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RECENT HEAVY PARTICLE DECAY IN A MATTER DOMINATED UNIVERSE

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Abstract

The cold matter scenario for galaxy formation solves the dark matter problem very nicely on small scales corresponding to galaxies and clusters of galaxies. It is, however, difficult to reconcile with a Universe with an Einstein-deSitter value of $\Omega = 1$. We will show here that cold matter and $\Omega = 1$ can be made compatible while retaining the feature that the Universe is matter dominated today. This is done by means of heavy (cold) particles whose decay subsequently leads to the unbinding of a large fraction of lighter clustered matter.

Current models of galaxy formation distinguish between three types of scenarios (Bond and Szalay 1983). They are referred to as the hot, warm and cold matter scenarios, and refer to the type of matter which is dominating the Universe around the onset of galaxy formation. Hot particles are defined to be those which are relativistic at the time their interactions decouple and are just going non-relativistic at the time of galaxy formation. A common example of this type of particle is a light neutrino ($m_\nu \lesssim 100$ eV). Warm particles decouple at a high enough temperature so that they go non-relativistic slightly before galaxy formation. Any light super-weakly interacting particle could be a warm particle candidate. Finally, cold particles are those which are non-relativistic very early. Such particles might include very massive neutrinos ($m_\nu >$ a few GeV), axions, gravitinos etc. Some scheme (annihilation, inflation...) must be invoked so that their mass density is acceptably low.

Taken alone, each of these scenarios has both its faults and virtues. Hot particles are very good at producing large scale structure such as filaments and voids (Melott 1983; Klypin and Shandarin 1983; Bond, Szalay and White 1983). The problem is that this scale is too large to be compatible with observations and not overclose the Universe (White, Frenk and Davis 1983). In addition, one faces the problem of getting the "hot" particles into galaxies. The minimum scale on which these particles can cluster is determined by the Jeans mass (Bond, Efsthatiou and Silk, 1980; Zeldovich and Sunyaev 1980)

$$m_J \approx 3 \times 10^{18}\ M_\odot/m_\nu^2(\mathrm{eV}) \qquad (1)$$

and thus for $m_\nu < 100$ eV, $M_J > 3 \times 10^{14} M_\odot >> M_G \sim 10^{11} - 10^{12} M_\odot$. Presumably, for this picture to work, some fraction of the hot particles must get left behind in galaxies during the violent shocks accompanying galaxy formation, as in the pancake scenario (Bond, Centrella, Szalay and Wilson 1983). One might think that warm particles, by virtue of their large mass, would become a good candidate for the dark matter (Olive and Turner 1982; and Bond, Szalay and Turner 1982). Since they go non-relativistic earlier they have acceptably small free-streaming lengths for large scale structure. However one still faces the problem of how to get the warm particles clustered on small scales. For example, if dwarf galaxies also contain large amounts of dark matter (Faber and Lin 1983; and Lake and Schommer 1983), one cannot get warm matter clustered on such small scales. For example, for $M_{warm} \lesssim 1$ keV, eq. (1) gives $M_J \gtrsim 3 \times 10^{12}\ M_\odot$ which is much greater than the mass of a typical dwarf, $M_D \sim 10^6 - 10^7 M_\odot$.

Cold matter seems to be the best choice for getting structure on small scales (Peebles 1982; Peebles 1984). The large scale structure would then proceed to form hierarchically. The problem is that since cold matter is so good at clustering the mass of galaxies and clusters would consist of primordial fractions of dark and baryonic matter. Since the overall mass densities at these scales corresponds to an Ω between 0.1 - 0.5, it would appear that cold matter is incompatible with an $\Omega = 1$ Universe.

In an attempt to save the neutrino scenario, Davis et al. (1981) and Hut and White (1983) considered a possibility involving (at least) two neutrino flavors. In this scenario, a heavy neutrino is unstable and decays into a lighter one. Before the heavy neutrino decays, the

Universe becomes matter dominated. This happens in such a way that the light neutrinos are able to clump on smaller scales than if they dominated the Universe themselves. The problem with such a picture is that in order to get sufficient growth of perturbations on scales of 5-10 mpc matter dominance must have occurred for temperatures of $T_{MD} \lesssim$ 100 eV. This requires the mass of the heavy neutrino to be either about 1 keV or 1 GeV. The lower value is not allowed if we require that the flavor changing decays are weak interactions. The heavy neutrino lifetime is then too long. The upper value is ruled out by direct experiment. If one substituted new particles for the heavy neutrinos the basic idea of these authors might be salvaged. However, the resulting scenarios would be warm matter scenarios with their small scale problems.

In two recent attempts to remedy the conflict with an $\Omega = 1$ Universe, Turner, Steigman and Krauss (1984) and Gelmini, Schramm and Valle (1984) consider late decays which leave the Universe radiation dominated today. In these scenarios a heavy particle (in their example taken to be a neutrino with non-weak decays) undergoes a non-radiative decay (i.e. into anything but photons) into a lighter one. The decay occurs late enough so that galactic scales go non-linear before the decay. In this case the greater part of Ω which was in the heavy particles is released into the light particles and remains unclustered. The baryons which will have already begun dissipative processes are left behind to form galaxies and clusters. The major problem is the age of the Universe which forces the Hubble parameter $h_o = H_o/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$ to be low; $h_o \lesssim 0.45$, whereas the observations imply $1/2 \lesssim h_o \lesssim 1$.

In what follows, we will look at a similar situation to that of the above scenario; however, our goal will be to retain a matter dominated Universe today. We will consider the case where the hot decay products of the heavy particles only contribute a fifth of closure density, leaving the rest to the primordial lighter particles. We will consider in some detail the effect of the decay of the heavy particles in unclustering a large fraction of the lighter particles with the benefit of leaving some behind to form the dark matter in galactic halos. It should be clear that what we have in mind is a hierarchical picture of galaxy formation -- not a pancake scenario.

We will not pin the scenario down to any specific model in particle physics but rather discuss the general properties these particles must obey. Specific models might be the heavy neutrino goes to light neutrino plus majoron as discussed by Gelmini, Schramm and Valle (1984) or supersymmetric models involving the non-radiative decay of the gravitino (Olive, Schramm and Srednicki 1984). For our purposes we will be mainly interested in four quantities: the masses of the heavy and light particles M_H and M_L, the density of heavies relative to photons Y, and the decay rate of the heavy particle Γ_D.

We begin the discussion with the light particle, L, as its role in this model is the simplest. As we have said we would like to have the density of primordial (i.e. those not produced in the decay of the heavies) lights be near closure density. The energy density of primordial light particles today can be expressed as

$$\rho_L = (3/4)\, M_L\, n_\gamma\, (T_L/T_o)^3 (g_L/2)$$

$$= (3/4)\, M_L\, n_\gamma\, (3.9/N(T^*))(g_L/2) \tag{2}$$

where T_o is the temperature of the microwave background radiation and T_L is the temperature of the light particles today. If L is a neutrino then $(T_L/T_o)^3 = (3.9/N(T^*)) = 4/11$ where $N(T^*) = 10.7$ is the number of degrees of freedom of relativistic particles at neutrino decoupling. For particles which have weaker interactions than neutrinos and decouple earlier, $N(T^*)$ is larger and hence T_L smaller (Olive, Schramm, and Steigman 1981). Finally, g_L is the number of degrees of freedom for L. If we express ρ_L in terms of a fraction of closure density

$$\Omega_L \equiv \rho_L/\rho_c \tag{3}$$

where $\rho_c = 1.88 \times 10^{-29} h_o^2$ g cm^{-3}, we have a relation between M_L and $\Omega_L h_o^2$

$$M_L = 9.1\ N(T^*)(2/g_L)\Omega_L h_o^2\ (2.7K/T_o)^3\ \text{eV} \tag{4}$$

In building a model, M_L will then be set according to the values of N, Ω_L and h_o. For example, for L a neutrino, $\Omega_L = 0.75$ and $h_o \simeq 1/2$ we would find $M_L \approx 18$ eV.

In addition to the mass density of the primordial light particles, we must also specify the density of decay products produced by the decay of the heavy particles. Before decay, the mass density of heavies is

$$\rho_H = M_H n_\gamma Y \tag{5}$$

where we have defined $Y \equiv n_H/n_\gamma$. If H were a neutrino $Y = 3/4 \cdot 4/11 =$

3/11. We have put eq. (5) in this form to leave open the possibility that H decoupled very early and that a period of inflation made Y very small. (Y would then be computed as the abundance produced in the reheating period after inflation.)

We can now express the energy density in decay products today by redshifting eq. (5) from the time of decay

$$\rho_D = \rho_c \Omega_D = .88 \; M_H Y n_\gamma \; T_o/T_D \tag{6}$$

or

$$M_H Y/T_D = 1.31 \times 10^5 \; \Omega_D \; h_o^2 \; (2.7K/T_o)^4 \tag{7}$$

where T_D is the temperature at $t_D \equiv 1/\Gamma_D$ and the factor of .88 comes from taking the exponential decay law into account (Scherrer and Turner 1984). Thus once we have specified Ω_D and h_o the quantity $M_H Y/T_D$ is fixed regardless of the particle physics involved.

In this scenario, the Universe becomes matter dominated by the heavy particles at a temperature $T_{MD} > T_D$. T_{MD} is determined by the condition that the energy density in matter is equal to that in radiation

$$\rho_{rad} = \rho_H + \rho_L + \rho_B \tag{8}$$

where ρ_B is the density in baryons and

$$\rho_{rad} = (\pi^2/30) \; N_{MD} \; T_{MD}^4 \tag{9}$$

where N_{MD} is the effective number of relativistic degrees of freedom at T_{MD}. For the temperature ranges of interest, $T_{MD} < 1$ keV, N_{MD} is determined by the contribution from photons and light (or massless) neutrinos,

$$N_{MD} = 2(1 + N_\nu (7/8)(4/11)^{4/3}) \tag{10}$$

where N_ν is the number of light neutrino flavors. For 3 light neutrinos, $N_{MD} = 3.36$ while if $T_{MD} \leq 10$ eV and all three flavors of neutrinos are non-relativistic N_{MD} can be as low as 2.0. At early times ($t \ll t_D$) the ratio of heavy particles to light plus baryons is

$$(\rho_L + \rho_B)/\rho_H = .88\ (T_o/T_D)\ (\Omega_L + \Omega_B)/\Omega_D \tag{11}$$

Combining eqs. (5 and 8-11) we find that T_{MD} is

$$T_{MD} = 0.74\ M_H Y\ (1 + .88\ (T_o/T_D)\ (\Omega_L + \Omega_B)/\Omega_D)/N_{MD} \tag{12}$$

The last quantity that will be specified is the decay rate, Γ_D, for H. In particular, the decays will occur when Γ_D is equal to one over the age of the Universe.

$$\Gamma_D = C_D[2.02\ M_H Y\ (1 + .88\ (T_o/T_D)\ (\Omega_L + \Omega_B)/\Omega_D)\ T_D^3]^{1/2}/M_p \tag{13}$$

where $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass. The constant C_D is to be found by numerical integration of the age of the universe. In the limit $\rho_H/\rho_L \to 0$, $C_D \to 3/2$; whereas for $(\rho_B + \rho_L)/\rho_H \to 0$ we find $C_D = 1.63$.

To a large extent, the scenario is fixed by the choices of Ω_L, Ω_D and the contamination from baryons Ω_B. Big Bang Nucleosynthesis puts constraints on Ω_B (Yang et al. 1984)

$$0.01 \leq \Omega_B h_o^2 \leq 0.049 \tag{14}$$

We will assume throughout that $\Omega = \Omega_L + \Omega_B + \Omega_D = 1$. The relevant quantity describing non-relativistic matter is the sum $\Omega_L + \Omega_B$. Once this quantity is specified, Ω_B only affects the value of M_L through eq. (4). We will refer to the sum as $\Omega_{NR} = \Omega_L + \Omega_B$. (If the neutrino species are also non-relativistic, their contribution to Ω must also be included in Ω_{NR}.)

Irrespective of the model from particle physics, the scenario proceeds as follows. The Universe is radiation dominated until the temperature T_{MD}. It then becomes matter dominated (by H) for a period

$$\frac{T_{MD}}{T_D} = 9.7 \times 10^4 (1+.88 \frac{\Omega_{NR}}{\Omega_D} \frac{T_o}{T_D}) \frac{\Omega_D h_o^2}{N_{MD}} (\frac{2.7K}{T_o})^4 \tag{15}$$

At T_D the Universe becomes radiation dominated again until a temperature T_E defined by

$$T_E = (\Omega_{NR}/\Omega_D)\ T_o \tag{16}$$

when the non-relativistic particles take over again and the Universe is again matter dominated. For $\Omega_{NR} > \Omega_D$ the Universe is matter dominated today.

The goal of our scenario is that during the first epoch of matter domination, there is some mass scale (hopefully large enough to encompass galaxies and small clusters of galaxies) which goes non-linear before the decay of H. These mass scales will contain the baryons and the primordial light particles. When the decay occurs a large fraction of the light particles will escape and become unclustered dark matter, as we will show below. The baryons, having gone through some dissipation, will be left behind. The latter period of matter domination will only produce clustered matter on the largest scales observed today.

As we just indicated, it will be crucial that some mass scale go non-linear, i.e. $\delta \equiv \delta\rho/\rho \sim 1$ on that scale, by the epoch of decay. Most of the growth in δ must occur in the 1st matter dominated epoch. If we assume a Harrison-Zel'dovich perturbation spectrum the amplitude at horizon crossing is scale independent. The maximum initial value for δ is governed by the limits on the anisotropy of the microwave background from the quadropole moment, $\delta T/T \lesssim 6 \times 10^{-5}$, which translates (see appendix) to $\delta_i \leq 1.2 \times 10^{-4}$ (Fixsen, Cheng, and Wilkenson 1982)†. Although the linear growth in δ occurs in the matter dominated phase there is some growth in the early radiation dominated phase by a factor A_λ which may be large depending upon when the scale λ crossed the horizon. This growth is due to the residual velocity field of the matter after it crosses the horizon (Blumenthal and Primack 1984; Bardeen 1984). At the time of decay the fluctuation at a given scale will have reached a maximal value

$$\delta_\lambda^D = A_\lambda \, \delta_i \, (T_{MD}/T_D) \tag{17}$$

In the hierarchical picture, the scale of structure at t_D will be that scale that has just gone non-linear at t_D. We call this scale λ_{NL}. In order to ensure that $\delta_{\lambda_{NL}} = 1$ we require that

$$A_{NL} = (1/\delta_i)\ T_D/T_{MD} \tag{18}$$

To determine the length scale which this corresponds to we find it convenient to express λ_{NL} in terms of λ_{MD}, the horizon scale at matter dominance. Today the comoving scale of λ_{MD} is

$$\lambda_{MD} = 7.4\ \left(\frac{T_o}{T_D}\right) N_{MD}^{1/2} \left(\frac{T_o}{2.7K}\right)^2 \left[(1+.88\ \frac{\Omega_{NR}}{\Omega_D}\ \frac{T_o}{T_D}) \Omega_D h_o^2\right]^{-1}\ \text{Mpc} \tag{19}$$

To determine $\lambda_{NL}/\lambda_{MD}$ from A_{NL} we have adapted the results of Peebles (1982). In Fig. 1 we plot A_{NL} as a function of $\lambda_{NL}/\lambda_{MD}$.

Now, we would like the scale λ_{NL} to roughly correspond to λ_{gg} the galaxy-galaxy correlation length. In order to pursue even an approximate calculation we need an estimate for the value of λ_{gg} based on observations of the distribution of baryonic matter in the universe. A reasonable estimate of the scale on which the galaxy distribution can be considered to have gone nonlinear is the diameter of a sphere sufficiently large that the variance of galaxies $\langle\delta N^2\rangle$ is equal to the square of the mean number within the sphere $\langle N\rangle^2$. This is found from the condition

$$\int e^{-x^2/\sigma^2}\, dx \int e^{-y^2/\sigma^2}\, dy\ \xi(x-y) = \lambda_{gg}^6 \tag{20}$$

where

$$\sigma^3 = \pi^{-3/2}\lambda_{gg}^3 \tag{21}$$

In eq. (20) $\xi(x-y)$ is the galaxy two particle correlation function, and σ is chosen so that the gaussian window function samples over an effective volume equal to that of a cube of side λ_{gg}. In eq. (20) we have used a gaussian window, rather than the usual sharp cutoff, in order to minimize the noise contributed by very small scales. Observations indicate that $\xi(x)$ is approximately given by

$$\xi(x) = (r_o/|x|)^{1.77} \tag{22}$$

where $r_o \approx 5.4\ h^{-1}$ Mpc (Davis and Peebles 1983). We conclude from eqs. (20-22) that λ_{gg} is about 9 h_o^{-1} Mpc.

Unfortunately, it is unrealistic to simply equate this length with the comoving scale on which nonlinearity has occurred in the pre-decay epoch. There are at least two effects, of opposite sign, which must be included in any detailed analysis of this model. First, if dissipative collisions among the protogalaxies are not effective then much of the structure built up before T_D on scales larger than individual galaxies will disperse. Second, when matter dominance is reestablished at a recent epoch, structure will start to grow again. The amount of growth to be expected in baryon density fluctuations at scales typical of galaxy clustering depends on the fraction of matter contained in galaxies. Based on dynamical considerations Davis and Peebles (1983) suggest that structure less than about 1 h_o^{-1} Mpc in radius have a mass to light ratio equivalent to Ω of about 0.2. Larger scales are more difficult to probe effectively, but may have a somewhat larger value.

Thus allowing for a small amount of recent growth in the amplitude of the correlation function it seems reasonable to adopt a value for λ_{NL} of $5h_o^{-1}$ Mpc. A much more detailed analysis would be necessary to decide if the first consideration would make this estimate unduly small. We note that this length corresponds to a baryonic mass

$$M_{NL}^{B} = \Omega_B \rho_c \lambda_{NL}^3 = 3.3 \times 10^{13}\ \Omega_B/h_o \quad M_\odot \tag{23}$$

which is of the order of a small cluster of galaxies.

Once we have fixed λ_{NL} it is possible to determine A_{NL}, λ_{MD} and T_D given Ω_D, Ω_{NR}, h_o, N and δ_i. The three undetermined variables are related by eqns. (18 and 19) and the implicit relationship between A_{NL} and $\lambda_{NL}/\lambda_{MD}$ expressed in Fig. 1. Once T_D is determined by this procedure we can calculate the particle physics properties (M_H Y, M_L, $N(T_*)$, Γ_D) necessary to make the model work.

Let us now look at a specific example with the choices $\Omega_D = 0.2$ and $\Omega_{NR} = 0.8$. We will require that $1/2 < h_o < 1$ in accord with the observational determinations. The age of the Universe will require that we are at the small end of the range in h_o. Remember, that even in a pure matter dominated Universe, $\Omega = 1$ and $h_o > 1/2$ implies that the age of the Universe is $t_u < 1.3 \times 10^{10}$ yrs, just barely consistent with the age of globular clusters. We will, therefore, choose $h_o = 1/2$. We will use $N_{MD} = 3.36$ correspondingly to three massless neutrino species (i.e. the standard model). We note however that if the limits on the quadropole anisotropy become much more stringent, it may be advantageous to consider a model with fewer massless neutrinos.

With $\Omega_D = .2$, $\Omega_{NR} = .8$, $h_o = 1/2$, $N_{MD} = 3.36$, $\lambda_{NL} = 10$ Mpc, $\delta_i = 1.2 \times 10^{-4}$, and $T_o = 2.7°K$ we find $A_{NL} = 4.6$, $T_D/T_o = 14$, $\lambda_{NL}/\lambda_{MD} = .65$, and $T_{MD} = 5.9$ eV. The age of the universe is 12×10^9 yrs for these values.

We can now give the particle physics properties necessary for our model. From eqn. (7) the product M_H Y is constrained to

$$M_H\, Y \sim 21(2.7K/T_o)^2 \text{ eV} \tag{24}$$

Eq. (13) fixes the value of the decay rate

$$\Gamma_D \sim 1.8 \times 10^{-31}\ (2.7K/T_o)^4 \text{ eV} \tag{25}$$

So far the treatment of the growth of density perturbations has been incomplete in that we have ignored the finite decay time of the heavy particles and its effect on the growth of perturbations. We have completed an exact calculation of the growth of a constant curvature perturbation spectrum. A description of this calculation is included in the appendix.

Figure 2 shows the parameter space in $M_H Y$ and Γ including the nonlinear structure constraint obtained from the appendix. We have indicated our suggested model parameters with a cross. We see that the constraint that λ_{NL} = 10 Mpc does not present our model with any difficulties. Not included in this figure is any constraint derived from requiring that the current structure in the galaxy clustering match the observed galaxy-galaxy correlation function. In the following pages we will discuss the difficulties involved in tracing the evolution of galaxy clustering past T_D. Here we will simply note that the extreme assumption that the clustering proceeds without significant streaming effects does not result in a useful constraint on the model parameters. It is clear that there is a reasonable area in parameter space in which the age of the universe is close to, or greater than, 12 billion years, and in which the various dynamical constraints of our model are satisfied. We will not concern ourselves here with the question of whether or not nature is obliging enough to present us with such particles. Instead, we will assume that they exist and explore the consequences for the formation of large scale structure.

A crucial feature of our model is that the decay of the heavy particles changes the structures that may exist in the Universe. This happens in two ways. First, structure that has already formed by t_D may disperse. Second, after t_D the Universe is dominated by radiation and free streaming matter. These effects considerably alter the spectrum of density perturbations at large scales. The details of dispersal have important implications for the subsequent growth of large scale structure. We work under the assumption that the preexisting structure consists of isothermal spheres with a radius comparable to λ_{NL}, the scale that has just gone non-linear at t_D. Although restrictive, the assumption of isothermal spheres is easy to treat and is supported by theoretical considerations of violent relaxation (Lynden-Bell 1967, Shu 1978).

We choose the initial mass within an isothermal sphere, M_i, to be the mass scale that has just gone non-linear by t_D. This mass is given by eq. (23) except that we must multiply by $(1+\Omega_L/\Omega_B + (\Omega_D/\Omega_B)\ T_D/T_o)$ to include the mass of non-baryonic particles. Each isothermal sphere has three mass components; heavy particles, light particles, and baryons. Initially, the predominant component is heavy particles, but as decay proceeds the light particles come to dominate. Being dissipationless, the heavy and light particles have the same spatial and velocity distributions. The baryons are assumed to have undergone some dissipation and have a much tighter spatial distribution, so that at the core of the sphere baryons may be the dominant mass fraction.

Isothermal spheres have a density profile that falls as $1/r^2$. If extended indefinitely this leads to a divergent mass. We will assume that the density profile cuts off sharply at a radius R_*, which should be comparable with λ_{NL}. The mass of the sphere is then

$$M_* = 4\pi\int_0^{R_*} \rho\ r^2\ dr = 4\pi\ \rho_*\ R_*^3 \tag{26}$$

where, $\rho_* = \rho(R_*)$, is the density at the outer edge of the sphere. Initially this sets

$$\rho_* = \bar{\rho}(\lambda_{NL}/R_*)^3/4\pi \tag{27}$$

where, $\bar{\rho}$ is the average density of the Universe at t_D. It is useful to introduce the dimensionless radial coordinate, x,

$$r = x\ R_* \tag{28}$$

Then, the mass contained within x is

$$M(x) = x\ M_* \tag{29}$$

The gravitational potential is independent of x. It follows that the velocity dispersion is also independent of x. From the virial theorem

$$\langle v^2 \rangle = GM_*/2R_* \tag{30}$$

As the heavy particles decay the mass of the isothermal sphere decreases. We define the fraction of initial mass, M_i, that has not yet decayed as y.

$$M_* = y\, M_i \tag{31}$$

If all the light matter remained in the sphere then the final mass of the sphere would be $M_f = y_f\, M_i$. However, as we shall see, a substantial amount of light material escapes the sphere so that M_f is substantially smaller. As the mass of the sphere decreases the gravitational binding energy decreases and it becomes possible for the remaining material to be lost. However, it is also possible that no material is lost and the sphere puffs up, reducing its thermal velocities in accord with the change in binding energy. The two different behaviors can be distinguished by whether or not the sphere is able to maintain virial equilibrium. If a typical orbit time is longer than the decay time, then the change in potential is rapid and the pre-decay velocity dispersion is preserved. With the decrease in potential typical particles will have escape velocity from the sphere and material is lost. On the other hand if orbit times are short compared to the lifetime of the decaying particle then the isothermal sphere can maintain virial equilibrium. In that case there is no mass lost from the system, only a change in radius

$$R_* = R_i M_i / M_* = R_i / y \tag{32}$$

As long as virial equilibrium is maintained the isothermal sphere increases in size proportionally with the amount of matter lost to decay.

Since the velocity dispersion is constant throughout the sphere, typical orbit times increase with radius. This means that as heavy particles decay the center of the sphere may be undergoing quasistatic expansion while at the same time the edges of the sphere are being lost due to a sudden decrease in the potential. Another effect enhances evaporation. As material is lost the mean velocity drops

$$v = y\ v_i \tag{33}$$

The drop in velocity increases the dynamical time scales, thus making the heavy particle decay appear even more sudden.

We can now describe the history of an isothermal sphere through a period in which a fraction of its mass decays. Let the time scale for decay be t_D, and the initial dynamical time scale for the sphere be $t_i \sim R_i/v_i$. We define a dimensionless dynamical time, w, by $t_i = w\ t_D$. The dynamical timescale as a function of position within the sphere is

$$t(x) = (xw/y^2)\ t_D \tag{34}$$

Material outside $x_f = y_f^2/w$ will have undergone sudden changes in potential by the end of the decay epoch. Therefore, x_f gives the fraction of light matter that remains bound in the isothermal sphere.

In our models y_f is given by (see eq. (11))

$$y_f = (\rho_L+\rho_B)/\rho_{tot}\ |_{t_D} = T_E/(T_E+1.15T_D) \tag{35}$$

Estimating w is difficult given the crude nature of our model, but roughly we expect w to be about 1 since the density within the sphere is not much enhanced over the average density of the Universe. Using eq. (27) we can determine

$$w \sim O(10)(R_*/\lambda_{NL})^{3/2} \tag{36}$$

As an example, for $T_D = 3.5\ T_E$ and $w \sim 1$ we find that x_f is roughly 1/25. Most of the light matter escapes from structured objects at t_D. This result may be modified if baryons are an important part of the cores of isothermal spheres, in which case Ω_B is a limiting value for the amount of material to remain behind.

The next issue to confront is the growth of structure in the post decay epoch. There are three main points. First, from t_D until t_E the Universe is radiation dominated, so there is no growth of structure during this period. Next, as the isothermal spheres disperse a considerable amount of material free streams across the Universe. The free streaming damps the amplitude of density perturbations on scales shorter than the free streaming length. Finally, the free streaming material is relatively hot, so the growth of perturbations cannot occur for scales shorter than the Jeans length of the free streaming material. These last two effects are complicated by the fact that material escaping the isothermal spheres later has a smaller escape velocity.

We start by considering the first material to escape. Let the initial velocity of this material be v_D. The peculiar velocity at later times will be

$$v_p = v_D\, T/T_D \tag{37}$$

The free streaming length is

$$\lambda_{fs} = a(t) \int_{t_D}^{t} (v_p/a(t'))dt' \tag{38}$$

where a(t) is the cosmic scale factor. If we substitute eq. (37) into eq. (38) it is not difficult to do the integral. The result is

$$\lambda_{fs} = \frac{v_D a_D a}{a_E^2} \sqrt{\frac{3}{8\pi G_N \rho_{NR}^E}} \quad I(\zeta) \tag{39}$$

where ρ_{NR}^E is the density of non-relativistic matter at t_E and

$$I(\zeta) = \ln \left[\frac{\sqrt{1+\zeta}\ -1}{\sqrt{1+\zeta}\ +1}\right]_{\zeta_D}^{\zeta} \tag{40}$$

and here $\zeta = a/a_E$.

The upper limit of the free streaming integral is to be determined by when the matter becomes cool enough to be trapped in potential perturbations on some scale $\lambda \leq \lambda_{fs}$. This happens when

$$v_p^2 < \phi_\lambda \equiv (1/\pi)\ G_N\ \delta_\lambda\ \rho_{NR}\lambda^2 \tag{41}$$

where G_N is Newton's constant, δ_λ is the density perturbation amplitude on the scale λ, and ρ_{NR} is the density of non-relativistic matter. In general, the gravitational potential depends on the scale λ in a non-trivial way. However, we shall see later that the relevant scale is λ_{fs}; i.e., the first scale at which free streaming material will be trapped is λ_{fs}. We can then use eqns. (37-40) in eq. (41) to derive that free streaming continues as long as

$$I(\zeta) < \sqrt{\frac{8\pi^2\zeta}{3\delta_{\lambda_{fs}}}} \tag{42}$$

It turns out that for the models we are considering eq. (42) is satisfied as long as the free streaming scale is behaving linearly. However, as soon as λ_{fs} becomes non-linear δ_{fs} grows and eq. (46) is no longer satisfied. So, the upper limit to free streaming is determined by when $\delta_{\lambda_{fs}} = 1$.

Before estimating v_D and $\delta_{\lambda_{fs}}$ we describe the effects of free streaming on the spectrum of density perturbations. This will justify our claim that λ_{fs} is the relevant scale in eq. (46). We are primarily interested in a Harrison-Zel'dovich spectrum. For a Harrison-Zel'dovich spectrum the primeval spectrum on large scales behaves as $\delta_p(\lambda) \sim 1/\lambda^2$. However, free streaming modifies this result. Suppose a fraction (1-x) of the matter in the Universe has a free streaming length λ' or greater. Then the perturbation amplitude for the scale λ' is damped to $\delta(\lambda') = x\ \delta_p(\lambda')$. For our case $x = 1$ for $\lambda' > \lambda_{fs}$ as given by eq. (44). However, for $\lambda' < \lambda_{fs}$, x decreases. Since the free streaming distance depends primarily on the escape velocity, $\lambda' \sim v_e$, we can easily estimate the fraction x for our isothermal sphere model. From eqs. (33 and 34) we see that the fraction of material with velocity less than v_e is $x = (v_e/v_D)^2$. Combining these results yields a flat density spectrum for $\lambda < \lambda_{fs}$.

$$\delta(\lambda') \sim x/\lambda'^2 \sim (v_e/v_D)^2 \, 1/\lambda'^2 \sim 1/v_D^2 \tag{43}$$

In the context of eq. (41) this implies that potential perturbations on scales smaller than λ_{fs} are unimportant.

Some comments are in order about eq. (43). First of all, at scales much less than λ_{fs}, δ must increase so as to match onto the non-linearity at λ_{NL}. Eq. (43) is only true for scales large enough to encompass several residual cores of the preexisting isothermal spheres. Second, the flat spectrum of eq. (43) may be modified by deviating from a Harrison-Zel'dovich spectrum. Finally, several small effects may tilt the spectrum one way or the other. For example: matter with smaller escape velocities escapes slightly later thus reducing its free streaming length; or, the primeval $\delta_p \sim 1/\lambda^2$ spectrum is softened since the scales in question come over the horizon near matter dominance. These two effects work in opposite directions.

The tilt of the perturbation spectrum, as determined by eq. (43), controls whether, subsequent to t_D, structure will form first at long or short wavelengths. Given a flat spectrum one would expect all scales to go non-linear at the same time. However, in our case the Jeans length of the free streaming material is long enough to suppress growth on small scales. The Jeans length is given by

$$\lambda_J = \frac{2\pi v_s}{(4\pi G_N \rho_{NR} x)^{1/2}} \tag{44}$$

where $v_s = 1/\sqrt{3}\ v_p$ is the speed of sound for the non-reacting light material and ρ_{NR} is the full non-relativistic density. However, ρ_{NR} must be multiplied by x, the fraction of material with velocities less than v_p. The point is that hotter material will have a longer Jeans length than cold material, will not cluster with cold material and does not contribute to the density used in calculating λ_J from linear perturbation analysis. Again, the fraction x is proportional to v_p^2 which results in a Jeans length which is independent of the peculiar velocity. We expect that a more careful analysis would introduce a logarithm but that is irrelevant to our point that the Jeans length of our model is described by that for the hottest component. So, λ_J is given by eq. (44) with x = 1 and $v_s = 1/\sqrt{3}\ v_D\ T/T_D$. As the free streaming matter slows down the Jeans length gets smaller. Scales at the long wavelength end of the spectrum are the first that are able to grow again at the beginning of the second matter dominated epoch. Given the flat spectrum of eq. (43) this implies that the first scale to go non-linear in the latter matter dominated epoch will be λ_{fs}. Rigorously, we must show that $\lambda_J > \lambda_{fs}$ at t_E for this to be true. At t_E

$$\lambda_J = \frac{2\pi}{\sqrt{3}} \frac{v_D T_E}{T_D} \frac{1}{\sqrt{4\pi G\rho_{NR}}} \tag{45}$$

Combining eqs. (39 and 45) gives

$$\lambda_J/\lambda_{fs} = 2.96/I(\zeta) \tag{46}$$

where $I(\zeta) < 2.8$ for $T_D = 3.5\ T_E$ so structure forms first at λ_{fs}. Note that although $\lambda_J > \lambda_{fs}$ allows growth on scales greater than λ_{fs} first, the density perturbation spectrum falls as $1/\lambda^2$ for $\lambda > \lambda_{fs}$ and the first scale to go non-linear is indeed λ_{fs}.

We summarize our discussion of density fluctuations in Fig. 3. The primeval spectrum is flattened by free streaming. After the Universe is matter dominated again, growth in the perturbation amplitude can occur for any scale longer than the Jeans length. One finds that the free streaming scale will be the first to go non-linear.

These results indicate that if any scale goes non-linear after T_E the first scale to do so is λ_{fs}. Furthermore, since no light material streams further than λ_{fs} structures forming late will feel the full value of Ω_L. If we want to have large scale dynamical measurements yield a small value of Ω, then all structure must have formed before t_D. However, as mentioned earlier, a small amount of growth in $\delta\rho_B$ at late times is still expected for scales $\lambda_{NL} < \lambda < \lambda_{fs}$. Because of this we set λ_{NL} equal to 10 Mpc, which is somewhat less than the present scale of non-linearity for baryons. Finally, we speculate that the proposed cluster-cluster correlation length $\lambda_{cc} \sim 25/h_o$ Mpc (Bahcall and Soniera 1983) may be a signal of λ_{fs} starting to go non-linear today.

For this picture to make sense we must calculate λ_{fs} and assure ourselves that λ_{fs} has not gone non-linear yet. To do this it is useful to rewrite eq. (39) as

$$\lambda_{fs}/\lambda_{MD} = 1.2\ v_D\ (T_{MD}/T_D)^{1/2} I(\zeta) \tag{47}$$

We approximate the velocity v_D as given by the square root of the gravitational potential at the time the isothermal sphere forms.

$$V_D \sim \phi^{1/2} \sim (\delta_i\ T_{MD}/T_{NL}\ A_{NL})^{1/2} \tag{48}$$

where the factor $T_{MD}/T_{NL}\ A_{NL} \leq 1$ is a suppression due to a lack of growth during the radiation era preceeding T_{MD}. It should be realized that an accurate value of v_D requires a detailed knowledge of the formation and dissipation of the isothermal spheres which in turn requires knowledge of the non-linear regime of our model. Combining eqs. (47 and 48) gives

$$\begin{aligned} \lambda_{fs}/\lambda_{MD} &\sim (\delta_i\ T_{MD}/T_{NL}\ A_{NL}\ T_{MD}/T_D)^{1/2}\ I(\zeta) \\ &\sim O(1) \end{aligned} \tag{49}$$

The perturbation amplitude for the scale λ_{fs} may be expressed as

$$\delta_{\lambda_{fs}} \approx A_{fs}/A_{NL}\ [1 + 3/2\ T_J/T_o] \tag{50}$$

where A_{fs}/A_{NL} normalizes $\delta_{\lambda_{fs}}$ relative to $\delta_{\lambda_{NL}}$ and the factor in brackets is the growth after λ_{fs} becomes longer than the Jeans length. From eqs. (39, 45 and 46) the temperature at which $\lambda_{fs} = \lambda_J$ is

$$T_J = T_E\ [I(\zeta)/2.96]^{1/2} \tag{51}$$

It is consistent to have free streaming continue till today, so we take $I(\zeta) = I(4) = 1.8$ for $T_D = 3.5\ T_E$. Our modification of Peebles' curve yields $A_{fs} \simeq 2.6$ for $\lambda_{fs} = \lambda_{MD}$, so we find $\delta_{\lambda_{fs}} \simeq 3.1$. This value is somewhat higher than we would want in order to explain the cluster-cluster correlation data but not have any well defined structures on scales greater than the galaxy-galaxy correlation length. We note that in order to have $\delta_{\lambda_{fs}} < 1$, as is required in this model, then A_{fs} must be $\lesssim 0.9$ and hence $\lambda_{fs} \gtrsim 2\,\lambda_{MD}$, which is consistent with the estimate expressed in eq. 49. On the other hand, if quadropole measurements force δ_i lower, A_{NL} (and hence λ_{MD}) will increase again lowering $\delta_{\lambda_{fs}}$. In this case, however, λ_{fs} would be lowered because of the smaller effect of the perturbations.

Let us now try to summarize our goals and accomplishments. We started out by wanting to bring consistency between the cold dark matter scenario and an $\Omega = 1$ Universe. To do so, we considered a heavy particle species which decays into a lighter one. Unlike previous scenarios involving decays, we try to retain the feature that the Universe is matter dominated today, giving us some help with the age of the Universe. In addition to having a present matter dominated era, our decay scenario allows for unclustered non-relativistic matter. It is also possible that the free streaming length is just going non-linear today, allowing for cluster-cluster clustering.

The consequences of the decay on any further growth of structure was considered in some detail. Not only do the decay products become completely unclustered (they are still relativistic today), but the loss of mass (by the decay) leads to the unclustering of the light primordial particles on the scales which have gone non-linear, λ_{NL}. As we saw, only a small fraction of the light particles remain behind and these are envisioned to play the role of the dark matter in galactic halos. The free-streaming of the light particles damps growth of any further structure on scales between λ_{NL} and λ_{fs}. Thus we would predict that the next scale to go non-linear would be λ_{fs}. All of the non-relativistic particles, however, would remain clustered on these scales. We expect therefore, that $\Omega \sim 0.8$ on the largest scales. We stress that it is absolutely necessary that $\lambda_{fs} > \lambda_{NL}$ so that Ω on the scale λ_{NL} is only ~ 0.2. It is also necessary that $\delta_{\lambda_{fs}} \lesssim 1$. Although it appears unlikely, there is no difficulty with having $\lambda_{fs} > \lambda_{cc}$.

We have avoided presenting an actual model from particle physics here. Instead, we have defined the model and determined the values of the relevant parameters and concentrated on the astrophysical consequences of the model. A model involving conventional neutrinos will be bound to previous constraints so that the true cold and hierarchical picture would not be possible. Supersymmetry offers a host of new candidates for the heavy and light particles. We expect that a suitable model can be described in that context (Olive, Schramm and Srednicki 1984).

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Footnotes

† (Pg 10) We use the limits on the quadrupole moment since we are interested in a Harrison-Zel'dovich spectrum and because of the uncertainties in interpreting the more recent small scale measurement of $\delta T/T$ (Uson and Wilkinson 1984).

APPENDIX

We wish to trace the evolution of a flat model universe containing stable matter (including baryons), unstable matter, radiation which is the product of particle decay, and primordial relativistic particles (like photons and neutrinos). In the following equations we have scaled all densities to the present critical density, the present scale factor of the universe to one, and all rates and times to the present Hubble constant or its inverse.

The density of each of the components of the universe is given by

$$\rho_{NR} = \Omega_{NR}/a^3 \tag{A1a}$$

$$\rho_H = 0.1504\ e^{-\Gamma t}\ M_H Y/a^3 \tag{A1b}$$

$$\rho_D = 0.1504\ M_H Y[\textstyle\int_0^t \Gamma\ a\ e^{-\Gamma t}\ dt]/a^4 \tag{A1c}$$

$$\rho_\gamma = 9.52 \times 10^{-5}(1 + N_\nu\ 0.2271)/a^4 \tag{A1d}$$

where $a(t)$ is the scale factor of universe; the subscript NR denotes stable non-relativistic matter; the subscript D denotes radiation produced by the decay of heavy particles; the subscript γ denotes primordial radiation; T_o is the present temperature of the microwave background; and N_ν is the number of distinct massless neutrino species. Hereafter we will assume $N_\nu = 3$, and $T = 2.7°K$.

The age of the universe corresponding to a particular value of the expansion factor is

$$t = \int_0^a (da/a)\ (\rho_{NR} + \rho_H + \rho_D + \rho_\gamma)^{-1/2} \tag{A2}$$

in units of the present Hubble time. We have the constraint that at the present time, t_o, the densities add up to the critical density, i.e.

$$(\rho_{NR} + \rho_H + \rho_D + \rho_\gamma)|_{t=t_o} = 1 \tag{A3}$$

Equations (A1) - (A3) give us a complete description of the evolution of one of our homogeneous models. For any choice of $M_H Y$ and Γ the constraint given by equation (A3) enables us to solve iteratively for Ω_{NR} and therefore to calculate all the properties of the model.

It remains for us to estimate the growth of perturbations in such a model. We will choose our normalization such that the variance of the mass contained within a gaussian window of size σ is given by

$$\langle(\delta M/M)^2\rangle = \int_0^\infty \langle|\delta(\underline{k})|^2\rangle k^2\ e^{-k^2\sigma^2/2} dk \tag{A4}$$

At long wavelengths the power spectrum is assumed to have the form

$$\langle|\delta(\underline{k})|^2\rangle = A|\underline{k}| \tag{A5}$$

where the constant A is related to the mean square amplitude of the quadropole components by the formula (Peebles 1982)

$$A = (12/\pi)\ \langle|a_2^m|^2\rangle \tag{A6}$$

Fixsen etal. (1982) estimate

$$\langle |a_2^m|^2 \rangle^{1/2} \lesssim 3.22 \times 10^{-5} \pm 1.11 \times 10^{-5} \tag{A7}$$

from which we derive a 2σ upper limit on A of

$$A \lesssim 1.2 \times 10^{-8} \tag{A8}$$

In calculating the linear evolution of perturbations we will use the formalism of Bardeen (1980) adapted for a multicomponent fluid. Each fluid has two degrees of freedom, density and velocity. We will restrict ourselves to the growing adiabatic mode. In this case the fractional density perturbations and velocities of the stable and unstable matter components are identical at all times. The initial perturbations in the primordial radiation are simply related to the matter perturbations and the initial perturbations in the decay products are irrelevant. The appropriate equations are

$$\dot{\delta}_m = -kv_m/a \tag{A9a}$$

$$\dot{v}_m = -(\dot{a}/a)v_m + (k/a)\Phi_A \tag{A9b}$$

$$\dot{\delta}_\gamma = (\dot{a}/a)\delta_\gamma - (4/3)\,(k/a)\,v_\gamma - (2/3)\,(\dot{a}/a)\,\pi_\gamma \tag{A9c}$$

$$\dot{v}_\gamma = -(\dot{a}/a)v_\gamma + (k/a)\Phi_A + (k/4a)\delta_\gamma - (k/6a)\pi_\gamma \tag{A9d}$$

$$\dot{\delta}_D = (\dot{a}/a)\delta_D - (4/3)(k/a)v_D - (2/3)(\dot{a}/a)\pi_D + \Gamma(\rho_H/\rho_D)(\delta_m-\delta_D) \tag{A9e}$$

$$\dot{v}_D = -(\dot{a}/a)v_D + (k/a)\Phi_A + (k/4a)\delta_D - (k/6a)\pi_D + \Gamma(\rho_H/\rho_D)(v_m-v_D) \tag{A9f}$$

where δ and v are the density and velocity perturbations for the fluid indicated by the subscript and the subscript m refers to both NR and H. π_γ and π_D are the traceless pieces of the pressure in the relativistic fluids. Φ_A is the gauge invariant potential perturbation given by

$$\Phi_A = -(3/2)(a/k)^2 \,[\rho_m\delta_m + \rho_D\delta_D + \rho_\gamma\delta_\gamma]-(a/k)^2(\rho_\gamma\pi_\gamma + \rho_D\pi_D) \qquad \text{(A10)}$$

We take the initial conditions to be

$$<|\delta_\gamma(\underset{\sim}{k})|^2>^{1/2} = (8/3)<|\delta_m(\underset{\sim}{k})|^2>^{1/2} = \frac{10}{9}\,\frac{|\underset{\sim}{k}|^{\frac{1}{2}}A^{\frac{1}{2}}a^2}{\Omega_\gamma} \qquad \text{(A11a)}$$

$$<|v_\gamma(\underset{\sim}{k})|^2>^{1/2} = <|v_m(\underset{\sim}{k})|^2>^{1/2} = -(5/6)\,(A/\Omega_\gamma)^{1/2}a\,|\underset{\sim}{k}|^{-1/2} \qquad \text{(A11b)}$$

Finally, in order to integrate equations (A9) forward in time we need to calculate π_γ and π_D. In point of fact, the exact value of these variables is almost irrelevant. The only important point is that they should lead to the damping of perturbations once they are inside the horizon. On this heuristic basis we choose

$$\pi_{\gamma,D} = v_{\gamma,D}\,4\sqrt{3}\left[\frac{(k/\dot{a})^2}{1+(k/\dot{a})^2}\right] \qquad \text{(A12)}$$

The line in figure 2, which indicates the possibility of forming structure on 10 Mpc scales, is the result of integrating equations (A9). Nonlinear structure is considered to have formed if the total density perturbation on that scale is equal to unity, i.e.

$$\int_0^\infty <|\delta(\underset{\sim}{k}|^2>\,k^2\,e^{-k^2\lambda^2/2\pi}\,dk = 1 \qquad \text{(A13)}$$

If we had used the upper limits on the small scale anisotropy in the microwave backround (Uson and Wilkinson 1984), instead of the limits on the quadropole moment cited above, then the primordial amplitude of the initial perturbations would have been smaller by a factor of three. As a result the line representing the formation of nonlinear structure at 10 Mpc scales would have moved to the left by a factor of about 5. This would have eliminated much of our favored region in parameter space. A calculation by Bond and Efstathiou (1984) indicates that a detailed consideration of the problem would make these limits even more stringent. We argue that this is not an appropriate limit to take in our model because the chance for early star formation is greatly enhanced, leading to the possibility of a reionized intergalactic medium smearing out all small scale anisotropies in the microwave background.

FIGURE CAPTIONS

Figure 1: The growth factor A_λ vs λ/λ_{MD} adapted from Peebles (1982). A_λ is defined by eq. (17).

Figure 2: Here we show the available parameter space in $M_H Y$ and Γ. We have assumed $h_o = 0.5$ and three massless neutrinos ($N_{MD} = 3.36$). The line labled $\lambda_{NL} = 10$ Mpc indicates those models which can just barely form structure on that scale before particle decay. Ω_{NR} and Ω_B are the current fractions of the critical density in non-relativistic matter and baryons respectively. The shaded region has $\Omega_D \geq 0.5$ at present. The lines labelled $t_{10} = 1.1$ and $t_{10} = 1.2$ indicate models with ages of 11 and 12 billion years respectively. The line labelled $T_D = T_E$ represents a lower limit to T_D. Models below this line were never dominated by the heavy decaying particle or that particle has not yet decayed. Finally the cross shows the location of the parameter choice used in the paper as an example.

Figure 3: Idealized behavior of the density perturbation spectrum as a result of free streaming. At t_D no free streaming has yet occurred, so $\delta \sim 1/\lambda^2$. After free streaming, δ = constant, for $\lambda < \lambda_{fs}$. At t_E δ can start to grow again for scales longer than the Jeans length, λ_J. At t_J, $\lambda_J = \lambda_{fs}$, so $\delta_{\lambda_{fs}}$ can start to increase. At later times smaller scales can undergo growth as λ_J decreases. We have depicted a small amount of growth in the flat part of the spectrum due to residual peculiar velocities from before t_D.

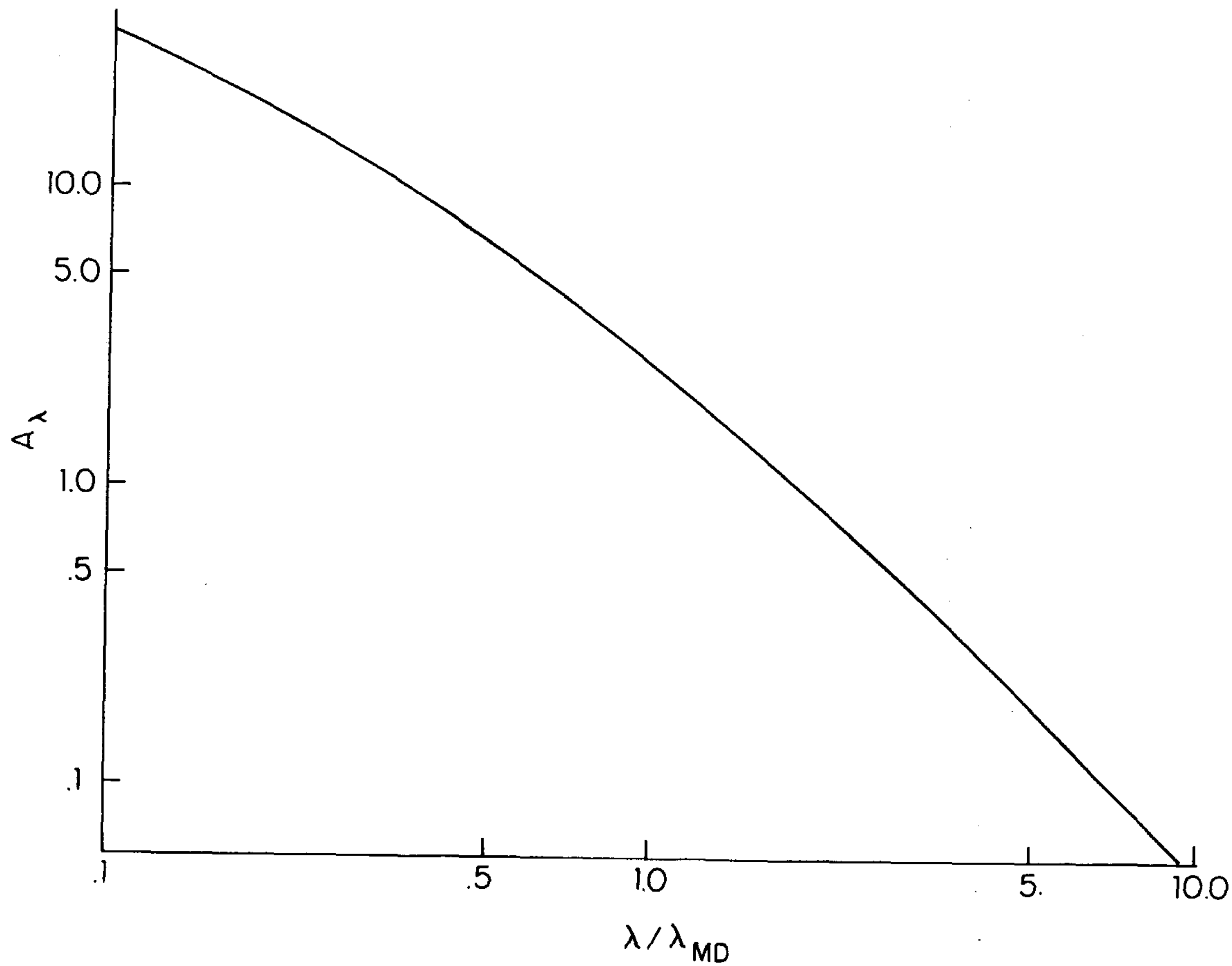
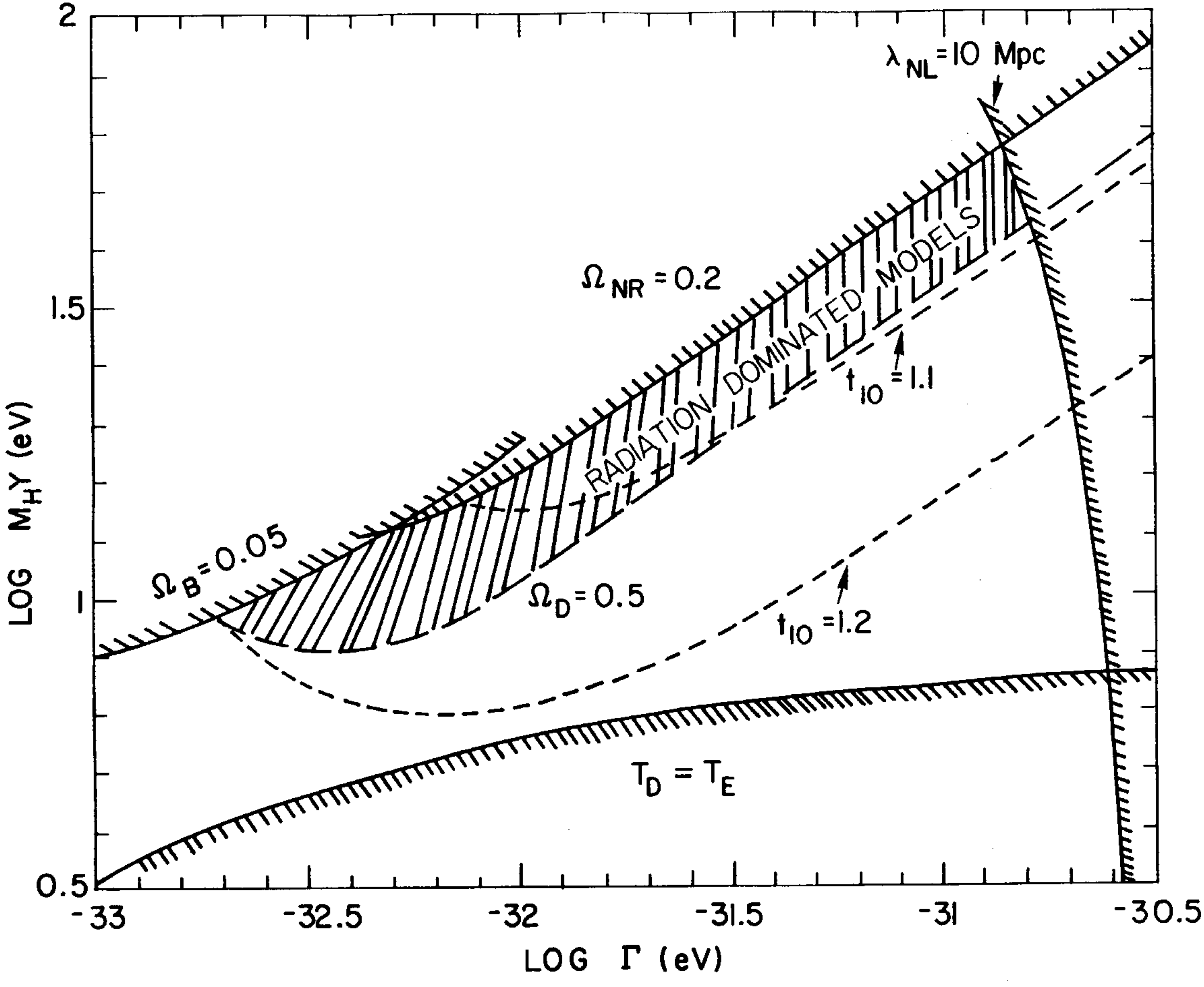


FIGURE 1



2
1.5
1
0.5
-33
-32.5
-32
-31.5
-31
-30.5
LOG Γ (eV)
LOG $M_H Y$ (eV)
λ_{NL}=10 Mpc
Ω_{NR} = 0.2
RADIATION DOMINATED MODELS
t_{10} = 1.1
Ω_B = 0.05
Ω_D = 0.5
t_{10} = 1.2
$T_D = T_E$

FIGURE 2

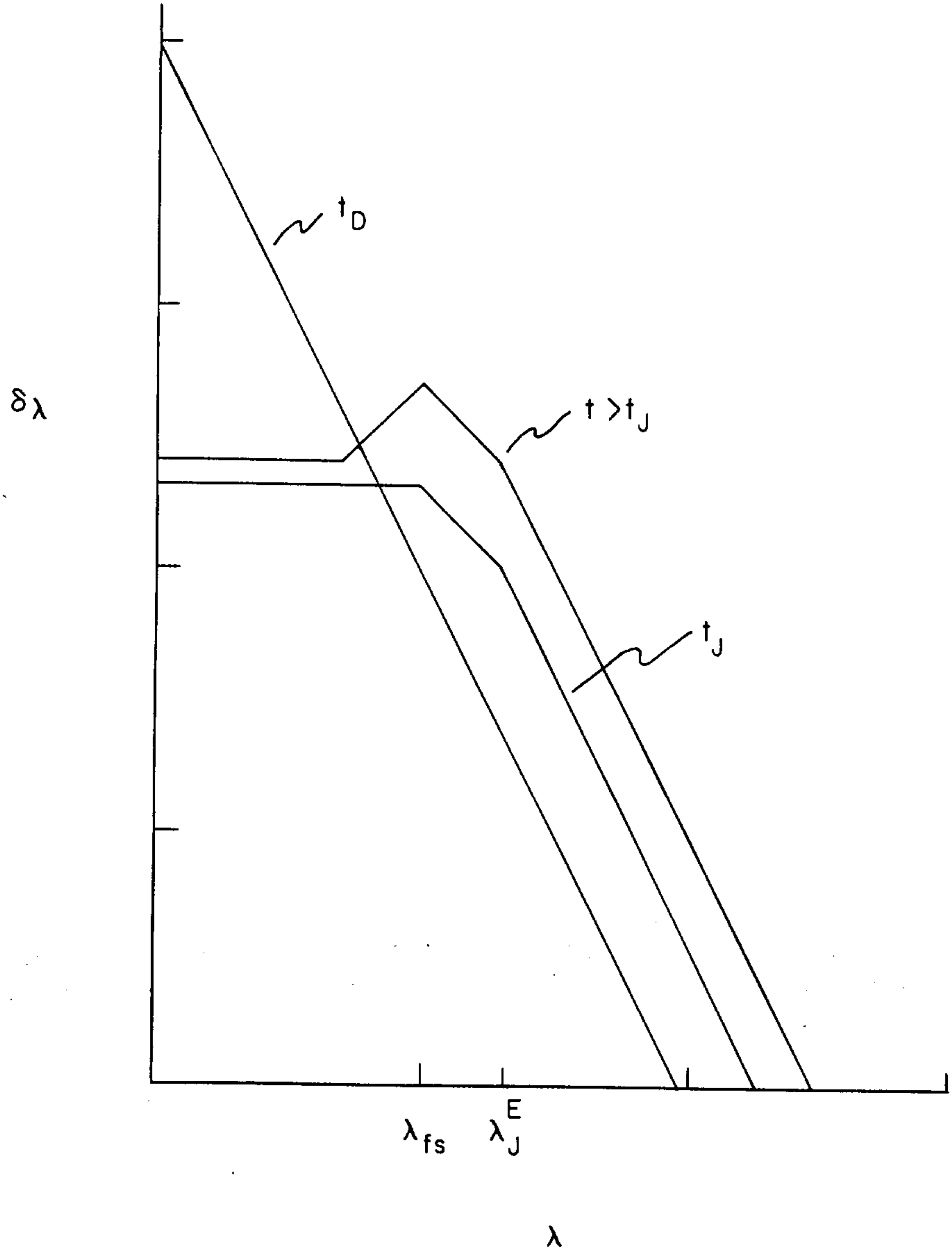


FIGURE 3