ULTRA HIGH ENERGY RADIATION FROM A BLACK HOLE ☆

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We give a model independent estimate of the flux of high energy Hawking radiation from a black hole, $E \gg 100$ GeV. We calculate the black hole contribution to ultra high energy cosmic rays and find that it is insignificant. We point out that the high energy radiation emitted by a real black hole is expected to be non-thermal.

Hawking radiation [1] strikingly illustrates the interplay between quantum mechanics and classical gravity. For the astronomer it is a signal for the direct detection of black holes, and for the particle physicist one of the few windows on the physics at energies approaching the Planck mass scale. The last statement follows from the surprising fact that the temperature of a black hole is *inversely* proportional to its mass. As a result the black hole grows hotter as it radiates, and can eventually attain extremely high temperatures.

In this letter we consider the possibility of observing this ultra high temperature Hawking radiation. Could it be a significant part of the high energy ($E \gg 100$ GeV) cosmic ray background? Based on some simple estimates of the high energy flux from a black hole, we find that the answer is no. Also we are able to show very simply that the energy spectrum of Hawking radiation is generally non-thermal. This is as expected for interacting theories, but contrasts with the case of a free field theory assumed valid up to infinite momentum scales.

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Unfortunately a precise calculation of the Hawking flux at temperatures $\gg 100$ GeV is not possible because we do not know the particle physics at these energies. As a result very different descriptions of a high temperature black hole have been proposed, with significantly different consequences. For instance Rees [2] has pointed out that if a black hole radiates rapidly or explodes after reaching a critical temperature, then this event should be observable from anywhere within our galaxy or even beyond. On the other hand, if we can extrapolate from our experience with low energy physics then the high energy radiation should be slow. In fact, by applying a few general principles which seem to hold at current energies, we find that we can roughly estimate the Hawking flux without knowing the precise form of the high energy theory.

Our basic assumption is the conventional picture of Hawking radiation as a thermal emission process characterized by a temperature T. We will ignore such issues as the validity of interpreting the effect as a thermodynamical one (versus a coherent process), and the potential problems associated with the UV behavior of the underlying field theory above the Planck scale.

Secondly, we assume that the effective couplings of the high energy theory are small for most energies below the Planck scale. Most unified theories satisfy this requirement, and moreover it reflects a set of theoretical prejudices. First of all, although there need not be a desert in the 10^2–10^{14} GeV region, we would expect on the order of 14 thresholds, new interactions and symmetries, etc., rather than 10^{14}. The onset of new physics is probably logarithmic in energy rather than linear – certainly this is true at familiar energies. Also this would be the result if thresholds are fixed by the renormalization group. Since couplings evolve logarithmically with energy, thresholds determined by this evolution will be exponentially far apart. For example a threshold could be associated with an effective coupling becoming of order one at some scale Λ. If this occurs at a high energy, the transition to strong coupling could signal the breakdown of a low energy theory and predict new physics at energies $E > \Lambda$. Alternatively, if the coupling gets big at low energies, Λ might set the scale for confinement, symmetry breaking, the masses of bound states, and so on. At lower energies ($E \ll \Lambda$) particles with mass $M > \Lambda$ would decouple, so that effectively one has a new and simpler theory below threshold.

One should be able to approximate the physics in the energy regions between thresholds by simplified effective theories. Because thresholds are rare, these effective theories should be valid over a large energy range and therefore (invoking Veltman's theorem and the Appelquist–Carazzone theorem [3]) should be renormalizable. Otherwise they are likely to be inconsistent; the non-renormalizable interactions in one loop and higher order will bring in uncontrollable contributions from the physics at the upper threshold. In particular it would be hard to understand why all the light particles in the effective theory do not get heavy masses on the order of this threshold.

These effective theories may be either asymptotically free or not. However non-asymptotically free theories probably are consistent only at energies where the running couplings are small. For example one expects $\lambda\phi^4$ and QED to become inconsistent at high energies when the coupling gets sufficiently big. Nevertheless these theories, like QED, may be extremely good approximations within the domain of their validity.

Thus this type of theory satisfies our requirement that the couplings be small. For asymptotically free theories the couplings are small at high energies. At a low energy the couplings do indeed become large, but this simply determines a threshold. Many particles will get masses on the order of this threshold, which is the dominant scale in the problem. Below threshold one can write down a new effective theory of the surviving light particles, to which the above discussion again applies, so that its couplings should also be small. For instance an effective theory of low mass Goldstone particles contains Adler zeros, and therefore these particles will interact weakly among themselves at energies far below the symmetry breaking threshold. We conclude then that our basic assumption should be valid except in the neighborhood of the rare thresholds.

Given the above, a black hole's particle emission rate should depend primarily (and approximately linearly) on the number of light elementary particles – the number of modes available for radiation. This number counts those states light compared to the temperature of the hole, since a black hole essentially does not emit more massive particles. Nevertheless we expect that the emission rate will be a roughly constant function of temperature. If we encounter more and more massive states as we go up in energy, we expect that at a sufficiently high energy there will be a simple substructure underlying this particle zoo, and that the relevant elementary excitations radiated by the black hole will be those of this substructure. We do not expect more than an order of magnitude increase in the number of states in this high energy theory beyond the number that obtains today. If anything we might expect increased simplicity and fewer states.

Our third assumption then is that the number of effectively light elementary particles is roughly constant with energy. This implies that the emission rate is also approximately constant, and slow. We can then show that Hawking radiation is very simple – the emission rate is just the right magnitude for particles to be emitted and decay approximately independently. To first approximation, the flux is independent of the detailed interactions since these are small.

Of course the decay and fragmentation of the radiated particles do depend on the details of the theory. Perhaps we could still make a reasonably model independent guess at the result, but in any case we will not need to consider this decay in detail.

We consider first the emission probability per unit time for a single free particle since the effective couplings are small. For spins 1/2, 1, and 2 this has been computed by Page [4]. The spin 0 emission probability is higher but not significantly so [5]. Explicitly the probability per unit energy for a (spherically symmetric) black hole of mass M is

$$\mathrm{d}P/\mathrm{d}t\,\mathrm{d}E = \Gamma_s(ME)/[\exp(8\pi ME) \mp 1]. \tag{1}$$

In natural units, with $k = h = G = c = 1$, the black hole temperature is $1/8\pi M$. Eq. (1) is just the usual black body spectrum of radiation for a body at this temperature. The $-(+)$ corresponds to the emission of a boson (fermion) with energy E. Γ_s, the absorbtivity, is the function of energy and spin computed by Page. A quantum mechanical black hole is thus in fact a grey body. It does not absorb all radiation incident upon it, but rather has a probability Γ of absorbing an incident particle. By the principle of detailed balance, Γ also determines the probability with which it emits a particle.

The total emission rate, assuming no interactions, is just the above probability summed over all modes. In the standard Weinberg–Salam (WS) model with three generations there are 102 modes (counting spin states and Higgs particles, but not the graviton). As explained above, we assume this is roughly the number of modes in the true high energy theory. We approximate the true rate by the WS rate, which is

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \sum_s N_s \int_0^\infty \mathrm{d}E \frac{\Gamma_s(ME)}{\exp(8\pi ME) - (-1)^{2s}},$$

$$N_0 = 4, \quad N_{1/2} = 90, \quad N_1 = 8, \quad N_2 = 2. \tag{2}$$

Summing over energies, we find the total number of particles emitted per unit time:

$$\mathrm{d}N/\mathrm{d}t \cong 10^{-2}/M. \tag{3}$$

Thus the time between particle emissions is $\mathrm{d}t \cong 100M$, assuming that the mass varies slowly, which it does except at late times.

We can now address the question of whether particles are radiated independently in a realistic high energy theory. For instance long range forces (e.g. color) could be important if particles (e.g. quarks) emerge from the hole infrequently, since then each particle travels a large distance from the hole before the next one is produced. In this case one would at least expect emissions to be correlated, and perhaps the picture of separate radiation of single particles would break down.

We consider therefore an asymptotically free theory, for which some force will become strong at a large distance scale $D = 1/\Lambda$. At black hole temperatures $T \gg \Lambda$, the time between emissions turns out to be much less than the scale D:

$$100/8\pi T = \mathrm{d}t \ll D = 1/\Lambda. \tag{4}$$

Also the effective size of the hole is small: the radius is $\sim 2M = 1/4\pi E \ll D$. Finally the typical energy of the radiated particles is large compared to Λ. Thus the situation resembles e^+e^- annihilation. Relative to the scale D, essentially one sees the simultaneous production of several high energy particles at a point. For this process, we know that the long distance iteractions have no effect on the short time propagation of the particles, but simply dress them into independent jets over a long time. At high temperatures the long range force does not effect the independent particle radiation of the black hole.

At very low temperatures $T \ll \Lambda$ the long range force is not relevant. The black hole is not hot enough to radiate isolated constituents or charged states, since the long range forces will give them a dynamic mass that is large and possibly infinite. The strong long range force is always screened. Instead low mass bound states and other light particles will be radiated, in accord with the effective theory describing their interactions. Again, by the same argument, this radiation represents independent emission. Thus black holes radiate sufficiently fast that long range forces are unimportant except for temperatures near the thresholds.

On the other hand if they radiate too fast they will produce a dense and interacting cloud of particles. Fortunately this too does not occur. We would expect particles with energy E to interact significantly only if they are emitted within a time $1/E$ of each other. Now these particles are emitted primarily when the temperature $T \sim E$, when the time between emissions is $\mathrm{d}t = 200/8\pi E \sim 8/E$. Since $\mathrm{d}t \gg 1/E$, a black hole at a given temperature radiates particles infrequently compared to their average energy, and these particles will tend not to interact before fragmenting.

We can elaborate this argument in a number of ways. A black hole in fact radiates particles with energy E throughout its lifetime, and at some point the time between emissions will shrink to less than $1/E$. Particles emitted later than this presumably do interact significantly before fragmenting. However, it is easy to compute by numerical integration the percentage of particles emitted earlier, and we find that more than 99.9% should be non-interacting.

Taking the finite size of the hole into account will also strenghten our conclusion somewhat. Particles emitted from opposite sides of the hole are less likely to interact than if there were no barrier between them. However this is not a strong effect. At a temperature $T \sim E$ the black hole effectively creates particles at a radius $\sim 3/(8\pi E)$ outside, but still close to, the horizon. This is a fraction of the effective size $1/E$ of the particle, so the overlap of two emitted particles is only slightly diminished.

We conclude therefore that except near thresholds a black hole essentially radiates free particles which then fragment in isolation. We now compute the total lifetime particle production of a black hole. We know already the characteristics of the radiation when the hole has a given temperature or mass. It remains to fold in the evolution with time of the hole, as its mass radiates away.

At a given time the decrease in mass is just the total power radiated:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\sum_s N_s \int \mathrm{d}E \frac{E\Gamma_s(ME)}{\exp(8\pi ME) - (-1)^{2s}}. \qquad (5)$$

Assuming that the effective high energy theory contains approximately the same number of modes as WS, we find

$$\mathrm{d}M/\mathrm{d}t \cong -10^{-3}/M^2. \qquad (6)$$

From one threshold to the next this equation is valid, and during this period the mass evolves like

$$M \sim K(t_i - t)^{1/3}, \quad t_i \text{ arbitrary},$$
$$K \sim (3 \times 10^{-3})^{1/3}. \qquad (7)$$

K is primarily determined by the number of light elementary particles in the effective theory. We assume, as stated above, that K is roughly constant, say within a factor of two, at all energies apart from thresholds.

Again ignoring thresholds, we compute the total spectrum of particles radiated by the black hole.

$$\frac{\mathrm{d}N}{\mathrm{d}E} = \sum_s N_s \int_{t_i}^{\infty} \frac{\Gamma_s(ME)}{\exp(8\pi ME) - (-1)^{2s}} \mathrm{d}t$$
$$= \sum_s N_s \int_0^{M_i} \frac{\Gamma_s(ME)}{\exp(8\pi ME) - (-1)^{2s}} \frac{M^2}{1000} \mathrm{d}M. \qquad (8)$$

Since we are interested in ultra high energy radiation, the initial time for the first integral, as long as it is early, is rather arbitrary. The black hole at early times is too cold to radiate high energy particles, and this region contributes negligibly to the integral. Thus for $E \gg t_i$, $EM_i > 1/8p_i$, and we can take the initial mass to be infinite. The spectrum is approximately

$$\frac{\mathrm{d}N}{\mathrm{d}E} \cong \frac{1}{E^3}\left(\sum_s N_s \int_0^{\infty} \frac{\Gamma_s(x)}{\exp(8\pi x) - (-1)^{2s}} \frac{x^2}{1000} dx\right). \qquad (9)$$

It scales like $1/E^3$, just as the ultra high energy cosmic ray background is observed to do above the so-called "knee" at $\sim 10^{15}$ eV [6]. However what we have so far computed is just the spectrum of almost free particles directly radiated by the black hole. For the actual spectrum of high energy hadrons, one must convolute this spectrum with the fragmentation of these particles into hadrons. The important point is that fragmentation will enhance the low energy end of the spectrum. The hadron spectrum will tend to fall off at high energy faster than $1/E^3$, and thus faster than the cosmic ray spectrum. Hawking radiation would thus be easier to see at low rather than at high energies, and ultra high energy cosmic rays can provide no useful limit.

The effect of thresholds on the spectrum should be insignificant except at the energy of the thresholds themselves. When the black hole temperature is at a threshold T_{thr}, the hole will be radiating particles with energy less than or equal to this temperature. But most of the mass of the hole has already been radiated away into particles with energy less than T_{thr}. The threshold contribution to the spectrum for energies less than T_{thr} is insignificant.

We now proceed to estimate the black hole contribution to the cosmic ray background. From eq. (9) the total contribution of a single black hole is approximately:

$$\mathrm{d}N/\mathrm{d}E \sim 10^{-7}/E^3. \tag{10}$$

Now of all black holes only those that have had time to evolve to high temperatures can contribute to the high energy background. This means we are dealing with small black holes, with much less than stellar mass. We know of no process that could produce holes of this size except the original Big Bang. So-called primordial black holes produced in the Big Bang with mass less than about 5×10^{14} g would either have completely evaporated by now or are just now evaporating, so these are the ones which contribute to our signal.

The best current bound on the primordial black hole density is that of Hawking and Page [7]. They have calculated that black holes should radiate gamma rays with a spectrum peaked around 100 MeV. By looking at the cosmic gamma ray spectrum they set an upper limit of $10^4/\mathrm{pc}^3$ on the density of these holes with masses near 5×10^{14} g.

Following their work, we can estimate an upper limit on the black hole contribution to the high energy cosmic ray background as observed on Earth.

$$\mathrm{d}J/\mathrm{d}E \cong \left[10^{16}\ \mathrm{eV}^2/\mathrm{m}^2\ \mathrm{s\ sr}\right]/(E\ \mathrm{eV})^3. \tag{11}$$

Comparing this to the observed background, which yields a flux

$$\mathrm{d}J/\mathrm{d}E \cong \left[6 \times 10^{24}\ \mathrm{eV}^2/\mathrm{m}^2\ \mathrm{s\ sr}\right]/(E\ \mathrm{eV})^3, \tag{12}$$

we find that the black hole signal is down by about 9 orders of magnitude, and thus unobservable. Even above the Greisen cutoff [8], we would be unlikely to detect such a background from nearer black holes.

We briefly comment on the spectrum of the radiation emitted by a black hole at a fixed temperature. The spectrum of particles emitted directly is given in eq. (1) – it is a simple grey body thermal distribution. Convoluting this spectrum with the fragmentation functions for these particles gives an hadronic spectrum

$$\frac{\mathrm{d}P}{\mathrm{d}t\,\mathrm{d}E} = \sum_j N_j \int_E^\infty \mathrm{d}E' \frac{\Gamma_j(ME')}{\exp(8\pi ME) - (-1)^{2s}} \times f_{ij}(E/E'), \tag{13}$$

i, j refer to particle type.

The hadronic spectrum will clearly be non-thermal. This is as expected [9] for a black hole emitting interacting particles into a zero temperature vacuum, but is particularly easy to see here. One could also calculate the hadron spectrum more explicitly as in ref. [10], where one of us found a simple form for the fragmentation function that accords with current theoretical and experimental expectations.

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