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A Class of SUSY Composite Models Based on $SU_H(N)$

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ABSTRACT

We analize a class of susy composite models based on $SU_H(N)$, by using the asymptotic freedom constraint as well as the anomaly consistency condition. A semirealistic model with three families is found. We conjecture that a combination of the ordinary $U_A(1)$ and the $U(1)$ associated to the R symmetry could be used to label the generations.

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SECTION I

The possibility that supersymmetry (SUSY) is the way out to some of the problems encountered in ordinary composite models has been emphasized by several authors [1]. Bardeen and Visnjic [2] suggested that quarks and leptons could be Goldstone fermions associated to the spontaneous breaking of susy. Massless fermions can also be identified as the superpartners of Goldstone bosons arising from the breaking of a global symmetry as discussed by Buchmuller et al. [3]. Finally in the conventional approach [4] and its generalization to susy [5], the preon theory has a chiral symmetry that protects the fermions from getting mass.

In this letter we consider, within the conventional approach, a class of susy composite models based on the confining hypercolor theory $SU_H(N)$. The motivations for this work can be summarized in the following three points i) In susy theories the number of elementary particles (preons) is larger than in conventional ones and as will see the asymptotic freedom property of the hypercolor theory implies a stronger constraint both on the possible chiral flavor symmetry as well as on the hypercolor group. ii) It is very likely that susy prevents the complete dynamical breakdown of chiral symmetry expected in ordinary gauge theories [6]. This in turn leads us to consider the anomaly matching condition at different levels opening new possibilities of obtaining realistic solutions. iii) susy models possess the R symmetry. We conjecture that this symmetry may play an important role in the labeling of the families.

SECTION II

We consider preons that are components of chiral superfields [7]. Φ^{α}_{-a} (Φ^{α}_{+a}) containing a complex scalar ϕ^{α}_{-a} (ϕ^{α}_{+a}) and a left handed ψ^{α}_{-a} (right handed $\bar{\psi}^{a}_{+\alpha}$ Weyl) spinor. These preons interact with each other through gauge bosons W^{a}_{μ} belonging to the real vector superfield V, which besides W^{a}_{μ} contains majorana fermions λ^{a} called gauginos. The indices a label the hypercolor symmetry group whereas $\alpha=1,2,\ldots n$ the global chiral symmetry $U_L(n)\times U_R(n)$. It is clear that n must be large enough such that

$$U_L(n) \times U_R(n) \supset SU_C(3) \times SU_L(2) \times U(1). \tag{1}$$

Let us consider now the following two cases;

1) Preons belonging to a representation r_0 of $SU_H(N)$ such that $r_0 \times r_0 \supset 1$. Given that we are dealing with a real representation it is enough to consider n left handed chiral superfields If N (related to the hypercolor symmetry group) is odd the adjoint is the representation of lower dimension. Then it is easy to show that the one loop coefficient of the β function is given by

$$\beta_{H_0} = N \cdot (3-n) , \tag{2}$$

so asymptotic freedom requires $n \leq 2$. If N is even the representation of lower dimension is represented by the Young tableau $\left.\begin{array}{c}\square\\ \vdots\\ \square\end{array}\right\} N/2$. In this case

$$\beta_{H_0} = N \cdot \left(3 - \frac{n}{8} \cdot \frac{N(N-2)!}{(N/2!)^2}\right) . \tag{3}$$

Possible values of N and n satisfying $\beta_{H_0} > 0$ and Eq. (1) are $N=2$ $n \leq 11$ and $N=4$ $n \leq 11$. However both solutions lead to nonrealistic models in the sense that it is not possible to find a pattern of spontaneous breakdown of the global symmetry such that massless states have the appropriate quantum numbers.[F1] Therefore there are no realistic model of the quarks and leptons if we consider preons in single real representation and demand asymptotic freedom of the hypercolor theory.

2) Preons belonging to a representation r_0 of $SU_H(N)$ such that $r_0 \times r_0 \times r_0 \supset 1$ but $r_0 \times r_0 \supset 1$. If we consider n left handed and n right handed superfields then

$$\beta_{H_0} = (3N - 2n\, T(r_0)) , \tag{4}$$

where $T(r_0)\delta^{ab} = Tr(\lambda^a(r_0)\lambda^b(r_0))$, $\lambda(r_0)$ being the generator of the group $SU_H(N)$ in the representation r_0. In this case only $N=3$ and $n \leq 8$ leads to a model satisfying both $\beta_{H_0} > 0$ and Eq. (1), so we will concentrate on this model in the following.

SECTION III

In order to identify the massless composite fermions we will apply the t' Hooft anomaly consistency condition as well as the idea of persistent mass condition [8]. So we will require preons that are constituents of massless composite fermions to the protected by some chiral symmetry from acquiring masses, even if the overall chiral flavor group G_F is broken to some subgroup G_F'.[F2] We will assume that this breaking occurs at the scale Δ_H (energy at which the hypercolor-theory confines) and therefore will apply the anomaly condition to the unbroken chiral symmetry group G_F'.

Let us consider the model based on the hypercolor group $SU_H(3)$ and with global chiral symmetry $U_L(n)\times U_R(n)\times U(1)_R$ where $n\leq 8$. The last $U(1)$ factor is associated to the 1 symmetry acting as follows on the physical fields

$$\phi_\pm \to \phi_\pm \qquad \psi_\pm \to e^{\pm i\alpha}\psi_\pm \qquad \lambda \to e^{i\alpha}\lambda \tag{5}$$

In order to have enough massless preons (protected by some chirality) to build quarks and leptons and satisfy Eq. (1) we must choose n=8. So after taking into account the effect of hypercolor instantons the global flavor symmetry of the model is $G_F=SU_L(8)\times SU_R(8)\times U_V(1)\times U(1)_{a.f.}$ where $U(1)_{a.f.}$ corresponds to the anomaly free current

$$J_\mu^{U(1)_{a.f.}} = -10\, J_\mu^{U_A(1)} + 16\, J_\mu^{U(1)_R} \tag{6}$$

Now we will assume that hypercolor is able to produce the scalar condensates[F3]

$$\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'} \langle \phi_-^{\alpha i}\phi^*_{+\beta i}\phi_-^{\alpha' i}\phi^*_{+\beta' i}\rangle = v \quad i = 1,2,3,4$$

not sum over i

$$\langle \phi_+\phi_+\phi_-\rangle = w \tag{7}$$

The first condensate produces the breaking $SU_L(8)\times SU_R(8)\times U_V(1)\rightarrow G_F' = SU(4)\times SU_L(2)\times SU_R(2)\times U_V(1)$ i.e. the symmetry group of the Pati-Salam model [10]. The second condensate breaks the chiral group $U(1)_{a.f.}$ down to a discrete subgroup Z_{10}. Preons representation under $SU_H(3)\times G_F'\times U(1)_{a.f.}$ are listed in Table 1. The reason we included $U(1)_{a.f.}$ is that Z_{10} and $U(1)_{a.f.}$ have the same quantum numbers.

Composite fermions come from three different kinds of configurations [5]. $\psi^*\psi^*\psi^*$, $\psi^*\phi^*\phi^*$ and $\psi\phi^*$. They form representation of the flavor group $G_F'\times Z_{10}$ and we specify and index for each of them. The values of these indices are restrained to satisfy the anomaly consistency condition that in this case comes only from the three point function $[SU(2)]^2\times U_V(1)$. We will assume that the indices associated with exotic particles are equal to zero, that is we assume that the exotic particles have masses of order Λ_H. In that case the anomaly consistency condition is

$$\ell_1 + \ell_2 + \ell_3 + \ell_4 = -1 \tag{8}$$

where the indices $\ell_1, \ell_2, \ell_3, \ell_4$ correspond to the same representation (4,2,1,1) under the group $SU(4)\times SU_L(2)\times SU_R(2)\times U_V(1)$ but with $U(1)_{a.f.}$ (Z_{10}) quantum numbers −18, 6, 14, −26 respectively. Obviously Eq. (8) admits a solution corresponding to the existence of three massless generations distinguished by different $U(1)_{a.f.}$ quantum numbers.

Other interesting possibilities is to assume, instead of Eq. (7a), the existence of the condensate

$$\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'} \langle \phi_-^{\alpha i}\phi_{+\beta i}^{*}\phi_-^{\alpha' i}\phi_{+\beta' i}^{*}\rangle = \begin{cases} v & i = 1,2,3 \\ v' & i = 4 \end{cases} \tag{9}$$

this produces the breaking $SU_L(8)\times SU_R(8)\times U_V(1)\rightarrow SU_C(3)\times SU_L(2)\times SU_R\times U_V(1)\times U'_V(1)$.[F4] Preon representations under $SU_H(3)\times G'_F U(1)_{a.f.}$ are given in Table 2. In this case we have two anomaly consistency equations: the $[SU(2)]^2\times U_V(1)$ and $[SU(2)]^2\times U'_V(1)$.

$$\begin{aligned} X_1 + 8X_8 + X'_1 + 3X_3 - 3X'_3 - 6X_6 &= 4 \\ X_1 + 8X_8 - 3X'_1 - 5X_3 + X'_3 + 2X_6 &= 0 \end{aligned} \tag{10}$$

where

$$X_i = \ell_{1i} + \ell_{2i} + \ell_{3i} + \ell_{4i}$$

$$X'_i = \ell'_{1i} + \ell'_{2i} + \ell'_{3i} + \ell'_{4i}$$

$i = 1,3,6$ and 8 are the dimension of color representations. The meaning of ℓ and ℓ' is more easily understood in Table 3. Notice that we have kept in Eq. (10) the indices corresponding to the representation 6 and 8 of color. If we are interested only in solutions where the absolute value of each index is less or equal to one and for which there is one to one correspondence between quarks and leptons generations then we obtain

$$\begin{array}{lll}
\text{A)} \quad X_1 = X'_3 = 1 & & X'_1 = X_3 = X_6 = X_8 = 0 \\
\text{B)} \quad X_1 = X'_3 = -X_6 = 1 & & X'_1 = X_3 = X_8 = 0 \\
\text{C)} \quad X_1 = X_6 = 1 & X'_3 = -3 & X'_1 = X_3 = X_8 = 0 \\
\text{D)} \quad X'_1 = -X_3 = -X_6 = 1 & & X_1 = X'_3 = X_8 = 0
\end{array} \tag{11}$$

The solution A corresponds to three generations of quarks and leptons without exotics. Solutions B, C, and D correspond to three generations of quarks and leptons plus one generation of color six quarks. As pointed out in Reference [11], solutions B, C and D are more interesting because the exotic color 6 could form the condensate $\bar{q}_{6R}q_{6L}(\phi_6)$ which dynamically breaks the weak gauge group $SU_L(2)\times SU_R(2)\times U(1)$ and produces Dirac masses for the quarks and leptons through the effective coupling

$$\frac{\tau_{\alpha\beta}}{\Delta_H^2}\,\bar{q}^{\alpha} q_6 \bar{q}_6 q^{\beta}\ .$$

The violation of parity could be achieved by the condensates [12]

$$\begin{aligned} &\langle \bar{q}_{6L}\gamma_{\mu} q_{6L}\bar{q}_{6L}\gamma^{\mu} q_{6L}\rangle = 0 \\ &\langle \bar{q}_{6R}\gamma_{\mu} q_{6R}\bar{q}_{6R}\gamma^{\mu} q_{6R}\rangle \neq 0 \end{aligned} \qquad (12)$$

The exotic 8 could be more interesting because it could be used in exactly the same way as the 6 and furthermore it could give a majorana mass to the neutrino. Unfortunately this solution does not exist and even if it existed it will violate the asymptotic freedom property of QCD.

We have a solution with three generations of quarks and leptons carrying different Z_{10} (U(1)a.f.) quantum numbers and one generation of color 6 whose Z_{10} charge can take any of the values -18, 6, 14 and -26. The labeling of families by using a discrete symmetry may be considered a natural susy generalization of the idea suggested by Harari and Seiberg [13]. We will leave for a future publication the study of the mass matrix based on the Z_{10} symmetry along the lines suggested in Ref. [14] as well as to use the complementary principle to investigate possible patterns of the chiral symmetry breaking [15].

SECTION IV

In conclusion we can obtain a semirealistic susy composite model with hypercolor group $SU_H(3)$ and flavor number n=8 which satisfies hypercolor asymptotic freedom, the persistent mass condition and the anomaly matching equations based on the broken chiral flavor group. We obtain three generations of quarks and leptons distinguished by the quantum number associated to the Z_{10} symmetry which survives the spontaneous breaking of a combination of the ordinary $U_A(1)$ and the U(1) related to the R symmetry. The breaking of the weak gauge group, parity and the production of Dirac quark and lepton masses can be attributed to the condensates of color 6-plets quarks.

Some of the problems that we have not considered are: The breaking of susy, the mechanism to produce q_6 masses and the existence of Goldstone bosons and their superpartners produced with the breaking of the global flavor group G_F. Let us remark however, that the Goldstone bosons can be heavy due to color and electroweak forces if the scale Δ_H is high enough [16]. Given that quark masses are of the order $\Delta^3 c_6/\Delta_H^2$ we can expect Δ_H~10 TeV if Δc_6~1 TeV.

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FOOTNOTES

[F1] For example: $SU_L(9) \to SU_L(6) \times SU_L(3)$ and $SU_L(6) \to SU_L(3) \times SU_L(2)$. So we could have $SU_L(9) \to SU(3) \times SU_L(2)$. In this case however we get both singlets and doublet of right handed triplets but not doublets of left handed color triplets. It seems that the minimum to get a realistic model is n=12.

[F2] This constraint is not fulfilled in some of the models previously considered in the literature [9].

[F3] Here we have changed the notation of the superfield index in the following way: $\phi_a^{\alpha=1,2,\ldots 8} \to \phi_a^{\alpha i}$ $\alpha=1,2$ and $i=1,2,3,4$.

[F4] Some combination of $U_V(1)$ and $U'_V(1)$ could be identified with baryon and lepton numbers but as we will see later this depends on the representation of quarks and leptons selected.

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Table 1. Preon representations under $SU_H(3)\times G_F'\times U(1)_{a.f.}$

	$SU_H(3)$	$SU(4)$	$SU_L(2)$	$SU_R(2)$	$U_V(1)$	$U(1)_{a.f.}$
ϕ_-	3	4	2	1	1	-10
ϕ_+	3	4	1	2	1	10
ψ_-	3	4	2	1	1	6
ψ_+	3	4	1	2	1	-6
λ	8	1	1	1	0	-16

Table 2. Preon representations under $SU_H(3)\times G_F'\times U(1)_{a.f.}$

	$SU_H(3)$	$SU_C(3)$	$SU_L(2)$	$SU_R(2)$	$U_V(1)$	$U_V'(1)$	$U(1)_{a.f.}$
ϕ_{3-}	3	3	2	1	1	α	-10
ϕ_{1-}	3	1	2	1	1	-3α	-10
ϕ_{3+}	3	3	1	2	1	α	10
ϕ_{1+}	3	1	1	2	1	-3α	10
ψ_{3-}	3	3	2	1	1	α	6
ψ_{1-}	3	1	2	1	1	-3α	6
ψ_{3+}	3	3	1	2	1	α	-6
ψ_{1+}	3	1	1	2	1	-3α	-6
λ	8	1	1	1	0	0	-16

Table 3. Assignment of indices.

$SU_H(3)$	$SU_C(3)$	$SU_L(2)$	$SU_R(2)$	$U_V(1)$	$U'_V(1)$	Indices	$U(1)_{a.f.}$
1	1+8+10	2	1	3	3α	$\ell'_{k1}, \ell_{k8}, \ell_{k10}$	18,-6,-14,-26 for k=1,2,3,4
1	1	2	1	3	-9α	ℓ'_{k1}	"
1	3	2	1	3	-5α	ℓ_{k3}	"
1	$3+\bar{6}$	2	1	-3	α	ℓ'_{k3}, ℓ_{k6}	-18,6,14,-26 for k=1,2,3,4