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MULTIJET FINAL STATES: EXACT RESULTS AND THE LEADING POLE APPROXIMATION

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Summary

Exact results for the process $gg \to ggg$ are compared with those obtained using the "leading pole approximation". Regions of phase space where the approximation breaks down are discussed. A specific example relevant for background estimates to W boson production is presented. It is concluded that in this instance the leading pole approximation may underestimate the standard QCD background by more than a factor of two in certain kinematic regions of physical interest.

Most of the perturbative QCD calculations encountered in the literature are based on the leading logarithm approximation. In the description of inclusive processes for example, large logarithms, resulting from gluon bremsstrahlung, counteract the logarithmic decrease in the running coupling and necessitate an all orders summation of at least the leading terms. A systematic procedure also exists for the inclusion of subleading terms. However, it is often of interest to study the structure of complete events, rather than just inclusive particle distributions. For this purpose Monte Carlo simulation programs have been constructed by a number of authors. These programs are often based on what is called the "leading pole approximation" (LPA). In this approximation, after integration over the four-vectors of final state particles, the results of the leading logarithmic approximation for inclusive distributions are obtained.

The leading pole approximation for a $2 \to n$ process is obtained by combining the cross section expression for the $2 \to (n-1)$ process with the appropriate Altarelli-Parisi[1] splitting function and dividing by the invariant mass squared (M^2) of the line which branches. In certain regions of phase space a single pole will dominate and the resulting expression closely approximates the exact result. Because the approximation involves only probabilities it can be inserted in Monte Carlo programs in an iterative way.

Several groups [2,3] have obtained exact expressions for the $2 \to 3$ processes encountered in QCD. Especially noteworthy are the results of ref. (3) where compact expressions have been obtained. These expressions can then be compared with the results obtained using the LPA. As a specific example, we shall consider $gg \to ggg$. The kinematics for this case can be conveniently discussed using the coordinate system presented in Ref.(2). The notation to be used for the four-vectors is $p_1 + p_2 \to p_3 + p_4 + p_5$, and $s_{ij} = (p_i + p_j)^2$. Also, it is convenient to introduce the scaled variables x_3, x_4, and x_5 where, in the parton-parton center-of-mass system, $x_i = 2\ E_i/\sqrt{s_{12}}$. The coordinate system is defined as the parton-parton center-of-mass frame with the z axis normal to the plane containing the three final state quanta. The x axis is chosen to lie along the direction of p_3. Particles 1 and 2 then are antiparallel along a direction specified by appropriate polar and azimuthal angles. A compact way of discussing $2 \to 3$ processes[2] is to employ a symmetric Dalitz plot with the axes labelled by x_3, x_4, and x_5. The symmetric final state and the requirement that $x_3 + x_4 + x_5 = 2$ yields a plot in the form of an equilateral triangle. For the process considered here there is a six-fold symmetry in such a plot.

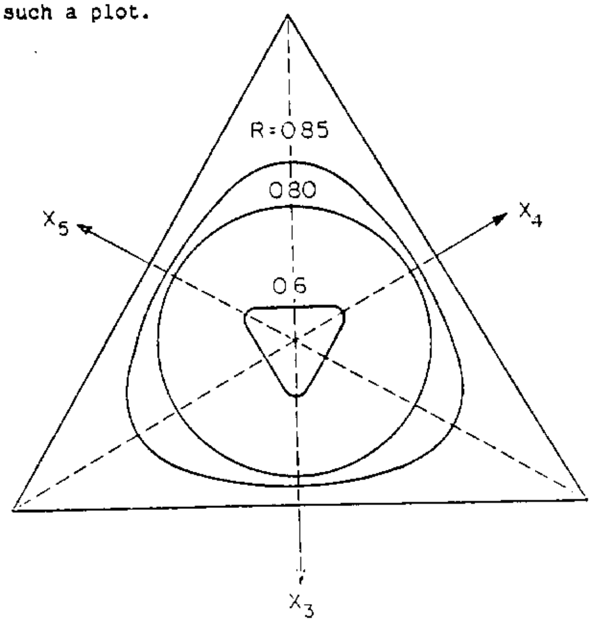

FIG 1. Dalitz plot for $gg \to ggg$ for the case where the initial state gluons are orthogonal to the plane containing the three final state gluons. The curves are contours along which the ratio R, defined in the text, is constant.

We are interested in the ratio R of the approximate to the exact expression for the process $gg \to ggg$. In Fig. (1) we show contours of constant R on a Dalitz plot as described in the preceeding paragraph. Here we have chosen the case where the initial state gluons are orthogonal to the plane containing the final state gluons. Near the edges of the plot one of the gg invariant masses becomes small and the leading pole approximation gives an adequate approximation to the exact result. This is easily seen by noting that $x_i = (1 - s_{jk}/s_{12})$, where i, j, and k are distinct and equal to 3, 4, or 5. As one moves away from the edges of the Dalitz plot, the ratio R defined above varies both above and below one. It is conceivable that on average the approximation might work very well. To test this we decided to perform a realistic calculation such as one might do with the Monte Carlo simulation programs. We considered the example of holding the mass of a pair of gluons fixed ($s_{45} = M^2$) and then looking at its transverse momentum distribution. The rapidities of both the gg system (4+5) and the recoiling gluon (3) have been fixed at zero. This calculation simulates a portion of the QCD jet background which would be encountered when looking for hadronic decays of the W into two jets. The quantity of interest here is $d\sigma/dy dy' dp_T dM^2$ where M is the mass of the gg pair produced with rapidity y' and transverse momentum p_T, and y is the rapidity of the recoiling gluon. We have calculated this quantity at $\sqrt{S} = 40$ TeV using the exact matrix element and also using the leading pole approximation which has the form

$$|T_3|^2 \sim |T_2|^2 P_{gg}(z)/s_{45} + O\left(\frac{1}{\sqrt{s_{45}}}\right) . \quad (1)$$

The exact results for the 2→3 and 2→2 subprocesses can be found in Refs. (3) and (4), respectively, and the splitting function $P_{gg}(z)$ can be found in Ref. (1). The variable z has been chosen as

$$z = (s_{14}+s_{24})/s_{12} \tag{2}$$

but any other definition which had the same $s_{45}\to 0$ limit would be acceptable. This ambiguity in the definition of z is always present in the LPA. A comparison of the exact results with those given by eqs. (1) and (2) shows that the corrections to the LPA are $O(1/\sqrt{s_{45}})$. The ratio of the approximate to the exact results, R, is shown in Fig. (2) for M=82 and 500 Gev/c². For the case of the leading pole approximation we have retained only the pole terms in M², i.e., we have not included pole terms corresponding to initial state bremsstrahlung. Therefore, this comparison follows the same algorithm as is used in ISAJET, one of the more widely used Monte Carlo simulation programs. To eliminate regions where two of the gluons become parallel or one of them becomes soft in the overall hadron-hadron center-of-mass system, a number of cuts have been imposed. These cuts require that each of the final state gluons have a momentum greater than 10 GeV/c in the hadron-hadron center-of-mass system and that each gluon be 0.5 radians away from the others and from the beam-beam axis. These requirements are typical of the cuts that would be used in a real experiment.

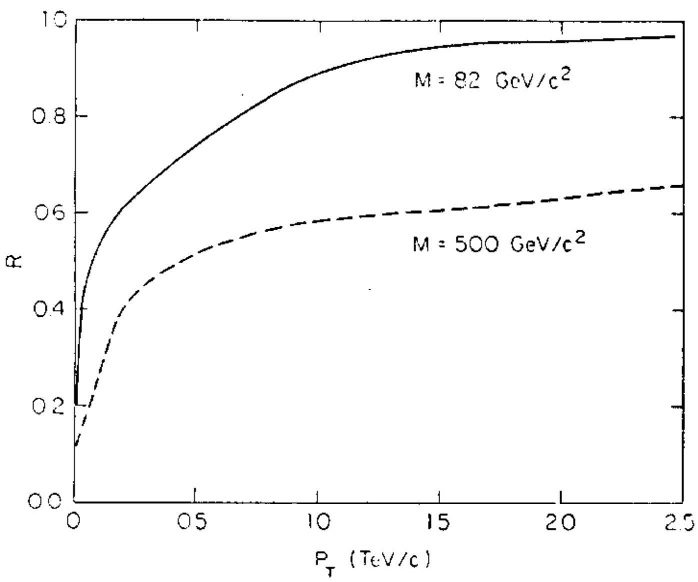

FIG 2. The ratio R of the leading pole and exact results for the calculation discussed in the text. Curves for two values of M are shown.

The results for the ratio R shown in Fig. (2) indicate that at sufficiently high p_T the leading pole approximation is indeed very good. In this region of phase space the pole in M² dominates because the gg invariant mass is the smallest of the possible invariant sub-energies s_{ij}. However, as one goes to lower values of p_T at fixed M or to higher values of M at fixed p_T the approximation becomes worse and the true answer is underestimated by a factor of two or more.

The preceeding discussion has shown that there are kinematic regions in which the leading pole approximation breaks down and that a factor of two or more difference in comparison with the exact results is not uncommon. The Monte Carlo programs as currently formulated may provide a useful means for obtaining estimates of qualitative features of event structures. However, when one is searching for precise quantitative features the above lesson must be remembered. As a general rule, it would seem advisable to include in the Monte Carlo programs as much of the exact results as are known, thereby minimizing the use of approximation wherever possible.

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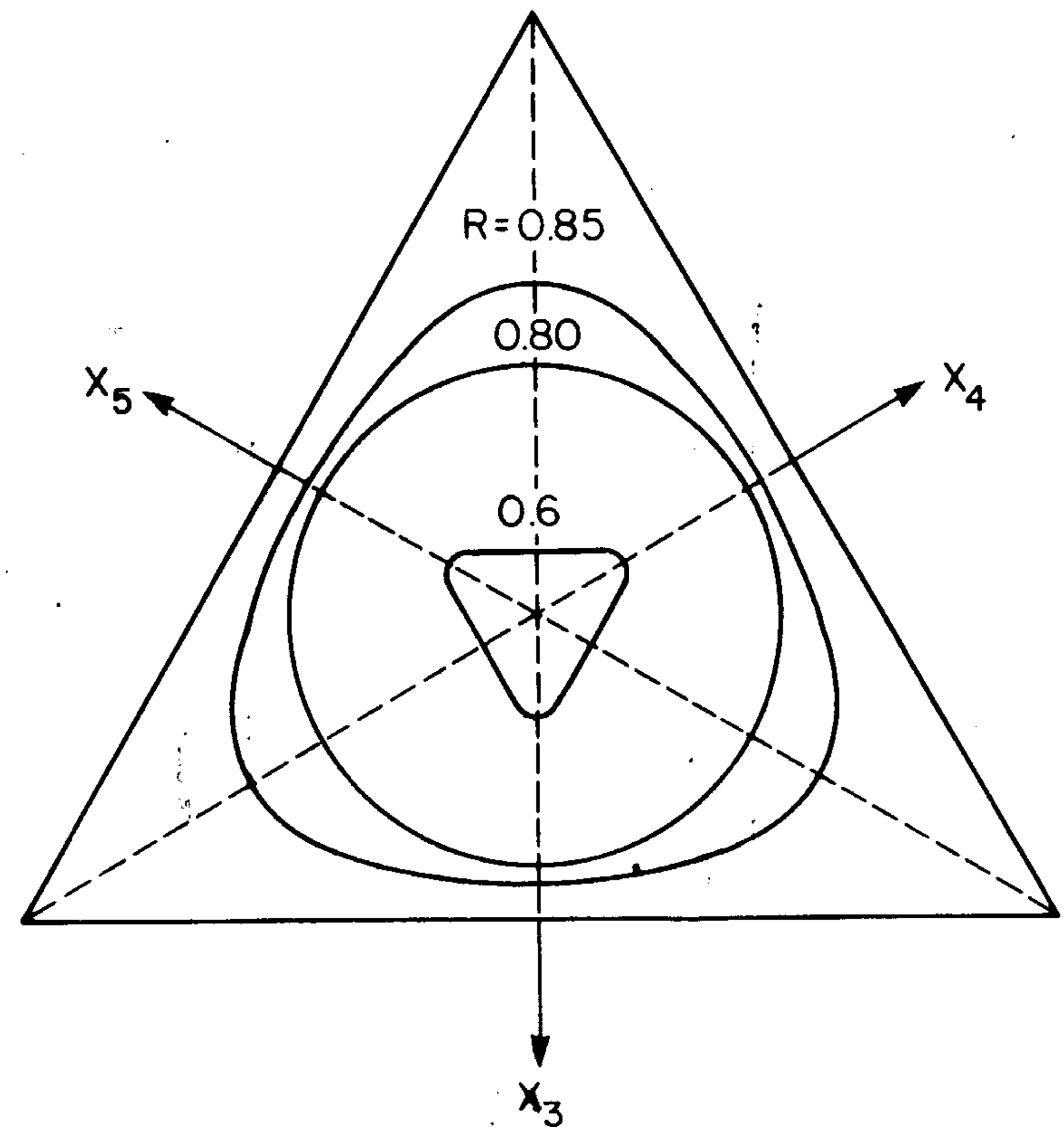
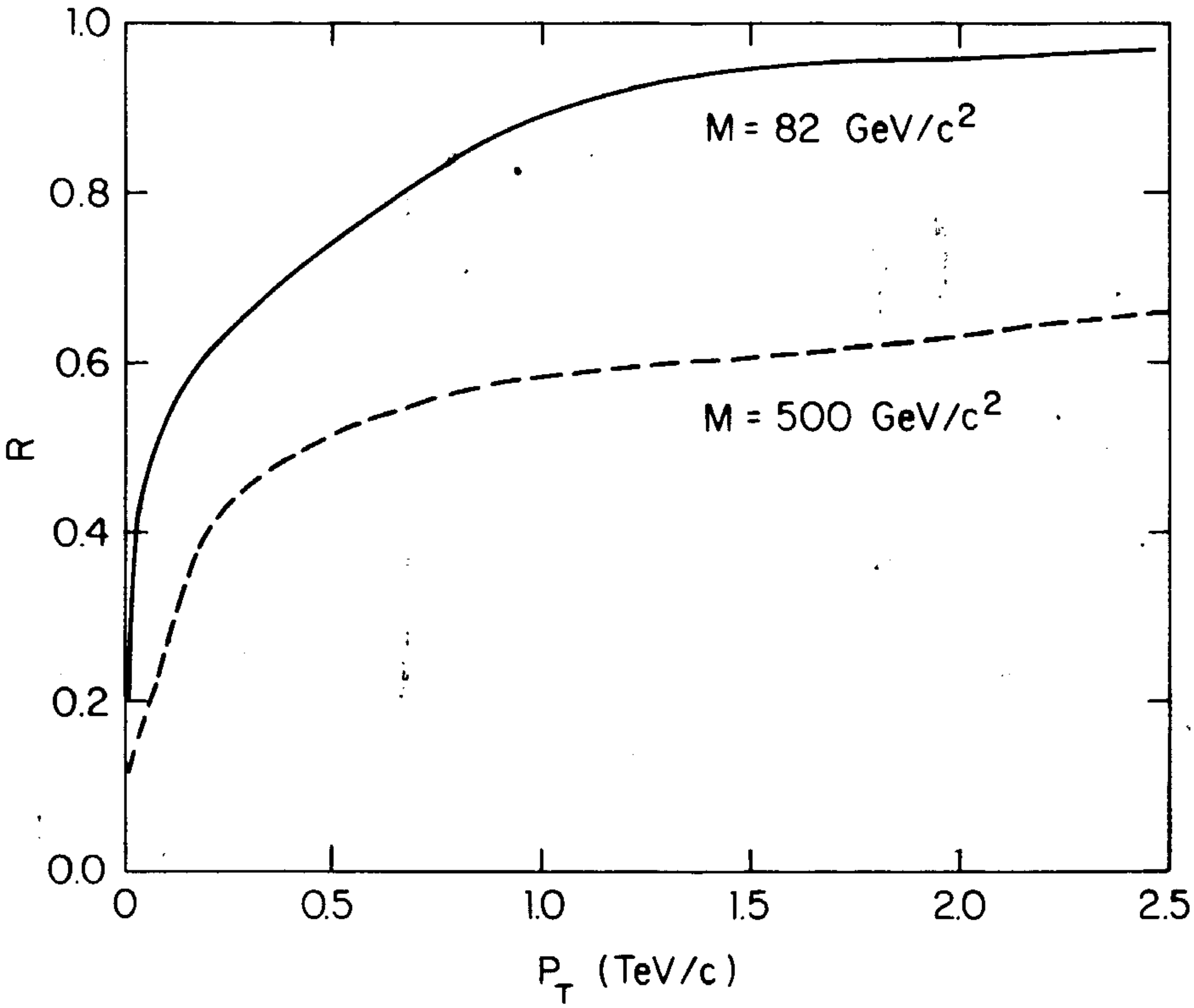

R=0.85
0.80
0.6
X5
X4
X3


1.0
0.8
0.6
0.4
0.2
0.0
R
0
0.5
1.0
1.5
2.0
2.5
P_T (TeV/c)
M = 82 GeV/c^2
M = 500 GeV/c^2