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Supersymmetric Effective Actions For Anomalous Internal Chiral Symmetries

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ABSTRACT

The effective action satisfying the anomalous axial current Ward identities in supersymmetric (susy) QCD theories is derived. It incorporates the consequences of these anomalies for low energy theorems governing the Goldstone boson interactions as well as interactions of their susy partners.

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Chiral anomalies [1] have many striking theoretical and experimental consequences. Their required cancellation or matching in chiral gauge theories puts tight constraints on model building for fundamental interactions [2] as well as composite models of quarks and leptons [3]. The anomalies also control the topological structure of these theories [4]. In addition, for theories such as QCD where the global chiral symmetries are spontaneously broken to a vector subgroup, the anomalies also govern contributions to the low energy theorems describing the interactions of the Goldstone bosons. Thus effective Lagrangians for QCD [5] contain terms allowing for the $\pi^0 \to \gamma\gamma$ decay.

In supersymmetric (susy) theories, anomalies play no less a pivotal role [6-8]. In this letter, we derive the contributions to the effective action for susy QCD theories which incorporate the low energy consequences of the axial current anomaly. The procedure is completely anologous to that employed by Wess and Zumino [5] in the construction of the non-supersymmetric effective action.

The underlying susy QCD action has a global internal flavor symmetry group $SU(n)_L \times SU(n)_R \times U(1)_V$ which is assumed to be spontaneously broken by condensate formation [F2] to a unbroken vector subgroup $SU(N)_V \times U(1)_V$. The $G=SU(N)\times SU(N)$ generators are denoted by T^A, $A = 1,\ldots,2(N^2-1)$. Generators, of the unbroken subgroup $H=SU(N)_V$ are generically labelled with a lower case letter from the beginning of the alphabet, $T_a, a=1,\ldots N^2-1$, while the broken generators are in 1-1 correspondence with the generators of the coset group G/H and are labelled with lower case letters from the middle of the alphabet, $T_i, i=N^2,\ldots,2(N^2-1)$. The algebra is given by

$$[T_A, T_B] = f_{ABC}\, T_C, \tag{1}$$

with f_{ABC} the totally anti-symmetric structure constants. Since the group parity operation (involutive automorphism) P is such that $T_i \overset{P}{\to} -T_i$, while $T_a \overset{P}{\to} T_a$, the algebra can be written as

$$\begin{aligned} [T_a, T_b] &= i\, f_{abc}\, T_c \\ [T_i, T_j] &= i\, f_{ija}\, T_a \\ [T_a, T_i] &= i\, f_{aij}\, T_j, \end{aligned} \tag{2}$$

with f_{abc} and $f_{ija} = f_{\tilde{i}\tilde{j}a}$ the SU(N) totally antisymmetric structure constants. Here we have defined the complimentary indices $\tilde{i}=i-(N^2-1)$ and $\tilde{a}=a+(N^2-1)$ so that for SU(3)×SU(3), for example; $\widetilde{11} = 3$, $\tilde{3} = 11$. (Alternatively, one could work in a left-right basis given by $L_a=1/2(T_a-T_{\tilde{a}})$, $R_a=1/2\ (T_a+T_{\tilde{a}})$, with $[L_a,L_b] = if_{abc}L_c$, $[R_a,R_b] = if_{abc}R_c$ and $[L_a,R_b] = 0$. We choose to work in the vector-axial vector basis since we will construct an action which is invariant and anomaly free under the vector $SU(N)_V$ subgroup).

The supersymmetric QCD action functional is denoted by $\Gamma[Q,\bar{Q},G,V,A]$, where $(\bar{Q})Q$ denote the quark (anti-)chiral superfields, G the gluon vector superfields, and $(A^i)V^a$ are the external Yang-Mills vector superfields associated with the (axial) vector symmetry. Letting $(\bar{\Lambda}^A)\ \Lambda^A$ be the (anti-) chiral superfields parametrizing the SU(N)×SU(N) gauge transformations, the G transformations of the fields are given by the action of the SU(N)×SU(N) Ward identity functional differential operators $\hat{\delta}(\Lambda,\bar{\Lambda})$ where

$$\hat{\delta}(\Lambda,\bar{\Lambda}) = \int dV[\hat{\delta}(\Lambda,\bar{\Lambda})\mathcal{V}^{A}]\frac{\delta}{\delta\mathcal{V}^{A}} \tag{3}$$

$$+\int dS[\hat{\delta}(\Lambda,\bar{\Lambda})Q]\frac{\delta}{\delta Q} + \int d\bar{S}[\hat{\delta}(\Lambda,\bar{\Lambda})\bar{Q}]\frac{\delta}{\delta\bar{Q}} \quad ,$$

with $dV=d^4xd^2\theta d^2\bar{\theta}$, $dS = d^4xd^2\theta, d\bar{S} = d^4xd^2\theta$. The Yang-Mills superfields $\mathcal{V}^a = V^a$, $\mathcal{V}^i = A^i$ transform as

$$\hat{\delta}(\Lambda,\bar{\Lambda})\mathcal{V}^A = \frac{1}{2}(\bar{\Lambda}^B + \Lambda^B)f_{BAC}\mathcal{V}^C + \frac{i}{2}(\bar{\Lambda}^B - \Lambda^B)[\mathcal{V}\coth\mathcal{V}]_{BA}, \tag{4}$$

where in the last term the matrix notation $(\mathcal{V})_{BC} = -i\mathcal{V}^A f_{ABC}$ has been employed. Letting $(Q_{\bar{N}})Q_N$ be quark superfields transforming as the $(\bar{N})N$ of $SU(N)_V$ then

$$\begin{aligned}\hat{\delta}(\Lambda,\bar{\Lambda})Q_N &= (i\Lambda^a\frac{\lambda_a}{2} - i\Lambda^i\frac{\lambda_{\tilde{i}}}{2})Q_N \\ \hat{\delta}(\Lambda,\bar{\Lambda})Q_{\bar{N}} &= (-i\Lambda^a\frac{\lambda_a^T}{2} - i\Lambda^i\frac{\lambda_{\tilde{i}}^T}{2})Q_{\bar{N}},\end{aligned} \tag{5}$$

while the anti-chiral quark superfields transform as the conjugate. The $\lambda_a/2$ are SU(N) representation matrices. The functional differential operators represent the SU(N)×SU(N) charges T_A and obey the algebra

$$\left[\hat{\delta}(\Lambda,\bar{\Lambda}),\hat{\delta}(\Lambda',\bar{\Lambda}')\right] = \hat{\delta}(\Lambda\times\Lambda',\bar{\Lambda}\times\bar{\Lambda}'), \tag{6}$$

where $(\Lambda\times\Lambda')_C = \Lambda^A\Lambda'^B f_{ABC}$

The anomalous Ward identities are obtained by considering the SU(N)×SU(N) variation of Γ and are given by

$$\hat{\delta}(\Lambda,\bar{\Lambda})\ \Gamma = G(\Lambda,\bar{\Lambda}) = G(\Lambda) + \bar{G}(\bar{\Lambda}), \tag{7}$$

where the chiral Adler-Bardeen axial anomaly is denoted by

$$G(\Lambda) = \int dS\ \Lambda^i G_i(V,A); \qquad \bar{D}_{\dot{\alpha}} G^i = 0 \tag{8}$$

and the anti-chiral axial anomaly is

$$\bar{G}(\bar{\Lambda}) = \int d\bar{S}\bar{\Lambda}^i\bar{G}_i(V,A); \qquad D_\alpha \bar{G}_i = 0, \tag{9}$$

with $\bar{G}(\bar{\Lambda}) = G^\dagger(\Lambda)$. The $G^i(V,A)$ is determined from the quark flavor $SU(N)_V$ representations and has the one-axial current form [F3]

$$G(\Lambda) = \frac{iN_c}{32\pi^2}\int dS\ \mathrm{Tr}\ \left[\Lambda_{(A)}W_{(V)}W_{(V)}\right], \tag{10}$$

with N_c being the number of colors. Here we have adopted a matrix notation so that $\Lambda_{(A)} = \Lambda^{\bar{a}}\lambda_a/2$, $W_{(V)} = W^a_{(V)}\lambda_a/2$. $W^a_{(V)\alpha}$ is the chiral field strength spinor for the $SU(N)_V$ Yang-Mills superfields which (in the one-axial current case) is given by

$$W_{(V)_\alpha} = -\frac{1}{4} \bar{D}\bar{D} \left[e^{-2V} D_\alpha e^{2V}\right], \tag{11}$$

with $V = V^a \frac{\lambda_a}{2}$.

The one-axial current contribution provides the overall normalization as well as the first term in the expression for the multi-axial current anomaly. The form of this multi-current anomaly is determined by the Wess- Zumino consistency conditions (5) which are secured by acting on Γ with eq. (6) and are given by

$$\hat{\delta}(\Lambda,\bar{\Lambda})G(\Lambda',\bar{\Lambda}') - \hat{\delta}(\Lambda',\bar{\Lambda}')G(\Lambda,\bar{\Lambda}) = G(\Lambda\times\Lambda',\bar{\Lambda}\times\bar{\Lambda}'). \tag{12}$$

Note that the one-current anomaly, eq. (10) is indeed a solution of this equation evaluated when the axial fields vanish. The determination of the multi-current form of $G(\Lambda,\bar{\Lambda})$ in a full susy covariant gauge will not be addressed there.

A great deal of information about the low energy behavior of susy QCD is embodied in the effective action describing the interactions of the (anti-) chiral Goldstone superfields ($\bar{\pi}^i$) π^i resulting from the $SU(N)_L \times SU(N)_R \to SU(N)_V$ spontaneous breakdown. This action should reflect the symmetries of the underlying susy QCD and hence obey the anomalous $SU(N) \times SU(N)$ Ward identities. For the effective theory, the $SU(N) \times SU(N)$ variations are given by

$$\hat{\delta}(\Lambda,\bar{\Lambda}) = \int dV \left[\hat{\delta}(\Lambda,\bar{\Lambda})\mathcal{V}^A\right] \frac{\delta}{\delta \mathcal{V}^A} \tag{13}$$

$$+\int dS \left[\hat{\delta}(\Lambda,\bar{\Lambda})\ \pi^i\right] \frac{\delta}{\delta\pi^i} + \int d\bar{S} \left[\hat{\delta}(\Lambda,\bar{\Lambda})\ \bar{\pi}^i\ \right] \frac{\delta}{\delta\bar{\pi}^i} \quad ,$$

with $\hat{\delta}(\Lambda,\bar{\Lambda})\mathcal{V}^A$ given by eq. (4) and with pion superfield transformations

$$\hat{\delta}(\Lambda,\bar{\Lambda})\ \pi^i = \Lambda^A A^i_A(\pi) \tag{14}$$

$$\hat{\delta}(\Lambda,\bar{\Lambda})\ \bar{\pi}^i = \bar{\Lambda}^A \bar{A}^i_A(\bar{\pi})$$

Here the (anti-) chiral superfield killing vectors $(\bar{A}_A{}^i)$ $A_A{}^i$ are defined to satisfy the Lie differential equations

$$A^j_A \frac{\partial A^i_B}{\partial\pi^j} - A^j_B \frac{\partial A^i_A}{\partial\pi^j} = f_{ABC} A^i_C \tag{15}$$

$$\bar{A}^j_A \frac{\partial\bar{A}^i_B}{\partial\bar{\pi}^j} - \bar{A}^j_B \frac{\partial\bar{A}^i_A}{\partial\bar{\pi}^j} = f_{ABC} \bar{A}^i_C$$

and when restricted to the invariant subgroup H, are given by the linear representations

$$A^i_a = f_{aij}\ \pi^j \tag{16}$$

$$\bar{A}^i_a = f_{aij}\ \bar{\pi}^j$$

These variations obey the SU(N) $\times$ SU(N) algebra of eq. (6). The effective action, Γ_{eff}, is now obtained as the solution to the anomalous functional differential equation

$$\hat{\delta}(\Lambda,\bar{\Lambda})\ \Gamma_{eff}\left[\pi,\bar{\pi},V,A\right] = G\ (\Lambda,\bar{\Lambda}) \tag{17}$$

The construction of the solution to the homogeneous equation was discussed in references [12 - 18] in the Wess-Zumino gauge [F4]. It is given by equation (3.47) of reference [18] in terms of the super Kähler potential K and the SU(N) × SU(N)superfield of currents $\hat{J}_A$ as

$$\Gamma_{eff}[\pi,\bar{\pi},V,A]_{hom} = \int dV\ \hat{K}\ (\pi,\bar{\pi},V,A), \tag{18}$$

$$\hat{K}(\pi,\bar{\pi},V,A) = K(\bar{\pi},\pi) + 2\hat{J}_A\mathcal{V}^A + 2\mathcal{V}^A\bar{A}^j_A g_{ji}(\bar{\pi},\pi)\ A^i_B\mathcal{V}^B,$$

where

$$\frac{\partial K}{\partial\bar{\pi}^i}\ \bar{A}^i_A + \frac{\partial K}{\partial\pi^i}\ A^i_A = \bar{F}_A\ (\bar{\pi}) + F_A\ (\pi)\ , \tag{19}$$

$$g_{ij}\ (\bar{\pi},\pi) = \frac{\partial^2}{\partial\bar{\pi}^i\partial\pi^j}\ K(\bar{\pi},\pi), \tag{20}$$

and

$$\hat{J}_A = \frac{i}{2}\ (\frac{\partial K}{\partial\bar{\pi}^i}\bar{A}^i_A - \frac{\partial K}{\partial\pi^i}\ A^i_A - \bar{F}_A + F_A), \tag{21}$$

with $(\bar{F}_A)$ F_A arbitrary (anti-) chiral superfields. Since

$$\hat{\delta}(\Lambda,\bar{\Lambda})\hat{K} = \Lambda^A F_A + \bar{\Lambda}\ ^A\bar{F}_A, \tag{22}$$

it follows that the action is invariant;

$$\hat{\delta}(\Lambda,\bar{\Lambda})\ \int dV\hat{K} = 0 \tag{23}$$

The particular solution yielding the anomaly can be found with a construction similar to the one used in reference [5]. Noting that

$$[\hat{\delta}(\Lambda_{(A)},\bar{\Lambda}_{(A)})]^n \Gamma_{eff}[\pi,\bar{\pi},V,A] = [\hat{\delta}_{\mathcal{V}}(\Lambda_{(A)},\bar{\Lambda}_{(A)})]^{n-1} \quad G(\Lambda,\bar{\Lambda}),$$

where

$$\hat{\delta}_{\mathcal{V}}(\Lambda,\bar{\Lambda}) = \int dV \; [\hat{\delta}(\Lambda,\bar{\Lambda})\mathcal{V}^A]\frac{\delta}{\delta \mathcal{V}^A} \tag{25}$$

transforms the vector superfields only and $\Lambda^i{}_{(A)} = \Lambda^i$ are the axial vector gauge parameters, it follows that

$$e^{\hat{\delta}(\Lambda_{(A)},\, \bar{\Lambda}_{(A)})} \; \Gamma_{eff} \quad [\pi,\bar{\pi},V,A] \tag{26}$$

$$= \Gamma_{eff} \; [\pi,\bar{\pi},V,A] + \left[\frac{e^{\hat{\delta}_{\mathcal{V}}(\Lambda_{(A)},\; \bar{\Lambda}_{(A)})}-1}{\hat{\delta}(\Lambda_{(A)},\bar{\Lambda}_{(A)})} \right] G(\Lambda,\bar{\Lambda})$$

Since $\hat{\delta}(\Lambda_{(A)},\bar{\Lambda}_{(A)})$ generates the axial transformations, the left hand side is simply the effective action expressed in terms of the axially transformed fields, $\Gamma_{eff}\;[\pi',\bar{\pi}',V',A']$. However, for every value of $(\bar{\pi})$ π, there exists a $(\bar{\Lambda}_{(A)}(\bar{\pi}))$ $\Lambda_{(A)}(\pi)$ such that $(\bar{\pi}' = 0)$ $\pi' = 0$. That is, we can always move in the coset space to the origin of coordinates, $\pi' = 0 = \bar{\pi}'$, by an axial transformation. For example, with the Killing vectors given by $A^i_j(\pi) = -i\;(\pi \coth \pi)_{ji}$, with $(\pi)_{ab} = -i\pi^i f_{Iab}$, the finite axial transformations of π and $\bar{\pi}$ become

$$e^{-2\pi'} = e^{-i\Lambda_{(A)}} e^{-2\pi} e^{-i\Lambda_{(A)}}$$

$$e^{-2\bar{\pi}'} = e^{i\bar{\Lambda}_{(A)}} e^{-2\bar{\pi}} e^{i\bar{\Lambda}_{(A)}} \quad , \tag{27}$$

where

$$\pi = \pi^i \frac{\lambda_I}{2}, \quad \text{etc.}$$

Thus for ($\bar{\Lambda}_{(A)} = -i\bar{\pi}$) $\Lambda_{(A)} = i\,\pi$, it follows that ($\bar{\pi}' = 0$) $\pi' = 0$. With the boundary condition that the anomalous action cannot be written as a function of V,A alone so that $\Gamma_{eff}\,[0,0,V',A'] = 0$, the particular solution is secured as

$$\Gamma_{eff}\,[\pi,\bar{\pi},V,A]_{inhom} = -\int_0^1 d\xi e^{\xi\hat{\delta}\,(i\pi,-i\bar{\pi})} G(i\pi,-i\bar{\pi}) \tag{28}$$

This effective action describes all the low energy manifestations of the anomalous chiral Ward identity. This includes the interactions of the true Goldstone bosons as well as the susy partner quasi-Goldstone fermions and bosons [16-19]. Focusing on the particular case of chiral SU(3) × SU(3), the above effective action governs the familiar decay of the neutral pion into two photons and, in addition, its decay into two photinos. The pion superfield is given in components by

$$\pi^i = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu} \left[(S^i + iP^i) + \sqrt{2}\theta\Psi^i + \theta\theta F^i\right]$$

$$\bar{\pi}^i = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}\left[(S^i - iP^i) + \sqrt{2}\bar{\theta}\bar{\Psi}^i + \bar{\theta}\bar{\theta}\bar{F}^i\right] \quad , \tag{29}$$

where the neutral pion π^0 is simply $f_\pi \tilde{P}^3$, f_π being the pion decay constant. Isolating the couplings of the neutral pion to two photons and two photinos we find close to the π^0 mass shell that

$$\Gamma_{eff} = - \frac{e^2 N_c}{16\pi^2} \frac{1}{6f_\pi} \int d^4x \; \{ \; \varepsilon^{\mu\nu\lambda\rho} \pi^0 F_{\mu\nu} F_{\lambda\rho} + 2\partial_\mu \pi^0 (\lambda\sigma^\mu\bar{\lambda} - \bar{\lambda}\bar{\sigma}^\mu\lambda) - 2im_{\pi^0} \pi^0 (\lambda\lambda - \bar{\lambda}\bar{\lambda}) \} + \ldots \tag{30}$$

Here $F_{\mu\nu}$ and λ_α are the electomagnetic field strength and photino fields respectively. In obtaining eq.(30), we have used the fact that close to the pion mass shell, the auxilary field $\tilde{F}^3$ has a contribution proportional to $\tilde{P}^3$ with proportionality constant $-im_\pi{}^0$. From eq. (30), we see that π^0 couples to two photinos $(\tilde{\gamma})$ with strength comparable to the two photon coupling. However, the limit on the $\pi^0 \to \nu\nu$ branching ratio, [20]

$$\frac{\Gamma(\pi^0 \to \nu\nu)}{\Gamma(\pi^0 \to all)} < 2.4 \times 10^{-5}$$

implies a similar limit on the $\pi^0 \to \tilde{\gamma}\tilde{\gamma}$ decay. This dictates that the $\pi^0 \to \tilde{\gamma}\tilde{\gamma}$ must be suppressed by phase space so that whatever mechanism is responsible for the susy breaking must give rise to a photino mass greater than half the neutral pion mass [21].

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Footnotes

[F1] We do not consider the $U(1)_A$ symmetry which is broken by anomalies containing the colored Yang-Mills gluon fields. We also do not include the global $U(1)_R$ symmetry which is not an internal symmetry since it does not commute with susy.

[F2] The question of whether chiral symmetry breaking condensates form in susy QCD theories is still an open one [6-9,10,11] and will not be further addressed here.

[F3] The one-axial current form of the anomaly corresponds to retaining those contributions to the $SU(N) \times SU(N)$ Ward identity arising from a single derivative with respect to the axial vector Yang-Mills superfield.

[F4] If only the Yang-Mills superfields of the invariant subgroup appear, the homogeneous solution for arbitrary gauge is given in ref. [18].

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