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THE DIMENSIONAL REDUCTION TRANSITION

Edward Kolb

Theoretical Astrophysics Group
Fermi National Accelerator Laboratory
Batavia, Illinois 60510, U.S.A.

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Edward KOLB

Theoretical Astrophysics, Fermi National Accelerator Laboratory, Batavia, Illinois 60510, U.S.A.[†]

If there are more than four space-time dimensions, dimensions small today may have been dynamically important in the evolution of the early Universe. The transition to four space-time dimensions may have produced physically significant phenomena observable today.

1. INTRODUCTION

In this conference on phase transitions in the early Universe I will discuss the transition from more than four space-time dimensions to four effective space-time dimensions; the dimensional reduction transition. The basic assumption is that the true dimensionality of space-time is more than four, and that at present the extra dimensions are compact and too small to be observable. The origin of the observed gauge symmetries from symmetries of a compact space has been proposed as a possible approach for the unification of particle physics and gravity.[1] The common assumption in all models with extra dimensions is that the unseen dimensions are compactified to a very small size, usually taken to be of the order of the Planck length,[2] $R_{pl} = m_{pl}^{-1} = 1.6\times10^{-33}$cm. To probe the structure of the extra dimensions would require energies of the Planck mass $E = m_{pl} = 1.2\times10^{19}$GeV, and the early Universe may be the only source of such energies.

If the Universe was ever at temperatures greater than the inverse of the size of the compact dimensions, then the Universe effectively had more than four space-time dimensions. When the Universe cooled to a critical temperature $T_c = R_D^{-1}$, where R_D is the physical size of the D compact dimensions, dimensional reduction is possible, and the Universe effectively becomes four

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dimensional. It should be remembered that although the physical size of the extra dimensions today is of the order of the Planck size, there is no reason to believe that in the early Universe the scale of the extra dimensions was R_{pl}. There are cosmological models with $R_D = 0$ as initial conditions,[3] and cosmological models with $R_D = \infty$ as initial conditions.[4] The dynamical evolution of R_D is coupled to the dynamical evolution of the three observed spatial dimensions through the Einstein equations. Of course as the size of the extra dimensions changes, the gauge couplings and gravitational constant change.[3,4] We believe that α and G_N were unchanged from present values as far back as primordial nucleosynthesis, so any change in R_D must have 'damped out' by the time the Universe reached $T \approx 1$MeV.

I will assume for initial conditions that all spatial dimensions are small, and that initially the Universe had $N = 3+D$ spatial dimensions. When the temperature fell below $T \approx R_D^{-1}$, where R_D is the physical size of the compact extra dimensions, the space-time dimensionality of the Universe underwent a reduction to effectively a 4 space-time dimensional Universe, and dynamical effects of the extra dimensions became unimportant. In this paper I will discuss three possible consequences of this cosmological dimensional reduction; entropy production (inflation), magnetic monopole production, and stable, massive particle production.

2. INFLATION WITH EXTRA DIMENSIONS

Inflation is the most active and the most promising area at the interface of particle physics and cosmology.[5] Inflationary cosmologies have been constructed in grand unified theories, supersymmetric grand unified theories, and supergravity theories. The existence of extra dimensions may also lead to inflationary cosmologies.

There have been two approaches to inflation with extra dimensions. The first approach is closely related to new inflation in the sense that it depends upon an effective cosmological constant to have the Universe go through a

de Sitter phase.[6] This approach is reviewed by Wetterich[7] in these proceedings. In any model with extra compact dimensions vacuum energy is necessary to keep the extra dimensions small. In general, the compact dimensions are stabilized by the interplay between forces tending to collapse the space, and forces tending to expand the space. If the size of the extra dimensions are away from the equilibrium size, there should be an unbalanced force, which may be interpreted as a four-dimensional cosmological constant in the dimensionally reduced Universe. This cosmological constant would cause a de Sitter expansion phase, leading to the standard inflationary Universe.

The second approach does not depend upon vacuum energy or a de Sitter phase. The success of inflation is the creation of a large amount of entropy, $S > 10^{88}$, in a smooth way in a causally-connected region. In the approach I will outline, entropy is created when the Universe goes through an epoch with 3 expanding dimensions with D contracting dimensions. An increase in temperature results if the mean volume increases while the expansion/contraction is adiabatic.[8-10]

To illustrate this effect, consider a toy model with one time and 3+D spatial dimensions and a metric with a symmetry $R^1 \times S^3 \times S^D$ $g_{MN} = \mathrm{diag}\,(1, R_3^2\bar{g}_{mn}, R_D^2\bar{g}_{\mu\nu})$, where R_3 and R_D are the cosmological scale factors for S^3 and S^D, and $\bar{g}_{mn}$ and $\bar{g}_{\mu\nu}$ are metrics for maximally symmetric 3- and D-dimensional spaces. For simplicity we can take the limit $S^3 \to R^3$. If a large amount of entropy can be converted from S^D to S^3, the limit $S^3 \to R^3$ would be reasonable as entropy creation leads to a "flat" Universe. We will assume the stress-energy tensor takes the 3+D-dimensional perfect fluid form, and is <u>isotropic</u>, i.e. it can be described by an energy density, ρ, and a pressure, p: $T_{MN} = pg_{MN} + (p+\rho)U_M U_N$. With the above metric and stress-energy tensor, the dynamical equations of motion for the two scale parameters are given by

$$3\,\ddot{R}_3/R_3 + D\,\ddot{R}_D/R_D = 8\pi\bar{G}[(2-N)\rho - pN]/(N-1) \tag{1a}$$

$$d/dt(\dot{R}_3/R_3) + \dot{R}_3/R_3\,(3\,\dot{R}_3/R_3 + D\,\dot{R}_D/R_D) = 8\pi\bar{G}(\rho-p)/(N-1) \tag{1b}$$

$$R_D^{-2} + d(\dot{R}_D/R_D)/dt + \dot{R}_D/R_D(3\,\dot{R}_3/R_3 + D\,\dot{R}_D/R_D) =$$

$$8\pi\bar{G}\,(\rho-p)/(N-1) \tag{1c}$$

where $\bar{G}$ is related to Newton's constant, G_N, by $\bar{G} = G_N V_D$ with V_D the volume of the compact space today.

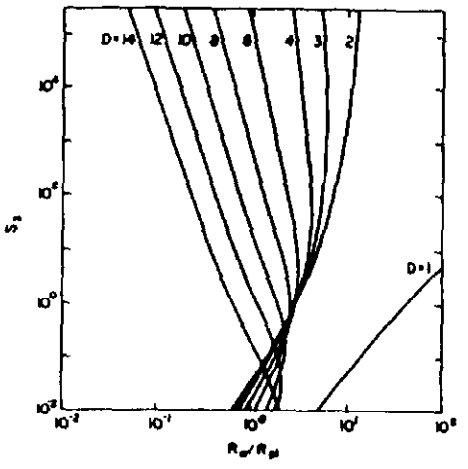
The program to calculate the entropy is to integrate the equations of motion until $R_D T = 1$, then calculate the entropy in the horizon 3-volume. If $R_D < T^{-1}$, then particles propagating along 3 directions will be massless, while particles propagating along D directions will have mass comparable to $R_D^{-1} > T$. Hence, the isotropy assumption breaks down. The horizon distance, ℓ_H, for S^3 is given by

$$\ell_H(t) = R_3 \int_0^t dt'/R_3(t') \,. \tag{2}$$

The energy density in N dimensions is $\rho_N = T^{N+1}$. At freeze out, all the degrees of freedom (entropy) in the extra dimensions are converted to degrees of freedom in S^3. If we denote the value of R_D when $R_D T = 1$ by R_*, then the effective 3-dimensional energy density at freeze-out is $\rho_3 = R_*^D \rho_N = R_*^{-4}$, since at freeze-out $T = R_*^{-1}$. The 3-dimensional entropy density is given by $s_3 = \rho_3^{3/4} = R_*^{-3}$, and the total 3-entropy is $S_3 = (\ell_H R_D^{-1})^3$.

The Einstein equations do not admit a solution with static compact dimensions. Therefore the compact dimensions reach a maximum, decrease, and eventually approach a singularity. As R_D approaches the final singularity, R_3 diverges. Because of collapsing extra dimensions the mean volume ($V^N \propto R_3^3 R_D^D$) decreases, which leads to an increase in temperature since in an adiabatic expansion (or collapse) $T^N V^N$ is constant. The increase in temperature while R_3 increases leads to an increase in the entropy in the 3-horizon volume.

Figure 1 is the entropy in the 3-horizon volume when $R_D T = 1$, for different numbers of compact dimensions. There is a one-parameter family of solutions for a given number of dimensions which is parameterized by R_*, the physical radius of the extra dimensions when $R_D T = 1$. We expect the freeze-out radius to be comparable to the radius of the extra dimensions today, hence we expect $R_* \gtrsim R_{pl}$.



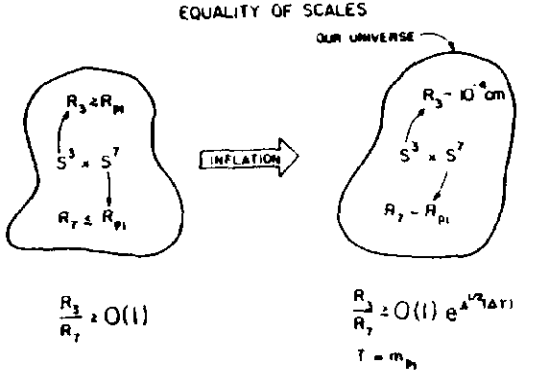
We believe that there are more than two extra dimensions and that the radius *of the extra* dimensions should be greater than R_{pl}, so the toy model failed. However the basic *mechanism worked -- entropy did* increase in the horizon 3-volume. Abbott, Barr, and Ellis[10] have pointed out that if *we relax the* freeze-out condition, it is possible to create more entropy. In fact they point out that if you go to a large number of extra dimensions the entropy contained within a volume of radius R_D increases with increasing number of extra dimensions, $S \sim (R_D T)^D$, and if D is large, $D > 44$, for $R_D T$ reasonable, say $R_D T > 100$, S would be greater than 10^{88}. In other words for a large number of dimensions a huge volume (hence entropy) can be characterized by a small *radius*.

It is not yet clear if the *approach outlined here can be made* to work. Inflation from extra dimensions may follow the above approach, or it may follow the scenario described by Wetterich.[7] The cosmological benefits of inflation are well known,[5] however it may be that inflation can provide answers to some problems in Kaluza-Klein.

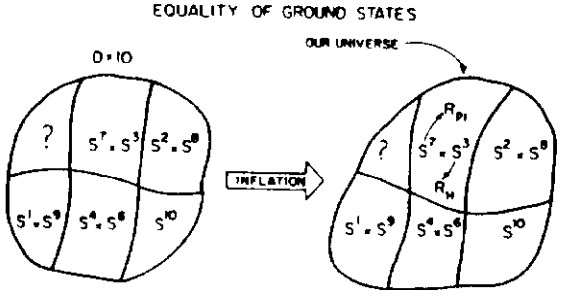
It is somewhat disturbing in the Kaluza-Klein approach that there are three large spatial dimensions with size comparable to the Hubble radius, $R_H \sim 10^{28}$cm, and that there are D compact dimensions with size comparable to the Planck radius, $R_{pl} \sim 10^{-33}$cm. As inflation explains why the Universe has expanded to its present size, it may explain why three dimensions are *so much* larger than the others. *Here, I propose that there are three huge* and D tiny *dimensions as a result* of the dynamics of the expansion of the early Universe.

Let us assume for simplicity that the Universe is closed. Then the three large spatial dimensions have the geometry of S^3, with physical radius $R_3 > R_H$. If there is some mechanism for inflating the physical size of the 3-sphere while keeping the physial size of the D-sphere constant (or at least not inflating as quickly), then the disparity in scales can be understood. This idea is illustrated in Figure 2 for D=7. All scenarios for Kaluza-Klein inflation assume that at some point *dimensional reduction is possible. In* order *for dimensional* reduction to take place it would be necessary for R_3 to be greater than R_D. However it is not necessary that $R_3/R_D \gtrsim 10^{61}$ as today, but only that $R_3/R_D \gtrsim O(1)$. If R_3 can expand to 10^{-4}cm and reheat to $T = m_{pl}$ (or "heat" in the Shafi-Wetterich model) and R_D stay stable at the Planck length, then the evolution of this Universe will give the one observed today, although as initial conditions the two scales were similar.

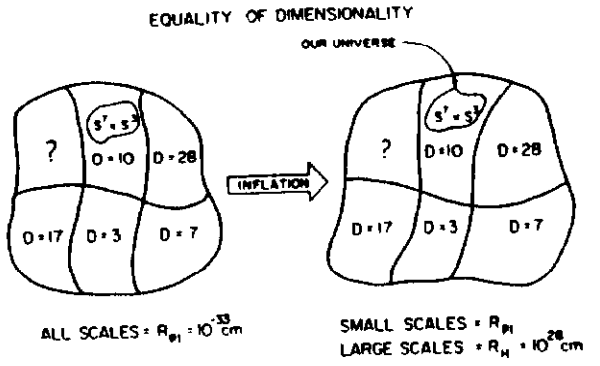
EQUALITY OF SCALES



The next question to ask is why $S^3 \times S^D$. It may be that the splitting of the 10-dimensional manifold into $S^3 \times S^D$ is due to the existence of a special type of Freund-Rubin field. However if one takes into account the dynamical evolution of the Universe, it may be that the original 10-dimensional space approximates $S^3 \times S^7$ only locally, and in a global sense there are other possible ground states. There may be a region in the 10-dimensional space that is flatter in three directions than in seven directions. If this region is Planck size, through inflation and the dynamical evolution of the Universe, it can become the Universe we observe. Outside of the Universe we observe there may be other combinations of large and small dimensions. Low-energy physics is much different in these other Universes, but they are outside of our horizon. There may well be other possible ground states for the geometry, and the question of why are there three large and D compact dimensions today may be that that just happens to be our Universe. This idea is illustrated in Figure 3.



As a final speculation on the subject, we might imagine that the total dimensionality of space can change. Again, as long as there is somewhere a local region with approximate geometry $S^3 \times S^D$ and scales characteristic of R_{Pl}, we can follow the evolution and end up in the Universe we observe. A final cartoon illustrating this idea is shown in Figure 4.



In conclusion, there are serious questions about the Kaluza-Klein approach, namely 1) Why are some dimensions large and some small, and 2) Why are three large. Since the dynamics of the evolution of the spatial dimensions are coupled through the Einstein equations, these questions may be answerable in a cosmological context.

3. KALUZA-KLEIN MONOPOLES

Massive, stable topological defects in the geometry of compactification would appear in four space-time dimensions as massive magnetic monopoles.[11]

Kaluza-Klein monopoles are produced when topological defects are frozen in as space is split into 3 large spatial dimensions, and D small, compact dimensions.[12] There is a difference between GUT monopole production and Kaluza-Klein monopole production. In the standard cosmology the Universe was in the symmetric phase as an initial condition so there were no GUT monopoles. The GUT monopoles first appear during the phase transition, and the number of GUT monopoles produced is independent of initial conditions. However if the splitting of the spatial dimensions is present as an initial condition, then Kaluza-Klein monopoles (i.e. the topological defects) appear as initial conditions, and it is impossible to calculate the expected number of monopoles. Therefore in cosmological models where the splitting is an initial condition,

we cannot predict the expected monopole abundance. However if the splitting of the spatial dimensions is dynamical, Kaluza-Klein monopoles would appear in this splitting,[12] and may be independent of the initial conditions. Magnetic monopoles are a potential source of information on the dynamics of dimensional reduction.

4. PYRGONS

In 1926 Klein[13] noticed that in the five-dimensional theory of Kaluza, the existence of a compact fifth dimension implies an infinite tower of four-dimensional particles corresponding to quantized excitations of the extra dimensions. These massive particles have been called pyrgons,[14] from the Greek word πυργοί, for tower. Klein's observation for the five-dimensional theory is true for any Kaluza-Klein theory with any number of compact dimensions.

If the Universe was ever at temperatures comparable to R_D^{-1} pyrgon states should have been present. If the Universe was ever at temperatures in excess of R_D^{-1}, the Universe would have been effectively N-dimensional, and since all the excitations of the geometry are "massless" in N dimensions they will be distributed over all the modes. If the maximum temperature of the Universe was less than but comparable to R_D^{-1}, the pyrgon states should have been pair produced. In either case, a reasonable guess for initial conditions is that pyrgons are about as abundant as photons.

The massive pyrgon states are not effective at annihilation, and disappear from the Universe only through decay.[14] If any of the pyrgon states are stable, or have a lifetime greater than the age of the Universe, the standard calculations predict that they should be present in the Universe today with an abundance comparable to photons. Since the massive pyrgon states are expected to have a mass comparable to the Planck mass, the energy density of the Universe would be about 4×10^{26} times the critical density.

The most likely mechanism to rid the Universe of pyrgons is decay. The reason they were stable in Klein's 5-dimensional example is that they carried a

charge not carried by the zero modes. However, there are somewhat more realistic models, such as 11-dimensional supergravity with vacuum $M^4 \times S^7$, that have stable pyrgons even though most of the 256 zero modes do carry the SO_8 quantum numbers of the symmetry of S^7.[14] It is possible to imagine several reasons for assuming some pyrgons might be stable. For example if all zero modes satisfy the usual electric charge/color relation, but some pyrgon does not. One might also imagine that there is an additional charge under which all the zero modes are neutral, but some pyrgon is not. In general, until a successful model is constructed and the mass spectrum computed, it is not possible to know whether there will be stable pyrgons.

5. REFERENCES

1) Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, Math. Phys. KI(1921) 966; O. Klein, Z. Phys. 37 (1926) 895.

2) Theories with the scale of the extra dimensions comparable to $m_W \simeq 100$ GeV have also been proposed, see e.g. G. Chapline and R. Slansky, Nucl. Phys. B209 (1982) 461.

3) P.G.O. Freund, Nucl. Phys. B209 (1982) 146.

4) A. Chodos and S. Detweiler, Phys. Rev. D21 (1980) 2167.

5) For a review, see M. Turner "Inflation Circa 1983", in Proceedings of IV Workshop on Grand Unification, eds. H.A. Weldon, P. Langacker, and P.J. Steinhardt (Birkhauser, Boston, 1983) p. 228.

6) Q. Shafi and C. Wetterich, Phys. Lett. 129B (1983) 387.

7) C. Wetterich, this volume.

8) E. Alvarez and M. Belen-Gavela, Phys. Rev. Lett. 51 (1983) 931; M. Bohm, J.L. Lucio, and A. Rosado, "Dynamical Compactification an Alternative to Inflation?" Preprint (1983); S. Sandev, Phys. Lett. 137B (1984) 155.

9) E.W. Kolb, D. Lindley, D. Seckel, Phys. Rev. D (1984) .

10) R.B. Abott, S.M. Barr, and S.D. Ellis, Phys. Rev. D (1984) .

11) D.J. Gross and M.J. Perry, Nucl. Phys. B226 (1983) 29; R. Sorkin, Phys. Rev. Lett. 51 (1983) 87.

12) J.A. Harvey, E.W. Kolb, M. Perry, Phys. Lett. B, to appear.

13) O. Klein, Nature 118 (1926) 516.

14) E.W. Kolb and R. Slansky, Phys. Lett. 135B (1984) 378.