Fermi National Accelerator Laboratory

FERMILAB-Pub-84/16-T
January 1984

TWIST-FOUR EFFECTS ON THE ASYMMETRY IN POLARIZED ELECTRON DEUTERON SCATTERING AND $\mathrm{SIN}^2\theta_W$

S. Fajfer*
Argonne National Laboratory
Argonne, Illinois 60439

and

Department of Physics and Astronomy, Northwestern University
Evanston, Illinois 60201

and

R.J. Oakes
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

and

Department of Physics and Astronomy, Northwestern University
Evanston, Illinois 60201

ABSTRACT

The twist-four, spin-two Quantum Chromodynamic corrections to the asymmetry parameter in polarized electron deuteron scattering and their effect on $\sin^2\theta_W$ have been calculated using the operator product expansion of the product of the weak and electromagnetic currents. The coefficients in the expansion were determined using perturbation theory techniques and the deuteron matrix elements of the operators were evaluated using the MIT Bag Model. The higher twist effects decrease the value of $\sin^2\theta_W$, as determined from polarized electron deuteron scattering, by about 1%, similar to the electroweak radiative corrections.

In the standard electroweak theory[1] the fundamental parameter $\sin^2\theta_W$ is of considerable interest and a great deal of effort has been devoted to its precise determination[2]. The asymmetry in polarized electron deuteron scattering, arising from the interference between electromagnetic and weak interactions, depends on $\sin^2\theta_W$ and polarized electron scattering data[3] have been used to measure $\sin^2\theta_W$. However, in extracting $\sin^2\theta_W$ from these data one must consider, in addition to the radiative electroweak corrections[4], the effects of Quantum Chromodynamics[5] (QCD).

We have calculated the nonperturbative twist-four, spin-two QCD corrections to the asymmetry parameter and $\sin^2\theta_W$ and have found their effect is, typically, to decrease $\sin^2\theta_W$ by about 1%. After a brief review of the asymmetry in polarized electron deuteron scattering we present our calculation of the twist-four, spin-two QCD effects and finally compute the magnitude of these QCD corrections to $\sin^2\theta_W$ for the present data.

The asymmetry parameter is defined in terms of the longitudinally polarized electron scattering inclusive cross sections on deuterium[6], $e_{R,L}(k) + D\ (p) \to e'(k') +$ anything:

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \tag{1}$$

This parity nonconserving asymmetry arises in the standard electroweak theory from the interference between the electromagnetic and weak neutral currents; i.e., the γ and Z° exchange diagrams.

The contribution coming from the axial vector coupling at the electron vertex is of the form[7]

$$\frac{A_{VV}(Q^2,\ x,\ y,)}{Q^2} \quad \alpha \quad \frac{\int e^{iqz} <D|[j^\mu(z)J^\nu(0) + J^\mu(z)j^\nu(0)]|D>d^4z\ \ell_{\mu\nu}}{\int e^{iqz} <D|j^\mu(z)\ j^\nu(o)|D>d^4z\ \ell_{\mu\nu}} \qquad (2)$$

where j^μ and J^μ are the usual electromagnetic and vector weak neutral currents, respectively, in the standard electroweak theory and $\ell_{\mu\nu} = \mathrm{Tr}\ (k'\gamma_\mu\ k\gamma_\nu)$. The kinematical variables are defined as usual: $q = k - k'$, $Q^2 = -q^2$, $\nu = pq$, $x = Q^2/2\nu$ and $y = pq/pk = (E-E')/E$. In the quark parton model, direct calculations shows

$$\frac{A_{VV}(Q^2,x,y)}{Q^2} = -\frac{G}{\sqrt{2}\pi\alpha}\left(\frac{9}{20}\right)\left[1 - \frac{20}{9}\sin^2\theta_W\right] \qquad (3)$$

Similarily, the contribution to the asymmetry parameter coming from the vector coupling at the electron vertex is of the form

$$\frac{A_{VA}(Q^2,x,y)}{Q^2} \quad \alpha \quad \frac{\int e^{iqz}<D|j^\mu(z)\ J_5^\nu(o)|D>d^4z\ \ell^5_{\mu\nu}}{\int e^{iqz}<D|j^\mu(z)\ j^\nu(o)|D>d^4z\ \ell_{\mu\nu}} \qquad (4)$$

where J_5^μ is the axial vector weak neutral current in the standard electroweak theory and $\ell^5_{\mu\nu} = \mathrm{Tr}\ (k'\gamma_\mu\ k\gamma_\nu\gamma_5)$. In the quark parton model one can easily show that

$$\frac{A_{VA}(Q^2,x,y)}{Q^2} = -\frac{G}{\sqrt{2}\pi\alpha}\left(\frac{9}{20}\right)(1 - 4\sin^2\theta_W)\left[\frac{1-(1-y)^2}{1+(1-y)^2}\right] \qquad (5)$$

To calculate the twist-four, spin-two QCD corrections to the quark parton model result for $A_{VV}(Q^2,x,y)$ we define the operator

$$V_{\mu\nu} = i\int e^{iqz}\, T\,[j_\mu(z)\, J_\nu(o) + J_\mu(z)\, j_\nu(o)]\, d^4z \tag{6}$$

In the physical region

$$A_{VV}(Q^2,x,y)\ \alpha\ \frac{1}{2\pi}\ \mathrm{Im} < D \mid V_{\mu\nu} \mid D > \ell^{\mu\nu} \quad ; \tag{7}$$

that is, the numerator in Eq. (2) above. Using the Wilson operator product expansion[8] (OPE) for the bilocal four-quark operators, which are of the typical form $\overline{q}_i(z)\gamma_\mu\, q_i(z)\, \overline{q}_j(o)\, \gamma_\nu\, q_j(o)$, where i and j denote u and d quarks, one obtains an expansion in terms of local operators with Q^2-dependent coefficients that obey the renormalization group equation. We shall choose the renormalization point μ^2 at Q^2 in these equations for the coefficients in the OPE, which amounts to neglegting their anomalous dimensions and allows the coefficients to be calculated using perturbative QCD techniques. One finds both diagonal contributions, involving two-quark-gluon operators, and nondiagonal contributions, involving four-quark operators. These can both be conveniently expressed in terms of the traceless, antisymmetric basis of operators $O_i^{\mu\nu}$, containing no contracted covariant derivatives, due to Jaffe and Soldate[9]:

$$V^{\mu\nu}_{\mathrm{diag}} = \frac{g^2}{q^6}\left[T^{\mu\nu}_{\mu_1\mu_2}\left\{\frac{1}{4}\, O_7^{\mu_1\mu_2}\ (0) + \frac{5}{2}\, O_9^{\mu_1\mu_2}\ (0)\right\} + \ (q^\mu q^\nu - g^{\mu\nu}q^2)q_{\mu_1}q_{\mu_2}/q^2 \left\{-\frac{3}{2}\, O_7^{\mu_1\mu_2}\ (0) + O_9^{\mu_1\mu_2}\ (0)\right\}\right] \tag{8}$$

and

$$V^{\mu\nu}_{\mathrm{nondiag}} = \frac{4g^2}{q^6}\, T^{\mu\nu}_{\mu_1\mu_2}\left\{O_2^{\mu_1\mu_2}\ (0) - 2\, O_4^{\mu_1\mu_2}\ (0) + O_6^{\mu_1\mu_2}\ (0)\right\} \tag{9}$$

In Eqs. (8) and (9) we have defined

$$T^{\mu\nu}_{\mu_1\mu_2} = \delta^{\mu}_{\mu_1}\delta^{\nu}_{\mu_2}q^2 - (\delta^{\mu}_{\mu_1}q^{\nu} + \delta^{\nu}_{\mu_1}q^{\nu})\, q_{\mu_2} + g^{\mu\nu}\, q_{\mu_1}q_{\mu_2} \tag{10}$$

and $g^2 = 4\pi\ \alpha_s\ (Q^2)$ is the QCD running coupling constant. We have verified that these results agree with both the previous calculations of the higher twist effects in electroproduction[9] and neutrino neutral current scattering[10].

In the case of $A_{VA}(Q^2,x,y)$ one can show there are no twist-four, spin-two interference terms and, consequently, the quark-parton model result, Eq. (5), is unaffacted.

The calculation is further simplyfied since the twist-four corrections to electroproduction[9], although model dependent, are nevertheless certainly small[11] (< 2%). Therefore, these higher twist corrections to the denominator of Eq. (2) can be neglected in comparison with the twist-four, spin-two effects in the numerator.

To calculate the matrix element of $V_{\mu\nu}$ between deuteron states (spin-averaged), we shall assume quark confinement models for which the quark wave function is of the form

$$q(\vec{r}) = \begin{bmatrix} f(r) \\ \vec{\sigma}\cdot\hat{r}\ g(r) \end{bmatrix} \chi \quad , \tag{11}$$

where χ is a two-component spinor; for example, the MIT Bag Model[12].

Direct calculation shows that the spatial part of the matrix element $\langle D|V_{\mu\nu}|D\rangle$ can be expressed in terms of the two integrals:

$$I_1 = \int [|f(r)|^2 + |g(r)|^2]^2\ d^3r \tag{12}$$

and

$$I_2 = \int |f(r)|^2|g(r)|^2\ d^3r \tag{13}$$

The matrix elements of the six different spin, color and flavor dependent four-quark local operators needed in the evaluation of $\langle D|V_{\mu\nu}|D\rangle$ are among the set of nine such matrix elements that have previously been given in the calculations of the twist-four, spin-two corrections to the neutrino neutral current cross section[10].

Matrix elements involving gluon fields will be neglected since they enter the OPE with coefficients smaller by an order of magnitude and, in addition, the gluon content of a nucleon (at rest) is also rather small.

Combining the above results one finds the total asymmetry $A = A_{VV} + A_{VA}$ to be

$$\frac{A(Q^2,x,y)}{Q^2} = -\frac{G}{\sqrt{2}\pi\alpha}\left(\frac{9}{20}\right)\left\{\left[1 + \frac{1-(1-y)^2}{1+(1-y)^2} + \right.\right.$$

$$\left. + \frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}\left(-\frac{85}{9}I_1 + \frac{440}{27}I_2\right)\right]$$

$$+ \sin^2\theta_W\left[-\frac{20}{9} - 4\,\frac{1-(1-y)^2}{1+(1+y)^2} + \right.$$

$$\left.\left. + \frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}\left(-\frac{262}{27}I_1 + 16\,I_2\right)\right]\right\} \qquad (14)$$

where u(x) and d(x) are the u and d (valence) quark distributions in nucleons and M is the nucleon mass.

To investigate the effect on the determination of $\sin^2\theta_W$ of the twist-four, spin-two corrections to the polarized electron deuteron asymmetry parameter $A(Q^2,x,y)$, we have used Eq. (14) to calculate

$$\delta\sin^2\theta_W = -\frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}\left[\left(\frac{85}{9}I_1 - \frac{440}{27}I_2\right) + \left(\frac{262}{27}I_1 - 16I_2\right)\sin^2\theta_W\right]\left[\frac{20}{9} + 4\frac{1-(1-y)^2}{1+(1-y)^2}\right]^{-1} \qquad (15)$$

The value $\sin^2\theta_W = 0.224 \pm 0.020$ has been determined from the polarized electron deuteron data[3] taken typically in the kinematic region $Q^2 = 1.67$ GeV^2, $x = 0.24$ and $y = 0.19$ ($E \simeq 20$ GeV). In the this regime[13] $x\,[u(x) + d(x)] \simeq 0.9$. Taking $\alpha_s(Q^2) = 0.27$ and using the MIT Bag Model values $I_1 = 20.36 \times 10^{-4}$ GeV^3 and $I_2 = 3.21 \times 10^{-4}$ GeV^3 one then finds from Eq. (15) that $\delta\sin^2\theta_W \simeq -0.001$. That is, the twist-four, spin-two effects on the asymmetry in polarized electron deuteron scattering result in a small decrease (< 1%) in the value of $\sin^2\theta_W$, similar to the electroweak corrections. It is interesting to note that the higher twist corrections to $\sin^2\theta_W$, as determined from neutrino current scattering[10], also decreased the value of $\sin^2\theta_W$ by about 1%.

We thank Chris Quigg for his kind hospitality in the Theoretical Physics Department at Fermilab. One of us (S.F.) is also grateful to Dennis Sivers and Alan White for the opportunity to visit the Theoretical High Energy Physics Division of Argonne National Laboratory. This work was supported in part by the National Science Foundation under NSF Grant PHY 82-09145 and YOR 81-020.

REFERENCES AND FOOTNOTES

* On leave from the University of Sarajevo, Sarajevo, Yugoslavia.

1. S.L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys.Rev.Lett. 19, 1264 (1967); A. Salam, in Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Symposium No. 8) edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968) p. 367.
2. For a review see J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev.Mod.Phys. 53, 211 (1981).
3. C.Y. Prescott, et al., Phys.Lett. 84B, 524 (1979); 77B, 347 (1978).
4. W.J. Marciano and A. Sirlin, Phys.Rev. D 22, 2695 (1980); Phys.Rev.Lett. 46, 163 (1981); Nucl. Phys. B159, 442 (1981) and J.F. Wheater and C.H. Llewellyn-Smith, Phys.Lett. 105B, 486 (1981); Nucl. Phys. B208, 27 (1982).
5. For excellent reviews see H.D. Poltizer, Phys.Rep. 14C, 129 (1974); A. Petermann, Phys.Rep. 53C, 157 (1979) and A.J. Buras, Rev.Mod.Phys. 52, 199 (1980).
6. The phenomenology of polarized electron scattering has been considered by J.D. Bjorken, Phys.Rev. D 18, 3239 (1978); R.N. Cahn and F. Gilman, W.J. Marciano and A.I. Sanda, Phys.Rev. D 18, 4341 (1978); L. Wolfenstein, Nucl. Phys. B146, 477 (1978); C. Cuthiell and J.N. Ng, Phys.Rev. D 16, 3255 (1977); E. Reya and K. Schilcher, Phys.Rev. D 10, 952 (1974); S. Bilenkii, et al., Yad. Fiz. 21 360 (1975) [Sov. J. Nucl. Phys. 21, 189 (1975)]; W. Wilson, Phys.Rev. D 10, 218 (1974); S. Berman and J. Primack, Phys.Rev. D 7, 2755 (1973) and E. Derman, Phys.Rev. D 7, 2755 (1973).
7. Here we are following, most closely, the analysis of J.D. Bjorken, ref. 6.

8. K. Wilson, Phys.Rev. 179, 1499 (1969).

9. R.L. Jaffe and M. Soldate, in the Proceedings of the 1981 Tallahassee Conference on Perturbative Quantum Chromodynamics, edited by D.W. Duke and J.F. Owens (AIP, New York, 1981), p. 60; Phys.Lett. 105B, 467 (1981) and Phys.Rev. D 26, 49 (1982).

10. S. Fajfer and R.J. Oakes, Fermilab-PUB-83/80-THY.

11. S. Fajfer and R.J. Oakes, Phys.Lett. 131B, 192 (1983).

12. T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys.Rev. D 12, 2060 (1975) and references cited therein.

13. See, for example, the analysis of A. Buras, Nucl.Phys. B125, 125 (1977).