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Mean Free Paths, Viscosity, and the Limitations of Perfect Fluid Hydrodynamics in the Description of the Quark-Gluon Plasma

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ABSTRACT

I shall discuss the applicability of a hydrodynamic description of high energy hadronic collisions. I shall review the results of recent computations of the mean free paths of quarks and gluons in a quark-gluon plasma, and the corresponding results for viscous coefficients. These quantities are employed to evaluate the limits to the application of perfect fluid hydrodynamics as a description of the time evolution of matter produced in various hadronic collisions.

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In very high energy hadronic collisions, matter is produced at very high energy densities. There are two schools of thought concerning the formation of this matter. The oldest school of thought is due to Landau and Fermi, and assumes that the initial state of the matter in a head on high energy collison consists of two overlapping pancakes at rest in the center of mass frame. The thickness of these pancakes is taken as R/γ where R is the rest frame hadron radius and γ is the center of mass Lorentz γ factor of the hadrons. This initial condition arises when the two Lorentz contracted hadrons, with width R/γ, stop in the center of mass frame. In this picture, the initial energy density grows as γ^2, and tremendous energy densities are achieved in ultra-relativistic collisions.

The other school of thought accepts the validity of the Landau-Fermi picture for sufficently low energy collisions, but at very high energies conjectures that the hadrons pass through one another.[1-2] This is plausible since very high energy hadronic probes should punch through a finite amount of hadronic matter. This second school of thought then further assumes that matter is formed in the region between the two hadrons after the two hadrons have passed through one another. The forming matter has longitudinal momentum, and it is assumed that in the rest frame of the formed matter, the formation takes place in a characteristic time scale $\tau_o \sim 1$ Fm/c. Due to Lorentz time dilation of the formation process, the matter with the smallest longitudinal momentum forms first in the center of mass frame. The matter therefore forms sequentially in the region between the two hadrons, and forms first at $x = 0$. The faster matter forms at a later time, and if it travels as a free particle during the time it is forming, forms at a time of $t \sim p''/m_o^2$ where $m_o \sim 200$ Mev $= 1$ Fm^{-1}, corresponding to a distance of $x \sim t$ from the collison point. The matter therefore forms in an inside- outside cascade.

The theoretical underpinnings of the inside-outside cascade description of hadronic collisions are in all field theoretical models of collison processes. In electromagnetic cascades, the Landau-Pomeranchuk-Migdal effect which predicts the lenghtening of the longitudinal development of ultra-relativistic electromagnetic showers is a consequence of inside-outside cascading. For this process, the cascade development is lengthened since the formation of electron-positron pairs and photons is strongly time dilated for particles with large momentum along the longitudinal cascade axis. If these particles do not form during the time an electron or photon traverses several nuclei as it penetrates a material, the cascade will not multiply at each collision with a nucleus, as it ordinarily would. The inside-outside cascade also properly describes matter formation when relativistic hadron pairs are formed in collisions described by the exactly solvable two dimensional massless Schwinger model.

In most of the remainder of this lecture, I shall assume the validity of the inside-outside cascade picture of the collision process. I shall also attempt to point out general results which apply for either description, and many of the conclusions which I draw are easily generalized to the Fermi-Landau picture. It is of course important to allow heresies in the inside-outside cascade picture, which has been described by P. Carruthers as The New Orthodoxy.[3] In particular, fluctuations in ordinary collisions may arise where hadrons, or some part of the hadrons, form in a much smaller volume than is appropriate for average hadron collisions. The matter might in this circumstance be better described by a Landau-Fermi picture. In average collisions, however, data from hadron-nucleus collisions and the theoretical considerations mentioned above seem to make a compelling case for an inside-outside cascade description.

The forming matter in a hadronic collision may in appropriate circumstances, which we shall attempt to outline in the remainder of this talk, form the initial conditions for perfect fluid hydrodynamic equations.[4-5] The hydrodynamic equations for a perfect fluid are extremely simple. Their solution describes the time evolution of the energy density, pressure and fluid velocity of matter given only the initial conditions and an equation of state which functionally relates the energy density and the pressure. (I shall ignore baryon number currents in my analysis, since this is applicable for most of the matter produced in an ultra-relativistic nuclear collisions. My conclusions do not strongly rely on this assumption, but the hydrodynamic equations are slightly more complicated, and the matter formation in the fragmentation region of the nuclei, where most of the baryon number current flows, is more involved.) The stress-energy tensor for a perfect fluid is written as

$$T^{\mu\nu} = \{\rho+P\}u^{\mu}u^{\nu}-Pg^{\mu\nu} \qquad (1)$$

where u^{μ} is the fluid four velocity vector, ρ is the energy density, and P is the pressure. The equation of motion for the fluid is conservation of energy-momentum,

$$\partial_{\mu}T^{\mu\nu} = 0 \qquad (2)$$

These four equations, an equation of state which relates energy density to pressure, and $u^2 = 1$ are sufficient to solve for the six components of u, ρ, and P given conditions at some time. As a consequence of the perfect fluid form for the stress-energy tensor, the entropy current is conserved,

$$\partial^{\mu}s_{\mu} = 0 \qquad (3)$$

where the entropy four current is taken as

$$s^{\mu} = su^{\mu} = \{\rho+P\}/T\ u^{\mu} \qquad (4)$$

This fact follows directly from the form of the stress-energy tensor, and dotting u^{μ} into the conservation equation, Eq. 2.

The situation is much complicated if the fluid dynamics is not perfect, and involves viscous flow. New parameters enter the hydrodynamic equations, the coefficients of shear and bulk viscosity, and the form of the equations are more involved. The viscous coefficients are difficult to estimate in QCD, but I shall soon review what is known of them. If these viscous corrections to the hydrodynamic equations are sufficiently large, then the approximation which reduces the hydrodynamic equations to local equations with the standard form of the viscous corrections to the perfect fluid hydrodynamic equations may itself breakdown, and the correct hydrodynamic equations may involve many more parameters. The point is that for our purposes, the viscous hydrodynamic equations are only reliable if the corrections arising from non-zero viscosity are small.

Another reason besides mathematical simplicity for wishing to apply hydrodynamics only for perfect fluids is that for a perfect fluid, the entropy is conserved. Entropy conservation relates particle multiplicities at early time to that at later times. If the expansion is isentropic, a window penetrates through the haze of hadronic interactions which allows us to reconstruct particle distributions at early times from those observed in the final state of the collision.

The stress-energy tensor, allowing for the effects of viscous flow is

$$T^{\mu\nu} = T_0^{\mu\nu} + \Delta T^{\mu\nu} \tag{5}$$

where T_0 is the stress-energy tensor for a perfect fluid, as given by Eq. 1, and ΔT is the correction which allows for entropy production, that is, for viscous flow. The most general form for ΔT may be extracted in an expansion in powers of gradiants of the energy density and fluid velocity vector times a characteristic scattering length. This characteristic scattering length is the mean free path for dilute systems such as gasses. This procedure for evaluating ΔT is discussed in Refs. 6-7, and I shall not repeat the derivation here. The result is

$$\Delta T^{\mu\nu} = \zeta\{g^{\mu\nu}-u^{\mu}u^{\nu}\}\nabla\cdot u + \eta\left\{\nabla^{\mu}u^{\nu}+\nabla^{\nu}u^{\mu}-\frac{2}{3}\{g^{\mu\nu}-u^{\mu}u^{\nu}\}\nabla\cdot u\right\} \qquad (6)$$

The derivative operator ∇ is a derivative orthogonal to the direction of fluid flow,

$$\nabla^{\mu} = \partial^{\mu}-u^{\mu}u\cdot\partial \qquad (7)$$

The coefficients of shear and bulk viscosity are η and ζ. For the zero baryon number density fluids which I consider, the heat conductivity is zero. This expression is only valid to first order in an expansion where spatial gradiants are weak, and if they are not, Eq. 6 is simply incorrect. For systems with sharp discontinuities, ΔT is more complicated and for practical purposes may not be computable. Put another way, when viscous corrections to the hydrodynamic equations become of the same order as the contribution associated with a perfect fluid, the framework of conventional viscous fluid hydrodynamics falls apart, and for practical purposes, we may say that hydrodyanamics is no longer applicable for a description of the dynamics. This means only that perfect fluid dynamics is inapplicable even when supplemented with viscous corrections. The full stress-energy tensor is of course conserved, but the form of this equation expressed in terms of ρ, P, and u is extremely complicated and in general non-local.

The criteria for the applicability of hydrodynamics may also be understood in terms of entropy generation. Unlike the case for perfect fluid hydrodynamics, entropy is produced in hydrodynamic expansion. The divergence of the entropy four current is

$$\partial\cdot s = \zeta/T\ \{\nabla\cdot u\}^{2} + \eta/2T\ \left\{\nabla^{\mu}u^{\nu}+\nabla^{\nu}u^{\mu}+ \frac{2}{3}\{g^{\mu\nu}-u^{\mu}u^{\nu}\}\nabla\cdot u\right\}^{2} \qquad (8)$$

This quantity measures the rate of entropy production per unit time or length in the system. If we let the characteristic time scale for expansion in the system be τ, which may also be thought of as a characteristic length scale for spatial gradiants, the criteria for small entropy generation is

$$\frac{ds}{d\tau}\Big|_{expansion} \sim \frac{s}{\tau} \gg \frac{ds}{d\tau}\Big|_{collisions} \sim \frac{s}{\tau_c} \qquad (9)$$

Before further investigating the validity of hydrodynamics for ultra-relativistic nuclear collisions, and considering in detail the form of the coefficients of shear and bulk viscosity, I shall review some of the crucial features of the inside-outside cascade model of ultra-relativistic nuclear collisions and aspects of the perfect fluid hydrodynamic description of these collisions which are relevant for determining the self-consistency of the analysis. In the initial collision, two Lorentz contracted pancakes of valence nuclear matter pass through one another. The transverse radius of these pancakes is the rest frame nuclear radius. I consider collisions of equal A nuclei at zero impact parameter for simplicity. Although the valence quarks of the nuclei are Lorentz contracted into a longitudinal distance scale of R/γ where R is the nuclear radius and γ is the center of mass lorentz γ factor, the low longitudinal momentum sea quark and gluon nuclear degrees of freedom do not become arbitrarily Lorentz contracted and give the nuclei a limiting thickness of τ_0. For times shorter than τ_0 in the rest frame of materializing sea quarks and gluons, the effects of interactions are assumed to be small. The sea quarks and gluons therefore have a velocity of

$$v = x/t \qquad (10)$$

since they are free particles. They 'form', that is, the effects of their interactions become important at a proper time of

$$\tau = \{t^2-x^2\}^{1/2} = \tau_0 \qquad (11)$$

The matter is formed in an inside-outside cascade since in the center of mass frame values of constant τ with $v = x/t$

correspond to slow particles forming first and fast particles forming last.

Convenient variables to analyze the cascade development and hydrodynamic evolution of the system are the proper time τ and the rapidity

$$y = \frac{1}{2} \ln \left\{\frac{t+x}{t-x}\right\} \tag{12}$$

For the free particles which act as initial conditions for the hydrodynamic equations, the space-time rapidity is also equal to the momentum space rapidity, since by Eq. 10, the space-time rapidity of Eq. 12 becomes

$$y = \frac{1}{2} \ln \left\{\frac{1+v}{1-v}\right\} = \frac{1}{2} \ln \left\{\frac{E+p''}{E-p''}\right\} \tag{13}$$

Notice that as a consequence of Eq. 10 for v, the matter forms with a large longitudinal momentum gradiant. At equal times, the matter farthest from the origin is moving most rapidly away from the origin. The matter is therefore expanding longitudinally as a consequence of the initial conditions. There is no transverse expansion initially, since the average transverse velocity is zero. After a time that a sound wave may penetrate from the surface inwards, transverse rarefraction of the matter begins. This is only important at late times for much of the matter if the nuclear size is large enough. For large nuclei, transverse rarefraction plays a role secondary to longitudinal expansion for cooling the hot matter formed in the collision.

A basic assumption of the inside-outside cascade description of nucleus-nucleus collisions is that the energy density of the matter measured in a local comoving frame is a constant ρ_0 which is approximately independent of the rapidity y. This assumption is justified by the slow variation of the central rapidity distributions of mesons produced in hadron-hadron collisions. Although this assumption is not essential to the basic framework of the inside-outside cascade description, and may be relaxed, we shall adopt it here as a semi-quantitatively good conceptual simplification.

The initial energy density is easily estimated since[4,8-9]

$$\rho_o \sim \frac{1}{\pi R^2} \langle m_t \rangle \frac{dN}{dx} |_{\tau_o} = \frac{1}{\pi R^2} \frac{dN}{dy} |_{\tau_o} \frac{\langle m_t \rangle}{\tau_o} \tag{14}$$

In this equation, $\frac{dN}{dy}$ is the initial rapidity density, and should be of the order of A-$A^{4/3}$ times the rapidity density appropriate for pp collisions. It is also approximately the value measured for the final state of the collision, although, depending upon how isentropic the hydrodynamic is, it may acquire some τ dependence and be slightly modified in the final state. The transverse mass is the value appropriate for the initial conditions, and probably decreases as a consequence of hydrodynamic expansion. As a consequence of the uncertainty principle, $\Delta E \Delta t > 1$, the transverse mass and formation time τ_o are constrained,

$$m_t \gtrsim 1/\tau_o \tag{15}$$

The energy density is therefore of the order of

$$\rho_o \sim \frac{1}{\pi R^2} \frac{dN}{dy} |_{\tau_f} \frac{1}{\tau_o^2} \tag{16}$$

For values of the rapidity density consistent with the results of the JACEE cosmic ray experiments,

$$\rho_o \sim 5 \text{ Gev/Fm}^3 , \ \tau_o \sim 1 \text{ Fm} \tag{17}$$

$$\rho_o \sim 500 \text{ Gev/Fm}^3, \ \tau_o \sim .1 \text{ Fm} \tag{18}$$

Such a range of values is consistent with what is known of pp and pA data. Also, the value of τ_o may be dependent upon A, decreasing as A increases, as is the case in string models of the collision.[14]

As the matter evolves from τ_o to later proper times, the standard theoretical analysis consists of applying perfect fluid hydrodynamic equations to evolve the matter from initial conditions to a final configuration which then freezes out to form final state rapidity distributions. The solution, ignoring transverse rarefaction is

$$y = Y \tag{19}$$

that is, the space-time rapidity equals the fluid rapidity, and

$$T = T_0 \left\{\tau_0/\tau\right\}^{v_s^2} \tag{20}$$

where v_s is the fluid sound velocity which is taken to be constant. The energy density is

$$\rho = \rho_0 \left\{\tau_0/\tau\right\}^{1+v_s^2} \tag{21}$$

The entropy four current is conserved during the expansion, so that $s \sim s_0/\tau$.

For systems of finite transverse extent, the 1+1 dimensional solutions of the hydrodynamic equations are inadequate. There is transverse rarefaction which generates a transverse expansion.[10-11] This transverse rarefaction arises as matter escapes into the vacuum in the transverse direction. If the system has a large transverse size, then the dominant expansion is by 1+1 dimensional longitudinal expansion, and the equations presented above describe the time evolution of the bulk of the matter distribution. This follows since in the initial state, the matter is not expanding in the transverse direction. Before matter a finite distance from the transverse surface may begin expnanding, a time which is greater than the time it takes a sound wave to travel this distance must elapse. For large nuclei, this time is large for matter not near the surface, and the most of the matter has cooled by longitudinal expansion before the transverse rarefaction is significant. For small nuclei and hadrons, the transverse rarefaction must certainly be more important. The transverse rarefraction may distort transverse momentum distributions, and the amount of this distortion may probe hydrodynamic behaviour and the equation of state of hadronic matter, as has been emphasized by Shuryak and by van Hove.[12-13] In some circumstances there may be sidewise splashing, and collective transverse flow, as has been suggested at Bevelac energies.[14-15]

Since transverse expansion provides a small correction to longitudinal expansion in many circumstances, and since the dynamics of 1+1 dimensional longitudinal expansion is conceptually simple, I shall ignore transverse expansion in the rest of this talk. I shall nevertheless discuss a criteria on the transverse size of the system which should be satisfied in order that a perfect fluid hydrodynamic expansion be valid.

The question which I shall attempt to address in the remainder of this paper is to what degree a perfect fluid hydrodynamical description provides a valid approximate description of the matter evolving after the nuclear collision. To begin this discussion it is useful to introduce a mean free path for quarks and gluons in hadronic matter. This length scale characterizes the surface thickness of the matter, and charatcerizes the length scale which must be compared to the length scale of gradiants in the matter distribution. If the surface thickness is small compared to the spatial size of the system, and if the mean free path is short compared to the scale sizes over which the matter distribution varies appreciably, then it is plausible that the perfect fluid hydrodynamic description is correct. Of course, it is possible that the naive considerations of mean free paths, which are rigorously valid for weakly interacting fluids, may be misleading when applied to hadronic matter where non-perturbative effects may be important. We shall therefore later more carefully formulate the issue of the applicability of perfect fluid hydrodynamics in terms of magnitudes of viscous coefficients. These coefficients may in principle be computed using the fluctuation-dissipation theorem and are defined outside the domain of weak coupling expansions.

The simplest estimate of the mean free paths uses the quark-parton additive cross-section model of hadronic interactions. The basic assumption of this extremely naive picture is that the quark-hadron cross section is 1/3 that of hadron-hadron,

$$\sigma_{qh} \sim \frac{1}{3}\,\sigma_{hh} \sim 13 \text{ mb} \tag{22}$$

This cross section will be treated as a constant and independent of the energy density of the matter through which the quark propagates. This assumption is in contradiction with the properties of quark interactions at very high energy densities when perturbative QCD may be used. We are assuming that the energy densities are sufficiently low that the effects of the matter do not significantly alter the basic two body quark interactions. We shall soon present perturbative QCD estimates.

The mean free path is

$$\lambda_{mfp} \sim 1/\sigma n \tag{23}$$

where n is the number density of hadrons. At ordinary nuclear matter energy densities, $\lambda_{mfp} \sim 5$ Fm. Assuming that the energy density scales as T^4 as it would for either an ideal gas of pions or a quark-gluon plasma, then $n \sim \rho^{3/4}$. The mean free free path is therefore

$$\lambda \sim .5 \text{ Fm},\ \rho \sim 1\text{-}2 \text{ Gev/Fm}^3 \tag{24}$$

$$\lambda \sim .01 \text{ Fm},\ \rho \sim 200 \text{ Gev/Fm}^3 \tag{25}$$

For either of these two energy densities, the mean free path is extremely small compared to the nuclear radius, and effects of transverse surface area are quite small for nuclei of reasonable size. In the last case, even for protons, the surface effects would be small. Also the effects of longitudinal expansion seem controllable in the first case, $\lambda \sim \tau_0$, and quite small in the second case $\lambda << \tau_0$, and viscous corrections to perfect fluid hydrodynamics seem manageable.

These additive quark model estimates must surely be modified for high energy densities where perturbative QCD adequately describes the dynamics. At these high energy densities, the quark and gluon cross sections become small, and approach zero as $\sigma \sim \alpha_s^2/q^2$, where α_s is the QCD interaction strength and q is some typical energy scale, $q \sim T$. The mean free path is

$$\lambda \sim 1/(\alpha_s^2 T) \tag{26}$$

At large temperatures, $\alpha_s \sim 1/\ln(T)$ and $\lambda \sim \ln^2 T/T$.

Two groups have independently computed the mean free paths of quarks and gluons in a quark-gluon plasma, along the lines previously advocated by Shuryak.[17-19] These different computations differ in the way that small angle scatterings are treated, where high order perturbative, and possibly non-perturbative, corrections are evaluated. Also, the value of the strong interaction coupling constant which is used in this evaluation is somewhat ambiguous since it is not precisely clear at what momentum scale the coupling constant is to be evaluated, that is, should the momentum scale be T or 10T. Finally, at the temperatures for which we shall apply their results, the effects of higher order perturbative corrections due to inelastic scattering should be important. The lowest order results only evaluate the effect of elastic scattering, and these higher order corrections should reduce the mean free path and increase the total cross section. Given these intrinsic ambiguities, it is impossible to draw any precise conclusion. What I shall do is give a range of values which spna the results of Hosoya and Kajantie, and of Danielowicz and Gyulassy and allow for some measure of the uncertainty in un-computed contributions.[17-18] I find for all values of energy density in the range of $\rho \sim 1\text{-}1000$ Gev/Fm^3

$$\lambda_g \sim 1/20 - 1/2 \text{ Fm}, \quad \lambda_q \sim 1/5 - 2 \text{ Fm} \tag{27}$$

The gluon mean free path is λ_g and that of the quark is λ_q in this equation. The variation in mean free path as the energy density varies over this wide interval is at most a factor of 2 in my estimates. The mean free path may therefore be effectively be regarded as a constant as the energy density varies over this wide range. The gluon mean free path is about a factor of three smaller than that of the quark as a consequence of the larger color charge of the gluon, which forces it to interact more strongly than the quarks.

For the mean free paths of Eq. 27, the surface to volume ratio should be small for large nuclei such as Uranium. For hadrons, the situation is much more difficult to resolve. There may be large surface effects for quarks in a plasma under the most pessimistic scenario, or moderate corrections due to finite hadron radius might be required under optomistic scenarios. For gluons, there might be large corrections due to finite hadron size under the most pessimistic scenario, or there might be only small effects under optomistic ones. The bottom line is that for large nuclei, the effects of finite nuclear size should be manageably small, but the situation is entirely unclear in hadron interactions. If the effects of finite size are large, a simple analysis using perfect fluid hydrodynamics is difficult to justify.

To what extent longitudinal expansion is modified by viscous expansion is resolved by using the condition that the rate of entropy production due to viscous terms be small compared to the change in the entropy density due to expansion. This criteria may be formulated precisely in terms of viscous coefficeints, but we shall here formulate the problem semi-quantitatively and qualitatively in terms of mean free paths. The change in the entropy due to expansion is given by the perfect fluid hydrodynamic equations as

$$ds/d\tau = -s/\tau \tag{28}$$

The change in the entropy density due to entropy productions is

$$ds/d\tau = s/\tau_c \tag{29}$$

where τ_c is the collision time. The criteria that perfect fluid hydrodynamics be valid is therfore simply

$$\tau_c/\tau << 1 \tag{30}$$

Since the collision time is roughly independent of energy density, and therefore of τ, after some time τ, the system always is capable of expanding to a good approximation as a perfect fluid. This is because as a consequence of the similarity solutions of the hydrodynamic equations, at later times the system is expanding more slowly.

The collision times given by Eq. 27 show that for proper times τ > 1/5-2 Fm, the quarks may expand isentropically, and the gluons for times τ > 1/20-1/2 Fm for the gluons. These numbers are not inconsistent with the assumption that after matter forms at a proper time $\tau_o \sim$ 1/10-1 Fm, the matter quickly thermalizes and expands to a fair approximation as a perfect fluid. At the earliest times, there is the greatest entropy production, and as time evolves, the system behaves more and more as a perfect fluid. To resolve this problem more precisely, it would be nice to have a non-perturbative estimate of the viscous coefficients.

The perturbative estimates of collision times have been used to estimate the coefficients of shear and bulk viscosity. Hosoya and Kajantie find[17]

$$\zeta = 0 \tag{31}$$

$$\eta = \frac{.2}{\alpha_s^2 \ln \alpha_s} \; T^3 \tag{32}$$

The evaluation of Danielowicz and Gyulassy gives a result which is a factor of three larger.[18] The hydrodynamic equations may be used to estimate the total amount of entropy production

$$s_{final} \sim s_{initial}\{1 + \tau_c/\tau_o\} \tag{32}$$

This equation illustrates the increasing effects of entropy production at increasingly early proper times.

At very early times there is entropy production due to a variety of effects, and it would be extremely valuable to have a controlled theoretical analysis of the pre-equilibrium quark-gluon plasma. It would seem that such an analysis is tractable since at early times the energy density is high and the effects of interactions are weak. Such an analysis would be required to rigorously derive the inside-outside cascade within QCD. The initial conditions for the hydrodynamic equations would follow from knowledge of the initial state nuclear wavefunctions, about which little is presently known. A spectrum of fluctuations could be derived, and the parameter τ_o could be computed. The magnitude and importance of coherent phenomenon could be deduced.

Another possible place where perfect fluid hydrodynamics might break down is when the quark-gluon plasma expands through a first order phase transition,[20-21] or if the quark-gluon plasma must be produced from hadronic matter by undergoing a first order phase transition.[22] In either of these possible scenarios, large scale density fluctuations might be produced, and a global hydrodynamic description might break down. The system might break apart into droplets of matter which might slowly burn, or explosively detonate the plasma. The possibility that the system might break up into slowly burning droplets has been proposed by van Hove, and would occur if the plasma spinodally decomposed.[20] If the plasma could supercool, then explosive detonation droplets might form.[21] If these large scale density fluctuations were not too strong, the matter might recombine in the hadron phase, and a viscous expansion would smooth out the density fluctuations. There would be some entropy production, but the final matter distribution might be considerably smoothed out. If the density fluctuations were too severe, the plasma might break apart into isolated droplets each of which might be treated hydrodynamically.

Finally, if the plasma does survive a confinement-deconfinement or chiral phase transition, then at some low energy density the system freezes out into non-interacting hadrons. This freeze out may be treated by standard methods, and the final state distributions of particles may be computed.

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Comments for the Round Table Discussion on Mean Free Paths

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I would just like to repeat the comment which I made in my talk that the reason that we would like typical scattering lengths to be small compared to the length scales associated with spatial and temporal variations, and with the spatial sizes of systems, is that it is difficult to construct a local fluid dynamics even supplementd by viscous corrections unless this is true. If it is true, then perfect fluid dynamics, supplemented by viscous corrections should provide an adequate description of the dynamics.

The actual criterion that viscous corrections to perfect fluid dynamics be small is a criterion which is more general than that of small scattering times. Although these criteria are the same for diffuse fluids, they are not necessarily true for strongly interacting fluids. These viscous corrections may be evaluated non-perturbatively by the fluctuation dissipation theorem. For a high energy density quark-gluon plasma, the interactions are weak, and the criteria are the same. It is also, in my opinion, implausible that the length scale associated with viscous coeeficients and that associated with a mean free path would be significantly different except in exceptional circumstances where there are radically different scales in the physical system being analyzed. I don't see how there could be such scales in a quark-gluon plasma at energy densities not near that of a second order or higher phase transition.