Fermi National Accelerator Laboratory

FERMILAB-PUB-84/ 28-T
February, 1983

Role of conservation laws in the Callan-Rubakov process with arbitrary number of generation of fermions

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ABSTRACT

It is shown that for a magnetic monopole in an SU(2) gauge theory, interacting with an arbitrary number of massless doublets of fermions, the global and the local conservation laws of the full four dimentional theory, together with the restrictions on the fermions to J=0 partial wave, uniquely determines the final state for a given initial state. As a result, the monopole induced baryon number violation is a necessary consequence of the conservation laws of the system. The modification of the conservation laws in the presence of the full SU(5) gauge interactions is discussed.

The subject of monopole induced baryon number violation, first introduced by Rubakov[1], and subsequently by Callan[2], has been of great interest in the past. In this letter we give a simple explanation of this phenomenon, based on the conservation laws of the system. We consider an SU(2) gauge theory, spontaneously broken down to U(1) by the vacuum expectation value of the adjoint higgs field ϕ. We also include n massless Dirac doublet of fermions in the theory, which we denote by $\begin{pmatrix}\psi_{i\uparrow}\\ \psi_{i\downarrow}\end{pmatrix}$ (i=1,..n). [Here $\psi_{i\uparrow}$ and $\psi_{i\downarrow}$ are eigenstates of the charge operator with eigenvalues +1/2 and -1/2 respectively]. The Lagrangian density of the system is given by,

$$-\frac{1}{4} F_{a\mu\nu} F^{a\mu\nu} + (D_\mu \varphi)^{+}(D^\mu \varphi) + \sum_{i=1}^{n} \bar{\psi}_i \, i \not{D} \, \psi_i$$

$$- \lambda \, (\vec{\varphi}^{\,2} - a^2)^2 \qquad (1)$$

The theory has n conserved vector charges, and n-1 anomaly free conserved chiral charges, given by,

$$S_i = \int d^3x \, (\bar{\psi}_{i\uparrow} \gamma^0 \psi_{i\uparrow} + \bar{\psi}_{i\downarrow} \gamma^0 \psi_{i\downarrow})$$

$$i = 1, \dots n \qquad (2)$$

$$S_{1i} = \int d^3x \, (\bar{\psi}_{i\uparrow} \gamma^0 \gamma^5 \psi_{i\uparrow} + \bar{\psi}_{i\downarrow} \gamma^0 \gamma^5 \psi_{i\downarrow}$$

$$- \bar{\psi}_{1\uparrow} \gamma^0 \gamma^5 \psi_{1\uparrow} - \bar{\psi}_{1\downarrow} \gamma^0 \gamma^5 \psi_{1\downarrow})$$

$$i = 2, \dots n \qquad (3)$$

There is also a conserved charge associated with the unbroken local U(1) symmetry, given by,

$$S = \int d^3x \left(\sum_{i=1}^{n} \left(\bar{\psi}_{i\uparrow} \gamma^0 \psi_{i\uparrow} - \bar{\psi}_{i\downarrow} \gamma^0 \psi_{i\downarrow} \right) \right) \tag{4}$$

The model posseses classical monopole solutions of the 't Hooft-Polyakov type[3]. We shall study the interaction of such a monopole with the fermions. We restrict the fermions to be in the $\vec{J}=0$ partial wave, since only the $\vec{J}=0$ partial waves have non-vanishing amplitude at the monopole core, and can interact with the monopole core without any suppression. [Here $\vec{J}$ denotes the total angular momentum operator]. If we work in a gauge where the monopole solution is spherically symmetric, then $\vec{J}$ is given by,

$$\vec{J} = \vec{L} + \vec{S} + \vec{T} \tag{5}$$

where $\vec{L}$, $\vec{S}$ and $\vec{T}$ denote the orbital angular momentum, the spin angular momentum, and the isospin operator respectively. The restriction to J=0 then implies that,

$$\hat{r} \cdot \vec{J} = \hat{r} \cdot (\vec{L} + \vec{S} + \vec{T}) = 0$$

i.e.

$$\hat{r} \cdot \vec{S} = - \hat{r} \cdot \vec{T} \tag{6}$$

since $\hat{r}\cdot\vec{L}=0$. Since $\hat{r}\cdot\vec{S}$ measures the radial component of the spin, and in the spherically symmetric gauge $\hat{r}\cdot\vec{T}$ measures the charge of the particle associated with the unbroken U(1) gauge symmetry, Eq. (6) implies that for outgoing particles the helicity must be negative if its U(1) charge is positive, and vice versa. For incoming particles, the helicity has the same sign as the U(1) charge. Formally, this says that in the final state,

$$\int \bar{\Psi}_{i\uparrow} \gamma^0 \Psi_{i\uparrow} \, d^3x = - \int \bar{\Psi}_{i\uparrow} \gamma^0 \gamma^5 \Psi_{i\uparrow} \, d^3x$$

$$\int \bar{\Psi}_{i\downarrow} \gamma^0 \Psi_{i\downarrow} \, d^3x = \int \bar{\Psi}_{i\downarrow} \gamma^0 \gamma^5 \Psi_{i\downarrow} \, d^3x \qquad (7)$$

$$i = 1, \cdots n$$

For a given initial state, we may compute the 2n conserved charges given by Eqs. (2-4). The final state must have the same value of these charges, since they are exactly conserved. These 2n conservation laws, together with the 2n constraints on the final state fermions given in (7), uniquely determines all the 4n charges,

$$\int \bar{\Psi}_{i\uparrow} \gamma^0 \Psi_{i\uparrow} \, d^3x, \quad \int \bar{\Psi}_{i\downarrow} \gamma^0 \Psi_{i\downarrow} \, d^3x$$

$$\int \bar{\Psi}_{i\uparrow} \gamma^0 \gamma^5 \Psi_{i\uparrow} \, d^3x, \quad \int \bar{\Psi}_{i\downarrow} \gamma^0 \gamma^5 \Psi_{i\downarrow} \, d^3x$$

$$i = 1, \cdots n \qquad (8)$$

in the final state, and thus uniquely determines the final state, upto the addition of fermion-antifermion pairs. For example, for an initial state $\psi_{1\uparrow L}+\psi_{2\downarrow R}$, the unique final state is $\psi_{1\downarrow L}+\psi_{2\uparrow R}$ (L and R denote positive and negative helicities respectively). For an SU(5) monopole, embedded in the 3-4 subspace, we may identify the first two doublets with[5],

$$\begin{pmatrix} u_1 \\ u_2^c \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} e^+ \\ d_3 \end{pmatrix}$$

If we ignore all the SU(5) gauge interactions other than the SU(2) gauge fields lying in the 3-4 subspace, the problem reduces to the case of an SU(2) monopole interacting with massless Dirac doublets of fermions. (We shall comment on these extra gauge interactions at the end of the paper). Then we see that for an initial state $u_{1L}+d_{3R}$, the final state is uniquely determined to be $e^+_R+u^c_{2L}$. Thus monopole induced baryon number violation is forced on us by the conservation laws of the system.

For some initial states, if we compute the charges given in (8) in the final state, using the conservation laws of the charges given in Eqs.(2-4), and the constraints (7), we find that the values of these charges turn out to be fractional. This happens, for example, if we just have $\psi_{1\uparrow L}$ in the initial state. In order to understand this we go back to the effective two dimensional boson field theory formulation of the problem, first introduced by Callan[2]. In this model, fermions are represented by solitons in the boson fields. If we send single soliton corresponding to $\psi_{1\uparrow L}$, we see that the outgoing state contains fractional solitons[6,7]. These fractional solitons are, however, stable only in the massless limit. If we give the fermions a mass m, then the fractional solitons evolve to full solitons in a time of order m^{-1}. This shows that for these processes, the zero mass limit is singular, and we must include the mass term for the fermions in order to determine the possible final states for a given initial state. But the inclusion of the mass term reduces the number of conservation laws of the system, and hence we cannot determine the final state uniquely for such processes.

Finally, let us note that a process of the form $\psi_{1\uparrow L}+\psi^c_{1\downarrow L}\to\psi_{2\uparrow R}+\psi^c_{2\downarrow R}$ is allowed when only two Dirac doublet of massless fermions are present, but not allowed when we include more doublets of fermions. This, in general, happens for processes which violate an anomalous chiral charge. In the presence of two Dirac doublet of fermions, there is no way to define an anomaly free chiral charge that is violated in this process. However, when we include more doubletsof fermions, we can define a non-anomalous chiral charge by subtracting twice the chiral charge carried by the third doublet from the total chiral charge carried by the first two doublets. This chiral charge is violated in the above process. In the case of SU(5) monopole, this means that the process $u_{1L}+u_{2L}\rightarrow e_R^+ + d_{3R}^c$ is allowed only if we ignore the presence of higher generations of fermions. But if, for example, we take the limit where $\begin{pmatrix}\mu^+\\ s_3\end{pmatrix}$ is also a massless doublet, then this process is not allowed. This particular feature is true only for those processes which violate a chiral charge and hence needs anomaly to play a non-trivial role in the process. Processes of the form $u_{1L}+d_{3R}\rightarrow u_{2L}^c+e_R^+$, which does not violate any chiral charge, and hence does not require anomaly to play a non-trivial role in the process[4,6], are allowed by the conservation laws even in the presence of higher generation of massless fermions.

In the realistic case of an SU(5) monopole, we must also include the effect of the gauge interactions other than those in the SU(2) subgroup spanning the 3-4 subspace. As a first approximation, this may be done by including only the three other linearly independent diagonal Abelian gauge fields corresponding to the SU(5) generators,

$$\begin{pmatrix} 1 &&&& \\ & -1 &&& \\ && 0 && \\ &&& 0 & \\ &&&& 0 \end{pmatrix}, \quad \begin{pmatrix} 1 &&&& \\ & 1 &&& \\ && 0 && \\ &&& 0 & \\ &&&& -2 \end{pmatrix}, \quad \begin{pmatrix} 1 &&&& \\ & 1 &&& \\ && -\frac{3}{2} && \\ &&& -\frac{3}{2} & \\ &&&& 1 \end{pmatrix}$$

These extra gauge interactions do not destroy any symmetry of the tree level Lagrangian, since the gauge fields couple to the linear combinations of the conserved charges given in Eqs.(2) and (3). They, however, may make some of the conserved currents J^μ, associated with the conserved charges given in Eqs.(2) and (3), anomalous, through triangle diagrams, with one vertex coupled to J^μ, and the other two vertices coupled to these new Abelian gauge fields. Triangle diagrams with one vertex coupled to J^μ, one to an Abelian gauge field, and the third to the SU(2) gauge field spanning the 3-4 subspace, vanish identically, since the generators of J^μ and the currents to which the extra Abelian gauge fields couple, commute with all the generators of the SU(2) subgroup. Thus the current J^μ may acquire an anomaly of the form $\varepsilon^{\mu\nu\rho\sigma}F^{(i)}_{\mu\nu}F^{(j)}_{\rho\sigma}$, where $F^{(i)}$ and $F^{(j)}$ are the field strength tensors associated with the new Abelian gauge fields. But the charge $\int J^0 \, d^3x$ is still conserved in a scattering process, since the vacuum does not have a non-trivial topological structure involving these Abelian gauge fields. (This is the same reason why the chiral charge of the electron is conserved in QED with massless electrons, although this charge is anomalous). As a result, all the conservation laws given in Eqs.(2-4) are unchanged in the presence of these new gauge fields[8]. Hence the conservation laws still uniquely determine the final state for a given initial state, thus forcing us to baryon number violating processes.

If we want to take all the SU(5) gauge interactions into account, we must study the conservation laws of the full SU(5) gauge theory. For n_g generations of massless fermions, the Lagrangian is symmetric under independent phase transformations of the fermions of each generation in the $\bar{5}$ and the 10 representations of the group. One of these symmetries is anomalous, hence we have $2n_g-1$ conserved charges. Besides these there are three more conserved charges associated with the three unbroken local U(1) symmetries, namely, the color isospin, the color hypercharge, and the electromagnetic charge. We thus have $2n_g+2$ exact conservation laws. (Note that we do not use the conservation of the weak hypercharge explicitly, hence the weak interaction symmetry breaking does not affect our discussion[9]). For one generation of fermions we have four conservation laws for two Dirac doublet of fermions, hence the conservation laws (which are the same as the ones derived in the SU(2) gauge theory) still uniquely determine the final state for a given initial state. For more than one generation of massless fermions, the $2n_g+2$ conservation laws form a subset of the $4n_g$ conservation laws of the SU(2) gauge theory. Although these conservation laws do not uniquely specify the final state for a given initial state, for an initial state of the form $u_{1L}+d_{3R}$, all possible final states, consistent with these $2n_g+2$ conservation laws, violate baryon number. Hence baryon number violation is still forced on us by the conservation laws of the system.

If we allow emission of gluons in the final state, we may have to relax the conservation of color isospin in the fermionic sector alone. (Gluons carrying a net color hypercharge takes the J=0 partial wave fermions out of the J=0 partial wave, hence we do not consider the emission of such gluons). As a result we may replace a final state u_1 by u_2 and vice versa. Since this does not change the baryon number of the final state, we are still forced into baryon number violating processes by the conservation laws.

The author would like to thank S. Das, A. S. Goldhaber and A. N. Schellekens for valuable discussions.

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[9]A different approach to the problem, using the conservation of weak hypercharge, has been discussed by Schellekens in Ref.8. See also A. S. Goldhaber, talk delivered at the fourth workshop on grand unification, held at Philadelphia, in April, 1983.