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Mass Formula for Pseudogoldstone Fermions in Broken Supersymmetry

William A. Bardeen, T. R. Taylor,* and Cosmas K. Zachos
Fermi National Accelerator Laboratory
P. O. Box 500, Batavia IL 60510

ABSTRACT

The goldstino of spontaneously broken supersymmetry develops a mass in the presence of perturbations which violate supersymmetry explicitly. In analogy to the chiral dynamics of pions, the general current algebra formula for this mass is derived. It is verified and illustrated in simple models at the tree level or through loop expansions. Some subtleties of the renormalization and the vacuum stability of the relevant O'Raifeartaigh-type models are examined.

*On leave of absence from the Institute of Theoretical Physics, Warsaw University, Warsaw, Poland.

I. INTRODUCTION

Spontaneously broken global supersymmetry implies the existence of a massless Nambu-Goldstone fermion, the goldstino.[1,2] In locally supersymmetric (supergravity) theories the degrees of freedom of this particle may be absorbed by the massive gravitino through the super-Higgs mechanism.[3] Even in the context of globally supersymmetric theories, the goldstino may develop a mass through the addition of a term in the hamiltonian which explicitly violates supersymmetry. Such a term will, in general, induce a mass for this pseudogoldstino.* This may happen in the tree approximation, in higher orders of perturbation theory, or even through some complicated dynamical mechanism. Such effects are analogous to the mass generation for pseudogoldstone bosons, like the pion masses in QCD.[4] A potential application of this mechanism may be found in ref.[5]; however, no satisfactory models exist which fully exploit these ideas.

The mass of the pseudogoldstino may be computed to first order in the explicit supersymmetry breaking perturbation ΔH, where the hamiltonian density is $H = H_0 + \Delta H$. The formula we find is:

$$m_\psi = \frac{1}{4f^2}\langle 0|\{\bar{Q}^\alpha, [\Delta H(0), Q^\alpha]\}|0\rangle , \qquad (1.1)$$

*We contrast this pseudogoldstone fermion to the "quasi-Goldstone fermions" in the literature, which are supersymmetry partners of Goldstone bosons of broken conventional symmetries in supersymmetric theories.

where f is the constant which measures the magnitude of spontaneous breaking. The Majorana spinor Q^a denotes the broken supercharge. The analogous formula in chiral dynamics is Dashen's formula[6] for the square of the pseudogoldstone boson masses:

$$m_{ab}^2 = \frac{1}{f^2} \langle 0 | [T_a, [T_b, \Delta H(0)]] | 0 \rangle \qquad (1.2)$$

where f is the decay constant and T_a, T_b are broken generators of the chiral symmetry.

We derive the mass formula (1.1) in Section II by paralleling Dashen's vacuum energetics approach,[6a] as well as his current algebra argument, [6b] which fixes the normalization of the constant f. We then proceed to verify and illustrate it in some detail through models in two and four dimensions.

In Section III, we first examine a scalar supermultiplet in two dimensions with spontaneous supersymmetry breaking. Upon introducing an appropriate supersymmetry violating perturbation which shifts the vacuum value of the scalar field, this model generates a goldstino mass at the tree level, much in analogy to the standard linear σ-model of PCAC. We next provide a more intricate illustration, where the goldstino develops a mass at the one loop level. To achieve this, we append an additional massive scalar multiplet to the previous system, along with a perturbation consisting of an additional mass term for the extra scalar field. The supersymmetry violation then passes on to the original supermultiplet radiatively. Use of the formula (1.1) involves a shorter calculation than the direct mass computation.

In Sections IV, we carry out the corresponding calculations in four dimensions; we study the large N limit of an O'Raifeartaigh model involving three types of supermultiplets, namely an O(N) singlet and two O(N) vectors. We discuss explicit symmetry breaking perturbations of mass dimension one and three, which exemplify the computational advantage of the mass formula over a direct calculation of the mass. Both examples, however, are somewhat pathological due to vacuum instabilities characteristic to models involving trough potentials with explicit supersymmetry breaking. We discuss these instabilities as well as the physical significance of the ultraviolet cutoff required for these theories in Section V. In Section VI we summarize our results and discuss the potential applications of the formula in the framework of dynamical supersymmetry breaking.

II. DERIVATION OF THE MASS FORMULA

The hamiltonian density can be split into a supersymmetric part H_0 and an explicit supersymmetry violating piece ΔH:

$$H(x) = H_0(x) + \Delta H(x) \tag{2.1}$$

However, supersymmetry is assumed here to be realized in the spontaneously broken mode, i.e. it is required that there be no vacuum state which is invariant under supersymmetry. Instead, supersymmetry operators connect the degenerate ground states $|\Omega(\alpha)\rangle$ of the unperturbed hamiltonian H_0 among themselves:

$$|\Omega\rangle = e^{i\bar{\alpha}Q/f} \; |0\rangle \qquad , \qquad (2.2)$$

where the Majorana spinor α^a is the fermionic coset parameter of supersymmetry.[7] The goldstino ψ is produced by quantizing the excitations parameterized by this Grassmann variable. We define $|\Omega(0)\rangle \equiv |0\rangle$, and leave the constant f to be interpreted below.

The vacuum energy density shift due to ΔH is computed to first order in this perturbation:

$$\Delta E(\alpha) = \langle\Omega|\Delta H(0)|\Omega\rangle = \langle 0|e^{-i\bar{\alpha}Q/f}\,\Delta H(0)\, e^{i\bar{\alpha}Q/f}|0\rangle \, . \quad (2.3)$$

The first derivative of this energy shift with respect to α vanishes due to Lorentz invariance. The second derivative yields

$$\left.\frac{\partial^2 E}{\partial\alpha^a\partial\bar{\alpha}^a}\right|_{\alpha=0} = \frac{1}{2f^2}\frac{\partial^2}{\partial\alpha^a\partial\bar{\alpha}^a}\langle 0|[\bar{\alpha}Q,[\Delta H(0),\bar{Q}\alpha]]|0\rangle =$$

$$= \frac{1}{f^2}\langle 0|\{\bar{Q}^a,[\Delta H(0),Q^a]\}|0\rangle \, . \quad (2.4)$$

Consequently the dependence of the effective action on the parameter α starts with a mass term proportional to $\bar{\alpha}\alpha$. We now note that the r.h.s. corresponds to two successive infinitesimal supersymmetry transformations with the same parameter α, namely:

$$\delta_{\bar{\alpha}}\,\delta_{\alpha}\,\Delta H(0) = [\bar{\alpha}Q\,,[\,\Delta H(0)\,,\bar{Q}\,\alpha]] \quad . \tag{2.5}$$

Moreover,

$$\frac{\partial^2(\bar{\alpha}\alpha)}{\partial\alpha\cdot\partial\bar{\alpha}} = 8 \qquad ,(4) \qquad , \tag{2.6}$$

in four (two) dimensions. Consequently, the pseudogoldstino mass is given by:

$$m_\psi = \frac{1}{8f^2}\,\frac{\partial^2}{\partial\alpha\cdot\partial\bar{\alpha}}\,\langle 0|\,\delta_{\bar{\alpha}}\,\delta_{\alpha}\,\Delta H(0)\,|0\rangle\,, \tag{2.7}$$

(and 2 times the r.h.s. in two dimensions), which is a more concise version of Eq.(1.1).

In order to identify the constant f (of dimension 2 in four spacetime dimensions, and dimension 1 in two dimensions) with the goldstino decay constant, we rederive Eq.(2.7) through a current algebra argument.

The supercurrent is linear in the goldstino field ψ:

$$S_\mu = i f \gamma_\mu \psi + \ldots \tag{2.8}$$

The constant f is defined as the strength with which the supercurrent couples the goldstino to the vacuum:

$$\langle 0 | S_\mu^{\,a} | \psi^b \rangle = if\, \gamma_\mu^{\,ab} \quad . \qquad (2.9)$$

It follows from current algebra that $f^2/2$ is the vacuum energy density. If the goldstino acquires a mass m_ψ, the supercurrent will no longer be conserved:

$$\partial_\mu S^\mu = f\, m_\psi \psi + \ldots \qquad (2.10)$$

We now evaluate the matrix element

$$q^\mu q^\nu \int d^4x\, e^{iq\cdot x} \langle 0 | T\left(\bar{\alpha}\cdot S^\mu(x), \bar{\alpha}\cdot S^\nu(0)\right)|0\rangle \quad . \qquad (2.11)$$

Combining translational invariance with the fact that

$$\partial_\mu S^{\mu a} = -i\, [Q^a, \Delta H] \quad , \qquad (2.12)$$

we take the limit $q \to 0$ in Eq.(2.11), to find:

$$\langle 0 | [\bar{\alpha} Q, [\bar{\alpha} Q, \Delta H(0)]] | 0 \rangle = i \int d^4x\, \langle 0 | \bar{\alpha}^a\, T(\partial^\mu S_\mu^a(x), \partial^\nu \bar{S}_\nu^b(0))\, \alpha^b | 0 \rangle . \qquad (2.13)$$

The right-hand side is evaluated through Eq.(2.10) and a Fierz transposition:

$$\langle 0|[\bar{\alpha} Q,[\Delta H(0),\bar{\alpha} Q]]|0\rangle = \frac{i f^2 m_\psi^2}{4}\int d^4x\, \bar{\alpha}\cdot\alpha \,\langle 0|\bar{\psi}(x)\psi(0)|0\rangle = f^2 m_\psi \bar{\alpha}\alpha \ . \tag{2.14}$$

Solving for m_ψ, we reproduce Eq.(2.7) with the identification of the constant $f^2/2$ as the vacuum energy density.

The mass formula (2.7) holds to first order in ΔH, but to all orders in the interactions of H_0. It may take nonzero contributions at the tree level, or in higher orders of perturbation theory, as illustrated below; conceivably, it can occur through a less direct nonperturbative mechanism.

We finally care to contrast the supersymmetric mass formula (2.7) to Dashen's corresponding formula for bosonic symmetry breaking, Eq.(1.2), for which the mass of the Goldstone particle is not proportional to the strength of the perturbation, but instead to its square root.

III. TWO DIMENSIONAL EXAMPLES

In two dimensions it is possible to break supersymmetry spontaneously with just one scalar supermultiplet. We consider the following lagrangian (for details and conventions see Ref.[8]):

$$\mathcal{L}_0 = \frac{1}{2}\int d\bar{\theta}\cdot d\theta\,[\tfrac{1}{2}\,\overline{D\Phi}\,D\Phi - 2f(\tfrac{1}{3}\Phi^3 + \Phi)] \qquad (3.1)$$

$$\Phi \equiv A + \bar{\theta}\psi + \tfrac{1}{2}\bar{\theta}\theta F$$

Integrating θ and $\bar{\theta}$ out and eliminating F, this reduces to the following lagrangian:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu A\,\partial^\mu A + \frac{i}{2}\bar{\psi}\not\partial\psi + fA\bar{\psi}\psi - \frac{1}{2}f^2(A^2+1)^2 . \qquad (3.2)$$

The supersymmetry transformations are:

$$\delta_\alpha A = \bar{\alpha}\psi \quad , \qquad \delta_\alpha\psi = [f(1+A^2) - i\not\partial A]\alpha \quad , \qquad (3.3)$$

and the supercurrent is

$$S_\mu = [\,if(1+A^2) - \not\partial A\,]\,\gamma_\mu\psi \; . \qquad (3.4)$$

Since $\langle A\rangle = 0$, the vacuum energy density is equal to $f^2/2$, and hence supersymmetry is broken:

$$m_\psi = 0 \quad , \quad m_A = \sqrt{2}\,f \; . \qquad (3.5)$$

This model is analogous to the linear σ-model of chiral dynamics.

Trivially, when the explicit breaking perturbation is a goldstino mass term itself, $\Delta H = \frac{m}{2}\bar{\psi}\psi$, the two dimensional version of Eq.(2.7) is readily verifiable at the tree level:

$$m_\psi = \frac{1}{4f^2}\frac{\partial^2}{\partial\alpha\cdot\partial\bar{\alpha}}\langle 0|m\bar{\alpha}\left(\partial_\mu A\partial^\mu A + f^2(1+A^2)^2 - fA\bar{\psi}\psi - \frac{i}{2}\bar{\psi}\not{\partial}\psi\right)\alpha|0\rangle = m \ . \tag{3.6}$$

Somewhat less obviously, we may instead add a linear term $\Delta H = \varepsilon A$, which shifts the location of the vacuum, as in the σ-model for PCAC.[4] To lowest order in ε:

$$\langle A\rangle = \frac{-\varepsilon}{2f^2} + O(\varepsilon^3) \ , \tag{3.7}$$

so that, at tree level:*

$$m_\psi = \frac{\varepsilon}{f} \ . \tag{3.8}$$

Again, the goldstino mass formula checks at the tree level:

$$m_\psi = \frac{\varepsilon}{4f^2}\frac{\partial^2}{\partial\alpha\cdot\partial\bar{\alpha}}\langle 0|\bar{\alpha}\left(f(1+A^2) - i\not{\partial}A\right)\alpha|0\rangle = \frac{\varepsilon}{f} \ . \tag{3.9}$$

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*This could also be readily read off from the PCSC relation $\partial^\mu S_\mu = \varepsilon\psi$, via Eq.(2.10).

The mass generation mechanism is however more interesting when the goldstino mass is first induced at the one loop level. To arrange this, we append to Eq.(3.1) another massive chiral supermultiplet:

$$\varphi \equiv A' + \bar{\theta}\psi' + \tfrac{1}{2}\bar{\theta}\theta F' \tag{3.10}$$

$$\mathcal{L}_0 = \frac{1}{2}\int d\theta \cdot d\bar{\theta}\left[\frac{1}{2}\overline{D\Phi}D\Phi + \frac{1}{2}\overline{D\varphi}D\varphi - 2f\left(\frac{\Phi^3}{3}+\Phi\right) + m\varphi^2 - g\varphi^2\Phi\right].$$

Integrating out $\theta,\bar{\theta}$ and eliminating F,F', we obtain

$$\begin{aligned}\mathcal{L}_0 = {} & \frac{1}{2}\left[\partial_\mu A\partial^\mu A + \partial_\mu A'\partial^\mu A' + i\bar{\psi}\not{\partial}\psi + i\bar{\psi}'\not{\partial}\psi'\right] \\ & - \frac{1}{2}\left[\left(f(1+A^2) + \frac{gA'^2}{2}\right)^2 + A'^2(gA-m)^2\right] \\ & - \frac{m}{2}\bar{\psi}'\psi' + fA\bar{\psi}\psi + \frac{g}{2}A\bar{\psi}'\psi' + gA'\bar{\psi}\psi' .\end{aligned} \tag{3.11}$$

We supplement this lagrangian by an explicit breaking perturbation $\Delta H = \frac{1}{2}M^2 A'^2$. For further use, we also note:

$$\delta_\alpha A' = \bar{\alpha}\psi' \quad , \quad \delta_\alpha \psi' = (-mA' + gAA' - i\not{\partial}A')\alpha . \tag{3.12}$$

The minimum of the potential, with or without the perturbation, is at $\langle A\rangle = \langle A'\rangle = 0$. Hence there is no ψ-ψ' mixing at tree level, and we may read off the masses

$$\begin{aligned} m_\psi &= 0 \qquad \text{(the goldstino)} \\ m_{\psi'} &= m \\ m_A &= \sqrt{2}\, f \\ m_{A'} &= \sqrt{m^2 + fg + M^2} \end{aligned} \tag{3.13}$$

Evidently, there is no tree level contribution to Eq.(2.7) in this case, since the double supersymmetry transform has no tree level vacuum value:

$$\delta_{\bar\alpha}\delta_\alpha\left(\frac{M^2 A'^2}{2}\right) = M^2 \delta_\alpha(A'\bar\alpha\psi') = -M^2\bar\alpha\alpha\left(\frac{\bar\psi'\psi'}{2} + mA'^2 - g\,AA'^2\right). \tag{3.14}$$

However, at the one loop level, the pseudogoldstino picks up a mass from the diagrams of Fig. 1 — there is no ψ loop tadpole since it is massless, nor is there any induced ψ-ψ' mixing. The mass is zero if there is no explicit breaking perturbation, $M = 0$, as expected. If there is explicit breaking, then the mass, to one loop, is given by:

$$m_\psi(\mathrm{I+II+III}) = M^2 \frac{m}{4\pi f^2} \frac{1}{1+M^2/fg} \log\left(1+\frac{fg+M^2}{m^2}\right) \tag{3.15}$$
$$= M^2 \frac{m}{4\pi f^2} \log\left(1+\frac{fg}{m^2}\right) + O(M^4) \quad .$$

Observe that when the coupling g vanishes, the goldstino multiplet Φ decouples from the sector of the theory containing the perturbation. As a result, the mass of the pseudogoldstino goes to zero.

The same answer is obtained more simply from the mass formula (2.7). To one loop, the last term in Eq.(3.14) does not contribute, since it involves a three line vertex. The sole contributions arise from a ψ' and an A' bubble, respectively:

$$m_\psi = -\frac{M^2}{f^2} \langle 0 | \frac{\bar{\psi}'\psi'}{2} + m A'^2 | 0 \rangle =$$
$$= \frac{-M^2}{f^2}\left(\frac{m f g}{(2\pi)^2}\, i \int \frac{d^2k}{(k^2-m^2)(k^2-m^2-fg)}\right) =$$
$$= \frac{M^2 m}{4\pi f^2} \log\left(1+\frac{fg}{m^2}\right) . \tag{3.16}$$

The approximation to first order in the perturbation parameter M^2 is valid for $M^2 \ll m^2, fg$.

IV. FOUR DIMENSIONAL O'RAIFEARTAIGH MODEL

In order to break supersymmetry spontaneously in four dimensions, we need at least three scalar superfields. We take the simplest O'Raifeartaigh model[9] described by the superpotential and potential, respectively:

$$W = \lambda \Phi_0 + g\, \Phi_0 \Phi_1^2 + m\, \Phi_1 \Phi_2 \quad , \tag{4.1}$$

$$2V = \left|\lambda + g A_1^2\right|^2 + \left|m A_1\right|^2 + \left|m A_2 + 2g A_0 A_1\right|^2 , \tag{4.2}$$

with $2\lambda g + m^2 > 0$. We further define:

$$\begin{array}{ll} a_i = \mathrm{Re}\, A_i & b_i = \mathrm{Im}\, A_i \\ f_i = \mathrm{Re}\, F_i & h_i = \mathrm{Im}\, F_i \end{array} \quad . \tag{4.3}$$

The tree level potential is characterized by a trough shape with continuously degenerate minima along the a_0 axis, all other fields vanishing. The vacuum energy density is $\lambda^2/2$, hence supersymmetry is spontaneously broken.

For formal expedience, we consider the Φ_1 and Φ_2 chiral superfields to be O(N) vectors – while Φ_0 is a singlet – with their indices saturated appropriately in Eq.(4.1). In the following we will make a consistent leading 1/N analysis of the loop expansion where g^2N and λg are fixed as $N \to \infty$.

Besides the kinetic part, and after eliminating F_1, F_2, and h_0, but not f_0, the lagrangian reads:

$$\begin{aligned}\mathcal{L} = & -\lambda f_0 + \tfrac{1}{2} f_0^2 - \tfrac{1}{2} m^2 \vec{a}_1^{\,2} - \tfrac{1}{2} m^2 \vec{b}_1^{\,2} - 2g^2 (\vec{a}_1 \cdot \vec{b}_1)^2 \\ & - g f_0 \vec{a}_1^{\,2} + g f_0 \vec{b}_1^{\,2} - \tfrac{1}{2}(m\vec{a}_2 + \mu \vec{a}_1 + 2g a_0 \vec{a}_1 - 2g b_0 \vec{b}_1)^2 \\ & - \tfrac{1}{2}(m\vec{b}_2 + \mu \vec{b}_1 + 2g a_0 \vec{b}_1 + 2g b_0 \vec{a}_1)^2 \\ & + \tfrac{1}{2} \bar{\vec{\psi}}_1 (-\mu - 2g a_0 - 2g i \gamma_5 b_0) \cdot \vec{\psi}_1 \\ & - \frac{m}{2} \left(\bar{\vec{\psi}}_2 \cdot \vec{\psi}_1 + \bar{\vec{\psi}}_1 \cdot \vec{\psi}_2 \right) - 2g \bar{\vec{\psi}}_1 \cdot (\vec{a}_1 + i\gamma_5 \vec{b}_1) \psi_0 \quad ,\end{aligned} \tag{4.4}$$

where a_0 has been shifted along the trough direction by an amount $\mu/2g$. The conserved supercurrent is:

$$S_\mu = i \partial\!\!\!/ (a_j + i\gamma_5 b_j) \gamma_\mu \psi_j - (f_j - i\gamma_5 h_j) \cdot \gamma_\mu \psi_j \quad . \tag{4.5}$$

At the tree level, the auxiliary field f_0 has a v.e.v. equal to the constant which parameterizes spontaneous supersymmetry breaking:

$$f_0 = \lambda \quad . \tag{4.6}$$

The goldstino ψ_0 is massless and, at this level, so are its superpartners a_0, b_0. The mass matrix of the remaining fermions has eigenvalues $\sqrt{\frac{\mu^2}{4} + m^2} \pm \mu/2$, which do not depend on the supersymmetry breaking scale f_0. We denote the corresponding mass eigenstates by ψ_+ and ψ_-

respectively, and chirally rotate ψ_- to reverse the sign of its mass, so that:

$$m_{\psi_+} = \frac{\mu}{2} + \sqrt{m^2 + \frac{\mu^2}{4}}$$
$$m_{\psi_-} = \frac{-\mu}{2} + \sqrt{m^2 + \frac{\mu^2}{4}} \quad . \tag{4.7}$$

Quantum effects will lift the degeneracy of the potential minimum with respect to shifts in a_0, and dictate $\mu = 0$ at the minimum;[10] in addition, the superpartners of the goldstino a_0 and b_0 will develop a nonzero mass. To see this, we calculate the effective potential along $a_0 = \mu/2g$, $b_0 = 0$, treating μ and f_0 as variational parameters.

For small* μ, we obtain the expansion:

$$\begin{aligned} V_{eff}(\mu, f_0) = {} & \lambda f_0 - \tfrac{1}{2} f_0^2 \left[1 + \frac{4g^2 N}{(4\pi)^2} \log \frac{\Lambda^2 e^{3/2}}{m^2} \right] \\ & + \frac{N m^4}{4(4\pi)^2} \left[(1+\rho)^2 \log(1+\rho) + (1-\rho)^2 \log(1-\rho) \right] \\ & - \left(\frac{\mu}{2g}\right)^2 \frac{g^2 N 2m}{(4\pi)^2} \left\{ 2 - \frac{1}{\rho} \left[(1+\rho)^2 \log(1+\rho) - (1-\rho)^2 \log(1-\rho) \right] \right\} + O(\mu^4), \end{aligned} \tag{4.8}$$

where

$$\rho \equiv \frac{2g f_0}{m^2} \quad .$$

As usual, we maximize this with respect to f_0, and minimize with respect to μ; at the minimum, we obtain

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*The large μ properties of V_{eff} and its ultraviolet cutoff Λ dependence are discussed in the next section.

$$\mu = 0$$

$$V_{eff} = \frac{1}{2} f_0^2 Z_G \qquad , \qquad (4.9)$$

where the factor

$$Z_G = 1 + \frac{2g^2N}{(4\pi)^2}\left[2 \log \frac{\Lambda^2 e^{1/2}}{m^2} + \frac{1-\rho^2}{\rho^2} \log(1-\rho^2)\right] \qquad (4.10)$$

turns out to be equal to the square of the goldstino wavefunction renormalization.

We now check that the goldstino ψ_0 remains massless beyond the tree approximation, in accord with the super-Goldstone theorem. To one loop, ψ_0 does not mix with other fields, and it could potentially pick up mass contributions from four diagrams (Fig. 2). However, it is clear that at the stable vacuum, $\mu = 0$, the intermediary fermions ψ_+ and ψ_- are degenerate, Eq.(4.7), and their relevant coupling is

$$\sqrt{2}\, g\, \bar{\psi}_0 (\vec{a}_1 + i\gamma_5 \vec{b}_1)\cdot(\vec{\psi}_+ - i\gamma_5 \vec{\psi}_-) . \qquad (4.11)$$

The structure of this coupling insures the cancellation of these diagrams, regardless of details of the scalar mass matrix. In particular, it is evident that some explicit breaking terms, like masses for a_1 or b_1 will not inhibit this cancellation to this order.

We now proceed to introduce explicit supersymmetry breaking perturbations which will induce pseudogoldstino masses.

i) Linear term $\Delta H = -\epsilon a_0$. Contrary to Section III, adding a linear term does not leave the tree potential, Eq.(4.2), (whose minimum is flat in the a_0 direction) bounded from below; however, the effective potential does have a local minimum near the origin. Hence we consider the addition of such a term as a formal exercise to illustrate the consistency of Eq.(2.7). The perturbation shifts the minimum of the effective potential to:

$$\mu = -\epsilon 2g \left[\frac{4m^2g^2N}{(4\pi)^2}\right]^{-1}\left\{2 - \frac{1}{\rho}\left[(1+\rho)^2\log(1+\rho) - (1-\rho)^2\log(1-\rho)\right]\right\}^{-1}. \tag{4.12}$$

We need to keep terms only to leading order in ϵ, since we will be considering the first order formula (2.7).

The fermion masses are now split apart by

$$m_{\psi_+} - m_{\psi_-} = \mu \tag{4.13}$$

and consequently the diagrams of Fig. 2 do not cancel. Through a straightforward but tedious calculation, they now yield a pseudogoldstino mass:

$$m_{\psi_0} = \frac{\epsilon}{f_0 Z_G}\,. \tag{4.14}$$

This same quantity is computable more directly through Eq.(2.7). The double transform of the perturbation is:

$$\delta_{\bar{\alpha}}\delta_{\alpha}(-\varepsilon a_0) = \varepsilon\,\bar{\alpha}\,(f_0 - i\gamma_5 h_0)\alpha\ . \qquad (4.15)$$

The constant f entering into the current algebra and Eq.(2.7) is identified as the square root of twice the physical vacuum energy density, Eq.(4.9), hence

$$f = f_0\sqrt{Z_G}\ , \qquad (4.16)$$

and the result of Eq.(4.14) is readily reproduced:

$$m_{\psi_0'} = \frac{1}{8f^2}\frac{\partial^2}{\partial\alpha\cdot\partial\bar{\alpha}}\langle 0|\delta_{\bar{\alpha}}\delta_{\alpha}(-\varepsilon a_0)|0\rangle = \frac{\varepsilon}{f_0 Z_G}\ . \qquad (4.17)$$

ii) Fermion mass term $\Delta H = \frac{-\varepsilon}{2}\bar{\psi}_2\cdot\psi_2$. In order to see how the mass formula works in a theory with a well-defined tree potential, we introduce a fermion mass perturbation $\frac{\varepsilon}{2}\bar{\psi}_2\cdot\psi_2$ to the lagrangian. This perturbation introduces an additional splitting for the tree level fermion masses:

$$m_{\psi_+} - m_{\psi_-} = \mu - \varepsilon . \tag{4.18}$$

This in turn modifies the effective potential by

$$\delta V_{eff} = V_{eff}(\varepsilon) - V_{eff}(0) = -\varepsilon\mu \frac{2Nm^2}{(4\pi)^2} \log \frac{\Lambda^2}{m^2} . \tag{4.19}$$

The minimum is now shifted from $\mu = 0$ to

$$\mu = -2\varepsilon \log\frac{\Lambda^2}{m^2}\left\{2 - \frac{1}{\rho}\left[(1+\rho)^2\log(1+\rho) - (1-\rho)^2\log(1-\rho)\right]\right\}^{-1} . \tag{4.20}$$

This perturbation is soft, in that it does not generate divergences more serious than logarithmic. Unfortunately, as discussed in the next section, it destabilizes the theory by generating an effective potential unbounded from below, as in case i). The minimum of Eq.(4.20) is only a local one.

The diagrams of Fig. 2 lead to the following pseudogoldstino mass:

$$m_{\psi_0} = \frac{4g^2N\varepsilon}{Z_G(4\pi)^2}\,\frac{1}{\rho}\left[2\log\frac{e\Lambda^2}{m^2} + \frac{1-\rho}{\rho}\log(1-\rho) - \frac{1+\rho}{\rho}\log(1+\rho)\right]. \tag{4.21}$$

Again, the same result can be derived more simply by applying Eq.(2.7):

$$m_{\psi_0} = -\frac{1}{8f^2}\frac{\partial^2}{\partial\alpha\cdot\partial\bar{\alpha}}\langle 0|\,\delta_{\bar{\alpha}}\,\delta_{\alpha}\left(\frac{\varepsilon}{2}\overline{\vec{\Psi}_2}\cdot\vec{\Psi}_2\right)|0\rangle =$$

$$= \frac{-\varepsilon}{f^2}\langle 0|\,m^2(\vec{a}_1^{\,2}-\vec{b}_1^{\,2}) + (\partial_\mu\vec{a}_2)^2 - (\partial_\mu\vec{b}_2)^2|0\rangle . \tag{4.22}$$

From inspection of the boson mass matrix in Eq.(4.4), we note that $\vec{a}_2$ and $\vec{b}_2$ are degenerate and thus they cancel each other's effects to one loop. The $\vec{a}_1$, $\vec{b}_1$ loops reproduce Eq.(4.21):

$$m_{\psi_0} = \frac{8\varepsilon g^2 N}{Z_G(4\pi)^2}\,\frac{1}{\rho}\int_0^{e\Lambda^2}\frac{dk^2\,k^2}{(k^2+1+\rho)(k^2+1-\rho)} . \tag{4.23}$$

V. VACUUM STABILITY AND RENORMALIZATION

In the previous section, we have given explicit examples of models which illustrate the power of current algebra in analysing the structure of the goldstino sector in the presence of explicit supersymmetry breaking. However, the models we have discussed contain certain pathologies related to the nature of the trough potential and to aspects of the ultraviolet behavior of chiral theories. In this section we will briefly discuss these features of the specific models considered in Section IV.

We would first like to discuss the vacuum stability of the four dimensional O'Raifeartaigh model. At the tree level, this model is indeterminate due to the existence of a trough in the potential along the direction where the field a_0 can develop a vacuum expectation value, $\mu/2g$. To leading order in 1/N, the loop effects remove this degeneracy and provide for a unique, stable ground state with $\mu = 0$. However, a remnant of the trough potential survives by the fact that the vacuum energy, or effective potential, grows only logarithmically for large μ. Hence the stability of the ground state is extremely sensitive to perturbations in the trough direction.

Our first model (i) in section IV is an obvious example of this sensitivity. The explicit breaking was a term linear in the field a_0, and $\Delta\mathcal{L} = \varepsilon a_0 = \varepsilon\mu/2g$. In the tree approximation, the symmetric effective potential is independent of a_0 and the perturbation clearly generates a vacuum instability when a_0 acquires a large vacuum value. In leading 1/N approximation, the effective potential for large μ has the form:

$$V_{eff} = \lambda f_0 - \frac{f_0^2}{2}\left\{1 + \frac{4g^2N}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}\right\} - \frac{\varepsilon\mu}{2g} =$$

$$= \frac{\lambda^2}{2}\left\{1 + \frac{4g^2N}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}\right\}^{-1} - \frac{\varepsilon\mu}{2g}. \qquad (5.1)$$

The perturbation again generates a large μ instability. Actually, the large N result does have a minimum for a value of μ near the Landau ghost pole, where the denominator of Eq.(5.1) vanishes. This large value of μ would generate physical masses of order the cutoff Λ. This is clearly not a completely acceptable scenario, but it also emphasizes the known fact[11] that these chiral models can only be regarded as effective field theories.

The chiral models are effective field theories because the logarithmic divergences, as seen in Eq.(5.1), cannot actually be renormalized consistently with the required positivity properties. The cutoff must be a physical limit on the domain of applicability of the effective field theory. Hence the results, as e.g. Eq.(5.1), only make physical sense when all masses and momenta are much less than the cutoff. These features are identical to those found for ϕ^4 field theory[12] or the Higgs sector of the Weinberg-Salam model. We note that the low energy physics may well place limits on how large this physical cutoff can be.[13] Beyond this cutoff, the physical theory must be modified to include new degrees of freedom, etc., and the effective theory ceases to apply. That small perturbations may generate large masses through these instabilities could be a useful physical mechanism for understanding the absence of low energy supersymmetry. However, a complete understanding of the broken symmetry theory would require knowledge of the physics at or above the cutoff.

The instabilities generated in our first example are obvious even at the tree level. In our second example (ii), the explicit breaking has no such effect at tree level. The addition of an explicit mass term for the fermions $\bar{\psi}_2$ is also an ultraviolet soft perturbation, since these fields have no dimension four interactions. However, in the leading N calculation of the loops, this explicit breaking will also generate a vacuum instability. In leading 1/N approximation, the effective potential for large μ has the form:

$$V_{eff} = \frac{\lambda^2}{2}\left\{1 + \frac{4g^2N}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}\right\}^{-1} - \frac{\varepsilon N m^2 \mu}{(4\pi)^2}\log\frac{e\Lambda^2}{\mu^2}, \tag{5.2}$$

where ε is the mass perturbation. As in the previous example, the instability will tend to generate a large vacuum value for μ and physical masses of order the cutoff. There remains, of course, the metastable ground state at small μ, which we used to study the systematics of the current algebra mass formula.

We should also note the explicit dependence on the cutoff Λ in this model both for the vacuum energy Eq.(5.2) and for the goldstino mass Eq.(4.21). This cutoff dependence could be removed by analysing the operator mixing structure of the theory. However, it really makes no sense to carefully renormalize the explicit breaking terms, as the symmetric theory cannot be renormalized to remove the logarithmic divergences. When treated as an effective field theory, these logarithmic divergences may correspond to real physical enhancements, and so on.

In this section, we have demonstrated the extreme sensitivity of models with trough potentials to explicit supersymmetry breaking. In addition, we have emphasized the effective nature of theories based on chiral superfields and the physical significance of the cutoffs which appear for these theories. Finally, we remark that this sensitivity may provide a mechanism for small perturbations to generate large scales for supersymmetry breaking in more realistic models than those considered here.

VI. SUMMARY AND DISCUSSION

On the basis of current algebra, we have derived a formula for the mass of pseudogoldstinos, Eq.(1.1), in terms of the relevant perturbation which breaks supersymmetry explicitly. In the context of perturbation theory, we have then illustrated the computational advantage of using this formula over a direct diagramatic calculation of the pseudogoldstino mass in several models involving scalar multiplets in two and four dimensions. We have, moreover, pointed out that the trough potentials widely used in model building are extremely sensitive to explicit supersymmetry breaking, because it tends to destabilize the vacuum. These instabilities introduce into the theory large scales of the order of the cutoff. In fact, the cutoff constitutes a physically significant scale in these models, which may only be regarded as low energy effective theories. We have pointed out some possible phenomenological applications of this effect.

The usefulness of the pseudogoldstino formula should extend beyond merely providing a computational shortcut. In the context of ref.[5], it could facilitate the systematic exploration of effective perturbations arising out of the low energy gauge interactions. In analogy to chiral dynamics, it could also be useful in the context of nonperturbative supersymmetry breaking. In that case, it might serve to estimate the mass of the composite pseudogoldstino, even though a direct calculation might not be feasible. Unfortunately, supersymmetry does not break dynamically[14] in all four dimensional models examined so far, although the construction of a successful theory may not be excluded.

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Fig. 1. One-loop contributions to the pseudogoldstino mass (Section III).

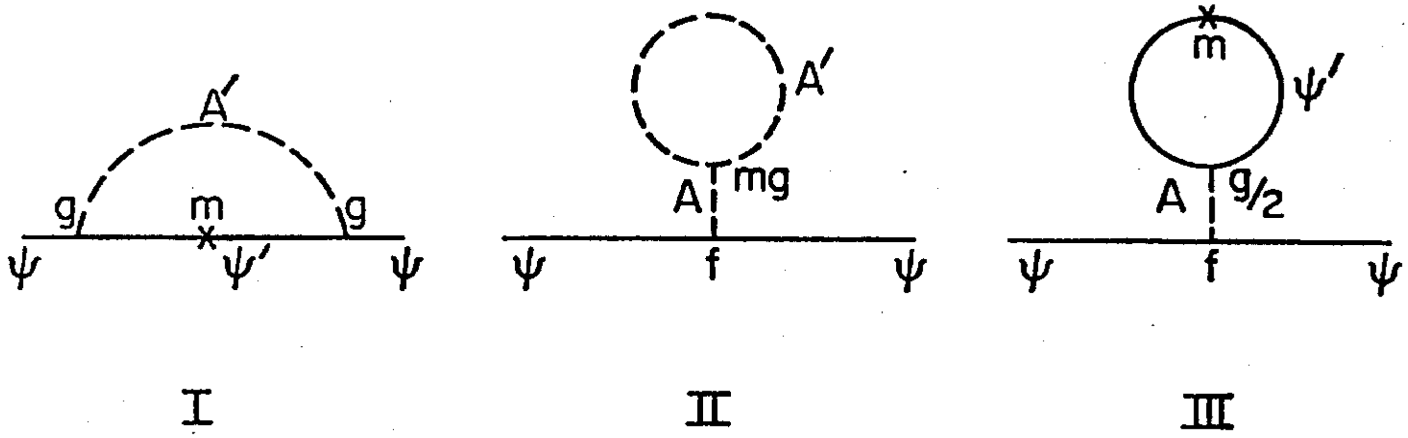

Fig. 2. One-loop contributions to the pseudogoldstino mass (Section IV).