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# THE ORIGIN OF COSMOLOGICAL DENSITY FLUCTUATIONS

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THE ORIGIN OF COSMOLOGICAL DENSITY FLUCTUATIONS

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The density fluctuations required to explain the large-scale cosmological structure may have arisen spontaneously as a result of a phase transition in the early Universe. There are several ways in which such fluctuations may have been produced, and they could have a variety of spectra, so one should not necessarily expect all features of the large-scale structure to derive from a simple power law spectrum. Some features may even result from astrophysical amplification mechanisms rather than gravitational instability.

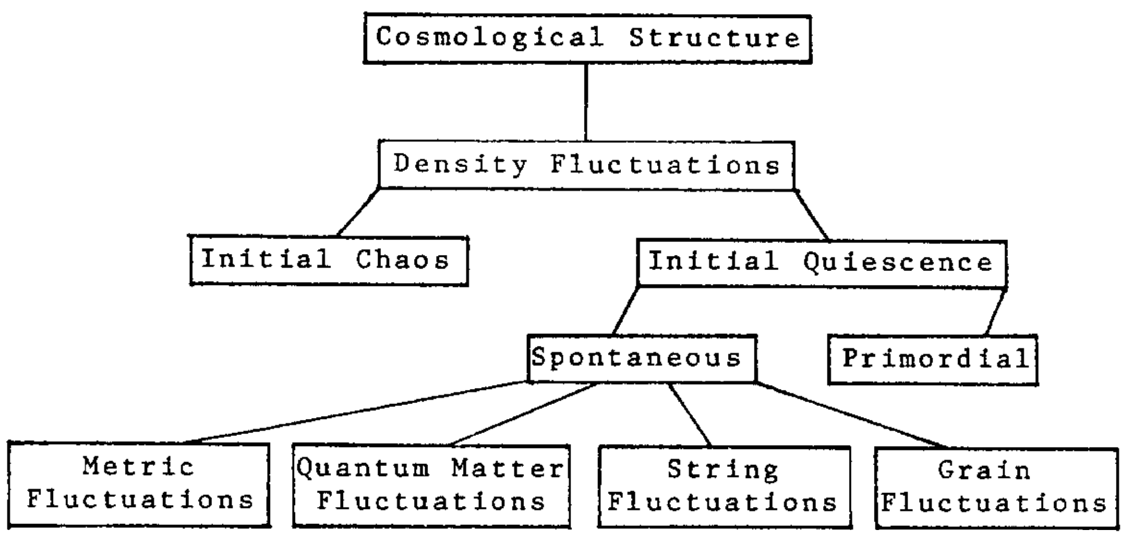
## 1. INTRODUCTION

Although the Universe is well described on large scales by the isotropic, homogeneous Friedmann model, the existence of galaxies and clusters implies that it must have contained density fluctuations at early times. A key issue in cosmology is to understand the origin of these fluctuations. One approach to the problem is to assume that the Universe started out chaotic, with large inhomogeneities and anisotropies on all scales,[1] and that dissipative processes at early times, such as particle production effects[2] or neutrino viscosity,[3] reduced it to nearly Friedmann form. In this picture the galactic scale inhomogeneities would be regarded as the small residual imprint of the initial chaos. Unfortunately, it now seems unlikely that this approach can work. It is difficult to smooth out a chaotic Universe anyway[4] and, even if it were possible, the dissipation involved would tend to produce far more entropy than is observed in the 3K background radiation.[5] This conclusion might be avoided by invoking an inflationary phase, but in this case the initial chaos would be irrelevant.

Most cosmologists therefore now subscribe to the view that the early Universe was quiescent, the deviations from Friedmann behaviour always being small. In this case, the inhomogeneities required for galaxies must either be primordial, in the sense that they are fed into the initial conditions of the Universe, or they must arise spontaneously at some finite time after the Big Bang. The latter possibility is obviously more appealing since, if one has to conclude that the inhomogeneities exist from the beginning, one is not really explaining anything. However, until a few years ago cosmologists were very pessimistic about the possibility of spontaneous fluctuations.[6] This pessimism was prompted by two facts: (1) in the standard Big Bang picture fluctuations cannot grow between entering the horizon and decoupling because of the effects of the 3K background; and (2) fluctuations cannot grow before entering the horizon (in a sense to be defined more precisely later) on account of causality.[7] Thus galactic-scale fluctuations must be of order $10^{-3}$ even at their inception, which is uncomfortably large.

In fact, this pessimism is not really warranted. Point (1) can be alleviated by invoking cosmological models which are matter-dominated for a period before decoupling[8] (since this allows an extra amplification factor of $10$-$10^3$); and point (2) takes on a different perspective if the Universe goes through an inflationary phase[9] (since the domain of causal interaction is much increased). In consequence, pessimism has waned in recent years and several schemes can now be entertained for the spontaneous generation of inhomogeneities. All of these depend on statistical fluctuations arising at some sort of cosmological phase transition. One possibility is that a phase transition may have endowed the Universe with graininess. In this case, there could be fluctuations associated with randomness in the grain distribution, although one needs fairly exotic grains to produce galaxies. Rather similar fluctuations could arise if strings were produced at a spontaneously broken symmetry epoch. Another possibility is that spontaneous fluctuations may have arisen through the non-adiabatic amplification of quantum fluctuations in any matter field present; this may occur whenever the Universe's equation of state ceases to be radiation-like and is particularly interesting if the Universe undergoes an inflationary phase (corresponding to a vacuum equation of state). Finally, it is possible that quantum gravity effects at the Planck time induce fluctuations. Although our understanding of quantum gravity effects is very rudimentary, one certainly cannot exclude this suggestion.

These possibilities for the origin of cosmological density fluctuations are summarized in the table below. The emphasis of this paper will be primarily on the spontaneous fluctuation scenarios. I will first assess what initial fluctuations might be required to explain the observed cosmological structure and I will then discuss what sorts of spontaneous fluctuations the Universe might be able to provide. Finally, I will emphasize the possibility that the observed cosmological structure may not entirely reflect the initial density fluctuations but may in part derive from astrophysical effects triggered by objects much smaller than galaxies. This could considerably reduce the amplitude required for the initial fluctuations.



## 2. WHAT FLUCTUATIONS ARE REQUIRED?

The Universe not only exhibits structure on the scale of galaxies. Observations indicate that the galaxies are themselves clustered and a clue as to the nature of the fluctuations required may be gleaned from the galaxy correlation function. On scales up to $20h^{-1}$Mpc this has the form[10]

$$\xi(r) = \left(\frac{r}{r_o}\right)^{-\gamma}, \ \gamma \approx 1.8, \ r_o \approx 10h^{-1}\text{Mpc} \tag{2.1}$$

where h is the Hubble constant in units of 50 km/s/Mpc; it then drops off more steeply. On the assumption that the clustering results from gravitational instability alone, one can in principle infer the form of the fluctuations necessary to explain eqn (2.1). In order to do this, the evolution of the fluctuations must be traced through three distinct phases: (1) the period prior to their entering the particle horizon; (2) the period between their entering the horizon and decoupling; and (3) the period after decoupling. Before discussing these phases, however, it must be stressed that eqn (2.1) may not tell the whole story. The 3-dimensional redshift surveys indicate that $\xi(r)$ falls off more slowly above $4h^{-1}$Mpc[11,12] and this may reflect the presence of another source of fluctuations.[13] There is also evidence that the Universe contains giant voids and filaments on scales up to 100 Mpc[14-16] and these may owe their origin to yet a third effect. It may therefore be naive to assume that any simple form for the initial fluctuations can explain all the features of the large-scale cosmological structure. It is not even obvious that all the features are gravitational in origin.

### 2.1 The evolution of fluctuations after decoupling

The baryons thermally and dynamically decouple from the 3K background radiation when the ionization drops at $t_d \approx 10^{13}\Omega^{-1/2}$s; here $\Omega$ is the total density of the Universe in units of the critical density. So long as they are non-linear, all fluctuations should grow like $(1+z)^{-1}$ between $t_d$ and the free-expansion epoch $(1+z=\Omega^{-1})$. If the decoupling fluctuations extend down to scales smaller than galaxies and have a simple power law form,

$$\left(\frac{\delta\rho}{\rho}\right)_d = \left(\frac{M}{M_1}\right)^{-\beta} \tag{2.2}$$

with $\beta > 0$, this means that ever larger bound structures should form through hierarchical clustering.[17] In this case there is a simple relation between $\beta$ and $\gamma$:[18]

$$\gamma = 6\beta \; (\xi < 1), \qquad \gamma = \frac{9\beta}{1+3\beta} \; (\xi > 1) \quad . \tag{2.3}$$

Eqn (2.1) then requires $1/3 < \beta < 1/2$ and $10^5 M_\odot < M_1 < 10^8 M_\odot$, the precise values depending on $\Omega$.[19] If the decoupling fluctuations do not extend down to galactic scales, the determination of $\xi(r)$ is less straightforward since galaxies can only form via fragmentation[20] and structure on scales below the initial cut-off has to build up through non-linear effects. In this case, the form of $\xi(r)$ evolves with time and numerical simulations are required in order to match it with eqn (2.1) at the present epoch.

2.2 The evolution of fluctuations before decoupling

Fluctuations on a baryon scale M enter the horizon at a time $t_H \approx 10^8 (M/10^{12} M_\odot)^{3/2}$s. This exceeds the time $t_e \approx 10^{10}\Omega^{-2}$s at which the Universe becomes matter-dominated if M is larger than $M_e \approx 10^{15}\Omega^{-2} M_\odot$. Since the radiation Jeans mass peaks at this value, scales above $M_e$ grow continuously. On the other hand, fluctuations with $M < M_e$ may be inhibited by a variety of effects. Their evolution depends on whether they are isothermal or adiabatic and on whether the dark matter which dominates the Universe's density[21] consists of baryons, neutrinos or axions.

Baryon-dominated Universe. In this case, any isothermal fluctuations which are subject to the drag of the 3K background will be unable to grow until $t_d$ on scales below $M_e$. This, in particular, applies to baryonic fluctuations. The fact that thermal and dynamical decoupling occurs only gradually at $t_d$ also results in the elimination of baryonic fluctuations on scales less than the baryon Jeans mass: $M_b \approx 10^6 \Omega^{-1/2} M_\odot$.[22,23] If the isothermal fluctuations are not dynamically coupled to the 3K background, they will still be unable to grow before $t_e$[24] but they will not be damped below $M_b$. On the other hand, the baryons themselves will be unaffected on scales below $M_b$ until a time $t_d(M_b/M)$

after decoupling due to the baryonic pressure. If the fluctuations are adiabatic, they will not be able to grow between the time they become smaller than the radiation Jeans length (which is of order $t_H$ before $t_e$) and decoupling. Instead they will appear as acoustical waves. These waves will be damped on scales below $M_s \approx 10^{13}\Omega^{-5/4}M_\odot$ by photon diffusion.[25] Their amplitude will also be reduced on scales below $M_e$ by a factor of $(t_e/t_d)^{1/6}$. The final amplitude will then determine the size of the baryonic fluctuations set up after decoupling. Note that it is sometimes claimed that the baryonic fluctuations with $M < M_e$ will exceed the original adiabatic fluctuations by a factor $(M/M_e)^{-1/3}$ due to kinematic effects.[26] However, it now seems likely that this boosting is just an artifact of assuming that decoupling occurs instantaneously.[27] A serious problem in the adiabatic scenario is that it is difficult to match eqn (2.1) without producing excessive anisotropies in the 3K background radiation.

Neutrino-dominated Universe. If the neutrino has a non-zero rest mass $m_\nu$, then the density of the neutrinos $[\Omega \approx (m_\nu/10^2 ev)]$ would exceed the baryon density for $m > 10eV$.[28,29] This has a crucial effect on the evolution of density fluctuations after $t_H$.[30-32] Any adiabatic fluctuations will be erased by neutrino free-streaming on scales less than $M_\nu \approx 10^{15}\Omega^{-2}M_\odot$. The photon and baryon fluctuations will also be erased on scales below $M_s$, but the latter will be re-established when the baryons fall into the neutrino potential wells at decoupling. Thus one expects the first objects to bind to be cluster-scale pancakes with neutrino halos. One advantage of this scenario is that it reduces the anisotropies expected in the 3K background radiation. Another advantage is that the large mass-scale associated with $M_\nu$ might be relevant to explaining giant voids.[33] However, a serious problem is that it is difficult to make galaxies early enough.[34] The growth of isothermal fluctuations will also be inhibited by the neutrinos: once they enter the horizon, they will not be able to grow until they become larger than the neutrino Jeans length. Since the neutrino Jeans mass falls of as $(1+z)^{3/2}$, this reduces the fluctuations by a factor of $(M/M_{\nu})^{2/3}$ below $M_{\nu}$. In addition, if the isothermal component is dynamically coupled to the 3K background, all fluctuations will be frozen until $t_d$. Nevertheless, providing the neutrinos are massive enough to cluster,[35] isothermal fluctuations could still lead to bound systems surrounded by dark halos.

Axion-dominated Universe. Some of the problems associated with the neutrino picture can be alleviated by supposing that the Universe is dominated by a particle whose mass is much larger than 10eV (thus reducing the free-streaming scale) and which decouples at an earlier time (so that the density is not too large). Candidates for such a particle might be the gravitino,[36,37] the right-handed neutrino,[38] the photino,[39,40] or the sneutrino.[41,42] However, a currently more popular solution is to invoke a "cold" particle like the axion which is not subject to free-streaming at all.[43-48] In this case, axionic fluctuations (which we will regard as adiabatic for present purposes) are never erased but they cannot grow after $t_H$ until the axions dominate the density at $t_e$. Although bound clouds of axions could form down to very small scales, baryons would be affected on scales below $M_b$ only when the baryonic Jeans mass has fallen sufficiently.[23] The situation is therefore very similar to the isothermal scenario in a baryon-dominated Universe. Note that the implied 3K anisotropy is consistent with observation provided $\Omega > 0.2$.[49,50] If the fluctuations are isothermal, the situation is similar except that they cannot grow until $t_d$ if they are subject to radiation drag and the axions fall into the baryon potential wells rather than vice-versa.

In all these scenarios, the baryonic fluctuations at decoupling are simply related to the total fluctuations at the horizon epoch, as indicated in Figure 1. The main difference is that one has a different lower cut-off in the scale of the surviving fluctuations, but the spectral shape is also affected. Note that, for isothermal fluctuations, one must distinguish between the fluctuation in the total density and the fluctuation in the baryon density. For surviving scales which enter the horizon before $t_e$,

$$\left(\frac{\delta\rho}{\rho_b}\right)_H = \left(\frac{\delta\rho}{\rho}\right)_H \left(\frac{t_H}{t_e}\right)^{-1/2} = \left(\frac{\delta\rho}{\rho}\right)_H \left(\frac{M}{M_e}\right)^{-1/3} . \qquad (2.4)$$

Thus the fluctuations in the total density at the horizon epoch are smaller than the baryonic fluctuations at decoupling by a factor of $(M/M_e)^{1/3}$.[51] If the isothermal fluctuations are decoupled from the 3K background, baryons can eventually clump even on scales below $M_b$.

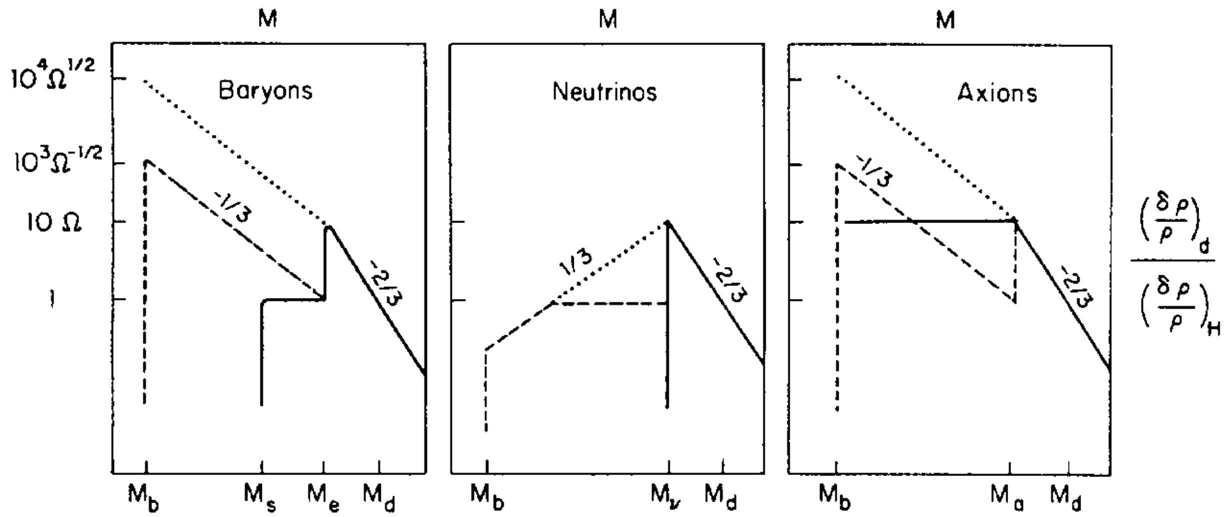



FIGURE 1

This shows the relationship between the fluctuations at decoupling and the horizon epoch for a Universe dominated by baryons, neutrinos or axions. The solid lines apply for adiabatic fluctuations; the broken (dotted) lines apply for isothermal fluctuations which are dynamically coupled (uncoupled) with the 3K background.

2.3 The growth of fluctuations before the horizon epoch

Before entering the horizon, fluctuations are not subject to any of the causal dissipative effects discussed above, but their evolution is tricky because of the problem of gauge ambiguity. Changing the gauge corresponds to changing the identification between points in the perturbed spacetime and points in the unperturbed background. Since the perturbation in some quantity depends upon how one makes this identification, the gauge ambiguity can add spurious (non-physical) contributions to the perturbation.[6] What one must do therefore is to refer only to gauge-invariant quantities which can be constructed directly from the metric and stress-energy tensor. Another problem is that there is no unique constant time hypersurface on scales larger than the horizon. However, some of the gauge-invariant quantities have a natural interpretation with respect to specific hypersurfaces. In particular, Bardeen[7] defines a gauge-invariant quantity $\epsilon_m$ which measures the amplitude of the density fluctuation relative to the "comoving" hypersurface, i.e. the hypersurface which is everywhere orthogonal to the matter flow lines. This is the description of the density fluctuations we will adopt for the purposes of the present discussion.

For primordial fluctuations, one can show that $\epsilon_m$ just grows as $\tau^2$. Here $\tau$ is conformal time, with respect to which the background metric is

$$ds^2 = a(\tau)^2 \, [d\tau^2 - d\underline{x}^2], \qquad a \propto \tau^{2/(3\Gamma-2)} \tag{2.5}$$

for an equation of state $p=(\Gamma-1)\rho$. With respect to synchronous time t, the metric is

$$ds^2 = dt^2 - a(t)^2 d\underline{x}^2, \qquad a \propto t^{2/3\Gamma} \tag{2.6}$$

and $\epsilon_m$ grows as $t^{(2-4/3\Gamma)}$; this corresponds to the usual growth law on scales smaller than the horizon. The situation is more complicated for spontaneous fluctuations because energy conservation requires that these can only arise via a stress perturbation $\eta$. If the stress perturbation turns on at some initial time $t_o$, Bardeen shows that the associated value of $\epsilon_m$ evolves as[7]

$$\epsilon_m = x^2 \int_{x_o}^{x} \frac{(\Gamma-1)(3\Gamma-2)}{(3\Gamma+2)} \, y^{-1} \eta(y) \, dy. \tag{2.7}$$

Here $x=k\tau$ measures the scale of the perturbation relative to the horizon size. The $x^2$ term in eqn (2.7) just corresponds to the $\tau^2$ growth associated with primordial perturbations. The integral term starts off as zero but, after an expansion time, is always of order the value of the current stress perturbation. This leads to two important features. Firstly, although a genuine density perturbation does develop after an expansion time, it is initially smaller than $\varepsilon_m$ by a factor of $(M/M_H)^{2/3}$; secondly, at the horizon epoch ($x=1$), $\varepsilon_m$ can be no larger than the stress perturbation at any preceding epoch. It is in this sense that the fluctuations cannot grow before entering the horizon: a common mistake is to assume that the initial density perturbation is of order $\eta$ and that it still grows like $\tau^2$ before entering the horizon.[52] We note that this conclusion pertains even if the Universe undergoes an inflationary phase[53]. In this case, the value of $\varepsilon_m$ on re-entering the horizon is still no more than the original stress perturbation, even though the scale of the original fluctuation is much enhanced.

Equation (2.7) is relevant to the issue of whether one expects the fluctuations at $t_H$ to be adiabatic or isothermal. In the spontaneous fluctuation scenario, there are no curvature fluctuations at $t_o$ and so (in this sense) the fluctuations are isothermal. However, Bardeen's analysis shows that adiabatic fluctuations inevitably develop and actually have comparable amplitude at the horizon epoch. Indeed, if the original stress perturbation has turned off by then, it is only the adiabatic fluctuations which survive. It is thus impossible to end up with pure isothermal fluctuations.

## 3. WHAT FLUCTUATIONS CAN WE EXPECT?

We will now review the various scenarios which have been proposed for the origin of cosmological density fluctuations. In all of these the density fluctuation (interpreted as $\varepsilon_m$) which develops after an expansion time can be expressed as

$$\left(\frac{\delta\rho}{\rho}\right)_o = \varepsilon\left(\frac{M}{M_o}\right)^{-\alpha} \tag{3.1}$$

where $M_o$ is the horizon mass at the time $t_o$ when the stress perturbation turns on. Eqn (2.7) implies that the horizon epoch fluctuation exceeds $(\delta\rho/\rho)_o$ by a factor $(M/M_o)^{2/3}$, independent of the preceding equation of state. Fluctuations with $\alpha=2/3$ are particularly interesting since their amplitude on entering the horizon is scale-independent. Various people have argued that such "constant curvature" fluctuations are best suited to explaining the cosmological structure[54,55] because otherwise one would produce too many black holes (or even close the Universe) on sufficiently large or small scales.[56] However, this conclusion applies only if the fluctuations extend to arbitrarily large or small scales, which need not be the case. There is no problem, for example, in producing galaxies with $\alpha>2/3$ fluctuations if the spectrum extends down to a mass-scale which is no smaller than $10^{-3/(\alpha-2/3)}$ times a galactic mass.

### 3.1 Quantum gravity effects

Harrison[54] has suggested that quantum fluctuations in the metric at the Planck time, $t_p \approx 10^{-43}$s, will naturally endow the Universe with scale-invariant fluctuations with $\varepsilon$ an arbitrary constant between 0 and 1. He begins by considering the Feynman propagator

$$\langle f_2 | f_1 \rangle = \sum_H \exp\,(iS_H) \tag{3.2}$$

where the f's are field configurations over some spatial hypersurface and the summation is over all possible fields, classical or otherwise. The action due to gravity alone is

$$S_H \sim (1/\lambda_p)^2 \int R^\mu{}_\mu \sqrt{-g}\, d^4x \tag{3.3}$$

and the variation due to field fluctuations is

$$\delta S_H \sim (1/\lambda_p)^2 \int \delta(R^\mu{}_\mu\sqrt{-g})\, d^4x \tag{3.4}$$

where $\lambda_p$ is the Planck length ($10^{-33}$cm). Corresponding to metric fluctuations $\delta g$ on a scale L, $\delta(R^\mu{}_\mu\sqrt{-g})$ will have terms of the form $L^{-2}(\delta g)$ and $L^{-2}(\delta g)^2$. Harrison takes the $d^4x$ integral in eqn (3.4) to be of order $L^4$, which implies

$$\delta S_H \sim (L/\lambda_p)^2 \times [O(\delta g) + O(\delta g)^2]\ . \tag{3.5}$$

The significant contribution to the Feynman propagator corresponds to $\delta S_H \approx \pi/2$, which gives

$$(\delta g) \sim (\lambda_p/L)^2. \tag{3.6}$$

Finally, he argues that the density fluctuation will have the same form as $\delta g$; this leads to eqn (3.1) with $\alpha=2/3$.

Harrison's argument is not very convincing.[57] The first worrisome point is that eqn (3.3) neglects the matter contribution to the action. If one included this contribution, the Einstein equations would guarantee that the order $(\delta g)$ term in eqn (3.5) disappears and this would give

$$(\delta g) \sim (\lambda_p/L) \ . \tag{3.7}$$

This indeed is the form for $\delta g$ originally suggested by Wheeler.[58] Of course, one might argue that Einstein's equations no longer apply at the Planck time but the physics is very unclear. The second problem is that Harrison does not specify what measure of density fluctuation he is identifying with $\delta g$. Bardeen's analysis shows that this identification applies only if one uses the gauge-invariant quantity $\varepsilon_g$ which describes the fluctuations relative to the zero-shear hypersurface. The quantity $\varepsilon_m$ is smaller than $\varepsilon_g$ by a factor $(L/\lambda_p)^2$. Thus the fluctuations which Harrison claims to predict are not constant curvature at all: eqns (3.6) and (3.7) would imply $\alpha=4/3$ and $\alpha=1$, respectively. Unfortunately, since $\varepsilon<1$, such fluctuations would be completely negligible on a galactic scale.

One way to regain constant curvature fluctuations would be to add a term in the action which is quadratic in R. Metric fluctuations $\delta g$ would then correspond to terms of the form $L^{-4}(\delta g)^2$ in eqn (3.4). This would give $\delta g$ independent of L, as required. The addition of a quadratic term in the action is not implausible because of renormalization effects.[59] Nevertheless, lacking a complete theory of quantum gravity, it is not clear that Harrison's approach to the problem of metric fluctuations is the correct one.

3.2 Quantum matter effects

Several authors[60-64] have invoked quantum fluctuations in the matter (rather than the metric) to induce spontaneous density fluctuations. To study this effect, one must first construct two gauge-invariant canonically conjugate scalars with which to describe irrotational matter perturbations in a Friedmann background.[61,62] One such scalar corresponds to the perturbation in the velocity potential $\Phi$. It can be written as

$$q = 3 \left(\frac{\dot{a}}{a}\frac{\Phi}{w} - \frac{1}{2} A\right) \tag{3.8}$$

where a dot denotes d/dt, w is the specific enthalpy of the fluid and A is related to the trace of the metric perturbation. The other scalar is associated with the density perturbation and can be written as

$$\tilde{q} = \frac{\Delta P}{3\alpha^2},\ P = \int a\Gamma q dt. \tag{3.9}$$

Here $\Gamma$ specifies the equation of state of the fluid and

$$\alpha = \left(\frac{a}{\beta}\right)\left(\frac{\Gamma}{3}\right)^{1/2},\quad \beta = \left(\frac{dp}{d\rho}\right)^{1/2}. \tag{3.10}$$

The evolution of q satisfies

$$q'' + 2(\alpha'/\alpha)\, q' - \beta^2 \Delta q = 0 \tag{3.11}$$

where a dash denotes $d/d\tau$. It can be shown that eqn (3.11) is conformally invariant only if $\alpha''=0$, which corresponds to an equation of state $p=\rho/3$. Providing this condition does not pertain, fluctuations can be amplified purely classically in the period before they enter the horizon and this gives rise to the possibility of spontaneous phonon production.

The form of the fluctuations produced depends on how the equation of state deviates from $p=\rho/3$. By applying the appropriate canonical quantization condition, one can show that the resultant horizon-epoch fluctuations are of the form[61,62]

$$(\delta\rho/\rho)_H = \left[\int_{k_H}^{k_o} |\beta_k|^2\, k^3 dk\right]^{1/2} \tag{3.12}$$

where $\beta_k$ is the amplification factor, determined by eqn (3.11), and the limits in the integral are wave-numbers corresponding to the initial and current horizon size. The fluctuations are maximized if the Universe starts off in a deSitter phase with $a \sim e^{Ht}$, corresponding to an equation of state $p=-\rho$, before entering its $p=\rho/3$ phase. In this case the horizon epoch fluctuations are given by[63]

$$\left(\frac{\delta\rho}{\rho}\right)_H \sim H\left[\int_{k_H}^{\infty} \frac{dk}{k}\right]^{1/2}, \tag{3.13}$$

so we get constant curvature fluctuations with an amplitude depending only on H. On the other hand, if the Universe starts off and ends with $p=\rho/3$, and only goes through an intermediate phase with $p\neq\rho/3$, we get[64]

$$\left(\frac{\delta\rho}{\rho}\right)_H \sim H\left[\int_{k_H}^{\sqrt{H}} (k+1)\, dk\right]^{1/2} \tag{3.14}$$

This corresponds to fluctuations with $\alpha=5/6$, steepening to $\alpha=1$ at small scales. These fluctuations are cut off before they reach unity and so can can be of order $10^{-3}$ on a galactic scale without making too many black holes on smaller scales. Note that the values of $\alpha$ in these cases are only "effective" values since the fluctuations do not really attain the relevant amplitude until they enter the horizon.

3.3 Inflation effects

In the original inflation scenario, the Universe undergoes an exponential expansion phase as a result of getting trapped in a false vacuum state when spontaneous symmetry breaking occurs at the GUT epoch.[9] This can be ensured providing there is a potential barrier separating the symmetric and non-symmetric state of the Higgs field $\phi$. Unfortunately, this scheme fails because the formation rate for the bubbles of broken symmetry is never large enough for the Universe to get out of the exponential expansion phase.[65] In the "new" inflationary scenario, the form of the potential is chosen to be very flat near the origin, so that $\phi$ rolls down to its new minimum only very slowly. This means that a single bubble of broken symmetry can grow large enough to contain the entire visible Universe.[66-68] When $\phi$ reaches its minimum, the Universe will be reheated by the dissipation of the field's kinetic energy, so this scheme retains the advantages of the original scenario (in that it solves the flatness problem, the horizon problem, and the monopole problem) without its disadvantage.

In standard SU(5) one expects the scalar potential to have the Coleman-Weinberg form[69]

$$V(\phi) = A\phi^4 \left[\ln(\phi^2/v^2) - (1/2)\right] + D\phi^2 + (1/2)Av^4 \qquad (3.15)$$

where A=0.05, D is an effective mass term, and v is the value of $\phi$ at the minimum. The evolution of $\phi$ is then determined by the equations

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + (\partial V/\partial\phi) = 0, \quad H^2 = (8\pi/3M_p{}^2)\left[V+(1/2)\dot{\phi}^2+\rho_r\right]. \qquad (3.16)$$

The $\ddot{\phi}$ term is negligible initially and the $\Gamma\dot{\phi}$ term (which is related to particle creation) only becomes important at reheating, so the time for the Universe to roll down to the new minimum is

$$\tau \approx 3H/\frac{\partial^2 V}{\partial \phi^2} \approx \frac{3H}{2D} \ . \tag{3.17}$$

This process will produce density fluctuations because the field will not roll down the potential homogeneously.[70-75] Some regions will roll down faster or slower than others because of quantum fluctuations in $\phi$. The time-spread is

$$\nabla\tau \approx \frac{\delta\phi}{\dot{\phi}} \approx \frac{3H^2}{4\pi^{3/2}}/\frac{\partial V}{\partial \phi} \tag{3.18}$$

and the associated horizon-epoch fluctuations are

$$(\delta\rho/\rho)_H \approx 2\sqrt{2}\ H\ \Delta\tau \approx \frac{3H^3}{\sqrt{2}\pi^{3/2}}/\frac{\partial V}{\partial \phi} \ . \tag{3.19}$$

(This spectrum applies both when the fluctuations go outside the horizon in the deSitter phase and when they re-enter it in the Freidmann phase.) If the potential is given by eqn (3.15), the fluctuations can be shown to have the nearly scale-invariant form

$$(\delta\rho/\rho)_H \approx \left[-\frac{32A}{3\pi^3}\ \ln\left(\frac{H}{v}\right)\right]^{1/2} \left(\ln\frac{H}{k}\right)^{3/2} . \tag{3.20}$$

On a galactic scale this is about 50, which is much too large to be consistent with observation.

One way round this problem is to invoke supersymmetry effects. In a broken supersymmetry theory, the mass splitting between boson and fermion states reduces the coefficient A in eqn (3.15) and hence the amplitude of the fluctuations.[76,77] The amplitude will be of order $10^{-3}$ (as required) providing the supersymmetry scale is around $10^{10}$GeV. On the other hand, if the inflation occurs at the GUT epoch, it turns out that the roll-over time can be long enough to be interesting only with very fine tuning of the parameters. This unnaturalness can be avoided if the inflation is presumed to occur before the GUT epoch (perhaps even near the Planck time). Such a "primordial" inflation scheme might in principle be able to produce scale-invariant fluctuations with the required amplitude. On the other hand, it may have difficulty ensuring enough reheating at the end of the inflation phase[78] and it does not eliminate the monopole problem.

3.4 Other spontaneously broken symmetry effects

Various effects at a spontaneously broken symmetry epoch could induce fluctuations even without inflation. For example, Press[79] has suggested that fluctuations of the form

$$\left(\frac{\delta\rho}{\rho}\right)_o \approx \left(\frac{T_o}{M_p}\right)^2 \left(\frac{M}{M_o}\right)^{-2/3} \tag{3.21}$$

could arise at the GUT phase transition ($T_o \approx 10^{15}$GeV) as a result of statistical gradients in the phase angle which prescribes the rotation of the vacuum state in its group space. One may regard such fluctuations are arising from patchiness in the conversion of the vacuum energy into thermal energy at the phase transition. However, there is a serious problem with this scenario: if both thermal energy and vacuum energy gravitate in the same way, the patchiness introduced would only represent a fluctuation in the stress tensor and not the energy density itself. In this case, eqn (2.7) suggests that $\alpha$ would be 4/3 rather than 2/3. Another criticism is that phase gradients of this kind are usually compensated by gauge fields.[80]

A more plausible way in which fluctuations could arise at a spontaneously broken symmetry epoch would be through the formation of domains.[81,82] The field will always tend towards uniformity if it is possible to go between states in neighbouring regions by a continuous transition in the group space. However, this possibility is precluded if the field contains trapped singularities of some kind. Depending on their dimensionality, these singularities correspond to either domain walls or strings or monopoles. The first case is excluded because it would produce unacceptably large anisotropies in the 3K background radiation;[83] the last case is excluded because the mass density associated with the monopoles would exceed that permitted by observations of the cosmological deceleration parameter unless inflation were invoked.[84,85] We therefore focus on strings.

Several types of fluctuations could be generated by strings. The strings will have mass per unit length $\mu \sim T_o^{\,2}$ and an equivalent tension. As they enter the horizon, this tension will tend to straighten them out and Zeldovich[80] has suggested that the dissipation involved will generate horizon-scale fluctuations whose amplitude is of order the ratio of the string density to the total density. This ratio is just $G\mu \sim (T_o/M_p)^2$, so one gets fluctuations of the form prescribed by eqn (3.21).

The physics behind this suggestion is rather vague but a more specific possibility arises if the strings form closed loops (either because they start off that way or because they generate loops through self-intersection).[86-92] One would expect the formation of roughly one loop per horizon volume at each epoch, so the associated horizon-scale density fluctuation would just be of order the loop mass over the horizon mass, which is again given by eqn (3.21). Since the loop perturbations are isothermal and uncoupled to the background radiation, the resultant fluctuations in the baryon density at decoupling will be as indicated by the dotted lines in Figure 1.[88] However, loops of initial size R also have the property that they will decay through gravitational radiation on a timescale $R/G\mu$. This means that only strings larger than $(G\mu)^2 M_e$ in a baryon-dominated Universe or $(G\mu)^2 M_\nu$ in a neutrino-dominated Universe will survive long enough to induce fluctuations in the rest of the matter. In both cases the lower cut-off in the resultant fluctuations will be at a mass of order $10^9 M_\odot$. In an axion-dominated Universe, there will be no such cut-off because the axions are affected by the strings even when the Universe is radiation-dominated, but the baryons themselves will still be unable to clump on scales below $M_b$. In all three cases the resultant fluctuations might be consistent with eqn (2.1) but only if $G\mu \sim 10^{-6}$-$10^{-5}$, so the strings have to form at the GUT epoch.

### 3.6 Axionic fluctuations

The possibility that axions could dominate the density of the Universe arises because the axion developes a mass $m_a$ due to QCD instanton effects at a temperature $T_I \sim 1\,\mathrm{GeV}$. The associated density is

$$\rho_a = 3\left(\frac{f_a{}^2}{M_p T_I}\right) m_a T^3 \langle \theta_a^2 \rangle \qquad (3.22)$$

where $f_a \sim 10^{12}$GeV is the scale at which the Peccei-Quinn symmetry is broken[126] and the last term is the mean-square value of the Peccei-Quinn angle. Since T is determined by the radiation density, this means that any adiabatic fluctuations would necessarily induce axionic fluctuations of the same form at the horizon epoch:[93]

$$\left(\frac{\delta\rho_a}{\rho_a}\right)_H = \frac{3}{4}\left(\frac{\delta\rho_r}{\rho_r}\right)_H \quad . \tag{3.23}$$

This conclusion applies even if the Universe undergoes an inflationary phase at $f_a$. However, in this case extra horizon-scale fluctuations of the form[45]

$$\left(\frac{\delta\rho_a}{\rho_a}\right)_H \approx \left(\frac{f_a}{M_p\theta_a}\right) \approx 10^{-7}\theta_a^{-1} \tag{3.24}$$

arise due to the inflation itself. These fluctuations are associated with quantum variations in $\theta_a$ and are too small to be interesting unless the Universe is within a domain where $\theta_a$ is smaller than average.[94]

A more exotic way of generating density fluctuations in the axion scenario invokes a combination of strings and domain walls[95]. The strings are produced at the Peccei-Quinn symmetry-breaking and then become attached to walls at the QCD phase transition. These walls will decay through gravitational radiation at around $10^4$s, just when they are beginning to dominate the density, and this means that one generates fluctuations of order unity on the mass-scale of the horizon at that time. This could result in the formation of $10^6 M_\odot$ black holes.

### 3.6 Grain fluctuations

At a certain level the Universe is bound to develop graininess, in particular, whenever it undergoes a phase transition. In the most natural situation the grains would have a mass of order the temperature of the phase transition, though in this case the associated fluctuations would be too small to be interesting.[96] However, in more exotic situations one could envisage grains which are as large as the particle horizon at the phase transition. For example, in the Guth inflationary scenario, one could produce black hole grains of $10^3$g through the collisions of bubbles of broken symmetry.[97,98] A similar effect at the Weinberg-Salam epoch could produce black hole grains of mass $10^{-5}M_\odot$. At the quark-hadron phase transition, the effects of colour-screening could generate density fluctuations large enough to produce black holes with mass up to $10^{-3}M_\odot$.[99] These effects all require 1st order phase transitions.

Since $10^{-5}$s is the latest time at which one could expect a phase transition in a hot Universe, one might presume that one could never get grains forming later than this. However, this conclusion could fail in less conventional cosmological models. In a cold Universe, for example, grains of mass $10^{-6}$-$10^{-1}M_\odot$ could form during the pion-nucleon phase transition at $10^{-4}$s[100,101] and the spontaneous shattering of metallic hydrogen at 1 s could produce fragments of $10^{-8}M_\odot$.[102,103] In the axionic wall-string scenario of Sec. 3.5, one could form black hole grains of $10^{6}M_\odot$ at around $10^{4}$s.[95] One could even conceive of situations in which the grains would be larger than the usual particle horizon size at formation. For example, with sufficiently fine tuning, bubbles of broken symmetry might arise at an inflationary epoch which are larger than the horizon size but smaller than the present Universe; these bubbles could then serve as grains.[96,104] In a cold Universe, a sufficiently slow quark-hadron phase transition could produce baryon clumps of $10^{6}M_\odot$ before the Universe is reheated by the associated radiation production.[105]

Whenever the Universe developes graininess like this, one would expect the spontaneous generation of statistical density fluctuations: if the fraction of the Universe which goes into grains is $f_*$ and the grain mass is $M_*$, the number of grains forming in regions of volume V would on average be $N=f_*V/M_*$ with a statistical fluctuation $\Delta N=\sqrt{N(1-f_*)}$. Since the grains form out of the background radiation, this does not in itself produce a fluctuation in the total density. For each region with a $\sqrt{N}$ excess of grains must have a corresponding deficit in its radiation content. The grain effect merely corresponds to a pressure perturbation[6]

$$\left(\frac{\Delta p}{p}\right)_o \simeq N^{-1/2} f_* (1+\tfrac{3}{4}f_*)^{-1}. \qquad (3.25)$$

However, as indicated by the discussion in Sec. 2.3, one expects this to lead to a genuine density perturbation

$$\left(\frac{\delta\rho}{\rho}\right)_o \approx \left(\frac{\Delta p}{p}\right)_o \left(\frac{M}{M_o}\right)^{-2/3} \approx N^{-7/6}\left(\frac{f_* M_\Omega}{M_*}\right)^{2/3} f_* \tag{3.26}$$

after an expansion time. Thus the effective value of $\alpha$ is 7/6. One might think that such fluctuations would not arise if the entire Universe went into the grains since $\Delta N=0$ if $f_*=1$. However, even in this case, fluctuations arise because non-linear effects will endow the grains with peculiar velocities. One can show that the consequent displacements will generate fluctuations after an expansion time of exactly the same form as given by eqn (3.26).[106,107] This result applies even on scales larger than the horizon, providing one measures the density fluctuations with respect to the comoving hypersurface.[96]

If the grains persist indefinitely, eqn (3.26) implies that there will be both adiabatic and isothermal fluctuations at decoupling of the form

$$\left(\frac{\delta\rho}{\rho}\right)_d \approx 10^{-3}(10\Omega)\left(\frac{M}{10^{12}M_\odot}\right)^{1/2}\left(\frac{M_*}{10^6 M_\odot}\right)^{1/2} f_e^{1/2} \tag{3.27}$$

where we have normalized M to the characteristic mass of a galaxy, $f_e$ is the fraction of the Universe's mass in grains at $t_e$, and the factor of $10\Omega$ is present only if the grains are uncoupled to the background radiation. If the grains disappear at some stage, eqn (3.27) still pertains except that $f_e$ must be replaced by the value which pertains when they last exist and the fluctuations will be purely adiabatic.The fact that one gets "white noise" fluctuations is consistent with the observed galaxy correlation function.[18] On the other hand, we see that galactic-scale fluctuations can have an amplitude of $10^{-3}$ (as required) only if $M_* > 10^6 M_\odot$, so one needs rather exotic grains unless one invokes the sorts of amplification processes discussed in Sec. 4. In a cold Universe, even grains as small as $10^{-4}M_\odot$ could generate galactic-scale fluctuations of $10^{-3}$, but the fluctuations on larger scales would fall off as $M^{-7/6}$, which is too steep to explain the correlation function.[96]

## 4. GENERATING GALAXIES VIA ASTROPHYSICAL AMPLIFICATION

As emphasized in the Introduction, the present cosmological structure need not directly reflect the initial density fluctuations if the first scales to bind are smaller than galaxies. This is because astrophysical activity associated with pregalactic objects could itself generate large-scale structure.[108,123] Figure 1 shows that one expects pregalactic objects to form unless the fluctuations are adiabatic and the Universe is dominated by neutrinos or baryons. We now discuss several scenarios for such astrophysical activity.

### 4.1 Peculiar velocity effects

In this scenario[108] the first pregalactic objects are born with large peculiar velocities; this might occur, for example, if the objects form black holes and collapse non-axisymmetrically. This generates $\sqrt{N}$ fluctuations on scales up to the distance the objects can traverse in a cosmological time. If the objects have a mass $M_1$ and form at a redshift $z_1$ with peculiar velocity $V_1$, the corresponding mass-scale is

$$M_2 \simeq 10^{12}\,(z_1/10^3)^{-3/2}\,(V_1/10^{-2}c)^3\,M_\odot\ . \tag{4.1}$$

Fluctuations below this scale are erased by "free-streaming", while those produced on scales above $M_2$ go like $N^{-7/6}$ rather than $N^{-1/2}$ for the reasons indicated in Sec. 3.6. Thus the second generation of objects to form have the mass $M_2$ and bind at a redshift

$$z_2 \simeq \left(\frac{M_1}{10^6 M_\odot}\right)^{1/2}\left(\frac{z_1}{10^3}\right)^{7/4}\left(\frac{V_1}{10^{-2}c}\right)^{-3/2}\ . \tag{4.2}$$

In order to have galactic scales or larger bind by the present epoch, we require

$$V_1 > 3000\left(\frac{z_1}{10^3}\right)^{1/2}\ \mathrm{km\ s^{-1}},\ M_1 > 10^6\left(\frac{V_1}{3000\mathrm{kms^{-1}}}\right)^2 M_\odot\,. \tag{4.3}$$

We note that the fluctuations on scales above $M_2$ fall off too steeply to be consistent with the galaxy correlation function, so this affect alone could not explain the observed large-scale structure. Perhaps the most interesting feature of the velocity effect is that the pregalactic objects only cluster on sufficiently large scales.

4.2 Statistical clustering effects

If the first pregalactic objects form in the radiation-dominated era (which probably requires that they be strings or black holes), then one expect $\sqrt{N}$ pressure perturbations to arise on all scales providing the probability of objects forming in different regions is uncorrelated.[52,109,125] This is just a variant of the grain fluctuation scenario and, if the objects dominate the density, one expects the fluctuations at decoupling to have the form

$$\left(\frac{\delta\rho}{\rho}\right)_d \approx 10^{-2}\left(\frac{M}{10^{12}M_\odot}\right)^{-1/2}\left(\frac{M_1}{10^6 M_\odot}\right)^{1/2}\Omega_1 \qquad (4.4)$$

where $\Omega_1$ and $M_1$ are the density and mass of the objects. Thus galaxies can bind by now providing $M_1 > 10^4\Omega_1{}^{-2}M_\odot$. Since one has $M^{-1/2}$ fluctuations, this might also explain the galaxy correlation function. We stress that the statistical clustering effect does not work for objects which form when the Universe is matter-dominated because one needs the objects to initially have a different equation of state from the background. We also note that the effect is distinct from the "statistical bootstrap" scheme, in which $\sqrt{N}$ fluctuations are continually generated even after decoupling;[127] such a scheme now seems implausible.[18,128]

4.3 The seed effect

If the first objects to form are sufficiently large, they could bind galactic scales merely by their gravitational Coulomb effect. This is because each object induces an effective density fluctuation $M_1/M$ on a surrounding region of mass M. Thus one can bind galaxies by today providing $M_1 = 10^9 M_\odot$. Such pregalactic objects might arise fairly naturally in the string scenario, where (as discussed in Sec. 3.4) the smallest surviving loops at decoupling have a mass of the required size.[86] Giant black holes might also serve as the necessary seeds.[108-111] (Such holes might even be generated by loops[92]). Since there is evidence that large black holes reside in at least some galactic nuclei (e.g. to power quasars[112,113] and explain certain dynamical observations[114,115]), it is attractive to suggest that these same holes might also produce the galaxies. A natural consequence of the seed scenario is that galaxies should have a density profile which scales as $r^{-9/4}$; this is fairly close to the $r^{-2}$ profile inferred for the dark halo distribution.[116] One can also predict a galaxy correlation function. If the seeds have a discrete mass spectrum, the effective decoupling fluctuations on scales larger than galaxies should go as $M^{-1}$, which is probably too steep. However, if the seeds have a mass spectrum which falls off as $M^{-\gamma}$, the decoupling fluctuations should have the form[108]

$$(\delta\rho/\rho)_d \propto M^{-\beta}, \quad \beta = \left(\frac{\gamma-2}{\gamma-1}\right) . \tag{4.5}$$

If the seeds are primordial black holes or loops, one expects $\gamma = 2.5$.[56] This implies $\beta$=1/3, which would be consistent with eqn (2.1).

4.4 The explosion scenario

Stars in the mass range $10\text{-}10^5 M_\odot$ can release explosive energy with an efficiency $\epsilon \simeq 10^{-4}$ at the termination of nuclear burning. Thus the first stars (or clusters of stars) could generate shockfronts which sweep up shells of gas.[117-120] The mass of these shells could exceed the mass of the original objects by a large factor: in the Compton cooling era (z>10) this factor is[120]

$$\xi \simeq 10^5 z_1^{-1.7}\left(\frac{\epsilon}{10^{-4}}\right)^{0.6}\left(\frac{M_1}{10^6 M_\odot}\right)^{-0.4} \tag{4.6}$$

so even a single explosive phase could amplify the scale of structure from $10^6 M_\odot$ to $10^{11} M_\odot$. Furthermore, it is possible that the shells would fragment into more exploding stars, in which case one could initiate a bootstrap process in which the shells grow ever larger. In this "Hierarchical Explosion Scheme" the shell mass would tend to an asymptotic limit[121]

$$M_\infty = \xi^{2.5} M_1 \simeq 10^{20} z_1^{-4.3} (\epsilon/10^{-4})^{1.5} M_\odot \tag{4.7}$$

providing the shells do not overlap first. After z=10, radiative cooling dominates and the shells would fragment into galactic-size objects rather than stars. Thus the end result of this picture might well be clusters of galaxies.[122] One might even generate giant voids: the final shell radius could be as large as $\sqrt{\epsilon}$ times the horizon size, which would just about be enough to explain Bootes.[121] Of course, this scheme works only if the fraction of the Universe in the initial seeds is very tiny; otherwise the shells would overlap too soon. It should also be stressed that invoking explosions to generate large-scale structure is very extravagant energetically. A somewhat less extravagant scheme would be to invoke the UV fronts produced by the stars in their main-sequence phase.[123,124]

This (by no means comprehensive) discussion illustrates that there are a variety of ways in which pregalactic objects could superpose new fluctuations on the ones that originally existed at decoupling. If the objects form at redshift $z_1$, Figure 2 illustrates what value of $M_1$ is required to generate galaxies: the lower boundaries are specified by eqn (4.2) with $z_2=1$ in the recoil scenario, by eqn (4.4) with $(\delta\rho/\rho)_d=10^{-3}$ and $\Omega=0.1$ in the clustering scenario, and by eqn (4.7) with $M=10^{12}M_\odot$ in the explosion scenario. For example, the $10^6 M_\odot$ objects which one would expect to bind first in many situations would suffice providing they formed in the period $z>10^3$ or $10<z<70$. Even the grain effect can produce such objects if the grain mass exceeds 1 $M_\odot$. In fact, providing the grains are uncoupled to the 3K background, the first objects could be somewhat smaller than $10^6 M_\odot$, so even the sort of black holes which might form at the quark-hadron phase transition could serve as the necessary grains.[125]

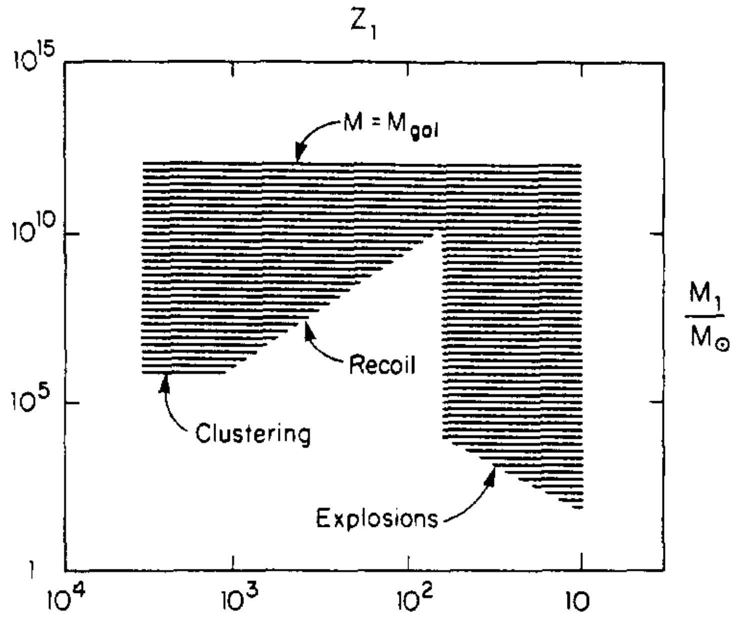



FIGURE 2

The shaded region shows the mass and redshift range in which pregalactic objects can generate galaxies by the various types of astrophysical activity discussed in the text.

## 6. CONCLUSIONS

In this talk I have emphasized the possibility that the density fluctuations required to explain the present cosmological structure may have arisen spontaneously at a phase transition in the early Universe. If one accepts this point of view, various consequences follow.

Firstly, it is clear that spontaneous fluctuations could have many different forms, so one should not necessarily expect the large-scale structure to originate from any single effect. The largest scale features (such as the giant voids and filaments) and the galaxy correlation function may well derive from completely different mechanisms. This is gratifying since it appears to be remarkably difficult to explain everything with a simple power law spectrum of fluctuations. Nor is it obvious that all features of the large-scale structure have to be gravitational in origin.

Secondly, it may be wrong to put too much emphasis on fluctuations with $\alpha=2/3$ (as most of the numerical simulations do). One can argue for the existence of fluctuations with $\alpha=1/3$, 1/2, 5/6, 1 or 7/6, all of which reproduce at least some feature of the large-scale structure. Admittedly, $\alpha=2/3$ fluctuations (such as arise in the inflationary scenario) are cleanest, in the sense that they avoid the complications of pregalactic objects forming, but the mechanisms for producing them all involve ultraspeculative physics and only seems to work with rather contrived parameters.

Thirdly, it is possible that part of the large-scale structure derives from relatively late phase transitions (like the quark-hadron transition), which are likely to be understood well before the more exotic phase transitions upon which cosmologists tend to focus at present. Admittedly, the schemes which invoke late phase transitions may have to be supplemented with astrophysical amplification mechanisms and these might be regarded as no less exotic than the physics of the very early Universe. Nevertheless, in my opinion at least, they should be taken just as seriously.

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