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Stability of Supersymmetric Ground State in Chiral Theories

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ABSTRACT

The effective action for the scalar propagator is studied in supersymmetric chiral gauge theories. The form of the scalar potential ensures the stability of the supersymmetric ground state, even for large values of the coupling constant. It is found however, that the gluino condensation tends to destabilize the system. This may signal dynamical supersymmetry breaking.

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The small mass behaviour of strongly interacting supersymmetric gauge theories with matter remains a puzzling aspect of supersymmetric dynamics. It is by now widely believed that for masses large compared to the scale of strong interactions the number of supersymmetric ground states equals the value of Witten's index [1]. This result is manifest within the effective lagrangian approach [2]. The effective lagrangian approach also reveals another feature of the ground states; as masses decrease, supersymmetric vacua correspond to larger values of some natural order parameters (like multiscalar condensates), leading in the zero-mass limiting case to models without a ground state. Many arguments ranging from current algebra [3] through complementarity [4] to instanton calculations [5] have confirmed this scenario. Yet none of the above techniques say anything about the possibility of dynamical supersymmetry breaking in chiral theories. The present paper is devoted to the search for a solution to the vacuum problem in genuinely massless supersymmetric chiral gauge theories.

We will follow closely the formalism of Cornwall et al. [6], which has proven to be quite successful when applied to an analogous problem in chiral dynamics [7]. Since this formalism does not insure the positive-definitness of the vacuum energy, its application to supersymmetric models is somewhat suspect. Nevertheless we hope that our work will give at least some hint about the infrared behavior of supersymmetric theories.

We will consider an arbitrary massless supersymmetric SU(N) gauge theory with gluons A, gluinos λ and matter belonging to chiral supermultiplets with scalars ϕ and fermions Ψ in complex representations. If chiral symmetry is unbroken, the generation of masses for scalars implies dynamical supersymmetry breaking. Let us consider the effective action for the scalar propagator following the formalism of Cornwall et al. [6,7,8]. We choose to work in the Wess-Zumino-Landau gauge, $\partial\cdot A = 0$, in which the composite operator $\phi(x)\phi^*(y)$ coincides in lowest order in perturbation theory with a gauge invariant operator [8]. Landau gauge has another remarkable virtue in the context of supersymmetry: radiative corrections do not induce mass terms for the scalar fields (at least to one loop), therefore they do not break supersymmetry explicitly.

Assuming unbroken gauge symmetry, $\langle 0|\phi|0\rangle = 0$, the effective action derived in [6] for the scalar propagator S(x,y) is given by

$$\Gamma(S) = i \,\mathrm{Tr} \log S^{-1} + i \,\mathrm{Tr}\, S\,(D^{-1} - S^{-1}) + \Gamma_2 \tag{1}$$

where

$$D(x-y) = -i \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{1}{p^2} \tag{2}$$

denotes the free scalar propagator and Γ_2 represents the sum of all 2PI diagrams with scalar propagators set to S(x,y). For clarity we consider ϕ in the fundamental representation and suppress gauge indices. It is very difficult to solve the extremum condition $\delta\Gamma/\delta S = 0$ for S exactly. Therefore we shall make the following ansatz

$$S(x-y) = -i\int\frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{1}{(1+Z)p^2 + \Sigma(p^2)} \tag{3}$$

where the factor Z represents corrections to the renormalised self-energy. We shall use the Hartree-Fock approximation. By expanding the effective action around $\Sigma = 0$ we get to the $0(g^2)$ order

$$\Gamma = iN\int\frac{d^4k}{(2\pi)^4}\left[Z\,\frac{\Sigma(k)}{k^2} + \left(\tfrac{1}{2} - 2Z\right)\frac{\Sigma^2(k)}{(k^2)^2}\right] + \Gamma_2 \tag{4}$$

where Γ_2 is given now by the sum of two-loop diagrams of Fig.1. Straightforward calculation gives

$$\frac{2}{N}\Gamma = i\int\frac{d^4k}{(2\pi)^4}\,\frac{\Sigma^2(k)}{[(1+Z)k^2]^2}$$

$$+ g^2\,\frac{N^2-1}{2N}\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4p}{(2\pi)^4}\,\frac{\Sigma(k)}{[(1+Z)k^2]^2}\,\frac{\Sigma(p)}{[(1+Z)p^2]^2}$$

$$- g^2\,\frac{N^2-1}{2N}\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4p}{(2\pi)^4}\,\frac{\Sigma(k)}{[(1+Z)k^2]^2}\,\frac{\Sigma(p)}{[(1+Z)p^2]^2}\,\frac{(k+p)^\mu(k+p)^\nu}{(k-p)^2}\left[g^{\mu\nu} - \frac{(k-p)^\mu(k-p)^\nu}{(k-p)^2}\right] \tag{5}$$

The second term comes from the diagram (1a) with a four-scalar interaction and Σ inserted on top and bottom lines. The third term comes from the one-gluon exchange diagram (1b) with similar Σ insertions. It is convenient to continue the effective action to the Euclidean space. After performing angular integrations one gets

$$-\frac{1}{8\pi^2 N}\Gamma_E = \int\frac{dk}{k}\Sigma^2(k) + \frac{g^2}{16\pi^2}\frac{N^2-1}{2N}\int\frac{dk}{k}\frac{dp}{p}\Sigma(k)\Sigma(p)\left[1-3\min\left\{\frac{k^2}{p^2},\frac{p^2}{k^2}\right\}\right] \tag{6}$$

where the irrelevant effects of self-energy corrections have been ignored by setting Z = 0.

The supersymmetric ground state at $\Sigma=0$ turns out to be more stable than the analogous chirally-symmetric one in the effective action for the helicity nonconserving part of a quark propagator in QCD [7]. The destabilizing attractive gauge force appears to be compensated by the repulsive four-scalar interaction which may be regarded as an exchange of an auxiliary component of the vector supermultiplet. As a consequence, the supersymmetric ground state remains stable even for values of the coupling constant, $\frac{g^2}{4\pi}\frac{N^2-1}{2N}$, considerably larger than the critical value, $\frac{\pi}{3}$, found for the case of chiral symmetry breaking in QCD [7].

Until now we disregarded the effects of a possible gluino condensation, $\langle 0|\lambda\lambda|0\rangle \neq 0$. A simple calculation shows that this condensation should occur for $\frac{g^2 N}{4\pi} \approx \frac{\pi}{3}$. It is not clear whether this sort of analysis is applicable in a supersymmetric theory, since for the values of the coupling constant large enough to trigger the $\lambda\lambda$ condensation the effective potential becomes negative[F1]. We do not attempt to resolve here any problems related to the positivity property of supersymmetric effective potentials. Even in non-supersymmetric models one encounters similar problems; for instance in a free theory the effective potential is unbounded from below.

We want to point out that the gluino condensation destabilizes our effective action. From the diagram (1c) with two subsequent helicity flips $\Lambda(p)$ on the gluino line we get

$$-\frac{1}{8\pi^2 N}\Gamma_E(1c) = -\frac{g^2}{2\pi^2}\frac{N^2-1}{2N}\int\frac{dk}{k}\frac{dp}{p}\,\Sigma(k)|\Lambda(p)|^2\left[1-\frac{1}{2}\min\left\{\frac{k^2}{p^2},\frac{p^2}{k^2}\right\}\right] \tag{7}$$

Clearly, this contribution destroys the stationary point at $\Sigma = 0$, unless $\Lambda = 0$. We see that gluino condensation may generate masses for scalars by enforcing a nonvanishing value of $\Sigma(0)$ at the minimum of the effective potential.

[F1] It may be that some nonperturbative effects are necessary to induce $\langle 0|\lambda\lambda|0\rangle \neq 0$ [9].

Since in a chiral theory there are no other two-particle gauge singlet channels for fermions and the nonsinglet channels are always less attractive, we expect matter fermions to remain massless at the energy scale of gluino condensation. Thus we conclude that supersymmetry breaks dynamically at the scale of gluino condensation.

Let us illustrate with a simple example. Consider the SU(5) model with one chiral matter supermultiplet in the $\underset{\sim}{5} + \underset{\sim}{\overline{10}}$ representation. We expect the gluino condensation to take place at an energy scale considerably higher than the confinement scale. This in turn would generate masses for matter scalars, breaking supersymmetry. The gluino condensate breaks also one of two non-anomalous U(1) symmetries, producing a Goldstone boson. At lower energies the model would look like a conventional SU(5) chiral model with massless fermions coupled to the goldstino field $\lambda\sigma^{\mu\nu}F_{\mu\nu}$ and the Goldstone boson. The model then evolves into the infrared in a way described in the classic paper of Dimopoulos et al. [10]. Similar conclusions, although on different grounds, have been recently reached by Affleck et al. [11].

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FIGURE

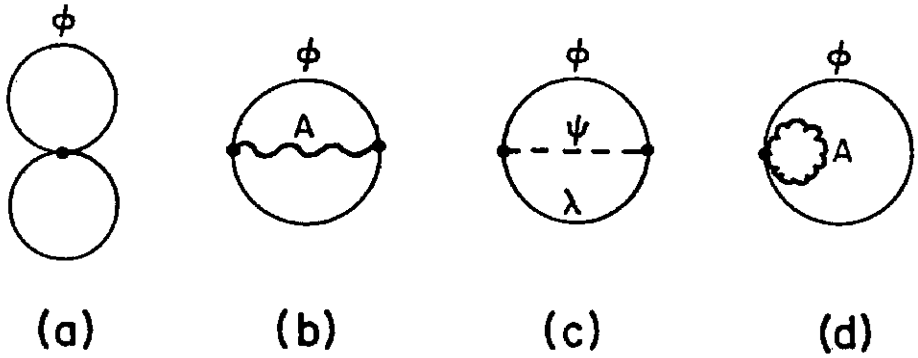

Fig.1.Two-loop contributions to the effective action.