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LEPTON-NUMBER VIOLATION IN COSMOLOGY AND ASTROPHYSICS*

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ABSTRACT

The cosmological and astrophysical implications of lepton number violation are discussed. The lepton number violation considered is due to the interactions of neutrinos with Nambu-Goldstone bosons.

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1. Introduction

At present there is no unambiguous evidence for non-conservation of lepton number. The standard low-energy weak interaction model has three conserved lepton numbers (L_e, L_μ, L_τ) associated with individual conservation laws for electron-lepton number, muon-lepton number, and tau-lepton number. Although L_e, L_μ, and L_τ are conserved in the standard model, there is no deep understanding why either the total lepton number, $L = L_e + L_\mu + L_\tau$, or the individual lepton numbers are conserved. Indeed, just about every extension of the standard model has violation of either individual or total lepton number. Terrestrial searches for violation of lepton number are difficult, so it is tempting to use cosmology and astrophysics as a laboratory to test the implications of lepton number violation.

In this paper I will review the cosmological and astrophysical consequences of lepton number non-conservation. The cosmological and astrophysical effects are strongly dependent upon the particle physics models, so in this paper I will concentrate upon a particular class of particle physics models, those with lepton number violation through the interactions of neutrinos with Nambu-Goldstone bosons. The Nambu-Goldstone bosons I will consider arise either from the breaking of the total global lepton number (Nambu-Goldstone boson = Majoron)[1,2,3] or from the breaking of a global family symmetry (Nambu-Goldstone boson = Familon).[4,5]

The energy scale for the spontaneous symmetry breaking is also model dependent. For Majorons, two energy scales have been considered. The original Majoron model[1] had $V \sim 10^{10}$ GeV for the scale of spontaneous symmetry breaking (SSB), while later models[2,3] with Majorons had $V \sim 1$ keV for the scale of SSB. Experimental bounds on $\mu \to eF$ or $K \to \pi F$ (F is a familon) imply that the scale for family symmetry breaking must be $V \gtrsim 10^{10}$ GeV. (If the family group also contains the Peccei-Quinn symmetry then astrophysical arguments suggest $V \lesssim 10^{12}$ GeV.[6])

The importance of the scale of SSB will be manifest in the first cosmological effect I will consider, neutrino decay. The decay width of a heavy neutrino of mass m_ν to a light neutrino with mass much less than m and a massless scalar particle is

$$\Gamma = h_{12}^2 \, m_\nu / 16\pi \tag{1}$$

where h_{12} is the coupling between the two neutrinos and the Nambu-Goldstone boson. One expects $h_{12} \sim m/V$ where V is the vacuum expectation value of the SSB of the global symmetry giving rise to the Nambu-Goldstone boson. Therefore the lifetime for neutrino decay is

$$\tau = 16\pi V^2/m^3$$

$$= 10^2 \, (100\text{keV}/m)^3 \, (V/10^{10}\text{GeV})^2 \text{ years.} \tag{2}$$

In the original Majoron model the Majoron coupling is diagonal,[3,7] and $h_{12} = 0$. However it is possible to complicate the model to give $h_{12} \neq 0$ by the introduction of two Higgs representations with different lepton numbers coupling to leptons and contributing to the (Majorana) neutrino mass matrix.[8,9] Such a complication is only viable in modes with $V \gtrsim 10^{10}$ GeV.

The lifetime as a function of m_ν for several values of V is shown in Figure 1. The curves marked 10^9, 10^{10}, and 10^{11} correspond to the lifetime given by equation (2) with $V = 10^9$ GeV, 10^{10} GeV, and 10^{11} GeV.

2. Neutrino Decay-Avoiding Cosmological Neutrino Mass Limit

Neutrinos decouple in the early universe at a temperature of a few MeV.[10] If we consider neutrinos of mass less than a few MeV, then they were relativistic at decoupling, and their number density was $n_\nu = g_\nu(3\zeta(3)/4\pi^2)T_D^3$, where g_ν is the number of neutrino spin states (which we shall take as $g_\nu = 2$) and T_D is the decoupling temperature. The photon number density at decoupling is given by $n_\gamma = (2\,\zeta(3)/\pi^2)T_D^3$, which results in $n_\nu/n_\gamma = 3/4$ at neutrino decoupling. Subsequent to neutrino decoupling, e^+e^- annihilation heats the photons increasing their temperature relative to the photon temperature[10] by a factor of $(T_\nu/T_\gamma)^3 = 4/11$. Therefore after e^+e^- annihilation, $n_\nu/n_\gamma = (3/4)(4/11)$. Today, the number density of photons in the universe is accurately known from the microwave background radiation. If we take the temperature of the microwave background radiation to be $T_\gamma = 2.7$K, then today $n_\gamma = 399$ cm^{-3}, and $n_\nu = (3/4)(4/11)n_\gamma = 109$ cm^{-3}.

If neutrinos have a mass greater than $T_\nu = 1.9$K, then the present mass density due to the relic massive neutrinos would be

$$\rho_\nu = m_\nu n_\nu$$

$$= (m_\nu/1\text{eV})\ 1.09 \times 10^2 \text{ eV cm}^{-3} . \qquad (3)$$

It is convenient to express this density in terms of $\Omega_\nu = \rho_\nu/\rho_c$, where ρ_c is the critical density expressable in terms of G (Newton's constant) and H_o (Hubble's constant): $\rho_c = 3H_o^2/8\pi G = 1.05 \times 10^4 h^2$ eV cm^{-3} where $1 \gtrsim h \gtrsim 1/2$ reflects the uncertainty in the observational determination of the Hubble constant. In terms of the neutrino mass, Ω_ν is given by

$$\Omega_\nu h^2 = 1.05 \times 10^{-2}\ (m_\nu/1\text{eV}) . \qquad (4)$$

A conservative upper limit on $\Omega_\nu h^2$ would be $\Omega_\nu h^2 \lesssim 1$, which from equation (2) requires $m_\nu \lesssim 100$ eV. Up to numerical factors this is the Cowsik-McClelland[11] bound on neutrino masses.

The Cowsik-McClelland bound can be avoided by making the neutrino mass greater than the decoupling temperature of a few MeV, thereby suppressing the number density at decoupling (hence the number density today) by a factor of e^{-m_ν/T_D} relative to the photon number density. For neutrino masses above about 5 GeV the exponential decrease in the number density results in $\Omega_\nu h^2 \lesssim 1$. However in this paper I will assume that the neutrinos have a mass much less than 5 GeV, in fact, I will assume the masses are less than 1 MeV.

A second way to avoid the cosmological neutrino mass limit is to have the neutrinos unstable, with a lifetime less than the age of the universe.[13] If a massive neutrino decays to relativistic particles, the energy density of the relativistic decay products will decrease in the expansion as R^{-4} (where R is the Robertson-Walker scale factor) compared to a R^{-3} decrease in the energy density of a massive particle. This "extra" decrease in the energy density of the decay products results in the massless decay products having an energy density today of

$$\Omega h^2 = 1.05 \times 10^{-2} \left(\frac{m_\nu}{1eV}\right) \frac{R_D}{R_o} , \tag{5}$$

where R_D/R_o is the ratio of the scale factor at decay to the scale factor today. It is customary to express this ratio of scale factors in terms of a cosmological red-shift z_D

$$\frac{R_o}{R_D} = 1 + z_D \tag{6}$$

where z_D is the "red-shift" of the neutrino decay ($z_D = 0$ corresponds to today).

If we require $\Omega h^2 \leq 1$, then from equation (4) and (5), we see that there is a relation between z_D and m_ν that must be satisfied for the decay products to have an energy density less than ρ_c

$$(1+z_D) \gtrsim 10^{-2} (m_\nu/1eV) \tag{7}$$

If we assume that the microwave background is primordial in origin and not the result of some other mechanism (such as photons produced in neutrino decay) then $T \propto R^{-1} \propto 1+z$, and the temperature of the universe at decay is given by

$$T_D = 2.7K(1+z_D) , \tag{8}$$

which must satisfy the inequality of eq. (7)

$$T_D \gtrsim 2.7 \times 10^{-2}K \, (m_\nu/1eV) \tag{9}$$

The limit (9) on the temperature of the universe at neutrino decay can be translated into a limit on the neutrino lifetime. The massive neutrinos decay when the age of the universe ($t_U = (2/3)\ H^{-1}$ if the universe is matter dominated when the neutrinos decay) is equal to the particle lifetime, τ. The expansion rate at the time of neutrino decay is given by ($m_{pl} = G^{-1/2}$)

$$H^2 = \frac{8\pi G}{3} \rho$$

$$= \frac{8}{3m_{pl}^3} m_\nu n_\nu \tag{10}$$

if the massive neutrino dominates the universe at the time of its decay.

Since at the temperature of neutrino decay $n_\nu = (3/4)\ 2\zeta(3)\ T_D^3\ (4/11)$, equating $t_u(T_D) = \tau$ results in the relation between τ and T_D:

$$\tau = \frac{m_{pl}}{m_\nu^4 T_D^{3/2}} . \tag{11}$$

Therefore the limit(9) on T_D results in the limit

$$\tau \leq 5.9 \times 10^3 \left(\frac{100\text{keV}}{m_\nu}\right)^2 \text{ yr}. \tag{12}$$

This upper limit as a function of m_ν is shown as the dashed line in Figure 1, and compared to the lifetime expected for neutrino decay into a light neutrino plus a Nambu-Goldstone particle. In order for the massless decay products to give $\Omega h^2 \leq 1$, the lifetime must be below the dashed line. From the figure we see that if $V \leq 10^9$ GeV the lifetime will always be short enough and there would be no limit on m_ν. If $V = 10^{10}$ GeV, then the lifetime would be short enough only if $m_\nu \geq 1.7$ keV, while if $V = 10^{11}$ GeV the lifetime will be short enough only if $m_\nu \geq 170$ keV. If $V \geq 10^{12}$ GeV, the lifetime will be too long, and the neutrino decay products will have $\Omega h^2 > 1$. Again, I emphasize that I am only considering neutrinos with masses less than a few MeV.

Other decay mechanisms for ridding the universe of massive neutrinos have been considered.[6,14] Most other mechanisms have a large fraction of the decay products as photons. It is very difficult to hide the photons from detection today. The advantage of the particle physics models considered here is that the decay products are "invisible" (i.e. the final state contains no detectable photons or charged particles). For neutrinos of mass less than a few MeV, the decays into Nambu-Goldstone particles with $V \leq 10^{10}$ GeV are the only models to give a short enough lifetime.

In the next section I will discuss the very intriguing possibility that the decay products of the neutrinos give $\Omega = 1$ today.

3. Neutrino Decay - A Flat Universe

Although observational evidence suggests (but not convincingly) that the universe is open, $\Omega \leq 0.5$, there is a strong theoretical prejudice that the universe should be spatially flat, i.e. have $\Omega = 1$. Indeed inflationary models predict that $\Omega = 1$ to a large degree of accuracy.[15] The observational determination of $\Omega \leq 0.5$ is sensitive to massive particles that clump on scales less than about 100h Mpc, and includes the "dark" matter needed to account for the dynamical masses of galaxies, groups of galaxies, clusters of galaxies, etc. The observational determinations of Ω would not be sensitive to an unclustered background, so a background of relativistic particles could provide the difference to give $\Omega = 1$.

This discrepancy between the observational data and the theoretical expectation led Dicus, Teplitz, and I to propose in 1977 that the massless decay products from the decay of a massive neutrino give $\Omega = 1$ today.[13] At the time we could find no reasonable particle physics model to give invisible decays fast enough to make the idea work for neutrino masses less than a few MeV. At the time we also noted two very serious problems with the idea.[14]

The first problem is that in a radiation-dominated $\Omega = 1$ universe the age is given by $t_U = (1/2)\, H_o^{-1} = 5 \times 10^9\, h^{-1}$ years (cf. $t_U = (2/3)\, H_o^{-1}$ for a matter dominated $\Omega = 1$ universe). Even with $h = 1/2$, $t_U = 10 \times 10^9$ years. The best estimate for the age of the universe is about 15×10^9 years, and the value of 10×10^9 years is barely acceptable. In order to get a universe of 15×10^9 years, we would need $H_o = 33$ km s^{-1} Mpc^{-1} which is about 3σ deviation from the value quoted by Sandage and Tammann.[16] Other determinations of H_o go as high as 100 kms^{-1} Mpc^{-1}, in which case the universe would be only 5×10^9 years old, in definite conflict with observation. In conclusion a radiation dominated universe implies either an uncomfortably young universe or an uncomfortably small value of H_o,[17] but is not in immediate contradiction.

The second problem with the scenario is that the growth of perturbations needed for galaxy formation is suppressed if the universe is dominated by relativistic particles.[18] Two groups have recently considered galaxy formation in detail in our scenario.[19,20] With similar assumptions, Turner, Stiegman and Krauss prefer a neutrino mass in the 70-200 eV range and a very recent neutrino decay, while Gelmini, Schramm and Valle prefer a neutrino mass in the 500-550 eV range and a somewhat earlier decay. The question of whether galaxy formation can occur in the above scenario (and for what values of m_ν and τ_ν) is a very difficult question, and more work should be done. Turner, Steigman and Krauss emphasize that it may be possible to divorce the idea of the universe dominated by relativistic particles produced by the decay of a massive relic particle from the idea that the decaying particle is a neutrino, and their analysis is more general.

A universe dominated by the decay products (in this case light neutrinos and Nambu-Goldstone bosons) of a massive neutrino is an attractive means to get $\Omega = 1$. Detailed calculations of galaxy formation seems to require a neutrino mass in the 70 - 550 eV range, which corresponds to a value of V in the $10^9 - 10^{10}$ GeV range.[21]

4. Light Majorons and Neutrino Annihilation

Although in the Gelmini-Roncadelli model the Majoron coupling to neutrinos is diagonal and the neutrino is stable at the tree-level against decay into a neutrino plus Majoron (M), in this model there is another way to rid the universe of massive neutrinos and allow neutrinos of mass greater than 100 eV to exist but not to contribute to the present mass density. Georgi, Glashow and Nussinov[3] pointed out that neutrino annihilation through Majoron exchange will reduce the neutrino density below the level necessary to give $\Omega_\nu h^2 < 1$.

In the Gelmini-Roncadelli (GR) model, neutrinos receive a Majorana mass by the introduction of a SU(2) Higgs triplet $\vec{\chi}$, which couples to the usual lepton doublets. The introduction of a Yukawa term in the Lagrangian of the form

$$L_\chi = -h_3\, \bar{\Psi}_L^c\, \vec{\tau} \cdot \vec{\chi}\, \Psi_L\,, \tag{13}$$

where Ψ_L is one of the usual lepton doublets (e.g. $\Psi_L^T = (\nu_e, e)_L$ for the first generation), $\vec{\chi}$ is the Higgs triplet, and $\vec{\tau}$ are Pauli matrices, results in a Majorana mass for the neutrino $m_\nu = h_3\, V_T$, where V_T is the

VEV of the neutral component of the triplet (constrained to be << 1 MeV).

In the GR model the Higgs potential is constructed to conserve a global lepton number. A lepton number of $L = -2$ can be assigned to χ, and when χ gets a VEV the global lepton number is spontaneously broken. The resulting Nambu-Goldstone boson is the Majoron. The Majoron coupling to neutrinos is of the form

$$L_{M\nu} = ih_3\, \bar{\nu}\gamma_5\nu M = i(m_\nu/V_T)\bar{\nu}\gamma_5\nu M \tag{14}$$

where ν is the neutrino Majorana field $\nu = \nu + \nu^c$. The Majoron coupling to charged particles is suppressed by a factor of V_T/V_D, where V_D is the VEV of the usual Higg doublet field, $V_D \sim 250$ GeV. Since this ratio is expected to be small the Majorons effectively decouple from charged particles and only interact with neutrinos.

Neutrinos can scatter by the exchange of the massless Majoron, and the neutrino-neutrino scattering cross section at energies greater than m_ν will be $\sigma_M \sim h^4/s$ where $\sqrt{s}$ is the center-of-mass energy and h is a Yukawa coupling. This is larger (for energies less than m_W) than the usual neutrino-neutrino scattering cross section mediated by the exchange of a massive W or Z, $\sigma_{W,Z} \sim G_F^2\, s \sim g^4 s/m_W^4$ (g is a gauge coupling), by the ratio $\sigma_M/\sigma_{W,Z} \sim h^4 m_W^4/g^4 s^2$. In the GR model neutrino-neutrino scattering is much larger than weak.

Through Majoron interactions the neutrinos will interact amongst themselves long after they have decoupled from the rest of the particles in the universe at the usual neutrino decoupling temperature of $T = 1$ MeV. Neutrinos can annihilate into Majorons, $\nu\nu \to MM$ with a cross section of[3)]

$$\sigma_{\nu\to M} \simeq h^4\, |\vec{v}|/48\pi s\ , \tag{15}$$

where $|\vec{v}|$ is the relative velocity and h is the coupling related to m_ν and V_T. The number density of neutrinos change due to annihilation and expansion according to

$$\dot{n}_\nu = -[n_\nu^2 - (n_\nu^{eq})^2]\ \sigma_A\ |\vec{v}| - 3\ \dot{R}/R\ n_\nu \tag{16}$$

where σ_A is the total neutrino annihilation cross section (for the lightest mass neutrino $\sigma_A = \sigma_{\nu\to M}$), and $\dot{R}/R$ is the expansion rate. If one solves (16), for V_T less than the red-giant limit[3)] neutrino annihilation into Majorons decreases the relic neutrino density enough to give $\Omega_\nu h^2$ today less than one.[22)]

In conclusion, in the GR model, as in the original Majoron model and the familon model, it is possible to evade the cosmological bound on the mass of neutrinos. The Nambu-Goldstone boson in the GR model also has other interesting astrophysical consequences discussed in the next section.

5. GR Majorons and Supernovae

The Majoron in the GR model couples to neutrinos and is responsible for lepton-number violating reactions such as the $|\Delta L| = 2$ reaction $\nu_L \nu_L \leftrightarrow MM$ or the $|\Delta L| = 4$ reaction $\nu_L \nu_L \leftrightarrow M \leftrightarrow \nu_R \nu_R$ (in this model the anti-neutrino is the right handed neutrino). Furthermore, the $\Delta L \neq 0$ ν-ν scattering cross sections are much larger than the usual weak interaction result. This suggests that one way to test the GR model is to do ν-ν scattering. Neutrino beams have been used since the 1950's, but neutrino targets are not yet available, at least terrestrially. The best place today in the universe to study ν-ν scattering is during the latter stages of the collapse of massive stars. If the GR model is correct it would have profound implications on models of gravitational collapse and Type II supernovae.[23-25]

The standard theory of Type II supernovae is described in these proceedings by E. Müller.[26] The crucial point for Majorons is that during the gravitational collapse of massive iron cores, electron neutrinos are copiously produced in electron capture and are "trapped" in the core, i.e. the mean free path between neutrino scatterings is much smaller than the size of the core and the time it takes for neutrinos to diffuse out of the core is longer than the time for the core to collapse.

At the point of neutrino trapping the mass density is about 10^{12} g cm^{-3}, the temperature is about 1.5 MeV, and the neutrino number density is about 10^{34} cm^{-3}! The existence of a large neutrino density means that in the standard collapse picture β-equilibrium is established in reactions of the type

$$e^- + p \leftrightarrow n + \nu_e \tag{17}$$

where the protons and neutrons are either free or bound in nuclei. The β-equilibrium condition is

$$\mu_e = \mu_\nu + \hat{\mu} + \Delta m \tag{18}$$

where μ_e (μ_ν) is the electron (electron neutrino) chemical potential, Δm is the neutron-proton mass difference ($\Delta m = 1.293$ MeV), and $\hat{\mu} = \mu_n - \mu_p$ is the difference of the neutron and proton chemical potentials. The existence of a large density of trapped neutrinos is necessary to allow ν-capture on neutrons to establish β-equilibrium.

The rapid establishment of β-equilibrium results in little entropy production, and the standard collapse scenario is a low-entropy collapse. In a low entropy collapse the nucleons remain bound in nuclei and the pressure is supplied by the relativistic electrons, which is insufficient to stop the collapse before reaching a few times nuclear matter density.

In the GR model however the scenario for collapse is much different. The Majorons mediate $\Delta L \neq 0$ reactions amongst the neutrinos. The rate for lepton-number violating reactions is greater than the weak rates trying to establish β-equilibrium. The lepton-number violating reactions will drive the neutrino chemical potential to zero so the β-equilibrium condition becomes $\mu_e = \hat{\mu} + \Delta m$. Therefore the value of μ_e in β-equilibrium is much less in the Majoron collapse than in the

standard collapse. The decrease in μ_e (i.e., in number of electrons) leads to a large entropy increase ($TdS = \mu dN$). The large entropy increase causes the nuclei to melt, increasing the number of non-relativistic particles and eventually leading to non-relativistic nucleons dominating the pressure. Thus, Majoron interactions will change the equation of state in the collapse.

In the standard collapse, after neutrino trapping the total lepton number of the core is conserved. The equilibrium obtained in the standard collapse is subject to the constraint of conservation of lepton number. The existence of a large neutrino chemical potential blocks electron capture, and the equilibrium conditions obtained in collapse are not too different than the conditions at neutrino trapping, resulting in a small increase in entropy. If lepton number is not conserved in the collapse, and the maximum entropy equilibrium state will be obtained with the lepton number zero. The de-leptonization of the core produces a large amount of entropy, changing the equation of state during collapse. Until detailed numerical calculations are done it is impossible to know whether the de-leptonization of the core helps or prevents the supernova explosion.

Finally, it should be emphasized that Majorons were only the agents for mediating lepton-number violation. The change in the collapse scenario only depended upon lepton number violation occurring rapidly during collapse. Any particle physics model with "fast" lepton number violating reactions amongst neutrinos would lead to the same effect.

6. Conclusion

Particle physics models with lepton-number violation through Nambu-Goldstone particles have several interesting cosmological and astrophysical effects. In these models it is possible to avoid the cosmological limit on neutrino masses. It is also possible that the light decay products of the massive neutrino provides the missing energy to result in $\Omega = 1$. If lepton number is a global symmetry spontaneously broken at a low energy scale as in the Gelmini-Roncadelli model, theories of stellar collapse and Type II supernovae will be drastically changed.

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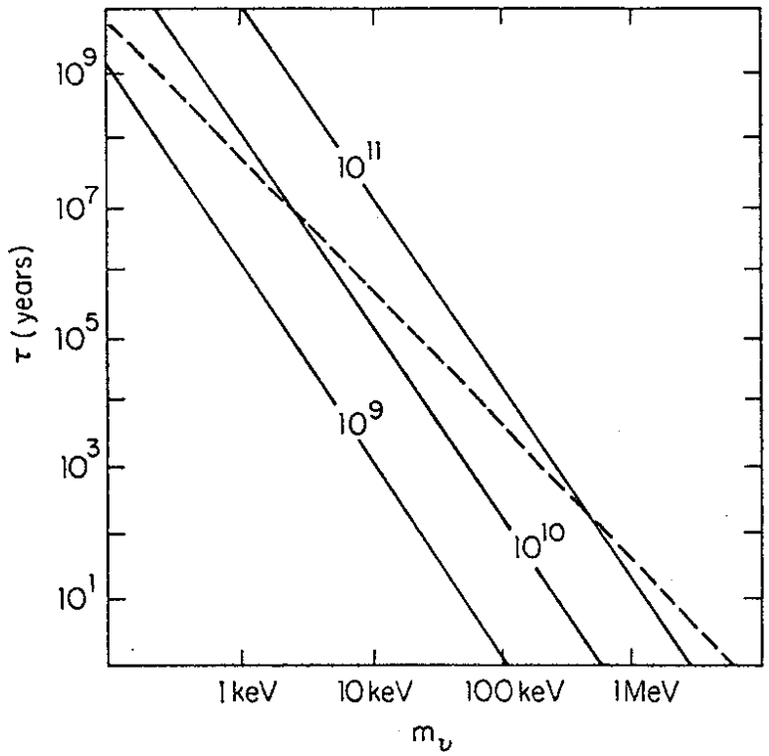

Figure 1

The neutrino lifetime (solid lines) for neutrino decay into a light neutrino plus a Nambu-Goldstone for different values of V = 10^9 GeV, 10^{10} GeV, 10^{11} GeV. The dashed line is the maximum lifetime for the decay products to give $\Omega h^2 \leq 1$ today.