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Multiphoton decays of positronium

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We compute the branching ratios for parapositronium and orthopositronium into four and five photons, respectively. We find $\Gamma(p\text{-Ps}\rightarrow 4\gamma)/\Gamma(p\text{-Ps}\rightarrow 2\gamma)=0.274(\alpha/\pi)^2\simeq 1.48\times 10^{-6}$ and $\Gamma(o\text{-Ps}\rightarrow 5\gamma)/\Gamma(o\text{-Ps}\rightarrow 3\gamma)=0.177(\alpha/\pi)^2\simeq 0.96\times 10^{-6}$. We also describe the energy spectra of photons from these decays.

In recent years there has been considerable progress in the study of positronium.[1] In particular, accurate measurements of the two- and three-photon decay rates of parapositronium and orthopositronium, respectively, have recently been achieved.[2] In this Brief Report we describe the results of a calculation of the branching ratio and momentum spectrum for the decay of parapositronium (p-Ps) into four photons and orthopositronium (o-Ps) into five photons. These decay channels may soon be directly observable. They also contribute to the decay widths of the states.

The leading-order amplitude for positronium decay to $n\gamma$ is given by

$$\mathcal{T}=\frac{\Psi_{\rm NR}(0)}{2m_e}\mathcal{M}_{e^+e^-\rightarrow n\gamma} .$$

Here $\Psi_{\rm NR}(0)$ is the nonrelativistic wave function evaluated at the origin $[\Psi_{\rm NR}(0)=(\alpha^3m_e^3/8\pi)^{1/2}$ for the ground state]. It is the amplitude for finding the electron and positron sufficiently close to annihilate. The amplitude for the annihilation to occur is $\mathcal{M}_{e^+e^-n\gamma}/2m_e$ (Fig. 1). This amplitude can be computed to leading order with the electron and positron on mass shell and at rest. Binding effects and atomic momenta are negligible since the decay occurs at distances very short relative to the size of the atom, i.e., $1/m_e$ vs $1/\alpha m_e$. This is not always the case for positronium decays. For example, one of the photons in the 3γ decay of a p state is typically very soft (with p_γ as low as $\alpha^2 m_e$), making binding effects very important. However, soft photon emission is strongly suppressed for s states (by $\langle v^2/c^2\rangle^2\sim\alpha^4$), as was evident from our calculation.[3]

All γ matrix manipulations in trace calculations were performed by the program REDUCE.[4] Phase-space integrals were evaluated numerically with use of Monte Carlo algorithms.[5] The results were cross checked by a calculation of all helicity amplitudes.[6] We obtained branching ratios of

$$\frac{\Gamma(p-\mathrm{Ps}\rightarrow 4\gamma)}{\Gamma(p-\mathrm{Ps}\rightarrow 2\gamma)}=0.274\left(\frac{\alpha}{\pi}\right)^2\simeq 1.48\times 10^{-6}$$

and

$$\frac{\Gamma(o-\mathrm{Ps}\rightarrow 5\gamma)}{\Gamma(o-\mathrm{Ps}\rightarrow 3\gamma)}=0.177\left(\frac{\alpha}{\pi}\right)^2\simeq 0.96\times 10^{-6} .$$

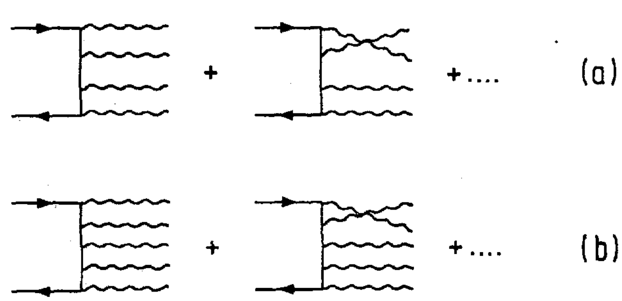

FIG. 1. Amplitudes contributing in leading order to $M(e^+e^-\rightarrow 4\gamma)$ and $M(e^+e^-\rightarrow 5\gamma)$. There are 24 and 120 diagrams, respectively, in all.

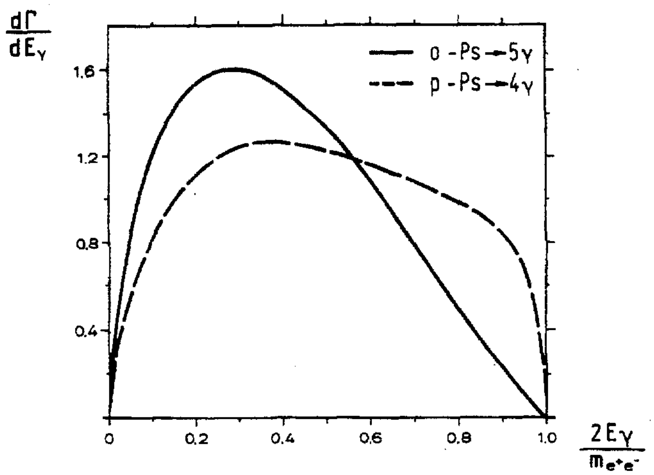

FIG. 2. Leading-order energy spectra of a photon in the decays p-Ps $\rightarrow 4\gamma$ and o-Ps $\rightarrow 5\gamma$. The spectra are normalized to have unit area.

The p-Ps branching ratio does not agree with the earliest result of 3×10^{-7}, due to McCoyd,[7] but is consistent with Refs. 8. We have also computed the photon energy spectra for these decays. The normalized spectra are plotted in Fig. 2.

We have recently learned of another calculation of these rates.[9] Our results agree.

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[1]For a review, see A. Rich, Rev. Mod. Phys. 53, 127 (1981).

[2]D. W. Gidley, A. Rich, E. Sweetman, and D. West, Phys. Rev. Lett. 49, 525 (1982), and references given in Ref. 1.

[3]This estimate is easily understood. For example, the s state might emit a soft photon ($p_\gamma \sim \alpha^2 m_e$) through an $E1$ transition to a virtual p state, which then decays into three hard photons. The amplitude for this to occur is $\sim \sum_{p\,\text{states}} T(p \rightarrow 3\gamma) \times \langle p,\gamma | (-e\vec{p}\cdot\vec{A}/m_e) | s \rangle / (E_p + E_\gamma - E_s)$. In the decay rate, the phase space for the soft photon cancels the energy denominators. Both the transition matrix element for $s \rightarrow p + \gamma$ and the p-state decay amplitude are suppressed by v/c. Thus this decay rate is down by $(v/c)^4 \sim \alpha^4$ relative to the decay into four hard photons. Other mechanisms for producing soft photons in an s-state decay are similarly suppressed. However, in p-state decays, soft photons can be strongly favored, primarily because a p-state decay into hard photons is already suppressed by $(v/c)^2$ [since $\Psi_{NR}(0) = 0$ for a p state]. This is evident from the same sort of analysis as given above.

[4]A. C. Hearn, Stanford University Report No. ITP-247 (unpublished).

[5]This included VEGAS as described in G. P. Lepage, J. Comput. Phys. 27, 192 (1978).

[6]K. Koller, K. Streng, T. Walsh, and P. Zerwas, Nucl. Phys. B206, 273 (1982).

[7]G. McCoyd, Ph. D. thesis (St. John's University, New York) (unpublished). We obtained this result through Ref. 1.

[8]A. Billoire *et al.*, Phys. Lett. 78B, 140 (1978); T. Muta and T. Niuya, Prog. Theor. Phys. 68, 1735 (1982).

[9]G. S. Adkins and F. R. Brown (unpublished).