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Transverse Momentum Distributions of Jets and Weak Bosons*

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The theoretical description of processes leading to events at large transverse momentum is reviewed. Numerical estimates are given for jet cross-sections and for W and Z production cross-sections. The influence which uncertainties in the input parameters have on the theoretical predictions is also discussed.

1. Jet Cross-Sections

The observation of clearly identified jets at the CERN Sps collider^{1,2)} opens a new era in the study of hadron structure. For the first time using hadronic probes we have irrefutable evidence for the parton substructure of the proton. The observed constituents scatter as the quarks and gluons of QCD should. Of course, in the interactions of objects as complicated as protons there are uncertainties, both theoretical and experimental, some of which will be described below. But before entering into these details it is important to remember that the gross features of the data are clearly in agreement with QCD.

The jet cross-section observed at the collider is four or more orders of magnitude bigger than the large p_T cross-section at the ISR. Despite this big change, the predictions³⁾ of the QCD improved parton model describe the data well, both in shape and in normalisation. This agreement with data requires the inclusion of a scale breaking gluon distribution. In addition to the p_T spectrum, the angular distribution of the observed jets is consistent with the exchange of a single massless vector gluon in the t channel⁴⁾. Apart from the scale breaking logarithms of QCD, the constituents of the proton appear to behave as point-like particles.

The cross-section for jet production is calculated using the parton model formula,

$$E \frac{d^3\sigma}{d^3p} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \left[\frac{p^0 d^3\sigma}{d^3p} \right] \quad (1)$$

where the sum on i, j runs over different types of partons. The parton cross-sections are calculable in perturbation theory and in lowest order are given by,

$$\frac{p^0 d^3\sigma}{d^3p} = \left(\frac{\alpha_s(Q)}{s} \right)^2 |M^{ij}|^2 \quad (2)$$

where M is the invariant matrix element. The contribution of the various sub-processes to the total cross-section is dependent on the size of the matrix element and the value of the distribution for the incoming partons. The influence of the former factor can be judged from Table 1 where the analytic forms of the matrix elements and their numerical values at 90° in the parton parton centre of mass are given. On the basis of the parton cross-sections alone it is clear that processes involving initial state gluons are favoured.

PARTON PROCESS	$ M ^2$	F_M
$qq' \rightarrow qq'$ $qq' \rightarrow qq'$	$\frac{4}{9} \frac{s^2+u^2}{t^2}$	2.22
$qq \rightarrow qq$	$\frac{4}{9} \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} \right) - \frac{8}{27} \frac{u^2}{st}$	3.26
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{t^2+u^2}{s^2}$	0.22
$q\bar{q} \rightarrow q\bar{q}$	$\frac{4}{9} \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} \right) - \frac{8}{27} \frac{u^2}{st}$	2.59
$q\bar{q} \rightarrow g\bar{g}$	$\frac{32}{27} \frac{u^2+t^2}{ut} - \frac{8}{3} \frac{u^2+t^2}{s^2}$	1.04
$g\bar{g} \rightarrow q\bar{q}$	$\frac{1}{6} \frac{u^2+t^2}{ut} - \frac{3}{8} \frac{u^2+t^2}{s^2}$	0.15
$q\bar{g} \rightarrow q\bar{g}$	$\frac{4}{9} \frac{u^2+s^2}{us} + \frac{u^2+s^2}{t^2}$	6.11
$g\bar{g} \rightarrow g\bar{g}$	$\frac{9}{2} \left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2} \right)$	30.4

Table 1. Parton matrix elements [averaged (summed) over initial (final) colours and spins]. F_M is the value of $|M|^2$ in the parton parton centre of mass at 90° , ($s = -t/2 = -u/2$). (Taken from ref. 3, second reference.)

Clean jets are observed at $\sqrt{s} = 540$ Gev for values of the transverse energy between 20 and 150 Gev. The parameter x_T therefore ranges between,

$$x_T = \frac{2E_T}{\sqrt{s}} \quad 0.07 \leq x_T \leq 0.56 \quad (3)$$

The variable x_T provides a good estimate of the value of x at which the parton distributions are probed. At lower values of x_T even the softer parton distributions such as gluons or antiquarks are important. Fig.1, taken from ref.(5), shows the contribution of the various subprocesses to the total jet production cross-section. Below E_T of 80 Gev the dominant processes are gluon initiated. Above this value of E_T the harder valence quark distribution makes the quark-quark scattering diagrams dominate. Low x_T jets thus provide an ideal place to study gluon jets.

Ignorance of the gluon distribution function, which is poorly determined from deep-inelastic scattering, does not lead to a large uncertainty in the jet cross-section, because of a correlation between the shape of the measured gluon distribution and the value of Λ . This is illustrated in Fig.2 where two phenomenologically acceptable gluon distribution functions taken from ref.(6) are shown at $Q^2 = 4\text{Gev}^2$ and $Q^2 = 2000\text{Gev}^2$. The narrower gluon distribution function (denoted D01) has $\Lambda = .2$ Gev, whereas the broader gluon distribution (D02) has $\Lambda = .4$ Gev. Despite the large differences at low Q^2 , at higher values (e.g. $Q^2 = 2000\text{Gev}^2$, the approximate scale relevant for high p_T jets), the two gluon distributions are practically identical. This is particularly true of the low x region in which the gluon distribution is most important.

A theoretical issue, related to the value of Λ , is the choice of Q , the scale in the parton distribution functions (eq.(1)) and in the running coupling constant (eq.(2)). In theory, this question could be resolved if an $O(\alpha_s^3)$ calculation had been performed. Different choices for the scale Q modify the form of the $O(\alpha_s^3)$ terms. Because the parton cross-section begins in order $O(\alpha_s^2)$ the truncated result without $O(\alpha_s^3)$ terms is quite sensitive to the choice of scale. Without an $O(\alpha_s^3)$ calculation we can at best make an educated guess of the correct scale using the only fragment of the complete calculation which has been performed^{7,8}).

$$q_i + q_j \rightarrow q_i + q_j + G \quad (4)$$

This calculation suggests that the most appropriate scale is

$$Q^2 = p_T^2 / 2 \quad (5)$$

With this choice of scale the corrections to the process in eq.(4) are small for most values of x_T . The complexity of the calculation of the radiative corrections to other partonic sub-processes, especially gluon-gluon scattering,

makes it probable that eq.(5) is the best estimate we shall have for some time. Furthermore our information on the gluon distribution is gleaned from deep inelastic scattering where it first enters at $O(\alpha_s)$. This information is not sufficient to provide a meaningful determination of the gluon distribution function including the $O(\alpha_s)$ terms. It is precisely these correction terms which are needed to give meaning to the $O(\alpha_s^3)$ terms in gluon-gluon scattering. So even if the calculation of radiative corrections to gluon-gluon scattering were technically feasible, it would still be hard to interpret.

2. Q_T Distributions of W and Z bosons.

The production of W and Z bosons proceeds via the Drell-Yan quark anti-quark annihilation mechanism. The total cross-section is calculated from,

$$\sigma = N \int dx_1 dx_2 \left[H(x_1, x_2, Q^2) \delta(x_1 x_2 - \tau) \right] + O(\alpha_s) \quad (6)$$

where N is an overall normalisation and H is the product of quark and anti-quark distribution functions evaluated at scale Q^2 , and weighted with the appropriate coupling factors. The $O(\alpha_s)$ terms have been calculated and give rise to a positive correction of about 40% for both W and Z production at $\sqrt{S} = 540$ GeV. Because the correction is large there is some uncertainty in the prediction for the total cross-section. However the $O(\alpha_s)$ correction is much smaller than it was for μ - pair production at lower energies, (because the coupling constant is smaller), and therefore the ambiguity in the overall normalisation of W and Z production (the so called K-factor) is reduced. The best theoretical values for the total production cross-sections at $\sqrt{S} = 540$ GeV are⁹⁾,

$$\sigma^{W^+ + W^-} = 4.2 \begin{matrix} +1.3 \\ -0.6 \end{matrix} \text{ nb}, \quad \sigma^Z = 1.3 \begin{matrix} +0.4 \\ -0.2 \end{matrix} \text{ nb} \quad (7)$$

Multiplying these numbers by the branching ratios into electrons,

$$B^{W^\pm \rightarrow e^\pm} = .089, \quad B^{Z \rightarrow e^+ e^-} = .032 \quad (8)$$

corresponding to $m_t = 40$ GeV and $\alpha_s/\pi = .04$ the predictions for the observed decay channels are,

$$(\sigma B)^{W^\pm \rightarrow e^\pm} = 370 \begin{matrix} +110 \\ -60 \end{matrix} \text{ pb}, \quad (\sigma B)^{Z \rightarrow e^+ e^-} = 42 \begin{matrix} +12 \\ -6 \end{matrix} \text{ pb}. \quad (9)$$

The transverse momentum distribution of the intermediate vector bosons is theoretically more complicated than the total cross-section. In the limit in which the transverse momentum q_T is of the same order as the mass of the vector boson Q, the transverse momentum should be well described by recoil against one

massless parton and the maximum transverse momentum A_T is controlled by the kinematics of the one parton emission diagrams.

$$A_T^2 = \frac{(S + Q^2)^2}{4S \cosh^2 y} - Q^2 \quad (10)$$

Decreasing q_T introduces a second scale into the problem and for small q_T we find that large terms are generated and must be resummed if we are to have a valid perturbative prediction. The emission of many gluons changes the form of the q_T distribution but should leave the total cross-section, which is quite reliably calculated in $O(\alpha_s)$, unchanged. The resummation was first attempted by DDT¹⁰⁾ and subsequently modified and consolidated¹¹⁾. A consistent framework for going beyond the leading double logarithmic approximation has been indicated by Collins and Soper¹²⁾. The work reported here⁹⁾ which was used to generate the numerical results has the following features.

a) At large q_T we automatically recover the $O(\alpha_s)$ perturbative distribution coming from one gluon emission, without ad hoc introduction of matching procedures between hard and soft radiation.

b) In the region $q_T \ll Q$ the soft gluon resummation is performed at leading double logarithmic accuracy.

c) Only terms corresponding to the emission of soft gluons for which the exponentiation can be theoretically justified are resummed.

d) The integral of the q_T distribution reproduces the well known results for the $O(\alpha_s)$ total cross-sections exactly.

e) The average value of q_T^2 is also identical with the perturbative result at $O(\alpha_s)$.

f) All quantities are expressed in terms of precisely defined quark distribution functions as measured in Deep Inelastic scattering.

g) The results constitute the first term in a systematic expansion.

The result contains a resummation of logarithmic terms in impact parameter space. This allows exact conservation of the transverse momentum of the emitted gluons. The form of the result is,

$$\frac{d\sigma}{dq_T^2 dy} = N \int \frac{d^2b}{(2\pi)^2} e^{-ib \cdot q_T} [R(b^2, Q^2) \exp S(b^2, Q^2)] + Y(q_T^2, Q^2) \quad (11)$$

where the form factor (including only terms which can be deduced from an $O(\alpha_s)$ calculation), is

$$S(b^2, Q^2) = \int_0^{A_T} \frac{d\mu^2}{\mu^2} [J_0(b\mu) - 1] \frac{4}{3} \frac{\alpha_s(\mu)}{2\pi} \left(2 \ln \frac{Q^2}{\mu^2} - 3 \right) \quad (12)$$

The complete $O(\alpha_s)$ expressions for Y and R are given in ref.(9) and are too complicated to reproduce here. The zeroth order term in R involves the parton distribution functions evaluated at a b-dependent scale

$$R = H(x_1^0, \frac{\tau}{x_1^0}, B^2) + O(\alpha_s), \quad x_1^0 = \sqrt{\tau} e^y, \quad B^2 \sim \frac{1.59}{b^2} \quad (13)$$

The function Y is completely finite as q_T tends to zero.

This result for the form factor is in agreement with the general form of Collins and Soper. Their result is written in terms of arbitrary parameters c_1 and c_2 which be used to modify the scale at which the separation between hard and soft contributions is made. After some manipulation their result for the form factor can be written as,

$$S_{CS}(b^2, Q^2) = -\frac{8}{3\pi} \int_{c_1/b}^{c_2 Q} \frac{d\mu}{\mu} \left[\ln \left(\frac{c_2^2 Q^2}{\mu^2} \right) A(\mu) + B(\mu) \right] \quad (14)$$

Making the natural choices for the parameters,

$$c_1 = 2 e^{-\gamma_E}, \quad c_2 = A_T/Q \quad (15)$$

we obtain in the \overline{MS} scheme,

$$A(\mu) = \alpha_s(\mu) + D \alpha_s^2(\mu) + O(\alpha_s^3), \quad B(\mu) = \alpha_s(\mu) \left\{ -\frac{3}{2} - 2 \ln c_2 \right\} \quad (16)$$

the term D is the higher order correction which is dominant in the low q_T region and is given by,^{13,14}

$$D = \frac{1}{2\pi} \left\{ \left(\frac{67}{18} - \frac{\pi^2}{6} \right) 3 - \frac{10 n_f}{18} \right\} \quad (17)$$

Eq.(14) is readily shown to be in agreement with eq.(12) in the approximation in which we replace the Bessel function by a θ function.¹⁵

The numerical consequences of eq.(11) are shown in Fig.(3). Also shown is a histogram of the 52 UA1 W events suitably normalised. The parton distributions used are those of ref.(6), which have two different choices for Λ . Both sets are compatible with low energy data. The principal uncertainty in eq.(11) is associated with the choice of Λ . From Fig.(3) we see that this leads to a variation of about 15%. The form of the parton distribution functions leads to a small uncertainty, since quark distributions, well determined in deep inelastic scattering, are most important. The behaviour of the strong coupling

constant in the very low momentum region has only a minor effect above q_T of 2 GeV. The influence of higher order corrections can be estimated by including the only term which has been calculated (eq.(17)) in our numerical analysis. The effect of the inclusion of D is numerically approximately equivalent to a rescaling of Λ by a factor of about 2. Thus the effect of higher order corrections cannot be distinguished from the uncertainty in Λ .

The differential cross-section for $W^+ + W^-$ production at zero rapidity is,

$$\left. \frac{d\sigma}{dy} \right|_{y=0} \sim 2.3 \text{ nb} \quad (18)$$

The corresponding result for Z production is,

$$\left. \frac{d\sigma}{dy} \right|_{y=0} \sim .8 \text{ nb} \quad (19)$$

The shape of the q_T distribution for Z production is very similar to the plot for W production shown in Fig.3. Here again the main uncertainty comes from the choice of Λ .

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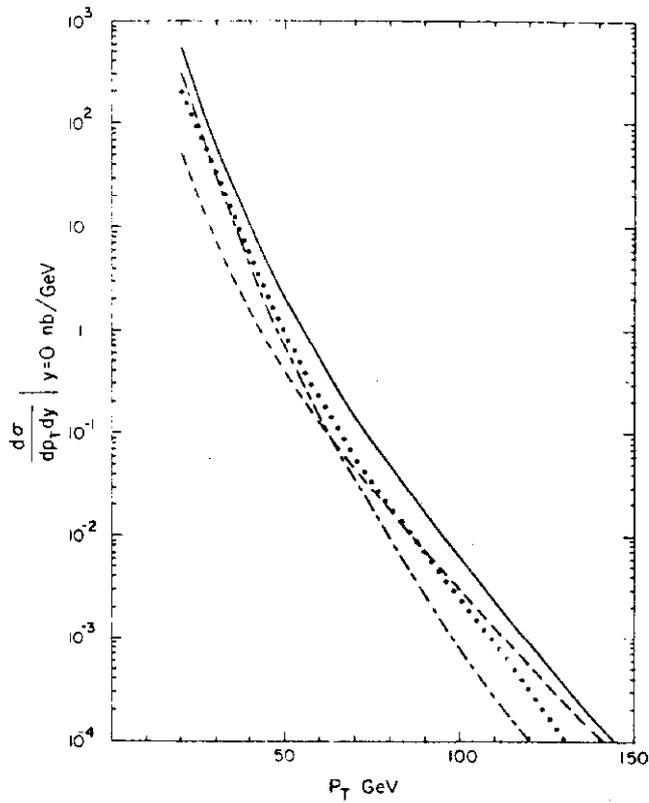


Fig. 1 Subprocess contributions to the jet cross-section (dashed line, qq scattering; dashed-dotted line, gg scattering; dotted line, qq scattering; solid line, total). The scale Q , (cf. eqs. (1,2)) is chosen so that $Q = p_T/2$ and $\Lambda = .2$ GeV.

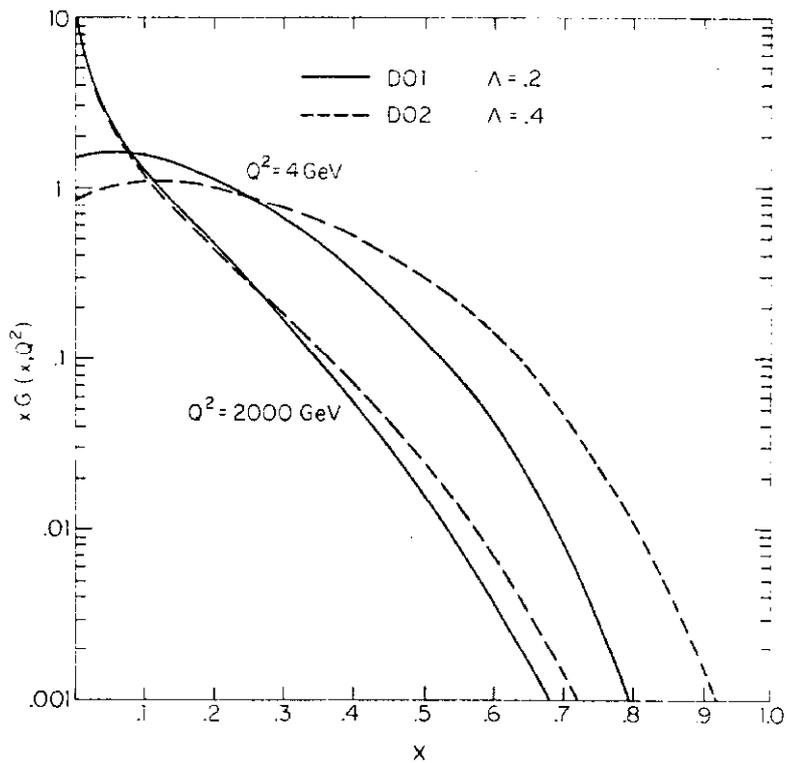


Fig. 2 Two parametrizations for the gluon distribution at $Q^2 = 4$ GeV² and $Q^2 = 2,000$ GeV².

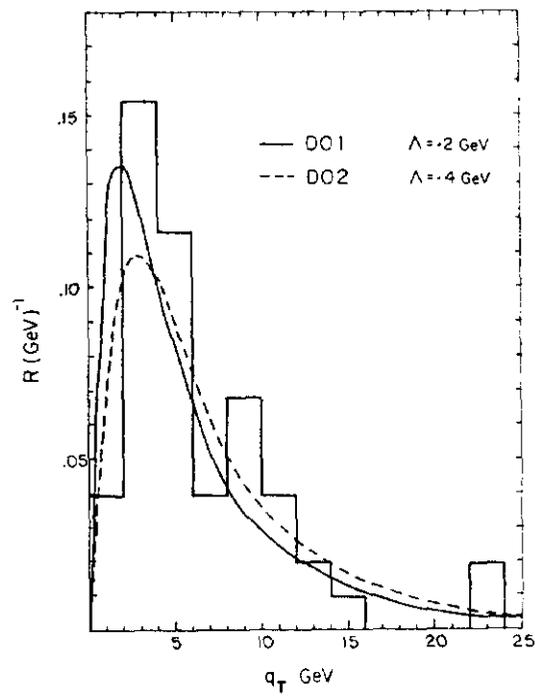


Fig. 3 The differential cross-section for the production of W bosons. The ratio

$$R = \frac{d\sigma}{dq_T dy} \bigg/ \frac{d\sigma}{dy} \text{ at rapidity } y=0 .$$