the whole of the observable universe must have been contained within the horizon at t_*. That means that any physically interesting scale today (e.g. galaxies, superclusters) must have originated from a region much smaller than the horizon at t_*. The evolution of a perturbation can then be broken into three pieces: a) from t_* until the perturbation crosses the horizon; b) the period of time that the perturbation spends outside the horizon; and c) the time at which the perturbation comes back into the horizon until today.

The growth of perturbations during part c) is fairly well understood and will not be discussed here. Up until a few years ago part b) of a perturbation's history was quite murky but recent work has clarified the behaviour of perturbations on scales longer than the horizon.[11,12] The crucial point is that no causal physics can act on scales larger than the horizon, so that the perturbation evolves kinematically. In the uniform Hubble constant gauge one finds[12]

$$\delta\rho/\rho = A[(HS/K)^2 + (2/9)\ 1/(1+w)]^{-1} \tag{21}$$

where A is a constant, $H = \dot{S}/S$, K is the comoving wave number (S/K is the physical wavelength), and $w = p/\rho$ describes the equation of state.

For almost all equations of state $1/1+w$ is a number of order 1 (specifically, during wall dominated inflation $w = -2/3$, $1/(1+w) = 3$) while the perturbation is outside the horizon $(HS/K) >> 1$ and the $1/1+w$ term can be neglected. In that case $\delta\rho/\rho \approx A(K/SH)^2$, so the perturbation has the same amplitude at both horizon crossings. Note that for a nearly flat universe, $H^2 = 8\pi G\rho/3$, so $\delta\rho/\rho \sim 1/\rho S^2 \sim \alpha$. Density perturbations outside the horizon grow in the same manner that curvature does. (The exception to this statement is that of normal inflation. Then $w \approx -1$ so that $1/1+w$ is the dominant term in eqn. (21) at the horizon crossing during inflation.)

The final epoch to consider is part a), the time the perturbation spends before it leaves the horizon. This period is far more complicated than the other two due to the possibility that dynamics can play a significant role for perturbations within a horizon. For example, it might be possible for pressure support to keep a perturbation from growing. The crucial quantity is the speed of sound, $v_s^2 = \delta p/\delta\rho$. Perturbations in the density of walls will have associated pressure perturbations, $\delta p = -2/3\ \delta\rho$, so $v_s^2 = -2/3$. However, one may also have energy perturbations due to the walls peculiar velocity and these perturbations will have $v_s^2 > 0$.

Another possibility is that relativistic motion of walls will effectively damp out perturbations whose scale is smaller than the horizon. However, even if we make this assumption there will still be Poisson fluctuations due to the discrete nature of the walls. If the walls are moving fast enough to damp perturbations by their essentially random motions then their positions should be uncorrelated. This will lead to minimal density fluctuations $\delta(\ell) \sim (\lambda/\ell)^{1/2}$, where ℓ is the scale of the perturbation. Note that the power law is 1/2, not 3/2, due to density correlations within a wall. (Even this simple power law is an approximation that ignores correlations due to bubbles or curvature of the walls.) For the rest of this discussion I will use this minimal spectrum, but it should be realized that the actual spectrum may be stronger.

The decision to use a minimal spectrum allows one to ignore the nature of the initial spectrum at the time the walls form. The point is that any interesting perturbation scale today must have been processed within a causal volume during the early stages of wall-dominated inflation. Nevertheless, I will say a few words that are relevant for fluctuations in the density of walls for scales larger than the horizon at t_w. There will be two types of perturbations in the wall density. The first type is due to the random initial positions of the walls, $\delta \sim (\lambda/\ell)^{1/2}$. These are perturbations in the equation of state: they cannot become true density perturbations until they interact dynamically. Such a perturbation cannot grow while outside the horizon, but upon reentering the horizon will act like a density perturbatiion of amplitude $\delta \sim (\lambda/\ell)^{1/2}$. The second type of perturbations in the wall density are those induced by pre-existing fluctuations in the radiation. A slightly denser region of space will cool to the critical temperature for wall formation at a slightly later time. As a result the correlation length in those regions will be a smaller fraction of the horizon, and the density of walls will be higher. This second type of perturbation is a true curvature perturbation and may evolve while outside the horizon. The amplitude of such a perturbation is a free parameter depending only on initial conditions.

Again, if the processing of perturbations early in inflation is incomplete then the initial spectrum may be important; however, I am assuming that processing is complete. The resulting spectrum is $\delta_H \sim (\lambda/\ell)^{1/2}$ upon leaving the horizon. Using eqn. (21) it is trivial to calculate that the perturbation amplitude upon reentering the horizon is 10/7 times the amplitude when the perturbation left, i.e. $\delta_H \approx (\lambda/\ell)^{1/2}$ for physically interesting perturbations.

Is such a spectrum acceptable? The exact form of the perturbation spectrum needed at horizon crossing is still under debate. Details of the spectrum depend on what kind of matter (baryons, neutrinos, axions...) is supposed to dominate the universe today and on the details of the galaxy formation process. In general, it is agreed that something similar to a Harrison-Zel'dovich spectrum is needed. Recall that this spectrum specifies that the perturbation amplitude, at horizon crossing is independent of scale. In order to agree with observations of the 3^{o}K blackbody spectrum[17] it is necessary to suppose that $\delta_H \approx 10^{-4}$-10^{-5}. If one abandons the Harrison-Zel'dovich spectrum one has problems. From galaxy formation and limits on the blackbody spectrum one constrains the amplitude at matter domination to be roughly 10^{-4}. If δ_H increases for larger scales then one would find too much structure at large scales. If δ_H increases on small scales then one must face the possibility of forming a large number of black holes for scales with $\delta_H > 1$.

Now consider the candidate spectrum $\delta_H \approx (\lambda/\ell)^{1/2}$. As described above, the scale is set by $\delta_H(T_m \approx 10\text{eV}) \approx 10^{-4}$. At horizon crossing the perturbation amplitude behaves as $\delta_H \sim (\lambda/t)^{1/2}$, so

$$\delta_H \approx 10^{-4}(T/10\mathrm{eV})^{1/2}, \quad T > 10 \text{ eV} \qquad (22)$$
$$\phantom{\delta_H \approx{}} 10^{-4}(T/10\mathrm{eV})^{1/4}, \quad T < 10 \text{ eV}$$

From eqn. (22) we see that there is no problem with excess large scale structure. However, when the walls are just coming back into the horizon, δ will be of order 1. To get galaxies to come out right requires a possibility that black holes may form at about $T_1 \approx 1$ GeV, when $\delta_H \approx 1$ and $\lambda \approx t$. Their mass would be about the horizon mass at that time, which turns out to be a solar mass. The number of black holes of one solar mass is bounded only by their contribution to the density of the universe. Once black holes form, their contribution to the energy density scales like matter, $1/S^3$. If the black holes dominate the matter of the universe today then the fraction of matter, f, that goes into black holes at T is bounded by $f \lesssim T_m/T$. For $T_1 = 1$ GeV and an $\Omega = 1$ universe this gives $f \lesssim 3 \times 10^{-8}$, which may or may not be an unreasonable number.

Up till now I have ignored perturbations on scales smaller than λ. This is because I do not expect any structure on scales smaller than λ to survive. These scales come into the horizon while the fossil walls are still thickening. In assuming that the walls thicken it is implicitly assumed that pressure support and/or free streaming are sufficient to keep the walls from collapsing back on themselves. There will also be perturbations within the plane of any given wall. These will naturally be associated with the scale of hole formation in the walls at the time the walls decay. Statistical fluctuations in the density of holes will be small on physically interesting scales; $\delta(\ell) \approx$

$[t_d/\ell]$, where t_d will be the typical size of a hole. The power law is 1 due to the two dimensional character of the walls and point like nature of the holes. One might expect some black holes of mass $M \sim \mu^3 t_d^2$ to result from this process for $\ell \approx t_d$, but no significant large scale structure can occur.

I can now summarize the picture of density perturbations in wall dominated inflation. The minimum perturbation amplitude is roughly $\delta_H \approx (\lambda/\ell)^{1/2}$. To not conflict with observations of the $3^{\circ}K$ background the temperature by which the walls must merge is roughly $T_1 \gtrsim 1$ GeV. Black holes of about one solar mass may result. The constraint on the walls merging is more stringent than that obtained for nucleosynthesis, $T_1 \approx T_N \approx 1$ MeV. Translated into an initial density of walls at the beginning of inflation ($\lambda_* = \varepsilon t_*$) this constraint becomes (see eqn. 13) $\varepsilon \lesssim 10^{-11}$.

Lastly, consider the monopole problem.[18] If one combines the standard model of cosmology with the concept of grand unification then one predicts that there should be many monopoles in the world today. A convenient measure is the ratio $r \equiv n_m/s$, where n_m is the number density of monopoles and s is the entropy density. Then, allowing for some early annihilation, the expected value for r today is $r_o \approx 10^{-10}$. Contrasted with this are many astrophysical[19,20] and cosmological[18] arguments that place limits of $r_o < 10^{-24}$. Some of the arguments are model dependent and some are much more restrictive, but it does not seem unreasonable to take this limit as indicative of the monopole problem.[21]

It should be clear that an inflationary epoch can help solve the monopole problem.[3] The number density of monopoles scales as $n_m \sim 1/S^3$. During an adiabatic radiation or matter dominated era $s \sim 1/S^3$ as well and r remains fixed. During wall dominated inflation the situation is dramatically different. At the end of inflation the energy stored in walls gets thermalized, so that $s_d = \rho_d^{3/4}$. During the inflation the effective entropy density scales as $s \sim \rho^{3/4} \sim (1/S)^{3/4}$. As a result r decreases, $r \sim n_m/s \sim S^{-9/4}$. The scaling laws give $S_*/S_d = \rho_d/\rho_* = T_d^4/T_*^4$. If one supposes enough inflation to solve the horizon and flatness problems and uses equations (10) and (16) then

$$r_d = r_*(S_*/S_d)^{9/4} \simeq r_* \, (T_m T_o/T_*^2)^{3/2} \simeq 10^{-72} r_* \tag{23}$$

where I have supposed that the walls are associated with the unification mass scale, $\mu \approx 10^{15}$ GeV, to derive the last equality. Even for r_* as high as 1 a satisfactory reduction can be achieved. One possible problem with this scenario is that when the walls decay there might be additional production of monopoles. As long as the grand unified symmetry is not restored this should not be a problem; however, if local temperatures near the decaying wall get hot enough one might worry about thermal production of monopoles.[22]

V. Summary

In this paper I have examined the possibility that a wall dominated era could be used to solve the horizon and flatness problems. During a wall dominated epoch the universe grows as $S \sim t^2$. As a result the size of a causal volume becomes much larger than the Hubble length, so the horizon problem may be solved. Furthermore, the effects of curvature decrease as $1/S$ during the wall dominated era, so the flatness problem may also be solved.

Unfortunately, it does not appear likely that this scenario can work. The basic problem is one of reheating the universe. It is supposed that the wall dominated epoch ends when the walls decay via hole formation. The decay products will eventually reheat the universe, but this takes some time. Specifically, the decay products are initially arranged in thin sheets. Directly after the walls decay the universe is not homogeneous on scales smaller than the typical separation between walls. The universe does not appear homogeneous on small scales until the wall scale comes back into the horizon. While the walls are thickening the benefits of inflation are dissipating.

To get a quantitative measure of this problem it is useful to define the parameter ε as the ratio of the wall scale to the horizon size at the beginning of the wall dominated era, $\lambda_* = \varepsilon t_*$. In order to solve the horizon and flatness problems and still have a homogeneous universe at the time of nucleosynthesis requires $\varepsilon \lesssim 10^{-8}$. A more stringent, but less reliable, limit comes from density fluctuations, $\varepsilon \lesssim 10^{-11}$. It is difficult to understand how such small values of ε could occur. If walls could annihilate efficiently one would expect $\varepsilon \approx 1$. A more optimistic estimate assumes no annihilation after formation. Then, the natural value for ε is $\varepsilon \approx \mu/M_p$, where the coherence length of the physics creating the walls is roughly $1/\mu$. For $\varepsilon \approx 10^{-8}$ this implies the walls form at a temperature of roughly $T_W \simeq \mu \simeq 10^{11}$ GeV. If you want to "solve" the flatness problem this is not good. The point is that one must now explain how the universe got to be that old without curvature being important.

Apart from the difficulty of producing a smooth homogeneous universe there do not appear to be any significant difficulties with wall dominated inflation. It seems feasible to produce enough baryons. The monopole problem is easily solved by the release of entropy when the walls decay. There is not a graceful exit problem in the sense that the inflationary epoch definitely ends. To get the inflation to end at an appropriate time requires a fine tuning of about one part in a hundred (see Appendix), which compares favorably with the fine tuning in models of "new" inflation.

Before ending, I would like to take a more optimistic attitude towards the study of walls. Even though wall dominated inflation does not work as presented in this paper that does not mean that walls are unimportant in the history of the universe. One possibility is that a wall dominated epoch in combination with some other non-standard (i.e. not radiation) era could account for our current universe. Besides using particle physics to explain the current state of the universe one may use the state of the universe to place constraints on models of particle physics. In this regard any grand unified theory should be examined for the possible existence of unstable walls.

ACKNOWLEDGEMENTS

I am grateful to Anthony Zee for his patience and advice during the preparation of this work. I would also like to thank Jim Bardeen, Steve Barr, and Michael Turner for useful discussions. This work was supported in part by NASA and the DOE at Fermilab.

References

1. Ya. B. Zel'dovich, I. Yu. Kobzavev and L. B. Okun, JETP 40 1. (1974).
2. L. Abbott and M. Wise, Cal. Inst. Tech. Preprint, CALT-68-1100 (1984)
3. A. Guth, Phys. Rev. D23, 347 (1981); A. Guth and E. Weinberg, Phys. Rev. D23, 876 (1981).
4. A. Vilenkin, and A. E. Everett, Phys. Rev. Lett. 48, 1867 (1982).
5. P. Sikivie, Phys. Rev. Lett. 48, 1156 (1982).
6. S. Coleman, Phys. Rev. D15, 2929 (1977); C. Callan and S. Coleman, Phys. Rev. D16, 1762 (1977).
7. A. Guth and E. Weinberg, Nucl. Phys. B212, 321 (1983); S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D26, 2681 (1982).
8. A. D. Linde, Phys. Lett. 108B, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); J. Ellis, D. V. Nanopoulos, K. A. Olive, and K. Tamvakis, Nucl. Phys. B221, 524 (1983).
9. P. J. Steinhardt and M. S. Turner, Phys. Rev. D29, 2105 (1984).
10. For a discussion of the standard big bang see S. Weinberg, Gravitation and Cosmology, Wiley and Sons, New York (1972); J. Peebles, Physical Cosmology, Princeton University Press, Princeton, N.J. (1971).
11. J. Bardeen, Phys. Rev. D22, 1882 (1980); A. Guth and S-Y. Pi, Phys. Rev. Lett., 49, 1110 (1982); A. Starobinsky, Phys. Lett. 117B, 175 (1982).

12. J. Bardeen, P. Steinhardt, and M. S. Turner, Phys. Rev. D28, 679 (1983); J. A. Frieman and M. S. Turner, Phys. Rev. D30, 265 (1984)

13. D. A. Kirzhnits and A. Linde, Phys. Lett. 42B, 471 (1972); C. W. Bernard, Phys. Rev. D9 3312, (1974); L. Dolan and R. Jackiw, Phys. Rev. D9 3320, (1974); S. Weinberg, Phys. Rev. D9 3357, (1974).

14. For a recent review see J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. A. Olive, Astrophys. J. (in press)

15. A. D. Sakharov, JETP Lett 5, 24 (1967); M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978); M. Yoshimura, Phys. Rev. Lett. 42, 746 (1978); S. Dimopoulos and L. Susskind, Phys. Rev. D18, 4500 (1978); A. Yu. Ignatiev, N. V. Krasnikov, V. A. Kuzmin and A. N. Tavkhelidze, Phys. Lett. B76, 480 (1978); D. Toussaint, S. B. Trieman, F. Wilczek and A. Zee, Phys. Rev. D19, 1036 (1979); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. B80, 360 (1979); For a review see E. W. Kolb and M. S. Turner, Ann. Rev. Nuc. Part. Sci. 33, 645 (1983).

16. Ya. B. Zel'dovich, Mon. Not. R. Astr. Soc. 160, 1, (1972); E. R. Harrison, Phys. Rev. D1, 2726 (1970).

17. D. J. Fixsen, E. S. Cheng, and D. T. Wilkenson, Phys. Rev. Lett. 50, 620 (1982); T. Uson and D. T. Wilkenson, Proceedings of Inner Space / Outer Space, to be published by Univ. of Chicago Press.

18. Ya. B. Zel'dovich and M. Y. Khlopov, Phys. Lett. 79B, 239 (1978); J. Preskill, Phys. Rev. Lett. 43, 1365 (1979); T. Goldman, E. W. Kolb, and D. Toussaint, Phys. Rev. D23, 867 (1981).

19. E. N. Parker, Ap. J. 160, 383 (1970); 163, 225 (1971); 166, 295 (1971); M. S. Turner, E. N. Parker, T. J. Bogdan, Phys. Rev. D26, 1296 (1982).

20. E. W. Kolb, S. A. Colgate, and J. A. Harvey, Phys. Rev. Lett. 49, 1373 (1982); S. Dimopoulos, J. Preskill, and F. Wilczek, Phys. Lett. 119B, 320 (1982); J. Harvey, Princeton University Preprint (1983).

21. For a comprehensive review of various limits see Proceedings of Monopole '83, ed. J. Stone, Plenum Press. (1983)

22. M. S. Turner, Phys. Lett., 115B, 95 (1982).

23. P. Sikivie, Univ. of Florida Preprint UF-TP-83-6.

24. J. Bardeen, Private Communication; J. Isper and P. Sikivie, Univ. of Florida Preprint UF-TP-83-18.

25. T. W. B. Kibble, Jour. Phys., A9, 1387 (1976).

Appendix: Creating and Destroying Unstable Walls

In the usual discussion of topological features of gauge theories the topological entities are stable. However, it is possible to create features that are not stable. An example is the domain walls of Sikivie[5] and Vilenkin and Everett[4] which are unstable to the formation of holes in the walls. In this Appendix I present a toy model in which walls form that are unstable to hole formation. The walls form without holes in them, except for a small number caused by thermal fluctuations. This is important as it allows the walls to conformally stretch with the expansion of the universe.

The model has scalar fields with an approximate SO(2) symmetry. The Lagrangian has linear, quadratic and quartic couplings.

$$L = 1/2(\partial_\mu\phi\cdot\partial^\mu\phi) + V(\phi) \tag{A1}$$

with $V(\phi) = (\lambda/4)\ ((\phi\cdot\phi)-v^2)^2 + \varepsilon(\phi\cdot\xi)$

where ξ is defined to be dimensionless and of unit length. The dimensions of ε are (mass^3). Without the $\varepsilon(\phi\cdot\xi)$ term V possesses an SO(2) symmetry. Although $|\langle\phi\rangle| = v$ is fixed the direction of $\langle\phi\rangle$ in the internal space is a free function of position. The shape of V is the familiar 'wine bottle'. When the $\varepsilon(\phi\cdot\xi)$ term is turned on the shape of the potential is tilted, the SO(2) symmetry is broken, and a unique value for $\langle\phi\rangle$ is picked out in the $-\xi$ direction. Now imagine that ε is a function of temperature so that the sign of ε changes as the temperature drops below some critical value.

$$\varepsilon(T) > 0,\ T > T_c \tag{A2}$$

$\varepsilon(T) < 0, \; T < T_c$

At the same time that ε is changing sign ξ is kept fixed. The tilt of the potential changes sign but not direction. As a result of the change of tilt, the minimum of V moves from $\langle\phi\rangle = -v\xi$ to $\langle\phi\rangle = v\xi$.

The path by which $\langle\phi\rangle$ moves from the old minimum to the new one is important. If the change in tilt is slow enough then $\langle\phi\rangle$ can be expected to stay in the ring of relative minima. Then there are two distinct paths (Fig. A1) by which $\langle\phi\rangle$ can move from its original to its final value. Since ξ is fixed these two paths are equally likely. The result is a broken Z_2 symmetry. As with other broken discrete symmetries one finds walls. Here, the walls are between regions of space where $\langle\phi\rangle$ develops along different paths. In the usual discussion of walls, the domains on different sides of the wall are different vacua. In this case, the domains on either side are actually the same vacuum. It is the history followed in getting to that vacuum that distinguishes the domains. To see the difference in structure of the two types of wall see Fig. A2.

As stated earlier, this type of wall is unstable. A hole can form in the wall by continuously letting ϕ go through 0. This is only possible because the domains on either side of the wall are the same. Although it is topologically possible to form a hole, it is classically forbidden because it entails a big cost in energy to drag ϕ over the hill in the potential at $\phi = 0$. However, holes can still form by quantum tunneling or by thermal fluctuations.

In order for a wall dominated phase to occur the thermal tunneling must be small. I assume this to be the case and concentrate on describing the quantum tunneling. In fact, there is a subtle point involving the parameter ε and thermal tunneling. If ε were zero at high T and turned on at some T_c then one would still get walls (this is precisely the case for axions).[23] However, these walls would end on strings. It is extremely unlikely to have an unbroken wall stretching over many horizons. One expects the walls to annihilate in a time proportional to the string spacing. As a result there is no inflation for $\varepsilon = 0$ models.

One can estimate the decay rate per unit area, Γ, due to quantum tunneling by estimating the action to form a critical size hole, A_c.

$$\Gamma \sim v^3 e^{-A_c} \tag{A3}$$

The critical radius for a hole is determined by competition between the wall energy missing from the hole and the string energy at the edge of the hole. The edge of a hole, where $\phi = 0$, is a string of the underlying approximate SO(2) symmetry. As such, the boundary of the hole has energy per unit length $\sigma_s \simeq v^2$. From dimensional analysis one finds that the wall energy per unit area is roughly $\sigma_w \approx v^{3/2}\varepsilon^{1/2}$. The critical size hole is then approximately $R_c \approx \sigma_s/\sigma_w \simeq (v/\varepsilon)^{1/2}$ and the critical action is $A_c \simeq v^3/\varepsilon$. Since the action shows up in an exponential one should worry about factors of order one. In general, this requires a numerical calculation, but in the limit that $\varepsilon << v^3$ one can perform a simple analytic calculation similar to Coleman's[6] thin wall approximation. In this "skinny" string approximation $R_c = 2(v/\varepsilon)^{1/2}$ and $A_c = 64\pi/3\ (v^3/\varepsilon)$.

Using this result with equations (A3, 15) one can estimate the lifetime of the walls.

$$t_d \simeq 1/v\ e^{A_c/3} \simeq 1/v\ e^{22.4\ v^3/\varepsilon} \tag{A4}$$

By making small adjustments in v and ε one can acquire large amounts of inflation. To end the inflation in a certain epoch requires only a small amount (10^{-2}) of fine tuning.

As an example, suppose one wishes to choose (v^3/ε) such as to solve the horizon and flatness problems and still preserve homogeneity at nucleosynthesis. Let the walls form at $t_w \approx m_p/v^2$. Then equation (A4) gives

$$t_d/t_w = v/m_p\ e^{22.4\ v^3/\varepsilon} \tag{A5}$$

Using equation (17) (with the temporary definitions $\mu \to v$, $\varepsilon \to \eta$) one can calculate

$$t_d/t_w = (\rho_w/\rho_d)^{1/2} = (T_w/T_d)^2 = 10^{20}\ \eta^{2/3}. \tag{A6}$$

Recall that the value of η for this scenario is $\eta \approx 10^{-8}$ and that the smallest reasonable value for η is $\eta \approx v/m_p$. Then

$$v^3/\varepsilon \simeq 1/22.4\ \ell n(10^{20}\eta^{-1/3}) \approx 2.4 \tag{A7}$$

To keep the reheat temperature accurate to within an e-folding one must fine tune v^3/ε to 4%. Before proceeding, note that $v^3/\varepsilon \approx 2.4$ is not large, so the skinny string approximation is not well justified.

So far, I have discussed a model with a Z_2 symmetry. It is not clear that the system of walls that form in this case will conformally expand. The problem is that the walls form bubbles. The different bubbles are isolated from each other. Consider a region between bubbles. Locally the stress-energy density is that of primeval background radiation. There is nothing to make the local region expand. This is in distinction to the normal inflationary scheme where the cosmological constant acts everywhere, so each local region of space expands in the same way. If one considers the expansion of a single spherical bubble in an asymptotically flat background one finds that the bubbles will eventually collapse under their self gravitational interactions.[24] This suggests that isolated bubbles cannot inflate space and fill the voids with their decay products. Fortunately, it is not difficult to see what modifications might avoid this problem. One needs a connected framework of walls. To construct such a framework one needs a model which allows three or more walls to join. The junction of the walls should have the topological characteristics of a string. To allow for the joining of three walls the model must allow for at least three types of vacuum domains. To use the walls for inflation, the walls must be unstable. It is not necessary for the strings to be unstable, since heavy stable strings are not in conflict with cosmology.[25] As the reader might guess it is not difficult to construct a scalar potential that has these properties, although finding a compelling and believable model is a more formidable task.

An interesting possibility is that one might find unstable domain walls in more familiar grand unified theories. For example, in an SU(5) model that progresses from SU(5) to SU(4) x U(1) and then to SU(3) x SU(2) x U(1) the second phase transition might produce unstable walls. As pointed out by Guth and Weinberg[3] there are two types of bubbles that may take SU(4) x U(1) into SU(3) x SU(2) x U(1). One type of bubble is SU(3) x U(1) x U(1) symmetric while the other has SU(2) x SU(2) x U(1) symmetry. Regions of space that tunnel via different bubble types should be separated by walls. Since the final state vacuum on either side of a wall is the same, the walls may decay. However, these walls are probably not important for cosmology because the action for the two types of bubbles to form is not degenerate.[3] One expects few bubbles of the SU(2) x SU(2) x U(1) type and it seems unlikely that those regions could percolate. Unless they do percolate, all such domains will be bounded and the domain walls will collapse instead of driving an inflationary phase.

Finally, I would like to suggest that it is not difficult to create other unstable topological features such as monopoles or strings. The key point of the model presented in this appendix is that between a unique initial state and a unique final state there are two classical histories that are equally likely. The choice of which history actually occurs breaks the discrete symmetry and as a result walls form. Similarly, call the set of histories leading to a unique final state H. If $\pi_1(H)$ is non-trivial unstable strings can form. If $\pi_2(H)$ is non-trivial unstable monopoles can form.

Figure Captions

Figure A1. Behavior of $\langle\phi\rangle$ as a function of temperature. As T changes from $T > T_c$ to $T < T_c$ the slope of the potential changes sign. As a result $\langle\phi\rangle$ rolls from $-\xi v$ to $+\xi v$. $\langle\phi\rangle$ may roll by either of two paths, labelled A and B, which lie in the ring of relative minima of $V(\phi)$.

Figure A2. Different structures of a) topologically stable domain walls and b) topologically unstable but classically stable walls. The arrows indicate the local value of ϕ. The walls lie perpendicular to the page in the y-z plane. The histories A, B are described in Fig. A1.

Fig A1

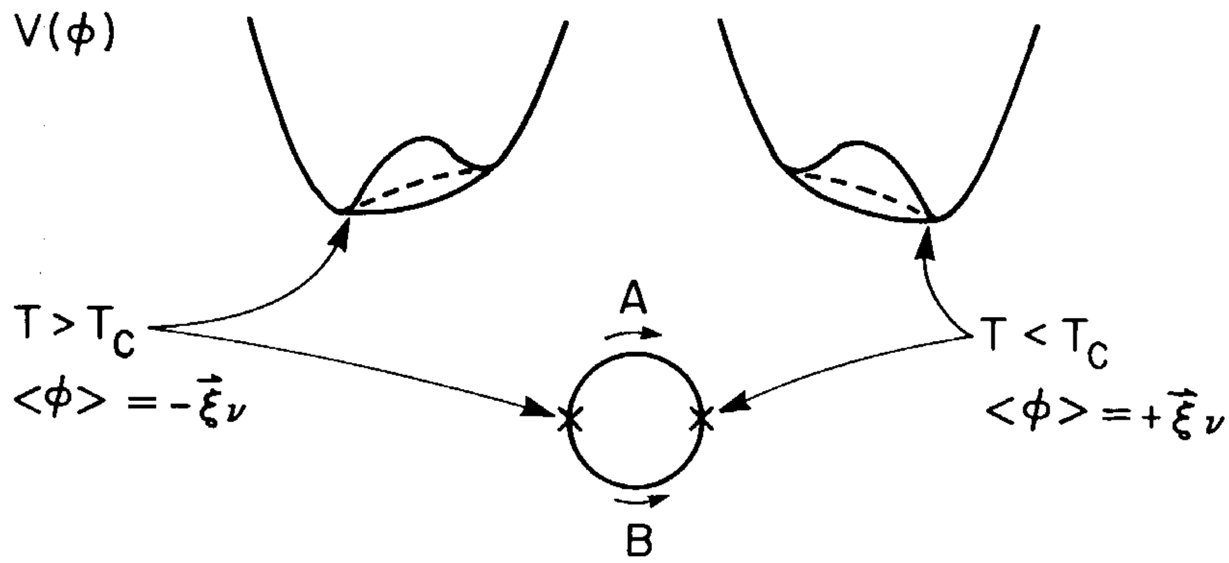

FIGURE A1

Fig A2

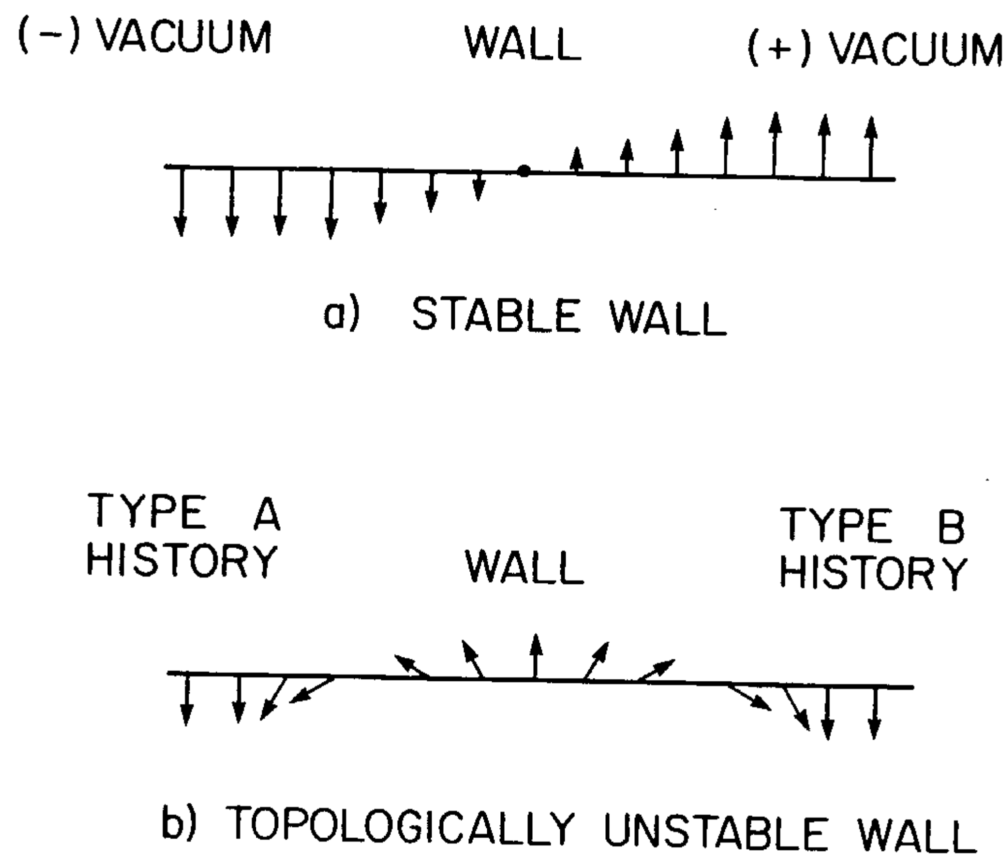

FIGURE A2