SOME COMMENTS ON α_s and $\Lambda_{\overline{MS}}$

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Summary

Some new determinations of the strong coupling constant α_s from hadronic and leptonic decay widths of quarkonia are added to the accumulation of data on α_s as a function of Q. When compared with the renormalization group prediction of $1/\alpha_s$ versus ln Q, parameterized by the QCD scale parameter $\Lambda_{\overline{MS}}$, these new points do very little to resolve whether α_s "runs" as predicted, and if so, on which $\Lambda_{\overline{MS}}$ curve.

Motivation

One frustration in predicting physics at SSC energies is our inability to pin down the strong coupling constant α_s and the QCD scale parameter $\Lambda_{\overline{MS}}$. If our data and our models were better, we would certainly have a good asymptotic estimate of $\Lambda_{\overline{MS}}$, since we have data ranging over more than two decades in Q, from roughly 0.3 GeV to 30 GeV. This problem even merits an Appendix in the most recent Review of Particle Properties.[1]

I will show later that the hadronic decay width Γ_{ggg} is proportional to α_s^4 and the leptonic decay width is proportional to α_s. Robert Knuteson and I[2] have used these widths from the strange, charm, and bottom quarkonium S-states to get values of α_s. We had hoped that our values would make it easier to determine the asymptotic $\Lambda_{\overline{MS}}$, but our conclusion is that we still cannot predict $\Lambda_{\overline{MS}}$ at Q = 1 TeV from the world accumulation of α_s data to better than a band ranging roughly from 50 MeV to 200 MeV. In fact, we show other determinations which would place $\Lambda_{\overline{MS}}$ between about 15 MeV and 35 MeV at Q = 1 TeV.

Since beginning this project in mid-1983, we have looked into the other values of α_s more carefully, as have many other people.[3] We include in our data sample only second order QCD models and only determinations from decay widths, ratios of decay widths, and jet analyses. We have some biases on which values of α_s are more likely to be correct, which we will justify in our longer paper.[2]

At each new flavor threshold, the value of $\Lambda_{\overline{MS}}$ changes for a given renormalization group curve. We now know enough about the top quark mass to make an educated guess at the six-flavor value of $\Lambda_{\overline{MS}}$. Certainly we cannot count on there being only three generations, but if no new quarks appear then $\Lambda^{(6)}_{\overline{MS}}$ is the value of interest for SSC energies and for the calculation of the proton lifetime, which is proportional to $(\Lambda^{(6)}_{\overline{MS}})^4$.

The Renormalization Group

The approximate solution to the renormalization group equation relating α_s to Q is

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_0} + \frac{b_0}{2\pi} \ln \frac{Q^2}{Q_0^2} + \frac{b_1}{\pi b_0} \ln \frac{\alpha_0}{\alpha_s} , \tag{1}$$

where

$$b_0 = \frac{11}{2} - \frac{n_f}{3} \tag{2}$$

and

$$b_1 = \frac{1}{4} \left(51 - \frac{19}{3} n_f\right) , \tag{3}$$

with n_f the number of lighter quark flavors. Thus we can generate a curve for $1/\alpha_s$ by choosing an α_s at a given Q_0 and knowing n_f for each Q. At each new quark threshold, the value of α_s at threshold becomes the new α_0, and b_0 and b_1 take on new values.

Equation (1) can be converted to a parameterization in terms of the QCD scale parameter $\Lambda^{(n_f)}_{\overline{MS}}$ instead of in terms of α_0 and Q_0,

$$\frac{1}{\alpha_s} = \frac{b_0}{2\pi} \ln \frac{Q^2}{\Lambda^{(n_f)2}_{\overline{MS}}} + \frac{b_1}{\pi b_0} \ln \ln \frac{Q^2}{\Lambda^{(n_f)2}_{\overline{MS}}} + \text{constant}. \tag{4}$$

The value of $\Lambda^{(n_f)}_{\overline{MS}}$ also changes at each quark threshold, to keep the $1/\alpha_s$ curve smooth.

In Fig. (1) I show eight curves calculated from Eq. (1), with the asymptotic value of $\Lambda^{(6)}_{\overline{MS}}$ labeling each curve. I have chosen "round" $q\bar{q}$ thresholds of 0.5 GeV for $s\bar{s}$, 3 GeV for $c\bar{c}$, 10 GeV for $b\bar{b}$, and 80 GeV for $t\bar{t}$. I do not extend most of the curves below the value of $1/\alpha_s = 3$, since $\alpha_s > 1/3$ is a large value for perturbation calculations. One can use Eq. (2) to see how $\Lambda^{(n_f)}_{\overline{MS}}$ "runs" with Q: $\Lambda^{(3)}_{\overline{MS}} \approx 100$ MeV below the charm threshold will dwindle to $\Lambda^{(6)}_{\overline{MS}} = 20$ MeV at 1 TeV.

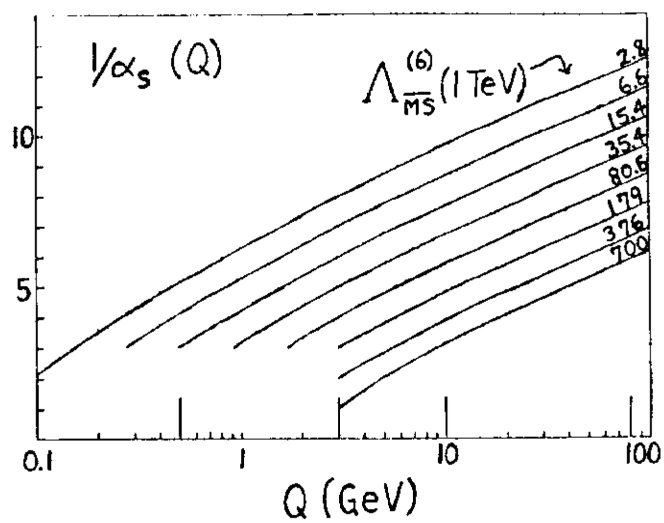

Fig. 1
The inverse strong coupling constant $1/\alpha_s$ plotted as a function of Q from Eq. (1). The marks at Q = 0.5, 3, 10, and 80 GeV are the thresholds for $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$, respectively. Thus n_f changes from 2 to 3 at Q = 0.5 GeV, etc. The eight curves shown are a family chosen to have $1/\alpha_s = 1, 2, . . . ,8$ at $Q_0 = 3$ GeV. The curves are labeled by the value of $\Lambda^{(6)}_{\overline{MS}}$ evaluated at 1 TeV. All values of $\Lambda^{(6)}_{\overline{MS}}$ are given in MeV.

Decay Width Formulas

The expressions for the decay widths of quarkonium in the reaction $e^+e^- \to q\bar{q} \to ggg$ or $e^+e^- \to q\bar{q} \to \ell^+\ell^-$, where g means gluon and ℓ means lepton, are used to extract α_s from measured decay widths Γ^{nS}, where nS refers to the nth S-state.

The gluon decay width is given by

$$\Gamma_{ggg}^{nS} = \frac{160}{81}(\pi^2-9)\,\alpha_s^3\,\frac{|\psi^{nS}(0)|^2}{M_n^2}(1-\Delta_{ggg}^n), \qquad (5)$$

where ψ^{nS} is the wave function for the nth S-state and $M_n = 2m_q + E_n = W$ is the mass of the nth $q\bar{q}$ bound state, or the total energy W of the e^+e^- pair.

The correction factor Δ_{ggg}^n is given by QCD, in the $\overline{MS}$ renormalization scheme,

$$\Delta_{ggg}^n = \frac{\alpha_s}{\pi}\{19.4-\tfrac{3}{2}(11-\tfrac{2}{3}n_f)[1.161+\ln(\frac{2M}{M_n})+O(\frac{v^2}{c^2},\frac{\alpha_s^2}{\pi^2})]\}. \qquad (6)$$

A feature to be noted about Γ_{ggg}^{nS} is that it is proportional to α_s^3, hence very sensitive to α_s. Another feature is $|\psi^{nS}(0)|^2$, which has customarily been regarded as a nuisance to be canceled out of the expression. We make use of a formula for $|\psi^{nS}(0)|^2$ and do not have to try to cancel this factor.

The leptonic width is

$$\Gamma_{\ell^+\ell^-}^{nS} = 16\pi e_q^2\,\alpha^2\,\frac{|\psi^{nS}(0)|^2}{M_n^2}\,(1-\frac{v^2}{3})(1-\Delta_{\ell^+\ell^-}^n), \qquad (7)$$

where e_q is the quark charge in units of e. The correction factor $\Delta_{\ell^+\ell^-}^n$ is

$$\Delta_{\ell^+\ell^-}^n = \frac{16\alpha_s}{3\pi}[1+.046v-v(1-v)^2]+(24.26-.115n_f)\frac{\alpha_s^2}{\pi^2}. \qquad (8)$$

We note that $\Gamma_{\ell^+\ell^-}^{nS}$ is not at first glance proportional to α_s at all. The fine structure constant α appears, but no α_s. Since $|\psi^{nS}(0)|^2$ also appears, it has been useful to previous authors[4] to take the ratio of decay widths to cancel the wave function at the origin. This leaves

$$\frac{\Gamma_{ggg}^{nS}}{\Gamma_{\ell^+\ell^-}^{nS}} \sim \alpha_s^3. \qquad (9)$$

Durand and Durand[5] have shown that the wave function at the origin satisfies

$$|\psi^n(0)|^2 \simeq [\frac{4\pi\alpha_s}{3v}\,\frac{1}{1-\exp(-4\pi\alpha_s/3v)}]\,v\frac{M_n^2}{16\pi^2}\,\frac{dM_n}{dn}. \qquad (10)$$

The factor in square brackets is the Coulomb factor (the Coulomb wave function at the origin). The velocity v is the relativistic velocity of a free quark with half the total energy M_n,

$$v_n = (1 - 4m_q^2/M_n^2)^{1/2}. \qquad (11)$$

This wave function formula is very accurate for $q\bar{q}$ potentials which can be solved analytically or numerically. It gives errors in the ground states of 5 - 10% and for all n > 1 the errors are less than 1%.

Having the relationship in Eq. (10), we can use both hadronic and leptonic decay widths to extract α_s. $|\psi^n(0)|^2$ is itself proportional to α_s. This means that

$$\Gamma_{ggg}^{nS} \sim \alpha_s^4 \qquad (12)$$

and

$$\Gamma_{\ell^+\ell^-}^{nS} \sim \alpha_s \qquad (13)$$

so that leptonic widths give us some information about α_s, and gluonic widths are more sensitive to α_s on their own than in ratios.

Our method involves fitting a smooth curve to the spectrum M_n for the S-states, so that we can get the factor dM_n/dn, the inverse density of states. We use generally accepted values of quark masses m_q, mainly from nonrelativistic potential model fits, $m_s = .225$ GeV, $m_c = 1.35$ GeV, $m_b = 4.7$ GeV. We choose $Q=M=M_n/2$ in Eq. (6) to make the ln factor zero. The constants in Eq. (8) are calculated for $Q=M_n$, so we plot leptonic width data at M_n.

The Data

In Fig. (2) on the next page we show a sample of our data. We have used the most recent Reviews of Particle Properties[1] for our values of Γ. The leptonic widths used were all e^+e^- decays of the four upsilon S-states. The errors on these widths are compounded from the errors on the total decay width and on the e^+e^- branching fractions, and are in the 25% to 50% range. These large errors convert to a huge spread in $1/\alpha_s$. Since Eq. (10) for $|\psi(0)|^2$ is not as accurate for the 1S state as for n > 1 (< 1% error), we correct all 1S states by a factor 1.057, the error in our formula used in a Coulomb plus linear potential.

The errors on hadronic widths have been taken as the simple percent error of the total width. That is, we have not made a compound error using all hadronic branching fractions. The four upsilon data points cluster nicely.

Both the ψ(1S) and ϕ(1S) points are probably not the best we could do: we have not separated out the radiative decays from the ψ width, and we probably should use a smaller value of m_s for the ϕ width. Both of these changes would lower $1/\alpha_s$. Of course $\alpha_s > 1/3$, or $1/\alpha_s < 3$, is not the best place to be using QCD formulas.

In Fig. (3) on the next page we show six values of $1/\alpha_s$ calculated from ratios of decay widths, and just four from recent DESY groups' jet analyses.

The point near Q = 0.5 GeV is calculated from the ratios $\Gamma_{\gamma gg}/\Gamma_{ggg}$ ($M = .157\,M_n$) for the ψ(1S) state.[4] The two points below Q = 1.5 GeV are from CUSB (upper) and CLEO (lower) Υ(1S) data,[4] and the point above Q = 1.5 GeV is from CUSB Υ(2S) data,[4] again $\Gamma_{\gamma gg}/\Gamma_{ggg}$ ($M = .157\,M_n$). The two points near Q = 4.5 GeV are from $\Gamma_{ggg}/\Gamma_{\ell^+\ell^-}$ ($M = .48\,M_n$) for Υ(1S) (lower) and Υ(2S) (higher).[4]

The two upper points near Q = 35 GeV are calculated using the independent fragmentation model and the two lower points using the string fragmentation model. The top point is from JADE, the next down from CELLO, the next down from JADE, and the bottom one from CELLO. In all cases the FKSS method with y cuts, and energy-energy correlation selection, were used.[6]

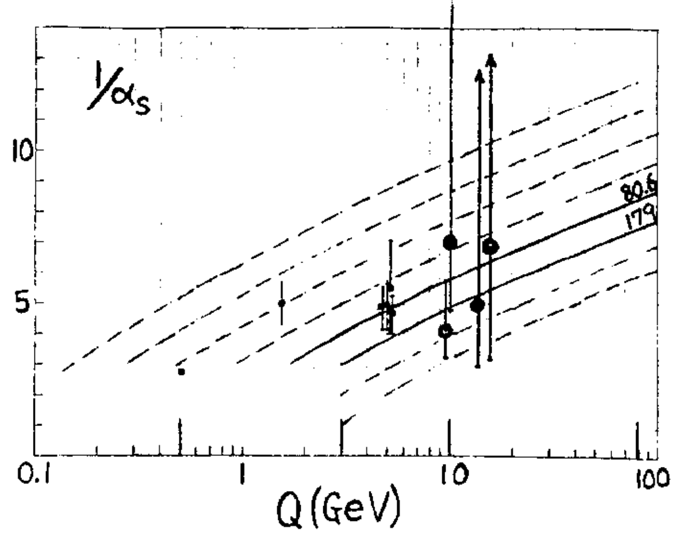

Fig. 2

The inverse strong coupling constant $1/\alpha_s$ calculated from four leptonic and six hadronic decay widths, all taken from the Review of Particle Properties.[1] The four points around 10 GeV are from $\Gamma^n_{e^+e^-}$ of the four upsilon S-states. $\Gamma_{leptonic}$ is roughly proportional to α_s, but Eq. (7) is sufficiently nonlinear in α_s to make a spread in the measured $\Gamma_{e^+e^-}$ cause a <u>very</u> large uncertainty in $1/\alpha_s$. The upper error bars on the 3S and 4S states are out of the room. The four points clustered around 5 GeV are from Γ^n_{ggg} of the upsilon S-states, calculated at $Q = M = M_n/2$. The two lower points at $Q \sim 1.5$ and 0.5 are from the hadronic decay widths of the $\psi(1S)$ and $\phi(1S)$, again plotted at $M_n/2$. The band of $\Lambda^{(6)}_{\overline{MS}}$ is my guess of a reasonable range from these data points, with the above comments taken into consideration.

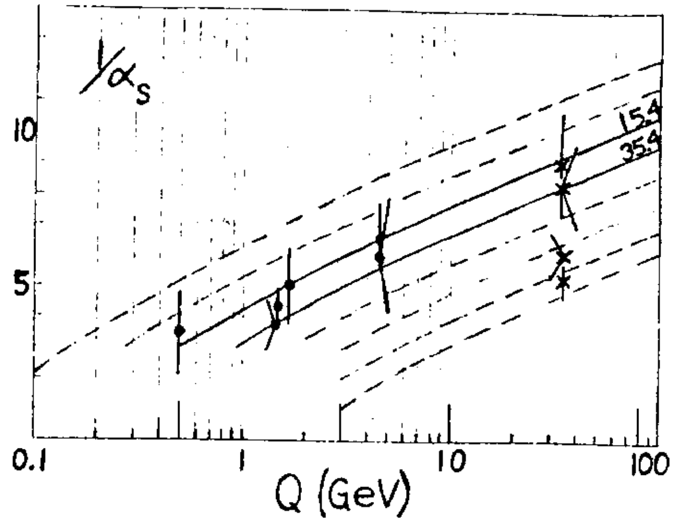

Fig. 3

The inverse strong coupling constant $1/\alpha_s$ calculated from six ratios of decay widths and four jet analyses. The four lowest points[4] are from $\Gamma_{\gamma gg}/\Gamma_{ggg}$ ($M = .157\ M_n$) for the $\psi(1S)$, two $\Upsilon(1S)$ and the $\Upsilon(2S)$ states. The two middle points[4] are from $\Gamma_{ggg}/\Gamma_{\ell^+\ell^-}$ ($M = .48\ M_n$ for the $\Upsilon(1S)$ and $\Upsilon(2S)$ states. The errors are quoted by the authors. The four highest points are my selection from recent second-order jet analyses.[6] In each pair near 35 GeV, the upper point is from JADE and the lower from CELLO. The upper pair use independent fragmentation and the lower pair use string fragmentation. The errors are my calculations from the maximum error quoted by the authors, who are not explicit on their sources of error.

The band sketched in Fig. 3 from $\Lambda^{(6)}_{\overline{MS}} = 15.4$ MeV to 35.4 MeV represents a reasonable limit on Λ from decay width ratios and independent fragmentation jet analyses. However, our past data points from Γ_{ggg} and $\Gamma_{\ell^+\ell^-}$ were in better agreement with earlier ratio data, and I am not certain that the most recent world averages of Γ's have been used for the six ratio points. We have not had time to recalculate these ratios since Snowmass and are relying on year-old references.

Since fragmentation is somewhere between independent and string, probably closer to string, I would say that the two lower points near 35 GeV give weight to our Fig. (2).

Conclusion

My most recent use of Eq. (10) for $|\psi^{nS}(0)|^2$ gives the ten points in Fig. (2) on the $1/\alpha_s$ *versus* Q graph. If I look at over 40 points I have collected recently, many from jet analyses, I can easily conclude that α_s does not run at all! My ten points, with the accompanying explanation of what I expect them to do in future iterations, favor the $\Lambda^{(6)}_{\overline{MS}}$ values from roughly 50 MeV to 200 MeV, and the two string fragmentation points in Fig. (3), selected from the most reliable recent data, support this conclusion.

Acknowledgments

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Footnotes and References

1. Rev. Mod. Phys. <u>56</u> (1984). There is an Appendix by I. Hinchliffe, p. S294, with caveats on α_s and Λ. The data we use for our width calculations of α_s are on pp. S18 - S22.
2. Bernice Durand and Robert O. Knuteson, Madison preprint MAD/TH/136 (1984), in preparation.
3. A recent preprint with a large collection of data points and references is L. Clavelli, Indiana preprint IUHET 94 (1984).
4. Paul B. Mackenzie and G. Peter Lepage, Phys. Rev. Lett. <u>47</u>, 1244 (1981), did the first ratio calculation from the (1S) state that I know of, using $\Gamma_{ggg}/\Gamma_{\mu^+\mu^-}$. Also the articles by P. M. Tuts (p. 284) and G. P. Lepage (p. 565) in D. G. Cassel and D. L. Kreinick, editors, *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies*, Cornell University, Ithaca, New York (1984), give new values from $\Gamma_{ggg}/\Gamma_{\ell^+\ell^-}$ for $\Upsilon(1S)$ and $\Upsilon(2S)$, and from $\Gamma_{\gamma gg}/\Gamma_{ggg}$ for $\psi(1S)$, $\Upsilon(1S)$, and $\Upsilon(2S)$, respectively.
5. B. Durand and L. Durand, Phys. Rev. D <u>25</u>, 2312 (1982) (WKB derivation of $|\psi(0)|^2$); Phys. Lett. <u>113B</u>, 338 (1982) (application to Γ's); Phys. Rev. D <u>28</u>, 396 (1983) (solution of relativistic Coulomb problem); and Phys. Rev. D <u>30</u>, (1984) (relativistic and nonrelativistic $|\psi(0)|^2$).
6. See, for example, Torbjörn Sjöstrand, DESY preprint DESY 84-023 (1984), for many references and a table of recent determinations of α_s using second order QCD.