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Lepton-Quark Scattering and Nucleon Spin Structure†

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ABSTRACT

Consideration is given to the asymmetries arising from the deep inelastic scattering of longitudinally polarized electrons and positrons with longitudinally polarized protons at e-p collider energies. Information from such measurements will provide means for testing models of nucleon spin structure. The Carlitz-Kaur model of spin structure is used as a guide for estimating the behavior of these asymmetries, which arise from the interference of the electromagnetic and neutral currents.

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Deeply inelastic lepton-nucleon scattering has provided us with an impressive amount of information in support of the quark-parton model of hadronic structure, and has played a key role in establishing QCD as the candidate theory of the strong interactions. All the information from deeply inelastic electron-nucleon scattering is contained in four independent structure functions, of which two are spin dependent, and reflect the spin distribution of the quark constituents inside the nucleon [1]. Knowledge of these spin dependent functions is important for testing models of nucleon spin structure, as well as testing the predictions of perturbative QCD concerned with the scaling behavior of the spin dependent structure functions. In addition, knowledge of the parton spin distributions will be essential for the interpretation of spin dependent high energy phenomena involving hadrons. Examples of such phenomena include polarized W or Z production from polarized proton-unpolarized proton collisions, and the polarized Drell-Yan process.

In this paper, we will be concerned with the asymmetries arising from the weak-electromagnetic interference in polarized e-polarized p scattering at e-p collider energies. The interference effects in the neutral current sector are proportional to $(q^2/q^2 + M_Z^2)$, and have the promise of becoming of order unity at projected collider energies. Consideration regarding the usefulness of polarization experiments at future e-p colliders have been presented in a recent report [2]. In addition to providing important information on the internal spin structure of the nucleon, experiments carried out at e-p colliders will also supplement the study of the effects of the W and Z propagators [3].

Before turning to a discussion of the asymmetries, it is useful to list the various lepton-proton differential cross sections from which they are constructed. Let $f_i^{\uparrow(\downarrow)}(x)$ denote the probability of finding, in the proton, a quark of flavor i with spin parallel (antiparallel) to the proton's spin and with fraction x of the proton's momentum. The four differential cross sections describing the scattering of polarized electrons with polarized protons are:

$$\frac{d^2\sigma(e_L^-,P_L)}{dx\,dy} = \frac{sx}{4\pi}\sum_i \left\{ f_i^{\uparrow}(x)\,\sigma_{LL}^{i} + (1-y)^2 f_i^{\downarrow}(x)\,\sigma_{LR}^{i} \right\}, \tag{1a}$$

$$\frac{d^2\sigma(e_L^-,P_R)}{dx\,dy} = \frac{sx}{4\pi}\sum_i \left\{ f_i^{\downarrow}(x)\,\sigma_{LL}^{i} + (1-y)^2 f_i^{\uparrow}(x)\,\sigma_{LR}^{i} \right\}, \tag{1b}$$

$$\frac{d^2\sigma(e_R^-,P_L)}{dx\,dy} = \frac{sx}{4\pi}\sum_i \left\{ f_i^{\downarrow}(x)\,\sigma_{RR}^{i} + (1-y)^2 f_i^{\uparrow}(x)\,\sigma_{RL}^{i} \right\}, \tag{1c}$$

and

$$\frac{d^2\sigma(e_R^-,P_R)}{dx\,dy} = \frac{sx}{4\pi}\sum_i \left\{ f_i^{\uparrow}(x)\,\sigma_{RR}^{i} + (1-y)^2 f_i^{\downarrow}(x)\,\sigma_{RL}^{i} \right\}. \tag{1d}$$

The helicity of the colliding particles is denoted by the subscript R (right-handed), or L (left-handed). The factors σ^i are proportional to the cross sections for the scattering of polarized electrons with polarized quarks of flavor i, and are given by

$$\sigma_{LL}^{i} = \left| \frac{Q_e^{em} Q_i^{em}}{q^2} + \frac{Q_{Le}^{Z} Q_{Li}^{Z}}{q^2+M_Z^2} \right|^2 , \tag{2a}$$

$$\sigma_{LR}^{i} = \left| \frac{Q_e^{em} Q_i^{em}}{q^2} + \frac{Q_{Le}^{Z} Q_{Ri}^{Z}}{q^2+M_Z^2} \right|^2 , \tag{2b}$$

$$\sigma_{RL}^{i} = \left| \frac{Q_e^{em} Q_i^{em}}{q^2} + \frac{Q_{Re}^{Z} Q_{Li}^{Z}}{q^2+M_Z^2} \right|^2 , \tag{2c}$$

and $$\sigma_{RR}^{i} = \left| \frac{Q_e^{em} Q_i^{em}}{q^2} + \frac{Q_{Re}^{Z} Q_{Ri}^{Z}}{q^2 + M_Z^2} \right|^2 . \tag{2d}$$

The Q^{em} denote the electromagnetic charges, and the weak charges are given by

$$Q_L^Z = \frac{e}{\sin\theta_W \cos\theta_W} \left(T_{3L} - Q^{em} \sin^2\theta_W \right) , \tag{3a}$$

and $$Q_R^Z = \frac{e}{\sin\theta_W \cos\theta_W} \left(T_{3R} - Q^{em} \sin^2\theta_W \right) . \tag{3b}$$

Here, T_{3L} and T_{3R} are the third component of weak isospin for left- and right-handed fermions, respectively. In the standard model assignment, all fermions are right-handed singlets: $T_{3R}=0$.

The corresponding set of cross sections pertinent to polarized e^+-polarized p scattering are obtained from Eqs. (1) by replacing $e^-_{L,R} \to e^+_{R,L}$ on the left hand side, and making the substitutions $1 \leftrightarrow (1-y)^2$ on the right hand side.

Although we will not be concerned with it here, one can also contemplate preparing the lepton beam polarized transverse to the beam direction. In this instance, the differential cross section formulae contain a different weighting of the spin distribution functions, relative to the longitudinal polarization case.

The double asymmetry for the scattering of polarized electrons with polarized protons is defined as

$$A(e^-p) = \frac{d^2\sigma(e_L,p_L) - d^2\sigma(e_L,p_R) + d^2\sigma(e_R,p_R) - d^2\sigma(e_R,p_L)}{d^2\sigma(e_L,p_L) + d^2\sigma(e_L,p_R) + d^2\sigma(e_R,p_R) + d^2\sigma(e_R,p_L)} , \tag{4}$$

and should be sensitive to the particular forms of the spin distributions, $f_i^{\uparrow\downarrow}(x)$, that occur in the cross section formulae from which A is defined. Thus, one may hope to test various models of nucleon spin structure through measurements of this asymmetry in different kinematic domains. Indeed, a recent measurement of the e-p asymmetry covering the kinematic range $0.18<x<0.70$ and $3.5<q^2<10.0$ $(\mathrm{GeV/c})^2$ has been carried out, and the data appear to single out two particular models [4]. Of these two models, only that of Carlitz and Kaur is a parton model, whereas the other is based upon Schwinger's source theory [5,6]. References for the other proposed models can be found in the review by Hughes and Kuti [7]. For the rest of our discussion, we shall focus on the Carlitz-Kaur picture, so we can make predictions about the double asymmetries one might expect to observe for e-p scattering at colliding beam facilities.

Our predictions involve the behavior of asymmetries at enormous energy scales ($s \approx 10^4 - 10^5$ GeV^2): one might question the reasonableness of using a particular model of spin structure, only known to be valid for low energies, as a reliable guide of spin structure at such extreme energies. We will in fact demonstrate that this is a reasonable thing to do, at least in the context of the Carlitz-Kaur model. In order to do so, it will be convenient to present a brief review of the essential features of this model.

In this model, the interactions between the valence quarks and the $q\bar{q}$ sea and gluons is taken into account by a phenomenological factor describing the spin dilution of the valence quarks. One introduces N(x) and M(x) as the spin-weighted density of gluons and $q\bar{q}$ pairs relative to the valence quarks, and $H_N(x)$ and $H_M(x)$ as the probability of interaction between the valence quarks and gluons and the valence quarks and $q\bar{q}$ pairs, respectively. Then, the probability of the transfer of spin from the valence quarks to gluons and $q\bar{q}$ pairs is

$$p(x) = \frac{\frac{1}{2}(H_N(x)N(x) + H_M(x)M(x))}{H_N(x)N(x) + H_M(x)M(x) + 1} \quad . \tag{5}$$

Based on the fact that gluons carry roughly 50% of the proton momentum, one expects that $H_N(x) >> H_M(x)$. In this approximation,

$$p(x) = \frac{\frac{1}{2}H_N(x)N(x)}{H_N(x)N(x) + 1} \tag{6}$$

A measure of the spin dilution induced by the transfer of spin to the gluons is given by the factor

$$\left(H_N(x)N(x) + 1\right)^{-1} \tag{7}$$

In finding the explicit form for N(x), appeal is made to Regge arguments for determining the $x \to 0$ behavior, and to dimensional counting rules for the $x \to 1$ behavior. Putting this information together leads to the parametrization $N(x) \sim x^{-1/2}(1-x)^2$. H_N is taken to be constant, so that $H_N(x)N(x) = H_0 x^{-1/2}(1-x)^2$. The crucial observation is to note that the factor in (7) is significantly different from unity only for small x--where N(x) is appreciable.

The final step is to express the nucleon's spin dependent structure function in terms of the spin-averaged distributions and the spin dilution factor, (7). In the scaling limit, this structure function is given by

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \{ f_i^{\uparrow}(x) - f_i^{\downarrow}(x) \} \quad . \tag{8}$$

Since g_1 measures the difference in the number of quarks with helicity parallel and opposite to the nuclear helicity (weighted by the square of the parton charge), it is given by the product of a term describing the asymmetries of the valence quark spins in the absence of interactions, together with the spin dilution factor:

$$2x g_1^p(x) = \left(H_0 x^{-1/2}(1-x)^2 + 1\right)^{-1} \left\{ \tfrac{4}{9} A_0 - \tfrac{2}{27} A_1 \right\}$$
$$2x g_1^n(x) = \left(H_0 x^{-1/2}(1-x)^2 + 1\right)^{-1} \left\{ \tfrac{1}{9}(A_0 - A_1) \right\} . \tag{9}$$

A_I is the contribution from a valence quark term in which the noninteracting valence quarks are in a state of isospin I:

$$A_0 = x u(x) - \tfrac{1}{2} x d(x)$$
$$A_1 = \tfrac{3}{2} x d(x) . \tag{10}$$

The constant H_0 is fixed by demanding that Bjorken's rum rule be satisfied [8]:

$$\frac{1}{3}\left(g_A/g_V\right) = 2\int_0^1 dx\,(g_1^p - g_1^n) = \frac{1}{3}\int_0^1 \frac{dx}{x}\, \frac{A_0 + \frac{1}{9}A_1}{H_0 x^{-1/2}(1-x)^2 + 1} \quad . \tag{11}$$

A knowledge of the spin-averaged valence quark distributions, u and d, together with the spin dilution factor (7), leads to a unique set of spin dependent distributions. Equating (8) and (9) leads to two relations for the four $f_i^{\uparrow\downarrow}$; two additional equations may be obtained by writing out the nucleon's spin averaged structure function. In the scaling limit, this is given by

$$F_2(x) = x\sum_i e_i^2 \left\{ f_i^{\uparrow}(x) + f_i^{\downarrow}(x) \right\} . \tag{12}$$

The explicit solutions are:

$$\begin{aligned} u^{\uparrow(\downarrow)}(x) &= \tfrac{1}{2}u(x) \pm \tfrac{1}{2}\left(H_0 x^{-1/2}(1-x)^2+1\right)^{-1}\left\{u(x)-\tfrac{2}{3}d(x)\right\} \\ d^{\uparrow(\downarrow)}(x) &= \tfrac{1}{2}\left(1 \pm \tfrac{1}{3}\left(H_0 x^{-1/2}(1-x)^2+1\right)^{-1}\right) d(x) \end{aligned} \tag{13}$$

In returning to the question of the possible applicability of this model to extreme high energy processes, we should first point out that up to now in our discussion, we have limited ourselves to pure parton model concepts. Indeed, since we are especially interested in evaluating various asymmetries over a considerably wide range in q^2, it would be proper to incorporate asymtotic freedom (scale-breaking) effects in our treatment. This can be done with the well-known method of Altarelli and Parisi [9]. In this context, the remarkable feature of the Carlitz-Kaur model is that, to a very good approximation, it allows one the option of <u>first</u> QCD-evolving spin-averaged distributions, before extracting the explicit spin dependence. The key to this assertion lies in the small x behavior of the spin dilution factor (7). Suppose we were to q^2-evolve $u^{\uparrow}(x)$; in the notation of Altarelli and Parisi, we would be faced with solving the equation

$$\frac{d}{dt}u^{\uparrow}(x,q^2) = \frac{\alpha(q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left\{\tfrac{1}{2}u(y) + \tfrac{1}{2}\left(H_0 y^{-\frac{1}{2}}(1-y)^2+1\right)^{-1}\left[u(y)-\tfrac{2}{3}d(y)\right]\right\}P\left(\frac{x}{y}\right), \qquad (14)$$

where $t = \log(q^2/q_0^2)$, and we used (13) for the form of $u^{\uparrow}$ at some reference value of $q^2 = q_0^2$. Because $x \leq y \leq 1$ in the above integral, and due to the behavior of the spin dilution factor noted earlier, it is a good approximation to factor the dilution term out of the integration. The error introduced comes only from the very small x region. It thus suffices to q^2-evolve the spin averaged distributions before applying the Carlitz-Kaur scheme to model the spin dependence. The error is controlled by H_0, which goes to zero for $H_0 \to 0$. For the distributions we have used, H_0 is of order 10^{-1}-10^{-2}. The point we would like to underscore here is the tremendous amount of effort saved by first building in the scale-breaking effects followed by the spin extraction. The alternate route would involve grappling with twice the number of (coupled) integro-differential evolution equations.

As we remarked previously, the Carlitz-Kaur model is the only parton model compatible with the existing e-p asymmetry data. We have also pointed out how their way of modeling spin structure essentially "commutes" with the process of evolving the distributions in q^2. Thus, our estimates of e-p asymmetries at very large values of q^2 should provide quite reasonable guides to what is happening in this regime. Our estimates make use of a recently developed set of q^2-dependent spin-averaged distributions [10].

To check the compatability of this set, and our method in general, we compare our evaluation of the asymmetry parameter to that measured by Baum et al., ref [4]. The results are given in Fig. 1. The agreement is impressive. In the comparison, we fixed the value of $q^2 = 6.30$

$(\mathrm{GeV}/c)^2$, which is the average q^2 of their data.

It is instructive to examine the behavior of $A(e^-,p)$ in the $M_Z \to 0$ limit, i.e., scattering mediated by photon exchange only. This limit is depicted in Fig. 2 for $q^2 = 10$, 10^3, and 10^4 $(\mathrm{GeV}/c)^2$. The q^2-dependence in this limit is rather weak, arising solely from the explicit q^2-dependence of the individual spin distributions. This weak dependence should not come as any surprise, since scaling is predicted to be broken only logarithmically--a fact in accord with present experiments. The slight widening apart of these curves for $0.05 \lesssim x \lesssim 0.40$ is a remnant of the fact that, when evolving a particular parton distribution to higher q^2, the most probable value of x decreases to zero. Hence, the only significant q^2 dependence in this asymmetry can come only from the small x region. The sharp drop of the asymmetry to zero for x near zero is due to the rapid turn off of the spin dilution factor in this range of x.

The interesting behavior comes about when we restore the coupling to the Z boson. The q^2-stability displayed in Fig. 2 breaks down due to the weak and electromagnetic interference effects, as shown in Fig. 3. The neutral current couplings tend to enhance the e-p asymmetry, and the relative increase in $A(e^-,p)$ from one momentum scale to the next should follow the basic pattern shown. This particular q^2 behavior of $A(e^-,p)$ is independent of the particular q^2 dependence of the structure functions and originates from the interference term, $(q^2/q^2 + M_Z^2)$. Of course, the x-dependence of A for a given q^2 will indeed depend on the explicit form of the structure functions.

As indicated by the cross section formulae, (1), the asymmetry A also has an explicit dependence on y, the relative energy loss of the lepton. We averaged over this variable in presenting Figs. 2 and 3. To see the dependence on y, we plot the asymmetry for y = .25, .50, and .75 for a fixed $q^2 = 10^4$ $(GeV/c)^2$ in Fig. 4.

Additional information can be obtained from polarized e^+-polarized p scattering. Actually, this assertion is only true for $y \neq 0$, otherwise we have $A(e^+,p) = -A(e^-,p)$, as shown for $q^2 = 10^4$ $(GeV/c)^2$ in Fig. 5. If one takes $M_Z \to \infty$ as well, these asymmetries vanish identically. If we average over y, we get the curves in Fig. 6., showing $A(e^+,p)$ for $q^2 = 10, 10^3$, and 10^4 $(GeV/c)^2$. The dependence on y for $q^2 = 10^4 (GeV/c)^2$ is shown in Fig. 7. The marked contrast in the behavior of $A(e^+,p)$ compared to that of $A(e^-,p)$ reflects the different weightings of the weak couplings at the lepton-quark vertex that are probed by using a positron versus electron beam.

For the sake of completeness, we should mention that the <u>single</u> asymmetry measurements (polarized lepton beam and unpolarized target) which have been carried out, do not provide any constraints bearing on the x-dependence of the quark distributions [11]. This is because these experiments have been performed using isoscalar targets, for which $u(x) = d(x)$, and the x-dependence drops out of the single asymmetry.

In this paper, we have tried to emphasize the wealth of information contained in the double asymmetry measurements carried out at collider energies. We have presented what we believe to be reasonable estimates of the asymmetries one might expect to observe. A direct calculation of the quark spin distribution inside the nucleon, starting from the equations of QCD is a difficult nonperturbative calculation, not presently amenable to a rigorous treatment. Thus, further insight to the nucleon's spin dependent structure functions can only come with additional testing of the phenomenological models. In addition to adding to our general knowledge of the nucleon structure, these asymmetry measurements, carried out with positrons as well as with electrons, will yield important information on the neutral current couplings of the standard model.

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FIGURE CAPTIONS

Fig. 1: Our estimate for the double asymmetry parameter A/D in polarized electron-polarized proton scattering. The data is taken from reference [4].

Fig. 2: The double asymmetry $A(e^-,p)$ in the limit of an infinitely heavy Z boson. The dependence on y has been integrated out.

Fig. 3: The double asymmetry $A(e^-,p)$ with the coupling to the Z restored. The dependence on y has been integrated out.

Fig. 4: The double asymmetry $A(e^-,p)$ for $q^2 = 10^4$ $(GeV/c)^2$ at three different values of y.

Fig. 5: The double asymmetries $A(e^-,p)$ and $A(e^+,p)$ at $y=0$ and $q^2 = 10^4$ $(GeV/c)^2$.

Fig. 6: The double asymmetry $A(e^+,p)$ averaged over y for three different values of q^2.

Fig. 7: The double asymmetry $A(e^+,p)$ for $q^2 = 10^4$ $(GeV/c)^2$ at three different values of y.

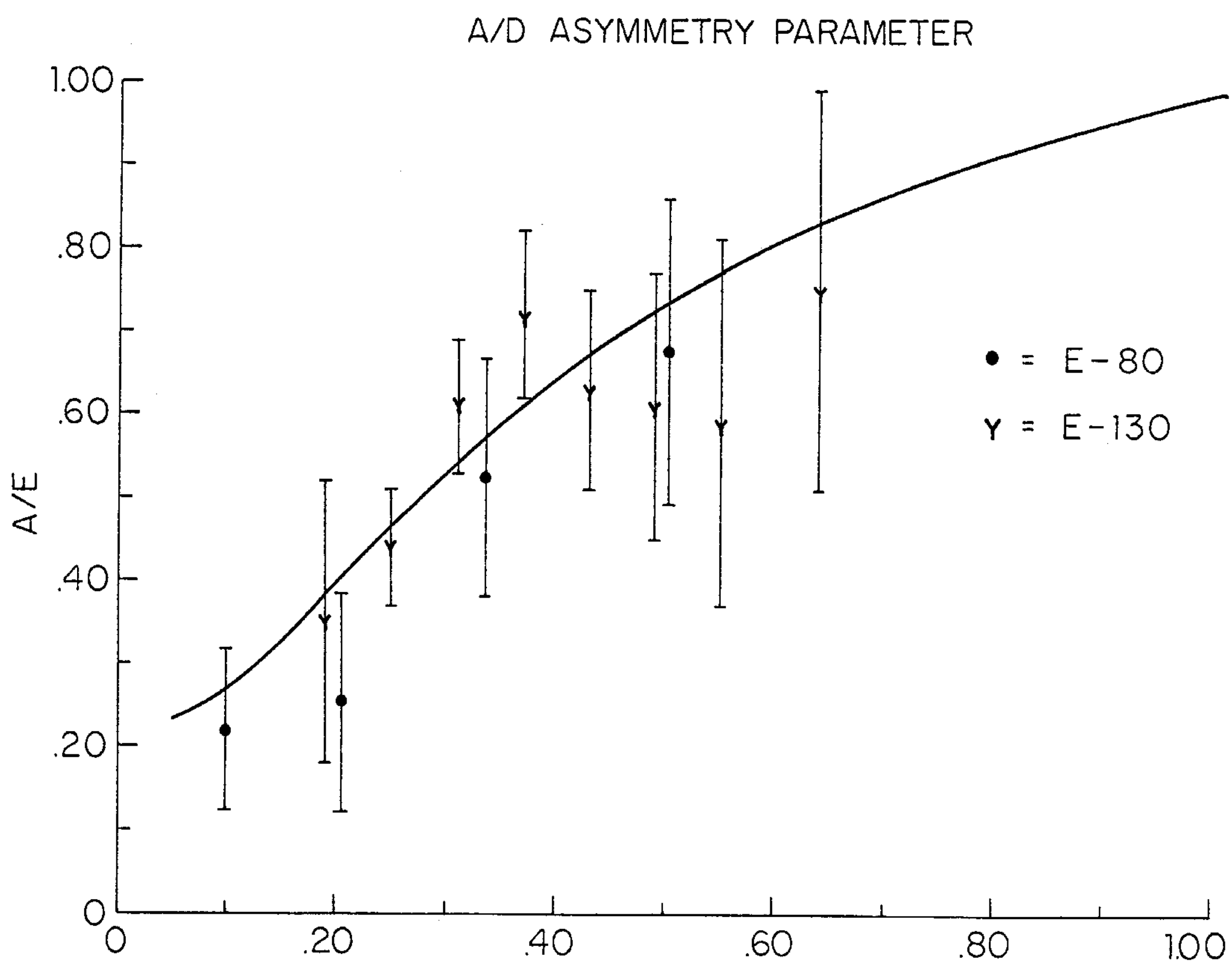

A/D ASYMMETRY PARAMETER
1.00
.80
.60
.40
.20
0
A/E
0
.20
.40
.60
.80
1.00
● = E-80
Y = E-130

Fig. 1

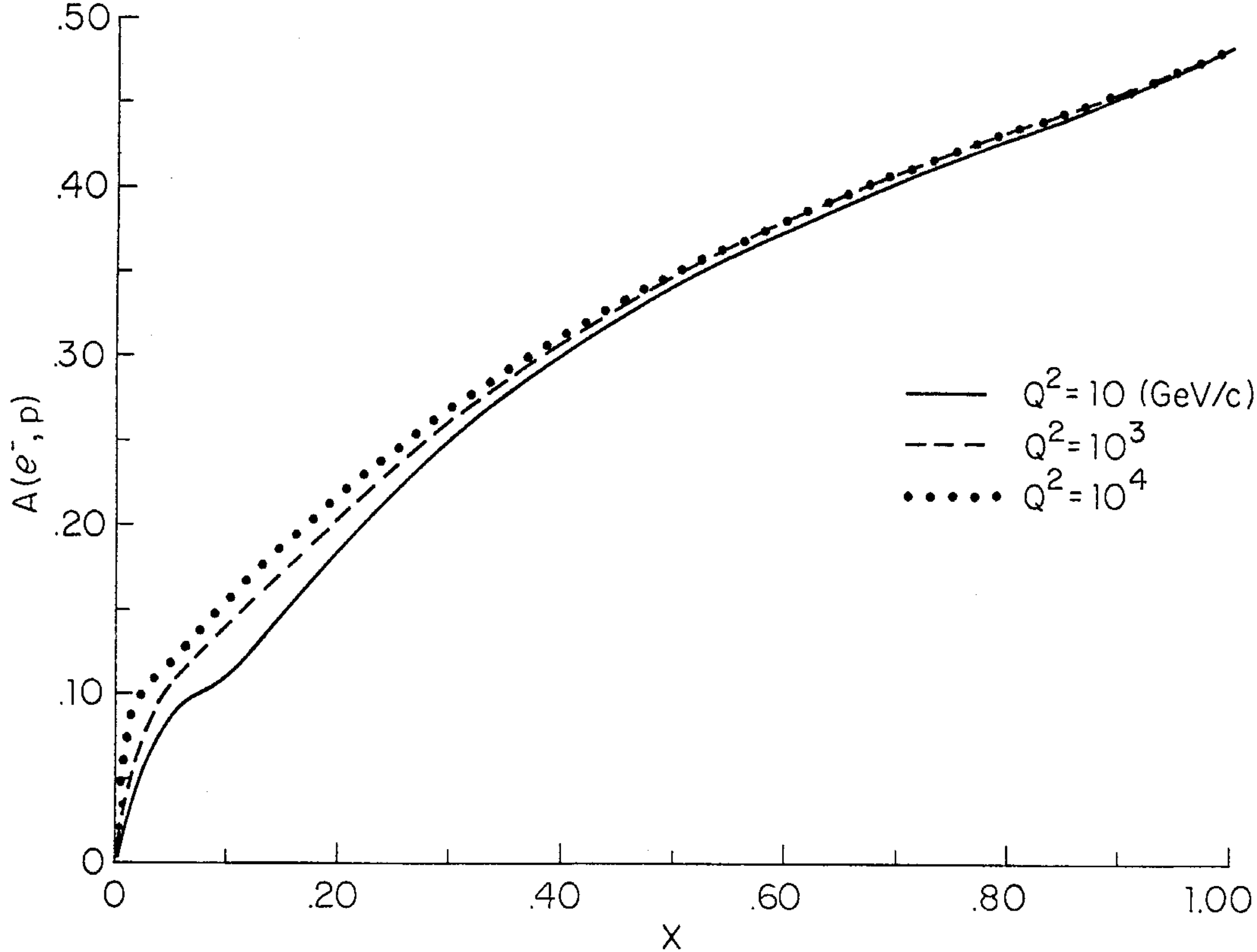


Fig. 2

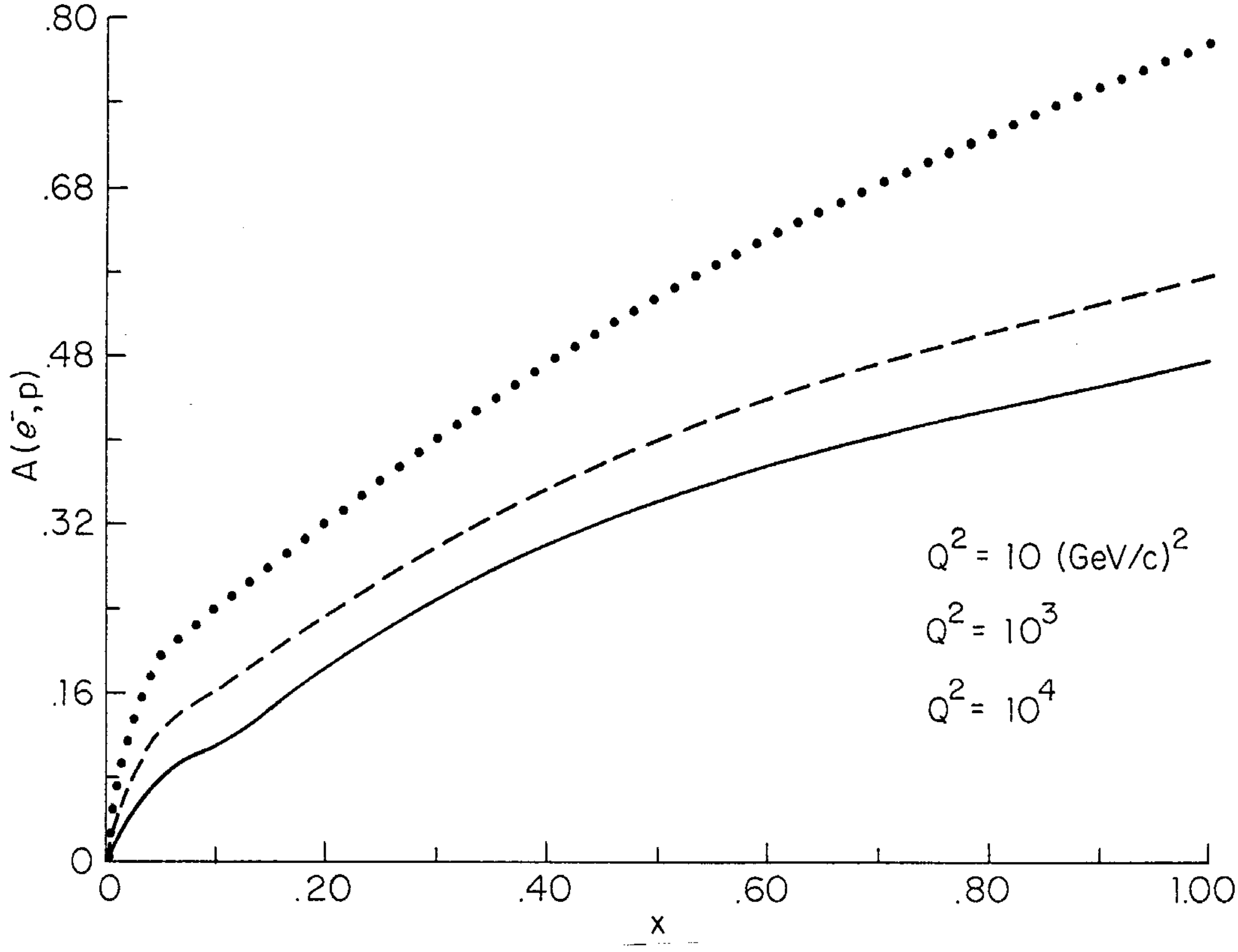


Fig. 3

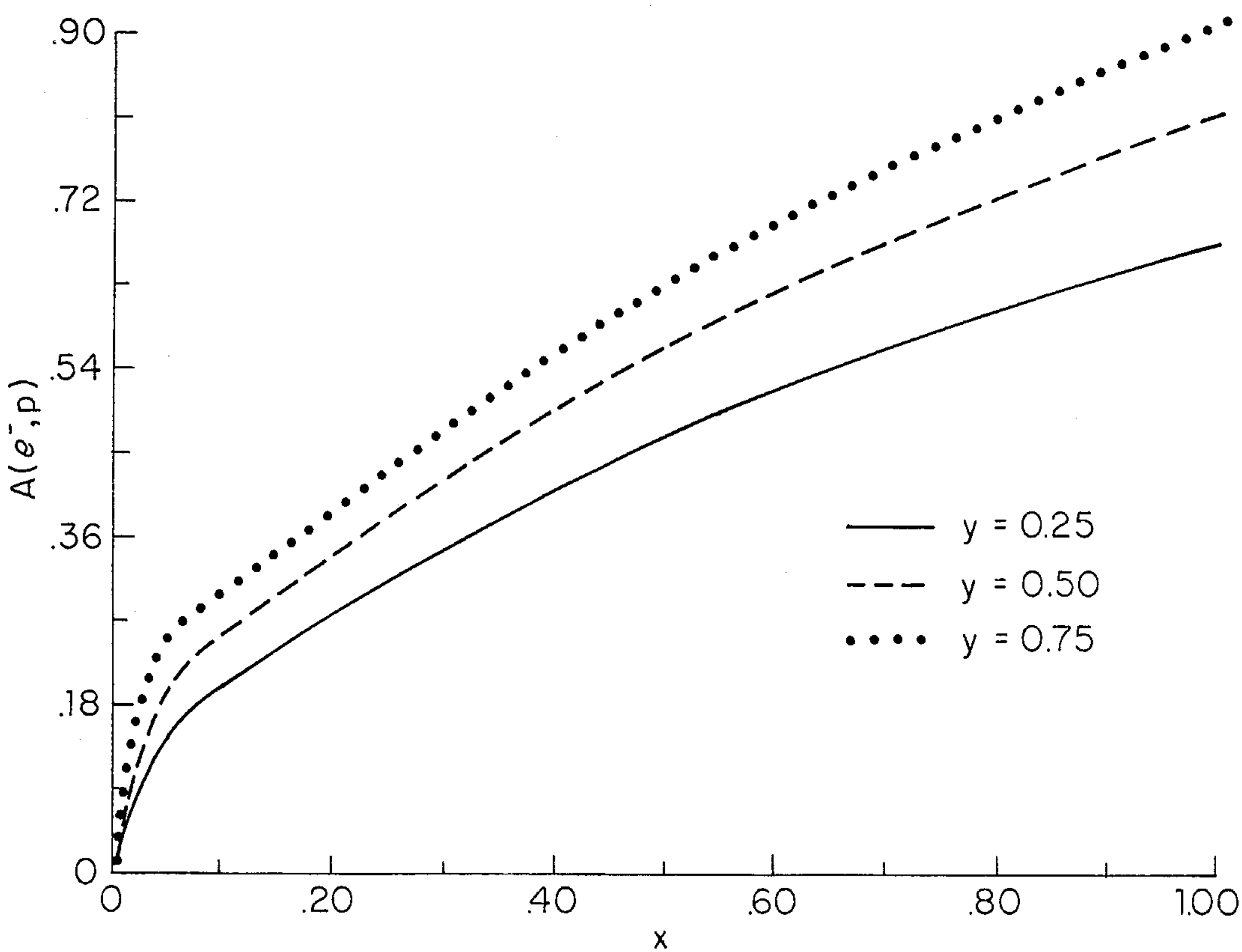


Fig. 4

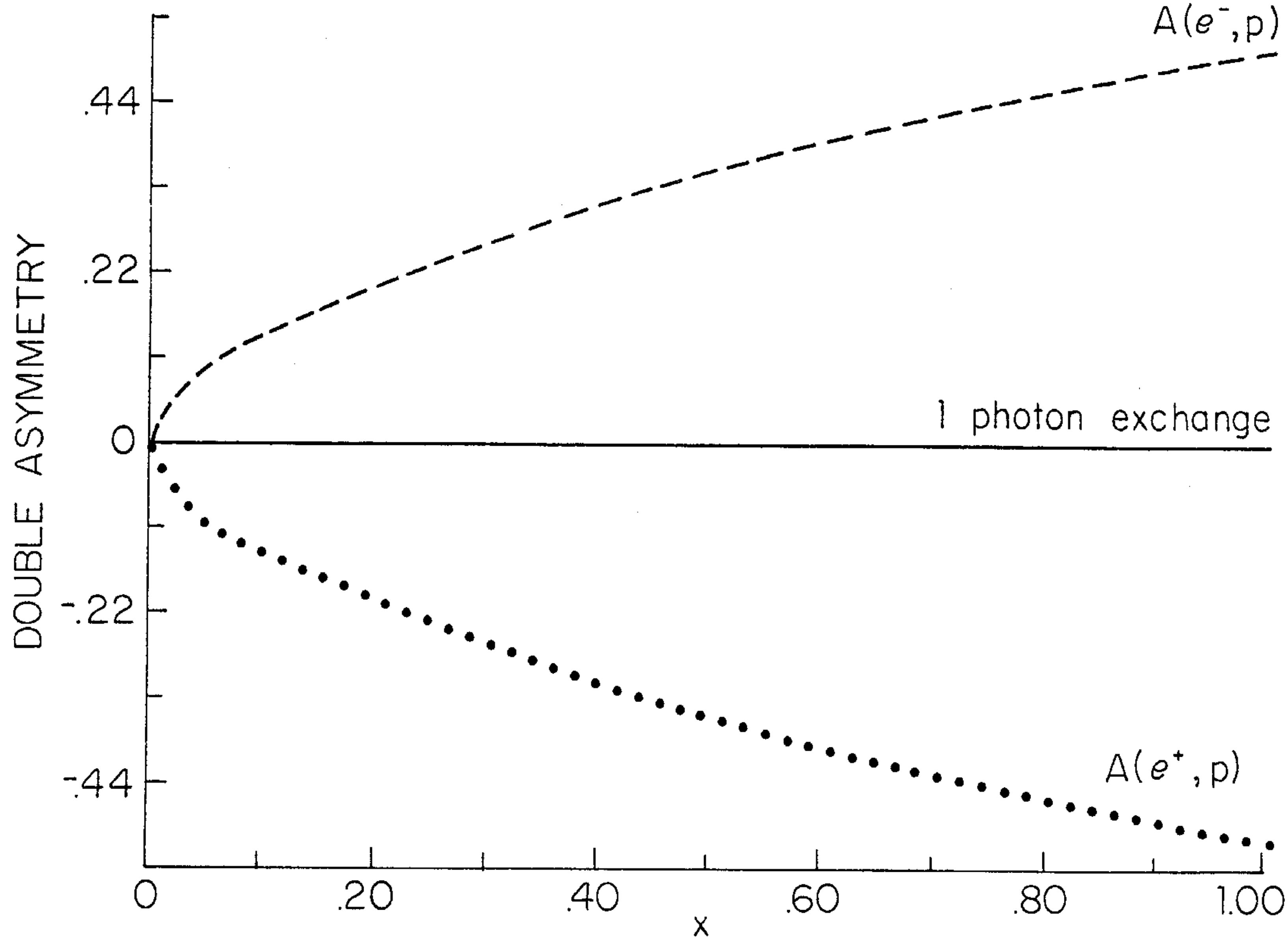
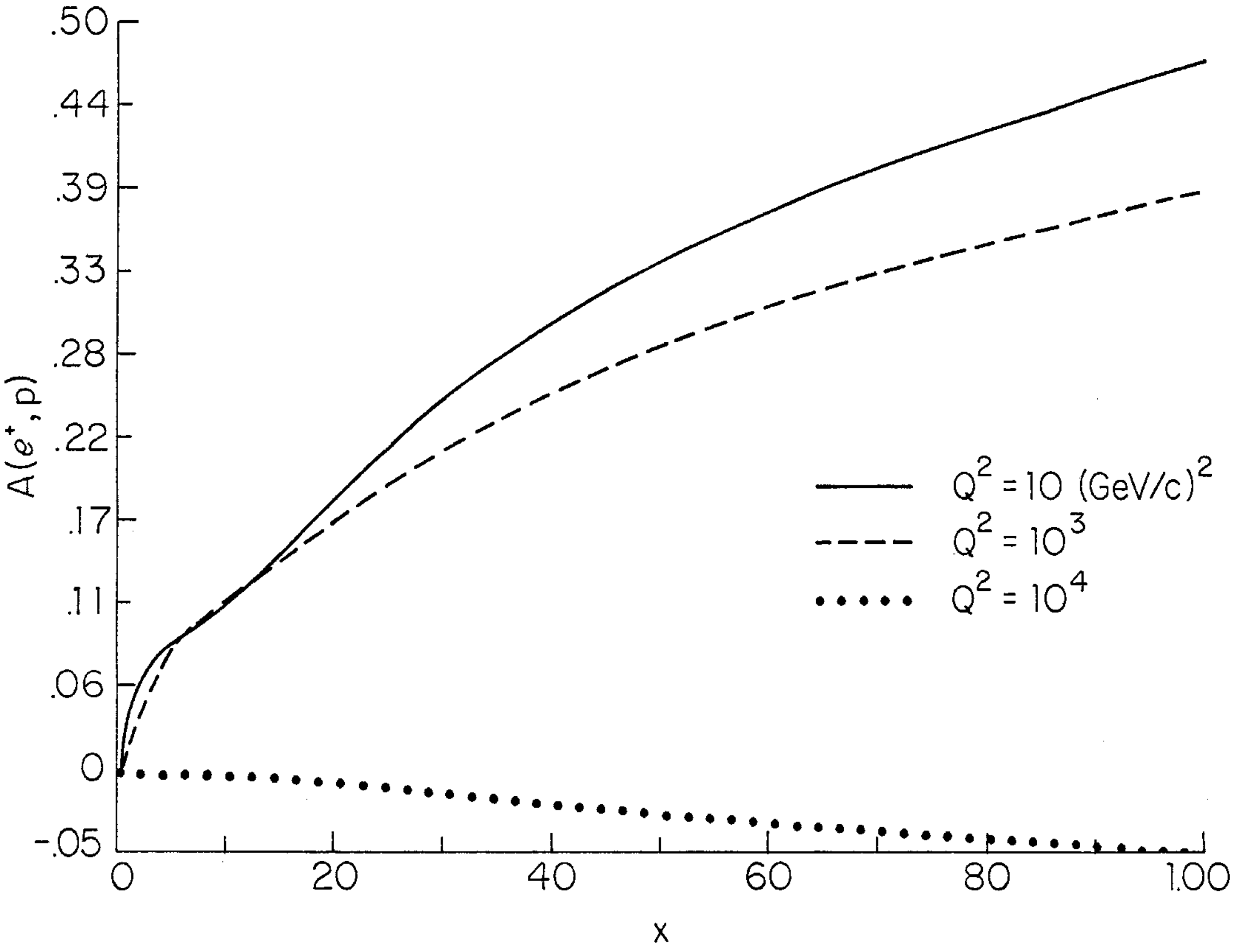
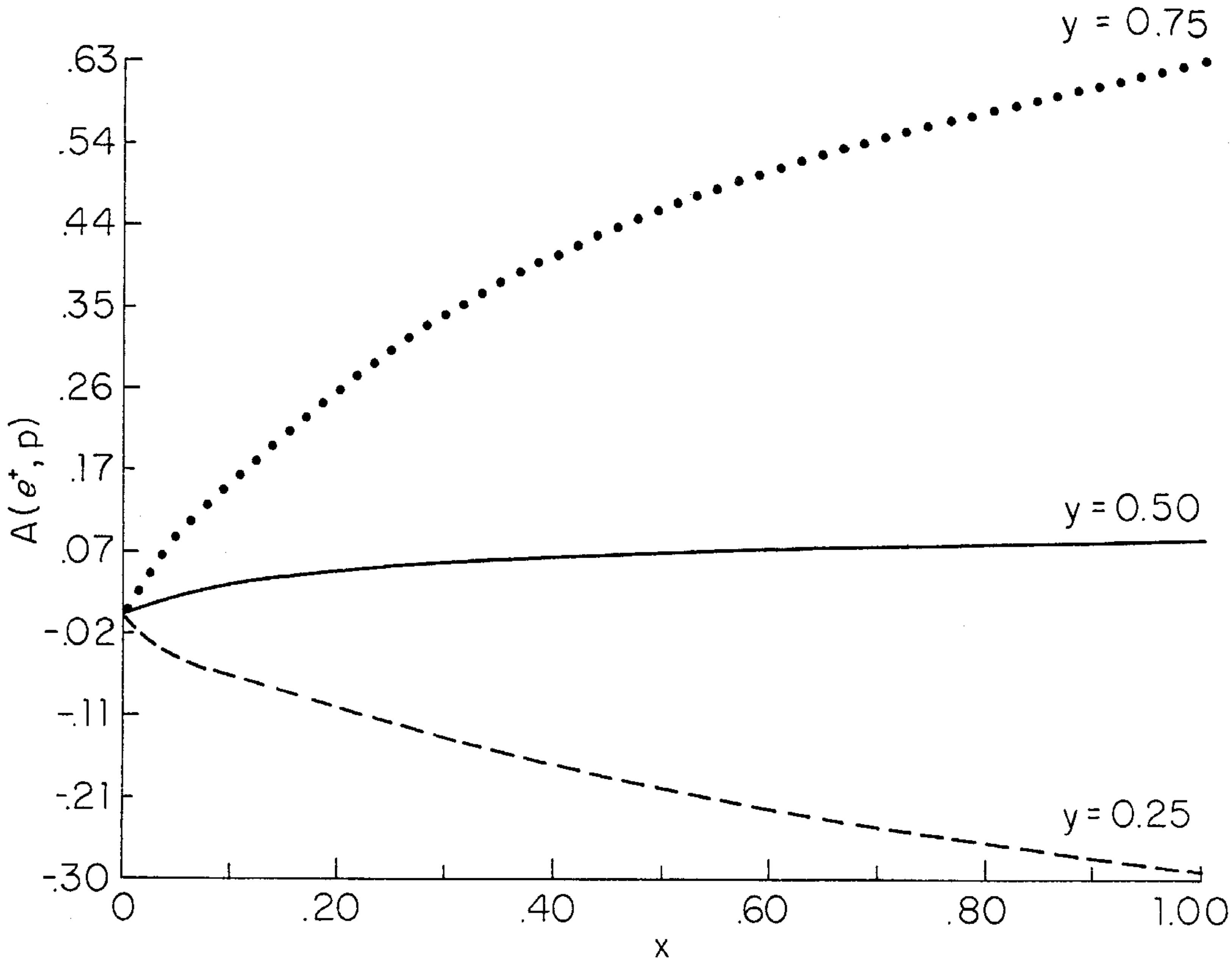

A(e⁻,p)
1 photon exchange
A(e⁺,p)
DOUBLE ASYMMETRY
.44
.22
0
-.22
-.44
0
.20
.40
.60
.80
1.00
X

Fig. 5



.50
.44
.39
.33
.28
.22
.17
.11
.06
0
-.05
A(e⁺,p)
0
20
40
60
80
1.00
X
Q² = 10 (GeV/c)²
Q² = 10³
Q² = 10⁴

Fig. 6



y = 0.75
y = 0.50
y = 0.25
A(e+, p)
.63
.54
.44
.35
.26
.17
.07
-.02
-.11
-.21
-.30
0
.20
.40
.60
.80
1.00
x

Fig. 7