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TWIST-FOUR EFFECTS IN NEUTRAL CURRENT NEUTRINO SCATTERING AND $\sin^2\theta_W$

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ABSTRACT

We have calculated the twist-four, spin-two corrections to neutral current neutrino scattering on isoscalar targets using the operator product expansion, determining the coefficients from perturbative Quantum Chromodynamics and evaluating the nucleon matrix elements of the operators in the MIT Bag Model. We find the higher twist effects decrease $\sin^2\theta_W$ by about 1%, considerably less than previously estimated, but comparable to the electroweak radiative corrections, which also decrease $\sin^2\theta_W$ by a few percent.

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One of the fundamental parameters in the standard electroweak theory[1] is $\sin^2\theta_W$ and its precise determination is a subject of considerable current interest, particularly since its value bears heavily on the issue of grand unified theories, such as[2] SU(5), in which the standard electroweak theory is embedded. The perturbative radiative electroweak corrections to $\sin^2\theta_W$, as extracted from various data, have been calculated[3] and typically amount to about 5%. In deep inelastic neutrino scattering, which can be used to measure $\sin^2\theta_W$, there are also nonperturbative Quantum Chromodynamic effects; namely, the higher twist corrections and these are, to some extent, intrinsically model dependent and therefore difficult to calculate reliably. Glück and Reya[4] have estimated that they could amount to as much as a 10% correction to $\sin^2\Theta_W$ and, in view of their uncertainty, such a large effect would then obscure the issue of the precise value of $\sin^2\theta_W$ at the level of the electroweak radiative corrections.

We have explored this question of higher twist effects in deep inelastic lepton scattering following the established systematic procedure[5]: The bilocal product of the two quark current operators is expanded into local operators using the Wilson operator product expansion[6] (OPE). The coefficient functions obey the renormalization group equations and the anomalous dimensions of the operators are calculated using perturbative techniques. The model dependence of the higher twist effects is then isolated in the nucleon matrix elements of the local operators and numerical results can be obtained using some quark confinement model for the nucleon wave function.

In the following we specifically consider the twist-four, spin-two effects in neutral current neutrino scattering and show that their effects on $\sin^2\theta_W$, as extracted from these data, are considerably smaller than estimated by Glück and Reya[4]. They are, however, comparable to the electroweak radiative corrections[3] and cannot be ignored in certain kinematical regions, but do become negligible at large Q^2, as expected.

In our calculations we shall essentially adopt the basic approach of Politzer[7], but we shall use the rather convenient local operator basis introduced by Jaffe and Soldate[8] in their analysis of electroproduction and also follow some of their techniques, as well as those in the related work of Luttrell, Wada and Weber[9] and Luttrell and Wada[10] in computing the coefficient functions. As in these previous analyses, we shall make the approximation that the difference between Q^2 and the renormalization scale μ^2 can be ignored, which is equivalent to neglecting the anomalous dimensions of the local operators; in fact, this is tantamount to computing the coefficient functions perturbatively in the quark-parton model using the running QCD coupling constant $\alpha_s(Q^2)$. And, finally, we shall use the MIT Bag Model[11] to evaluate the nucleon matrix elements of the local operators.

We begin by briefly reviewing some standard definitions and formulas we shall need later[5]. The inclusive cross section for neutral current neutrino-nucleon scattering $\nu(k) + N(p) \rightarrow \nu(k') +$ anything is of the form

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{G^2}{4\pi ME} \ell^{\mu\nu} W_{\mu\nu} \tag{1}$$

and the standard kinematical variables are $q = k - k'$, $Q^2 = -q^2$, $\nu = pq = M(E-E')$, $x = Q^2/2\nu$ and $y = pq/pk = \nu/ME = (E-E')/E$. The leptonic tensor is simply

$$\ell_{\mu\nu} = -g_{\mu\nu} kk' + k_\mu k'_\nu + k_\nu k'_\mu - i\varepsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \quad . \tag{2}$$

The hadronic tensor is defined by

$$W_{\mu\nu} = \int dz\, e^{iqx} \langle p | [J_\mu(z), J_\nu(0)] | p \rangle \tag{3}$$

and can be written in terms of the familiar structure functions $\nu W_{L,2,3}(\nu, Q^2) = F_{L,2,3}(\nu, Q^2)$:

$$\begin{aligned} W_{\mu\nu} &= (g_{\mu\nu} - q_\mu q_\nu / q^2)[\nu W_L(\nu, Q^2)/2x] \\ &- [g_{\mu\nu} + p_\mu p_\nu q^2/\nu^2 - (p_\mu q_\nu + p_\nu q_\mu)/\nu][\nu W_2(\nu, Q^2)/2x] \\ &- i\epsilon_{\mu\nu\alpha\beta}(p^\alpha p^\beta/\nu)[\nu W_3(\nu, Q^2)] \end{aligned} \tag{4}$$

where an average over nucleon spins is understood in Eq. (3). The neutral current in the standard model is

$$\begin{aligned} J_\mu &= \bar{u}\, \gamma_\mu [(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_w)(1 - \gamma_5) - \tfrac{2}{3}\sin^2\theta_w\, (1 + \gamma_5)]u \\ &+ \bar{d}\, \gamma_\mu [(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_w)(1 - \gamma_5) + \tfrac{1}{3}\sin^2\theta_w (1 + \gamma_5)]d \end{aligned} \tag{5}$$

where we have kept only the light quarks, u and d. The twist-two contributions to $W_{\mu\nu}$ correspond to the free quark-parton model and the structure functions $W_{L,2,3}$ then depend only on the Bjorken scaling variable $x = Q^2/2\nu$ and can be written in terms of the (valence) quark distribution functions.

Due to the Quantum Chromodynamic (QCD) interactions of the quarks there are two kinds of Q^2-dependent corrections to the scaling limit; logarithmic scaling violations, which can be calculated in QCD using perturbation theory, and "higher twist" or nonperturbative QCD corrections, which decrease as powers of Q^2. Phenomenologically, these are difficult to disentangle in the present data[12], thus hampering precise tests of perturbative QCD.

To estimate the higher twist effects we consider the Wilson operator product expansion[6] (OPE) of

$$T_{\mu\nu} = i \int dz\, e^{iqz}\, T\, [J_\mu(z)\, J_\nu(0)] \,, \qquad (6)$$

where the connected part is understood. $T_{\mu\nu}$ can be decomposed into scalar structure functions $T_{L,2,3}$ completely analogously to $W_{\mu\nu}$ and in the physical region they are related by

$$W_i = \frac{1}{2\pi}\, \mathrm{Im}\langle p|T_i|p\rangle \qquad (7)$$

where, again, an average over nucleon spins is understood. In the OPE of $T_{\mu\nu}$ we shall use the very convenient basis of local operators $O_i^{\mu_1 \cdots \mu_n}$ due to Jaffe and Soldate[8], which is completely symmetric and traceless, as well as free of contracted, convariant derivatives:

$$\begin{aligned} T_{\mu\nu} = \sum_{n,i} \Big[&(g_{\mu\nu} - q_\mu q_\nu/q^2) q_{\mu_1} q_{\mu_2} C^i_{L,n}\Big(\frac{Q^2}{\mu^2}, g^2\Big) \\ &- (g_{\mu\mu_1} g_{\nu\mu_2} q^2 - g_{\mu\mu_1} q_\nu q_{\mu_2} - g_{\nu\mu_2} q_\mu q_{\mu_1} + g_{\mu\nu} q_{\mu_1} q_{\mu_2}) C^i_{2,n}\Big(\frac{Q^2}{\mu^2}, g^2\Big) \\ &- i\epsilon_{\mu\nu}{}^{\alpha\beta} g_{\alpha\mu_1} q_\beta q_{\mu_2} C^i_{3,n}\Big(\frac{Q^2}{\mu^2}, g^2\Big)\Big] q_{\mu_3} \cdots q_{\mu_n} \Big(\frac{2}{Q^2}\Big)^n O_i^{\mu_1 \cdots \mu_n} \,. \end{aligned} \qquad (8)$$

In Eq. (8) n is the spin (dimension-twist) of the operators in the basis, while $i = 1,2,\ldots$ simply enumerates them. The coefficient functions $C^i_{j,n}(\frac{Q^2}{\mu^2}, g^2)$ $(j = L,2,3)$ obey the renormalization group equation and their values at $Q^2 = \mu^2$ can be determined using perturbation theory, which is sufficient since we shall neglect their anomalous dimensions, $\gamma^i_{j,n}$. As noted above, the twist-two terms in Eq. (8) correspond to the free quark-parton model.

We shall be concerned here with the twist-four, spin-two corrections to $T_{\mu\nu}$ (or $W_{\mu\nu}$) and there are contributions coming from both diagonal diagrams like Fig. 1 and nondiagonal diagrams like Fig 2. It is convenient to consider the combination of products of the V - A and V + A currents VV and AA in the OPE. (The VA and AV interference terms do not contribute to twist-four, spin-two). The VV and AA contributions coming from diagonal diagrams like Fig. 1 are the same; viz.,

$$T_1^{\mu\nu}(VV) = T_1^{\mu\nu}(AA)$$

$$= \frac{g^2}{q^6}\Big[T^{\mu\nu}_{\mu_1\mu_2}\{\tfrac{1}{4}\, O_7^{\mu_1\mu_2}(0) + \tfrac{5}{2}\, O_9^{\mu_1\mu_2}(0)\} \tag{9}$$

$$+ \{(q^\mu q^\nu - g^{\mu\nu}q^2)q_{\mu_1}q_{\mu_2}/q^2\}\{-\tfrac{3}{2}\, O_7^{\mu_1\mu_2}(0) + O_9^{\mu_1\mu_2}(0)\}\Big]$$

where

$$T^{\mu\nu}_{\mu_1\mu_2} = \delta^\mu_{\mu_1}\delta^\nu_{\mu_2}q^2 - (\delta^\mu_{\mu_1}q^\nu + \delta^\nu_{\mu_1}q^\mu)q_{\mu_2} + g^{\mu\nu}q_{\mu_1}q_{\mu_2}. \tag{10}$$

Similarly, the corresponding twist-four, spin-two contributions coming from nondiagonal diagrams like Fig. 2 are

$$T_2^{\mu\nu}(VV + AA) = \frac{8g^2}{q^6}\, T^{\mu\nu}_{\mu_1\mu_2}\{O_2^{\mu_1\mu_2}(0) + O_6^{\mu_1\mu_2}(0)\} \tag{11}$$

and

$$T_2^{\mu\nu}(VV - AA) = -16\,\frac{g^2}{q^6}\, T^{\mu\nu}_{\mu_1\mu_2}O_4^{\mu_1\mu_2}(0). \tag{12}$$

We have checked that Eq. (9) and the sum of Eqs. (11) and (12) agree with the results for electroproduction[8].

Next we turn to the evaluation of the matrix elements of the local operators $O_k^{\mu_1\mu_2}$, which must be of the form

$$\langle p|O_k^{\mu_1\mu_2}(0)|p\rangle = A_k(p^{\mu_1}p^{\mu_2} - \frac{1}{4} M^2 g^{\mu_1\mu_2}). \qquad (13)$$

The coefficients A_k can be evaluated in the nucleon rest frame using some quark confinement model wave function for the nucleon. Since we are considering only transverse components, using rotational invariance, one finds

$$A_k = \frac{2}{M} \langle N|O_k^{00}(0) + \frac{1}{3} O_k^{ii}(0)|N\rangle \qquad (14)$$

where $\langle N|\ldots|N\rangle$ represents the isoscalar nucleon matrix element; that is, the average of the proton and neutron matrix elements.

We shall consider quark confinement models for which the four-component ground state quark wave function is of the form

$$q(r) = \begin{pmatrix} f(r) \\ \vec{\sigma}\cdot\hat{r}\ g(r) \end{pmatrix} \chi \qquad (15)$$

where χ is a two-component spinor. Both the diagonal and nondiagonal matrix elements of the four-quark operators can be expressed in terms of only two, model dependent, integrals over the space coordinates:

$$I_1 = \int d^3r[|f(r)|^2 + |g(r)|^2]^2 \qquad (16)$$

and

$$I_2 = \int d^3r|f(r)|^2|g(r)|^2 \ . \qquad (17)$$

In addition, one needs the matrix elements of nine different spin, color and flavor dependent four-quark operators in evaluating the complete matrix elements of the operations $O_k^{\mu_1 \mu_2}$. For isoscalar targets these are the following:

$$\langle N|\ (Q^2 \lambda^a/2) \cdot (\lambda^a/2)\ |N\rangle = -10/9 \tag{18}$$

$$\langle N|\ (Q\,\vec{\sigma}\,\lambda^a/2) \cdot (Q\,\vec{\sigma}\,\lambda^a/2)\ |N\rangle = -14/9 \tag{19}$$

$$\langle N|\ (Q^2\,\vec{\sigma}\,\lambda^a/2) \cdot (\vec{\sigma}\,\lambda^a/2)\ |N\rangle = 10/3 \tag{20}$$

$$\langle N|\ (I_3\,\lambda^a/2) \cdot (I_3\,\lambda^a/2)\ |N\rangle = 1/3 \tag{21}$$

$$\langle N|\ (I_3\,\vec{\sigma}\,\lambda^a/2) \cdot (I_3\,\vec{\sigma}\,\lambda^a/2)\ |N\rangle = -5/3 \tag{22}$$

$$\langle N|\ (I_3^2\,\vec{\sigma}\,\lambda^a/2) \cdot (\vec{\sigma}\,\lambda^a/2)\ |N\rangle = -1 \tag{23}$$

$$\langle N|\ (I_3\,Q\,\lambda^a/2) \cdot (\lambda^a/2)\ |N\rangle = -1 \tag{24}$$

$$\langle N|\ (I_3\,Q\,\vec{\sigma}\,\lambda^a/2) \cdot (\vec{\sigma}\,\lambda^a/2)\ |N\rangle = 1 \tag{25}$$

$$\langle N|\ (I_3\,\vec{\sigma}\,\lambda^a/2) \cdot (Q\,\vec{\sigma}\,\lambda^a/2)\ |N\rangle = -5/3 \tag{26}$$

Here, of course, Q and I_3 denote the quark charge and weak isospin while $\vec{\sigma}$ and λ^a are the usual quark spin and color matrices. The matrix elements of gluon operators will be neglected since they enter with a coefficient smaller by an order of magnitude and the gluon content of the nucleon (at rest) is negligible.

Combining our results above, one finds the neutrino neutral current cross section, including twist-four, spin-two effects to be

$$\begin{aligned}\sigma_{NC}/\sigma_{CC}^{parton} = {} & \frac{1}{2} + \left(-\frac{56}{27}\, I_1/M + \frac{1600}{81}\, I_2/M\right) \frac{\alpha_s(Q_0^2)}{Q_0^2} \\ & + \left[-1 + \left(-\frac{1360}{27}\, I_1/M + \frac{7040}{81}\, I_2/M\right)\frac{\alpha_s(Q_0^2)}{Q_0^2}\right]\sin^2\theta_w \\ & + \left[\frac{20}{27} + \left(\frac{2096}{81}\, I_1/M - \frac{128}{3}\, I_2/M\right) \frac{\alpha_s(Q_0^2)}{Q_0^2}\right]\sin^4\theta_w\end{aligned} \tag{27}$$

where, solely for convenience, we have normalized σ_{NC} to the quark-parton model value of the charged current neutrino cross section for isoscalar targets

$$\sigma_{CC}^{parton} = G^2 ME/2\pi \simeq 0.79 \times 10^{-38} \left(\frac{E}{GeV}\right) cm^2 \tag{28}$$

and have integrated over the entire range of y and values of $Q^2 > Q_o^2$. Of course, in analyzing any specific experimenal data one should take into account the appropriate kinematical cuts as well as the experimental sensitivity.

To numerically illustrate the effect of the twist-four, spin-two corrections on $\sin^2\theta_W$, we shall that assume all other corrections have already been included in σ_{NC}. That is, we shall equate $\sigma_{NC}/\sigma_{CC}^{parton}$ to the naive result $1/2 - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$ evaluated at $\sin^2\theta_W = 0.229 \pm 0.010$, the world average[13]. One then finds $\sigma_{NC}/\sigma_{CC}^{parton} = 0.310$. Using the MIT Bag Model values[8] for integrals $I_1 = 20.36 \times 10^{-4}$ GeV^3 and $I_2 = 3.21 \times 10^{-4}$ GeV^3 one finds then from Eq. (27) that $\sin^2\theta_W = 0.226$ for $\alpha_s(Q_o^2) = 0.27$ and $Q_o^2 = 2$ GeV^2, which corresponds to present neutral current neutrino scattering data in the regime where the twist-four effects might be expected to be the most significant. At higher values of Q_o^2 these effects are yet smaller.

We conclude that the effect of twist-four, spin-two corrections to the neutral current neutrino cross section on isoscalar targets is to decrease $\sin^2\theta_W$ by about 1%. Clearly, this is certainly much smaller than previously estimated[4]; indeed, quite small compared to the present experimental uncertainties. Nevertheless, these higher-twist effects will have to be taken into account when $\sin^2\theta_W$ can be determined to the precision of the electroweak radiative corrections[3], which also decrease $\sin^2\theta_W$ by a few percent.

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Addendum: After this work was completed we learned that the higher twist effects on $\sin^2\theta_W$ have been estimated by C. Llewellyn-Smith (Oxford University preprint) to be quite small, in agreement with our calculations. We thank Jorge Morfin for providing us with the unpublished manuscript of this work as well as for several very helpful discussions of the various experimental determinations of $\sin^2\theta_W$.

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FIGURE CAPTIONS

Fig. 1. Typical higher-twist diagonal diagrams.

Fig. 2. Typical higher-twist nondiagonal diagrams.

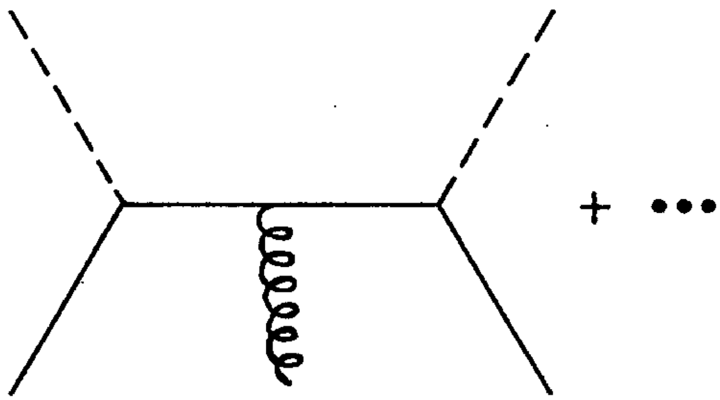

Fig. 1

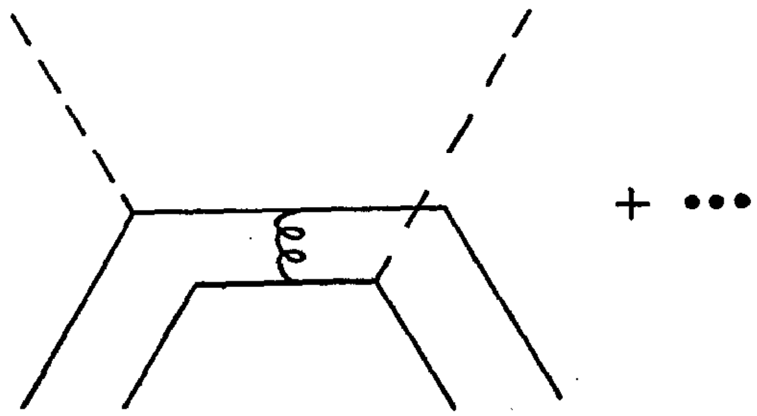

Fig. 2