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Monopole-Fermion Interactions: The Soliton Picture

S. Dawson[a)] and A. N. Schellekens[b)]
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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ABSTRACT

Monopole catalysis of proton decay is examined in the soliton picture and monopole-soliton scattering is studied numerically by constructing time histories of scattering events. We study the effects of finite fermion masses and the coupling constant dependence of the interactions in both an SU(2) model and the grand unified SU(5) model. All relevant Abelian Coulomb energies, (including the electroweak energy at distances less than $1/M_Z$), are included and we find that the qualitative nature of a scattering process is unchanged by the inclusion of these interactions.

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I. Introduction

Magnetic monopoles have fascinated theorists for over fifty years, although their observation remains elusive. Recently, Rubakov[1] and Callan[2] have demonstrated that in grand unified theories, magnetic monopoles have the surprising property that they catalyse proton decay at strong interaction rates. Many other authors[3,4] have extended their work, although unfortunately little quantitative work has been done. In this paper, we attempt to fill a part of this gap and study proton-monopole scattering numerically. We gain a physical understanding of the catalysis mechanism by constructing time histories of scattering processes.

Rubakov and Callan have approached the problem of monopole-fermion interactions in two different ways, both of which involve the scattering of a fermionic J=0 partial wave from a 't Hooft-Polyakov monopole. In Rubakov's approach, the problem reduces to a massless two-dimensional Schwinger model, which is exactly solvable. In this approach, however, the effect of a finite fermion mass is unclear. In Callan's approach, the theory is written as an equivalent boson theory and fermions are written as soliton states. This picture is most suitable for constructing the time history of a baryon number violating process and to see the effect of finite fermion masses. It is this bosonized version of the fermion-monopole dynamics which we will use in this paper. We emphasize however that Rubakov's and Callan's work describes the same physics and a combination of concepts from both approaches can be extremely useful.

Most of our results are obtained for the SU(2) model studied by Rubakov and Callan. We introduce this model in Section II. In Section III, we formulate the discrete version of the model which we use for our numerical analysis, with special attention to the region near the monopole core.

In Section IV, we discuss the kinematics of the problem. This results in a list of all possible final states for a given initial state. In the massless case, there is at most one final state per initial state and the problem is completely determined. For non-zero fermion masses, additional processes are allowed. The simplest of these are helicity-flip scattering processes, which do not lead to a violation of any charge.

The competition between different final states can only be studied numerically and this is one of the purposes of this paper. Starting with a set of solitons moving toward the monopole, we integrate the equations of motion to find the complete time evolution and the final state. The effect of the boundary conditions and the Adler-Bell Jackiw anomaly[5] become transparent by studying the behavior of the boson fields near the core.

Our numerical results are presented in Section V. For the most interesting initial states, we show under which conditions the baryon number violating processes dominate over the processes with only helicity flip. We investigate the dependence on the velocity of the incoming solitons, their spatial separation, and the gauge coupling constant.

In Section VI, we address the question of whether the SU(2) model adequately describes proton decay catalysis by SU(5)[6] monopoles. We consider the effects of Abelian color and electromagnetic interactions not yet included in the SU(2) model and find that their effect is unimportant. We also include the effects of the Z_0 electrostatic energy for distances less than $1/M_Z$ from the monopole.[7] We discuss in Section VII the effects of including an extra heavy generation of fermions. In the Appendix, we derive the effective Lagrangian for the SU(5) model, with particular emphasis on the Coulomb interaction terms.

II. The SU(2) Model

In this section, we study an SU(2) model in which an SU(2) gauge field is coupled to an I=1 Higgs field and an even number N_f of left-handed Weyl spinors. The Higgs potential is arranged in such a way that the symmetry is broken to U(1), resulting in a monopole which is coupled to the fermions in the doublet representation of the SU(2) group corresponding to the monopole, (which has generators $\vec{T}$). We denote the spinors which couple to the monopole by,

$$\psi_{iL} = \begin{bmatrix} \psi^+_{L,i} \\ \psi^-_{L,i} \end{bmatrix} \qquad i=1,N_f \quad , \tag{2.1}$$

where + and - indicate the T_3-charge of the particle. (In this section "charge" refers to this T_3-charge.) In the J=0 partial wave, the upper components $\psi^+_{L,i}$ and their anti-particles, $\psi^-_{R,i}$ are only allowed as incoming states, while the lower components $\psi^-_{L,i}$ and $\psi^+_{R,i}$ are only allowed as outgoing states.

We will study this system with nonvanishing fermion masses. In principle, a mass term can be constructed by pairing the upper component of a doublet ψ_i with the lower component of a doublet $\psi_{i'}$. In the SU(5) and SO(10) grand unified models,[3] the doublet assignment for the first generation is

$$\begin{bmatrix} e^- \\ \bar{d}_3 \end{bmatrix}_L \qquad \begin{bmatrix} d_3 \\ e^+ \end{bmatrix}_L \qquad \begin{bmatrix} \bar{u}_1 \\ u_2 \end{bmatrix}_L \qquad \begin{bmatrix} \bar{u}_2 \\ u_1 \end{bmatrix}_L \tag{2.2}$$

(Note that the T_3 charge has the <u>opposite</u> sign from the electromagnetic charge.) This has the property that if $\psi^+_{L,i}$ is paired with $\psi^-_{L,i'}$, then $\psi^-_{L,i}$ is paired with $\psi^+_{L,i'}$. This is not necessarily true in general, but we will assume this in the rest of the paper. The mass terms combine the N_f doublets in 1/2 N_f pairs. We denote each pair generically as,

$$\begin{bmatrix} a_L^+ \\ b_L^- \end{bmatrix} \qquad \begin{bmatrix} b_L^+ \\ a_L^- \end{bmatrix} \quad . \tag{2.3}$$

The bosonization of the spherically symmetric sector of the theory can be done in two ways: one which respects the pairing of the fermions in SU(2) T_3 eigenstates and one which respects the pairing into mass eigenstates. The two bosonizations are connected by a canonical transformation.[2] In the second bosonized form, each mass-eigenstate is represented by a field, (ϕ_a and ϕ_b for Eq. (2.3)). The pairing into doublets is represented by a boundary condition at the origin. This boundary condition is all that remains of the non-Abelian properties of the fields in the core of the monopole in the limit of vanishing core size.

The second bosonization is the most convenient one for the soliton picture and will be used in the rest of this paper.

III. The Discrete Lagrangian

To study the problem numerically we discretize the r-dependence of the Lagrangian. In the continuum limit, the Lagrangian is, (see Ref. 2 and the Appendix):

$$L = \int_0^r dr \; [1/2 \sum_{i=1}^{N_f} (\dot{\phi}_i^2 - \phi_i'^2) - m_i^2 \cos(2\sqrt{\pi}\phi_i)]$$

$$- \frac{F}{2r^2} \left(\sum_{i=1}^{N_f} \phi_i \right)^2 \quad , \tag{3.1}$$

for N_f flavors of Weyl spinors in the original fermion Lagrangian. (The "dot", ("prime"), denotes differentiation with respect to t,(r)). The constant F is related to the gauge coupling, g, as follows:

$$F = \frac{g^2}{16\pi^2} \tag{3.2}$$

The coefficient m_i is related to the fermion mass.[8]

The boundary conditions on the bosons at r=0 are

$$\left.\begin{aligned} \phi_i(0) &= \phi_{i+1}(0) \\ \phi_i'(0) &= -\phi_{i+1}'(0) \end{aligned}\right\} \quad \text{for odd } i. \tag{3.3}$$

We need a discrete version of L which implements the boundary condition. For this purpose we use the following Lagrangian,

$$L = -\frac{1}{2a^2} \sum_{i\ \mathrm{odd}}^{N_f} [(\phi_{i0} - \phi_{i1})^2 + (\phi_{i0} - \phi_{i+1,1})^2]$$

$$+ \sum_{j=1}^{N-1} \sum_{i=1}^{N_f} (\phi_{ij} - \phi_{i,j+1})^2$$

$$+ \sum_{j=1}^{N} \sum_{i=1}^{N_f} [1/2\dot{\phi}_{ij}^2 - m_i^2 \cos (2\sqrt{\pi}\phi_{ij})]$$

$$-\sum_{j=1}^{N} \frac{F}{2j^2a^2} (\sum_{i=1}^{N_f} \phi_{ij})^2 \quad , \tag{3.4}$$

where $\phi_{ij}=\phi_i(ja)$, N is the number of lattice points for the r variable and a is the lattice spacing.

Since there is no canonical momentum for ϕ_{i0}, we obtain a constraint equation for that variable,

$$\phi_{i0} - \phi_{i1} = -(\phi_{i0} - \phi_{i+1,1}) \tag{3.5}$$

This is the second equation in Eq. (3.3). The first is satisfied by construction. We impose no boundary condition at r=Na, since we will assume that all fields stay in one of the vacua of the sine-Gordon theory for all times at large r.

Our boundary conditions at r=0 differ from those proposed recently by Callan.[9] He chooses to enforce charge (T_3) conservation by a boundary condition $\sum_i \phi_{i0} = 0$. In our case, such a condition is enforced dynamically on the fields at r=a. For sufficiently small a, the combination $\sum_i \phi_{i1}$ must be very small to keep the total energy finite. Then Eq. (3.3) leads to

$$\lim_{a\to 0} \sum_i \phi_{i0} = 0 \tag{3.6}$$

Thus in the continuum limit the different boundary conditions are equivalent.

IV. Allowed Processes

In this section we will investigate which final states can be obtained from a given initial state. (Some of the material in this section can be found in Refs. 1, 2, and 14). Initial states are defined at $r=\infty$ and $t=-\infty$. In that limit the Coulomb term in Eq. (3.1) can be ignored and the solutions are just the familiar sine-Gordon solitons;

$$\phi_i = \frac{2}{\sqrt{\pi}} \tan^{-1}(\exp(A_i))$$

$$\dot{\phi}_i = -\frac{2m_i\beta_i}{\sqrt{1-\beta_i^2}} \frac{1}{\cosh(A_i)} \tag{4.1}$$

where

$$A_i = \frac{2\sqrt{\pi}m_i}{\sqrt{1-\beta_i^2}} (r-r_i^0) \quad .$$

(r_i^0 is the position of ϕ_i at $t=-\infty$ and β_i is its velocity).

The charges, q_i, and helicity, λ_i, of a soliton can be determined from the correspondence with the fermion Lagrangian;

$$q_i = \frac{1}{\sqrt{\pi}}\int \phi_i'(r)\ dr. \tag{4.2}$$

λ is -1 (1) for an incoming (outgoing) positively (negatively) charged soliton. Note that the charge is normalized to one. We summarize the possible asymptotic states in Figure 1.

A. Charge Conservation

In the absence of the Coulomb term the time evolution of the solitons can be solved exactly until they reach $r=0$. The solution is simply Eq.(4.1) with r replaced by $r-\beta t$, (where β is positive for a soliton moving toward the monopole). In fact even the behavior near $r=0$ is exactly solvable. This becomes clear when we define a boson field,

$$\Phi_i(r) = \begin{cases} \phi_i(r) & r>0 \\ \phi_{i+1}(-r) & r<0 \qquad \text{for } i \text{ odd.} \end{cases} \tag{4.3}$$

This automatically respects the boundary conditions. Now we have a sine-Gordon Lagrangian for all r.

A soliton ϕ_i coming towards the monopole from $r=\infty$ will pass through the origin unchanged and continue to move towards $r=-\infty$. According to Eq. (4.3), a soliton on the negative r-axis should be interpreted as a ϕ_{i+1} type soliton, which is obtained by a reflection with respect to the point $r=0$. Therefore we conclude that the boundary conditions alone allow the process $\phi_i \to \phi_{i+1}$, for i odd. In fermion language, this process corresponds to,

$$a_L^+ \to b_L^- \quad . \tag{4.4}$$

(We note that this process will NOT occur when the gauge coupling constant is non-zero).

This is the process discussed in Ref. 10 and more recently in Ref. 11. It is a consequence of the fact that the helicity operator commutes with the Hamiltonian for a gauge theory monopole, (even inside the core and even for non-zero fermion mass), as long as only the magnetic (monopole) field is taken into account. In soliton language, this corresponds to the fact that in the absence of a Coulomb term a soliton moves with constant velocity, without changing its shape.

In the presence of the Coulomb term, the situation is completely different. In that case, a process like Eq. (4.4) would leave a T_3 charge on the monopole core, changing the monopole into a dyon, and the Coulomb energy due to that charge is inversely proportional to the size of the core. This would violate conservation of energy! Therefore T_3 charge will be conserved within the fermion sector alone, as long as the kinetic energy of the fermions is much less than the excitation energy of a dyon. Charge (non) conservation in the field of a monopole has been the subject of many recent papers.[12]

B. The Selection Rule in the Massless Case

The allowed processes can now be derived using charge conservation and the peculiar kinematics of J=0 fermions in the monopole. According to Fig. 1, a set of incoming fermions with total charge Q must have total helicity -Q. (Here Q is the sum of the q_i's, defined in Eq. (4.2)).

Since charge is conserved we conclude, again using Fig. 1, that the total outgoing helicity is +Q. Therefore any process with total incoming charge Q must have a total change in helicity equal to 2Q. This is true in the massive as well as in the massless case.

In the massless case, the only source of helicity violation is the Adler-Bell-Jackiw anomaly, which requires a very specific change in helicity, $\delta H = nN_f$, for any integer n. (The anomaly manifests itself through the Coulomb term because all helicity changing processes disappear if that term is removed). Therefore we obtain the selection rule,

$$Q = 1/2 \ n \ N_f \quad , \tag{4.5}$$

where Q is the total incoming charge and N_f the number of SU(2) Weyl- doublets.

Since there has been much confusion about the role of anomalies, some clarification may be helpful. We distinguish two anomalies:

(1.) It is well known that baryon number has an anomaly with respect to the weak interaction gauge group $SU(2)_L$. The kind of proton decay discussed in Refs. 1 and 2 is however not due to this anomaly. Effects of this sort should vanish if $M_W \to \infty$ and therefore we expect such effects to be small.

(2.) Chiral fermion number has an anomaly with respect to $SU(2)_M$, the subgroup in which the monopole is embedded. Rubakov uses this anomaly to explain the presence of chirality non-conserving processes. In a theory with massless fermions, this anomaly is the only source of chirality violation. Baryon number does not have an anomaly with respect to the weak interaction gauge group $SU(2)_M$. The source of B-violation is not an anomaly, but the fact that in the core fermions with different B-eigenvalues are combined into doublets. This is a consequence of the grand unification.

C. Helicity Conserving Processes

The simplest case to discuss is n=0. In that case the total incoming charge is zero and helicity is conserved. (Such processes were first mentioned in Ref. 13.) In one special case, this problem is exactly solvable: if at some time t_0 all solitons have equal position and velocity. It is convenient to consider the linear combination $\sigma(r)=\sum_i \phi_i(r)$ and the N_f-1 combinations of fields orthogonal to it. Of these new fields, only σ is sensitive to the Coulomb term.

But since $\sigma(r,t_0) = \dot{\sigma}(r,t_0) = 0$, $\sigma(r,t) = 0$ for all t. This implies that the Coulomb energy plays no role in the problem and we have a situation which we have already discussed. We conclude that such a process will be a combination of processes like that given in Eq. (4.4), in such a way that the total charge in the initial and final states vanishes.

There is another simple way to arrive at the same conclusion and that is the observation that in the absence of a Coulomb term and a mass term, the N_f flavors of SU(2) doublets in the original theory are completely decoupled from each other. Therefore each incoming fermion can only go to an outgoing fermion of the same doublet. This allows us to consider the case where both an "a" soliton and "b" (anti) soliton are approaching the monopole. This corresponds, in terms of the field Φ defined in Eq. (4.3), to two solitons coming from $r=\infty$ and $r=-\infty$ and scattering at $r=0$.

The possible processes of this type are,

$$a_L^+ + b_L^+ \rightarrow a_L^- + b_L^- \tag{4.6}$$

$$a_L^+ + b_R^- \rightarrow a_L^- + b_R^+ \ . \tag{4.7}$$

(Process (4.6) can obviously only occur if there is an (almost) simultaneous other process with a compensating change in electric charge). a_L^+ and b_L^+ are in different doublets; b_R^- is the anti-particle of b_L^+ and hence a_L^+ and b_R^- are also in different doublets. Thus Eqs. (4.6) and (4.7) are just the superposition of two processes like Eq. (4.4). The same results can be obtained using the well known scattering behaviour of sine-Gordon solitons.

D. Helicity Violating Processes in the Massless Case

The simplest helicity violating process has $n=1$, (see Eq. (4.5)), and total incoming charge $Q = 1/2\ N_f$. This process can only occur because of the presence of the anomaly and its selection rules can be obtained by means of a simple extension of Rubakov's calculation[1] to an arbitrary number of flavors, (each Weyl doublet of fermions defines a flavor).

If for simplicity we allow at most one incoming soliton per flavor, then we obtain the following basic processes in the zero mass limit:

$$\psi^+_{Li} + \psi^-_{Rj} \to \psi^-_{Li} + \psi^+_{Rj} \tag{4.8}$$

$$\psi^+_{Li_1} + \ldots \psi^+_{Li_m} \to \psi^+_{Rj_1} + \ldots + \psi^+_{Rj_m} \quad , \tag{4.9}$$

where $m = 1/2\ N_f$ and $i_a \neq j_b$ for any a or b. (Hence $i_i \ldots i_m$, $j_i \ldots j_m$ are just a permutation of the N_f flavor indices.) The first process conserves helicity, the second is the simplest helicity violating process generated by the anomaly. Any combination of these processes is of course also allowed.

E. Conservation Laws

The conservation laws for these processes have been discussed recently by Sen.[14] His conclusion is that any charge Y is conserved if it satisfies the following requirements,

$$\text{(a)} \quad [T_a, Y] = \alpha\, i\, \varepsilon_{a3b}\, T_b \tag{4.10a}$$

and

$$\text{(b)} \quad \mathrm{Tr}\,(T_a\, T_b\, Y) = 0 \quad , \tag{4.10b}$$

where $\vec{T}$ is an SU(2) generator and α an arbitrary real number. Obviously, T_3 satisfies these requirements and we have already seen that T_3 is conserved.

The validity of Eq. (4.10) can be demonstrated as follows. For any charge other than T_3, Eq. (4.10) implies,

$$Y = \tilde{Y} - \alpha\, T_3 \qquad \text{with} \qquad [T_a, \tilde{Y}] = 0 \quad . \tag{4.11}$$

Therefore the upper and lower component of each doublet have the same $\tilde{Y}$, and hence $\tilde{Y}$ is conserved in process (4.8). The second condition, Eq. (4.10b), implies that $\sum_{i=1}^{N_f} \tilde{Y} = 0$, (i.e. $\tilde{Y}$ has no anomaly with respect to SU(2)), and hence it is also conserved in process (4.6). Therefore $\tilde{Y}$ and Y are conserved in every process.

Conditions (4.10a) and (4.10b) are satisfied in the standard SU(5) model for all Abelian gauged symmetries, (color isospin, color hypercharge, electric charge, and the weak Z_0-charge), and for B-L. (Baryon number of course does not satisfy Eq. (4.10)). Condition (4.10b) must be satisfied for any gauged symmetry, because of the absence of anomalies, but there may exist rather contrived models in which Eq (4.10a) is not satisfied for some gauge charge. In that case the Coulomb energy of such a charge has to be included to enforce conservation of this charge.

F. The Allowed Processes for Non-Zero Mass

To obtain the complete set of allowed processes in the massive case we have to include the basic helicity flip process allowed by the mass term,

$$\psi^{+}_{Li} \rightarrow \psi^{+}_{Ri} \quad . \tag{4.12}$$

This interaction conserves all Abelian charges as long as they are vectorial. (The Z_0-charge is not conserved by a process such as Eq. (4.12). We will discuss this process in detail in Section VI.)

In the massive case the allowed processes can be completely specified by requiring the conservation of $1/2\ N_f + 1$ charges. These charges can be chosen to be T_3 plus a vectorial flavor charge assigned to each of the $1/2\ N_f$ pairs of doublets. In the notation of Eq. (2.3),

$$\begin{bmatrix} a_L^+ \\ \\ b_L^- \end{bmatrix} \text{ has charge } +1, \qquad \begin{bmatrix} b_L^+ \\ \\ a_L^- \end{bmatrix} \qquad \text{has charge } -1.$$

Conservation of these 1/2 N_f + 1 charges is a necessary condition; whether it is sufficient will be checked in Section V.

For $N_f = 2$, the only processes that can occur for any number of incoming particles are uninteresting ones. They can all be generated by the mass terms alone. The simplest examples are,

$$\begin{aligned} a_L^+ &\to a_R^+ \\ a_L^+ + b_L^+ &\to a_R^+ + b_R^+ \\ a_L^+ + b_R^- &\to a_R^+ + b_L^- \quad . \end{aligned} \tag{4.13}$$

The anomaly contributes to the same helicity flip amplitude as the mass term and it is not very relevant which of the two is at work.

For $N_f = 4$, corresponding to one generation of fermions in standard models, the situation is more interesting. In Table 1, we list the processes with one or two incoming solitons and at most one soliton per flavor. In general there is more than one final state per initial state. Among the final states there is always one which does not violate any charge except helicity. Whether such processes dominate the other ones, (which all lead to violation of baryon number in grand unified models), is a dynamical question which we will study in the next section.

V. Numerical Results for the SU(2) Model

The results we give in this section are obtained with the Lagrangian of Eq. (3.4) for $N_f = 4$. The Weyl doublets are taken to be,

$$\begin{bmatrix} a^+ \\ b^- \end{bmatrix}_L \qquad \begin{bmatrix} b^+ \\ a^- \end{bmatrix}_L \qquad \begin{bmatrix} c^+ \\ d^- \end{bmatrix}_L \qquad \begin{bmatrix} d^+ \\ c^- \end{bmatrix}_L , \qquad (5.1)$$

where (a,b) and (c,d) satisfy the boundary conditions of Eq.(3.3). The results depend on the following parameters,

- the initial positions r^0 and velocities β^0 of the incoming solitons,
- the masses of each of the four soliton types,
- the coupling constant F,
- the number of points, N, and the total length, R, and
- the time increments Δt used to solve the equations of motion.

All results satisfy an obvious scaling law--they are invariant under the transformation,

$$
\begin{aligned}
&m_i \to \frac{1}{\eta} m_i && \text{for all } i \\
&R \to \eta R \\
&r^0 \to \eta r^0 \\
&\beta_0 \text{ fixed} \\
&\Delta t \to \eta \Delta t ,
\end{aligned}
$$

where N is fixed. This invariance is exact even for finite a and Δt. Unless otherwise specified, we choose the following standard set of parameters: $m_i = 1$ for all i, (compared to the length scale shown on the r-axis of the plot), F = .1, N = 100, Δt=.003, and R is the maximum length shown in the plots.

The parameter Δt must be sufficiently small to prevent numerical fluctuations. The advantage of the Lagrangian of Eq. (3.4) and in particular the fact that the boundary conditions follow from the equations of motion is that the total energy is exactly conserved for non-zero a and for $\Delta t \to 0$. Therefore unphysical, numerical fluctuations due to a value of Δt which is too large can easily be recognized since they lead to non-conservation of energy. In practice, requiring that energy conservation is satisfied with an accuracy better than .1% turns out to be sufficient to make all results virtually independent of Δt. Such an accuracy could be obtained rather easily.

A. One Incoming Soliton

If the masses of the four flavors are chosen to be equal, the helicity flip process $a_L^+ \to a_R^+$ is energetically more favorable than $a_L^+ \to b_L^- + c_R^+ + d_R^+$, which requires the creation of two additional solitons. One can try to overcome this barrier by giving the incoming soliton a large kinetic energy, but we have not been able to produce the second possible final state in that way. What we find is the appearance of "half-solitons" which, according to Refs.(9) and (14) should occur in the massless (or extremely relativistic) limit. This final state is shown in Fig. 2. These half-solitons, after they have moved some distance away from the origin, choose to become an a_R^+ soliton rather than three solitons. (Of course, half-solitons can not move out to infinity since that would cost an infinite amount of potential energy).

When the three particle final state is made more attractive by lowering the masses of b,c, and d with respect to a, the third process does occur. We have studied this for $m_b=m_c=m_d=.5$ and $m_a=.5\ \mu$, with an incoming "a" soliton with $r^0=2$. As is shown in Figure 3, the transition between the two processes occurs slightly above the kinematic threshold, $\beta = \sqrt{1 - \mu^2/9}$.

B. *Incoming* $a_L^+ + b_L^+$

This is the process that has been most extensively discussed in the literature. The competition is here between *a process that is due to the mass term,* $a_L^+ + b_L^+ \rightarrow a_R^+ + b_R^+$, and a process that is possible because of the anomaly, $a_L^+ + b_L^+ \rightarrow c_R^+ + d_R^+$. Since the mass term should become unimportant if the soliton is extremely relativistic, we expect the anomaly induced process to dominate at sufficiently large velocities. We find indeed that a threshold velocity, β_{th}, exists above which the second process, (corresponding to proton decay in the SU(5) model), and below which the first process, (helicity flip), occurs. This threshold depends on many parameters: the relative velocity and distance of the two incoming solitons at the initial time, the distance from the origin, the masses, and the strength of the Coulomb interaction. In Fig. 4, we show an example of a process that is above threshold, for a rather arbitrary choice of initial conditions.

In addition to a dependence on physical parameters, β_{th} depends also on numerical approximations. The main effect is the discreteness of the r variable. Since we do not continue the Coulomb term to $r = 0$, the maximum of the Coulomb term is *finite even if the sum of all fields at* $r = 0$ is different from zero. Near the origin, the Coulomb term grows as N^2, where N is the number of lattice points for a given length R. The threshold velocity will therefore grow with N, and we have to investigate whether for $N \to \infty$, β_{th} approaches 1 or a value smaller than 1. (If the limit is 1, we would conclude that anomaly induced proton decay like processes occur only in the massless theory and vanish for any non-zero mass.[15])

The dependence of β_{th} on the number of lattice points is shown in Fig 5. The asymptotic value is clearly smaller than 1. The approach to the asymptotic value appears to be exponential as a function of $\sqrt{N}$. The threshold velocities are obtained with an initial state consisting of two solitons at $r^0 = 2$. The threshold increases somewhat for larger r^0 and has a very strong dependence on the initial separation between the two solitons. Clearly, for large separation the solitons will scatter independently and choose the helicity-flip mode. This is shown in Fig. 6, a plot of β_{th} versus δr, the spatial separation of the incoming solitons, (both solitons have the same initial velocity and their initial distances to the origin are 2 and $2+\delta r$). Clearly, the separation has to be less than roughly the inverse mass of the solitons for the baryon number violating process to occur.

Finally, the threshold velocity depends on the strength of the coupling, F. This dependence is shown in Fig. 7. A realistic value for F $(=g^2/16\pi^2)$ would be $\sim 10^{-3}$. To approach that value in a realistic way we have to make sure that $Er_c < F$, where r_c is the core size and E the energy of the incoming solitons. If this condition is not satisfied, it is possible to excite dyon states. For SU(5) monopoles, Er_c << F for any reasonable value of E. In our case, however, r_c is effectively equal to the parameter a, the size of a lattice cell. It is impossible to make a extremely small and therefore F must be restricted to values greater than ~.05.

Although Fig. 7 gives a rough idea about the behavior of β_{th} for small F, the small coupling limit, ($F \sim 10^{-3}$), is numerically inaccessible. The threshold velocities in this figure are obtained with N=100; Fig. 5 shows that the results for small F are more sensitive to an increase in N than the ones for large F. In particular, the threshold velocity for F = .05 increases to a much larger value, (between .5 and .6) if N approaches the continuum limit.

C. Incoming $a_L^+ + c_R^-$

This process is very simple to describe if a_L^+ and c_R^- are always at the same point. Then the Coulomb energy does not contribute and the discussion of Section IV.B applies. We conclude that in this case the final state will always be $b_L^- + d_R^+$. When there is a finite distance in phase space between a_L^+ and c_L^-, the Coulomb energy can have an effect and force the final state to be $a_R^+ + c_L^-$. This will clearly happen if the two incoming states have a large difference in their arrival time at the monopole core. An example of the process $a_L^+ + c_R^- \to b_L^- + d_R^+$ is shown in Fig. 8. This process occurs almost independently of velocity if the initial separation between the solitons is less than ~.8. For larger separations the helicity flip process occurs.

D. Incoming $a_L^+ + c_L^+$

Since there is only one allowed final state this process is all ready completely described by the arguments in Section IV.F.

E. Incoming $a_L^+ + b_R^-$

There is no new physics here: the possible final states are trivial combinations of the ones discussed in section V.A. The processes with double solitons in the final state, (see Table 1), can be obtained by separating the arrival times of solitons a_L^+ and b_R^- and choosing appropriate masses.

VI. The SU(5) Model

We consider the minimal SU(5) GUT with one family of fermions in the $[\bar{5}]$ and [10] representations and Higgs fields in the [24] and [5] representations. The vacuum expectation value of the [24] breaks the symmetry to SU(3) x SU(2) x U(1) at a scale $M_X \sim 10^{14}$ GeV, thus producing monopoles with a mass near M_X/α_{GUT}. The embedding of the monopole into the SU(5) group and the transformation of the fermions under the SU(2) group corresponding to the monopole are described in the Appendix.

As in the SU(2) model of Section II, each Dirac fermion is replaced by a bosonic field which satisfies the boundary condition,

$$\phi_e(r=0) = \phi_{d_3}(r=0) \qquad \phi_e'(r=0) = -\phi_{d_3}'(r=0)$$
$$\phi_{u_1}(r=0) = \phi_{u_2}(r=0) \qquad \phi_{u_1}'(r=0) = -\phi_{u_2}'(r=0). \qquad (6.1)$$

The derivation of the effective boson Lagrangian and the inclusion of the Coulomb interaction terms requires more careful thought than in the SU(2) model. In the Appendix, we discuss in detail the bosonization of the SU(5) theory and the construction of the effective Lagrangian. We include a discussion of turning on the electroweak interactions at distances less than $1/M_Z$ from the monopole.

At distances greater than $1/M_Z$ from the monopole, the gauge symmetry is SU(3) x $U(1)_{em}$. As in Section III, we discretize the Lagrangian and also enforce the boundary conditions of Eq. (6.1). The effective Lagrangian is then, (see Eqs. (3.4) and (A17)),

$$L_1 = L - \sum_{j=1}^{N} \frac{g^2}{32\pi^2 j^2 a^2} \left[(\phi_{1j}-\phi_{2j})^2 + 1/2(\phi_{3j}-\phi_{4j})^2\right] \qquad (6.2)$$

where $\phi_1 = \phi_{u_1}$, $\phi_2 = \phi_{u_2}$, $\phi_3 = \phi_{d_3}$, $\phi_4 = \phi_e$, and g is the SU(5) gauge coupling constant. (Note that we neglect all renormalization group scaling of the gauge coupling constants).

At distances less than $1/M_Z$ from the monopole, the effective gauge theory is $SU(3) \times SU(2)_L \times U(1)$ and is described by the Lagrangian,

$$L = L_1 + \delta L \quad , \tag{6.3}$$

where δL describes the couplings of the bosonic fields to the Z_0 gauge field. In the $A_0 = 0$ gauge, the discrete form of δL is, (see Eqs. (A15) and (A16)),

$$\begin{aligned}\delta L = &- \frac{1}{2\sqrt{10\pi}}\Big\{ \sum_{j=1}^{N} (\dot{\phi}_{4,j} - \dot{\phi}_{3,j}) A_{r,j} + \frac{2}{a} \sum_{j=1}^{N-1} \Big[\sum_{i=1}^{2} (\phi_{i,j+1} - \phi_{i,j}) \\ &- \sum_{i=3}^{4} (\phi_{i,j+1} - \phi_{i,j}) \Big] A_{r,j} \Big\} - \frac{2\pi a^2}{g^2} \sum_{j=1}^{N} j^2 (\dot{A}_{r,j})^2 \quad . \\ &- \frac{1}{5\pi} \sum_{j=1}^{N} (A_{r,j})^2\end{aligned} \tag{6.4}$$

where $A_{r,j}$ is the Z_0 field in the r direction at a distance $r=ja$ from the monopole. (We omit the superscript 3 used in the appendix on A_μ.)

In this gauge, the variables $A_{r,j}$ can be treated as dynamical variables and their equations of motion can be integrated if the initial values of $A_{r,j}$ and $\dot{A}_{r,j}$ are given. Since initially the soliton is at a distance $>>1/M_Z$ from the core, we can take $A_{r,j}(t_0) = \dot{A}_{r,j}(t_0) = 0$. As soon as the soliton enters the region $r<1/M_Z$, the weak field is generated with the correct strength.

This can be checked in the $A_r = 0$ gauge, in the small coupling limit. This case was considered in Ref. 7, with the result,

$$A_0' = \frac{g^2}{8\pi r^2} \int \frac{J_0^3(s)}{\sqrt{\pi}} \, ds \tag{6.5}$$

$$J_0^3 = -\frac{1}{\sqrt{40}} \quad [(\phi_e' - \phi_d') + 2(\dot{\phi}_{u_1} + \dot{\phi}_{u_2} - \dot{\phi}_e - \dot{\phi}_d)] \tag{6.6}$$

which can be derived from Eq. (A15) by ignoring the last term. For free solitons with velocity β,

$$\dot{\phi}_i = \beta \, \phi_i' \quad , \tag{6.7}$$

which is valid if the gauge coupling is sufficiently small.

The field strength in the two gauges, ($\dot{A}_r$ and A_0' respectively) should be identical in the limit $m \to 0$, $\beta \to 1$, since the mass term breaks the gauge invariance of the weak interaction. Using Eqs. (6.6) and (6.7), we can calculate the integral in Eq. (6.5) and compare A_0' with the dynamically calculated field strength $\dot{A}_r$. For incoming solitons starting at position r_0, we find that, in the limits specified above, the correct field strength is built up for all radii smaller than r_0, (we use $M_Z = 0$ here). For larger radii, we do not get the correct field because of the initial condition $\dot{A}_{r,j}=0$, but this is irrelevant. Alternatively, we can remove the "axial" terms in the current, (the $\dot{\phi}$ terms in Eq. (6.6)), in which case Eq. (6.5) is exact and the results in the two gauges agree even for finite fermion masses. We do not use Eq. (6.5) however, since it requires a boundary condition at r=0, which is hard to implement when the solitons are near the core.

In the next subsection, we describe a numerical study of the Lagrangians of Eqs. (6.2) and (6.4).

B. Numerical Results

At large r the theory is that of four uncoupled solitons with the solution given by Eq. (4.1). The effective discrete Lagrangian is shown in Eq. (6.2) and the equations of motion are easily integrated numerically as in Section V. However, when a distance $\sim 1/M_Z$ from the monopole is reached, the Lagrangian of Eq. (6.4) must be used for numerical integrations, as described above. We make the simplifying assumption that the Z_0 interaction is turned on with a theta function at $r=1/M_Z$.

The fermion masses are all taken to be equal and are set equal to zero when the Z_0 interactions are turned on. This is because the $SU(2)_L$ gauge symmetry forbids fermion mass terms in the Lagrangian.[16] We take the coupling constant F to be .1, although we find similar results for $.05<F<1$.

1. $u_{1R} + u_{2R} \to \bar{d}_{3L} + e^+_L$

The effect of the additional strong and electromagnetic Abelian Coulomb interactions on the physics near the core is expected to be small since the corresponding charges vanish near the core. We find that even quantitatively there is hardly any visible difference for F<.1. (We use the unified SU(5) coupling constant for all interactions.) In Figure 9, we show the effect of these additional interactions, (Eq. (6.4)), on the final state of the process $u_{1R} + u_{2R} \to \bar{d}_{3L} + e^+_L$ for F=.2, an unrealistically large value.

The effect of the Z_0 field is equally small, but the fact that we set $m_i=0$ for $r<1/M_Z$ is more important. We find that solitons enter the region around the core very easily, but are sometimes trapped within a radius $1/M_Z$. This happens because the sphere at $r=1/M_Z$ acts as a barrier. At $r=1/M_Z$, the massless solitons moving away from the core have to become massive, which costs a finite amount of energy. Often the solitons do not pass this barrier, but non-topological radiation is emitted during the process. After a while the total energy of the field configuration within $r=1/M_Z$ drops below the threshold for production of any of the allowed final states, and the massless solitons are trapped.

The monopole core is not left in one of the ground states, but in a state with four oscillating soliton fields. The conserved charges of the original incoming solitons are *present within* $r=1/M_Z$, *but not on the core.* The correct physical interpretation of this phenomenon in the full quantum theory is difficult, but since it depends crucially on the r-dependence of the soliton mass near the core the physical relevance is questionable.

Despite this phenomenon, the processes $u_{1R}+u_{2R}\rightarrow\bar{d}_{3L}+e^+_L$ and $u_{1R}+d_{3L}\rightarrow\bar{u}_{2R}+e^+_L$ can still be observed above a certain velocity threshold and for small radial separations of the incoming solitons. For example, Figure 9 shows the final state of the first process with F=.2 and $1/M_Z=.5$. We find that Figure 6, showing β_{th} as a function of the separation, is essentially unchanged. The threshold velocity is in fact slightly lower in the presence of the weak interaction effects. A plot very similar to Figure 6 also describes the threshold velocity for the second process mentioned above. Below these thresholds we do not observe the helicity flip process, but the phenomenon described in the previous paragraphs.

2. $u_{1R} \rightarrow u_{1L}$

The processes with one incoming soliton are expected to proceed via an intermediate state of half solitons, [9,14] which should materialize into full solitons at $r=1/M_Z$. The half solitons are very easily visible, but we find that they are more likely to remain trapped than full soliton final states. The three soliton final state is reached only for special choices of the four masses and a large velocity of the incoming soliton, (e.g. m_1=3., m_2=1., m_3=1., m_4=.1, and β=.9).

These results were obtained with F=.1 and $1/M_Z$=.5. The full range of relevant mass scales (from m_e to M_Z) can not be studied numerically. It is however clear that the results of this section will approach those of Section V if we could increase M_Z/m_e to a much larger value, so that the size of the weak interaction region is much smaller than the size of a soliton.

VII. Heavy Flavors

Our results thus far involve only one generation of fermions. Additional generations could be important since they are coupled to the first generation by the anomaly. In the massless case this is evident in the selection rule (4.5). (Notice that the helicity conserving processes are not expected to be sensitive to additional generations, since they do not proceed via the anomaly.) However,if the additional fermions are sufficiently heavy ,we expect them to decouple.

We have studied the effect of additional heavy fermions from extra generations by adding two solitons, e and f, to the model of Section V. There are several ways to choose masses for the six solitons (see, for example, Ref. 9), but we have concentrated on just one possibility. In our approach, the solitons a,b,c,d all have equal masses, and the masses of e and f are equal to each other, but different from the others.

The solitons e and f are coupled by the boundary condition at r=0, just as are (a,b) and (c,d). (In realistic models, e and f could correspond to charm quarks). We study the final state as a function of $\rho=m_e/m_a$, with two incoming solitons of type a and b. There are three possible final states:

$$a_L^+ + b_L^+ \to a_R^+ + b_R^+ \qquad (1)$$

$$\to c_R^+ + d_R^+ \qquad (2)$$

$$\to e_R^+ + f_R^+ \qquad (3)$$

For $\rho \gg 1$, (and sufficiently large velocity of the incoming solitons), we expect to see final state (2), since the heavy particles e and f should decouple from the problem. The system reduces then to the one discussed in Sec. V.B. For obvious reasons, we expect final state (3) if $\rho \ll 1$. If $\rho=1$, modes (2) and (3) are equally attractive, and since the system is deterministic, the final state must be invariant if the pairs (c,d) and (e,f) are interchanged. In other words, modes (2) and (3) are not allowed, since the system has no way of choosing between them, and therefore, for any velocity, the final state will be (1).

A numerical analysis is necessary to determine the final state in a given scattering process. We find that mode (2) is chosen for $\rho>1.05$ and mode (3) for $\rho<.95$, for incoming solitons a and b with $r_0=2$, $\beta=.9$, and $F=.1$. Thus the system behaves very much like a four flavor system at values of ρ rather close to 1, long before the heavy flavors actually decouple. The details of the final state and the threshold velocity are of course influenced by the heavy flavors over a much larger range of ρ.

The region around $\rho=1$ has a very complicated structure, consisting of many small intervals of ρ-values in which one of the three final states is chosen,bordering on intervals in which another final state is preferred for a given initial state. There even exists an interval for $\rho>1$, in which final state (3) is chosen instead of (2) ,although (3) requires the formation of heavier solitons.

This interesting region shrinks with increasing N,and may disappear in the continuum limit. Since this fascinating phenomenon has no relevance for catalysis of proton decay we have not investigated this in more detail.

VIII. Conclusion

We have studied the interactions of fermions with a 't Hooft-Polyakov monopole in a classical soliton approximation. Our results can be interpreted as classical cross sections for production of the different final states. We find the dynamical thresholds for the occurrence of a given process. The main features of these thresholds as functions of several parameters are presented in Sec. V.

Before we summarize our main conclusions, let us first discuss the limitations of this approximation. In the absence of the Coulomb term, (see Eq. 3.4), our approximation reduces to a system with exactly the same physical content as the Dirac equation for fermions in the field of a magnetic monopole. This is the system studied in Refs. 10, 11 and 17, (apart from the physics at the core). Our approach includes the Coulomb fields produced by the quarks, which turn out to be extremely important, and goes one step beyond solving the J=0 part of the Dirac equation. Therefore a calculation of the cross-section in the soliton approximation is at least as justified as using the Dirac equation for that purpose.

Quantum corrections are obviously absent in this approximation, but that is not the most serious limitation. In the SU(2) model, quantum corrections will presumably modify the theta function thresholds by an exponential tunneling behavior below threshold and a resonance-like behavior above threshold. In any case, they will not suppress a process that can be observed classically.

The more serious limitation is the incomplete description of weak and strong interaction physics, which can only partly be implemented in the SU(2) model. (We are only able to include Abelian interactions-- non-diagonal interactions are beyond our approximation). Related to this is the fact that we have only studied free quarks, ignoring the fact that they are confined inside a proton.

Our main conclusions are:

1. All of the processes expected to occur in the field of a monopole, (see Table 1), have been observed. We have understood which processes occur due to the presence of the anomaly and which are due to the boundary conditions at the monopole core. (See also Ref. 14.) No processes have which violate electromagnetic charge have been observed for finite values of the coupling constants.
2. The most important baryon number violating processes for one generation which we observed are,
   a. $u_{1R} + u_{2R} \rightarrow e^+_L + \bar{d}_{3L}$
   b. $u_{1R} + d_{3L} \rightarrow \bar{u}_{2R} + e^+_L$
   c. $u_{1R} \rightarrow \bar{u}_{2R} + e^+_L + \bar{d}_{3L}$
   d. $d_{3L} \rightarrow e^+_L + \bar{u}_{1R} + \bar{u}_{2R}$.

   Processes (c) and (d) can only occur above a kinematic threshold discussed in Section V.
3. The inclusion of finite fermion mass terms in the Hamiltonian does not affect the qualitative nature of

the scattering processes -- monopole catalysis of proton decay proceeds as predicted.

4. We have studied the dependence of the scattering processes on the coupling constant, the velocities and the spatial separation of the incoming solitons. The main features are given in Sec. V.

5. We have included the additional weak, electromagnetic and strong Coulomb energies relevant for SU(5) monopoles. We find that their effect on the final state is very small. The r-dependence of the effective fermions masses due to the breaking of the weak interaction symmetry (or due to QCD-effects) is more important, but less well understood.

6. Additional generations will not have an important effect on catalysis of proton decay.

Although a reliable estimate of the cross-section (or rate) for catalysis is still lacking, we have not found any effect that would suppress it by many orders of magnitude. Processes a and b of point 2 occur in all the circumstances which we have studied. The occurrence of c and d is more sensitive to the (current or constituent) masses of the fermions, and we do not have meaningful conclusions about them.

All B-violating processes require the quark velocities to be above certain thresholds. For a small ($\beta \sim 10^{-3}$) relative velocity of the proton and the monopole, the relevant velocities are those of the quarks in the proton.

The cross section depends on the probability - determined by the proton wave function - for the quarks in the proton to be in the right region of phase space for one of the aforementioned processes. This is a complicated model dependent question, which we have not tried to address.

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APPENDIX

In this appendix we derive the bosonized two dimensional Lagrangian, including all Abelian Coulomb interactions, for the standard SU(5) model. The derivation of the SU(2) part of the Lagrangian can be found in Refs. 1 and 2 and expressions for the additional Coulomb energies are given in Refs. 2 and 7. Since a complete derivation has not been presented before and since there are several small differences between the available results, we think it is useful to present our complete derivation.

We start from the SU(5) Lagrangian,

$$L = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu b} + i\bar{\psi}_5 \gamma^{\mu} D_{\mu} \psi_5 + i\bar{\psi}_{10} \gamma^{\mu} D_{\mu} \psi_{10} \tag{A1}$$

with
$$D_{\mu} = \partial_{\mu} + A_{\mu}$$
$$A_{\mu} = -ig A^{a}_{\mu} T_a$$
$$F_{\mu\nu} = [D_{\mu}, D_{\nu}] = -ig F^{a}_{\mu\nu} T_a$$

and ψ_5, ψ_{10} are left-handed Weyl-fermions in the $[\bar{5}]$ and [10] representations. The normalization of the generators is

$$Tr(T_a T_b) = \frac{1}{2} C_1 \delta^{ab} \tag{A2}$$

with $C_1 = 1$ for the fundamental representation. The simplest eg = 1/2 monopole is embedded in the SU(2) subgroup generated by

$$\vec{T} = \frac{1}{2}\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \vec{\tau} & \\ & & & 0 \end{bmatrix} \tag{A3}$$

All other generators can be classified according to representations of this SU(2) subgroup. Since the bosonization method is not very suitable for off-diagonal interactions, we have to restrict ourselves to diagonal ones. These generators can be chosen in the following way,

$$\begin{aligned} M_1 &= \frac{1}{2}\,\mathrm{diag}(\ 1,\ -1,\ 0,\ 0,\ 0) \\ M_2 &= \frac{1}{\sqrt{8}}\,\mathrm{diag}(-1,\ -1,\ 1,\ 1,\ 0) \\ M_3 &= \frac{1}{2\sqrt{10}}\,\mathrm{diag}(1,\ 1,\ 1,\ 1,\ -4) \end{aligned} \tag{A4}$$

We will only consider the J=0 part of the fermions in Eq. (A1). First consider the gauge fields. We choose the following parameterization, consistent with spherical symmetry with respect to J=L+S+T,

$$A_0 = -i[\ \vec{T}\cdot n\mathscr{A}_0^0 + \sum_{i=1}^{3} M_i \mathscr{A}_0^i\ ]$$

$$A_i = A_i^{cl} - i\ n_i[\vec{T}\cdot n\mathscr{A}_i^0 + \sum_{i=i}^{3} M_i \mathscr{A}_1^i$$

$$A_i^{cl} = i\ (\vec{T}\ x\ n)\ \frac{1 - F(r)}{r} \tag{A5}$$

where $n=\vec{r}/|r|$, $F(0) = 1$, and $F(\infty) = 0$.

When Eq. (A5) is substituted into the action one obtains,

$$-\frac{1}{4} F_{\mu\nu}^{a} F^{\mu\nu a} = \frac{1}{2g^2}[\ \sum_{k=0}^{3} (\dot{\mathscr{A}}_1^k - \mathscr{A}_0^{\prime k})^2]$$

$$+ \frac{2F^2}{r^2}\ [(\mathscr{A}_0^0)^2 - (\mathscr{A}_1^0)^2] \tag{A6}$$

This can be simplified if it is written in a two dimensional notation,

$$x^\mu = (t,r)$$

$$A_\mu^k = (\mathscr{A}_0^k, \mathscr{A}_1^k)$$

$$F_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k \tag{A7}$$

where our metric is (+, -). Then we obtain,

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} = -\frac{1}{4g^2}\sum_{k=0}^{3}F^k_{\mu\nu}F^{\mu\nu k} + \frac{2F^2}{r^2}A^0_\mu A^{0\mu} \quad . \tag{A8}$$

This last term vanishes in the limit $M_X\to\infty$, (zero monopole size), and will be dropped from now on.

Now consider the fermions. We are only interested in the fermions that interact with the monopole. Since the generators M_k are SU(2) singlets, the fermions in one SU(2) doublet have the same M_k eigenvalue,

$$\begin{array}{lccccc}
\text{Doublet:} & & \begin{bmatrix} e^- \\ \bar{d}_3 \end{bmatrix} & \begin{bmatrix} d_3 \\ e^+ \end{bmatrix} & \begin{bmatrix} \bar{u}_2 \\ u_1 \end{bmatrix} & \begin{bmatrix} \bar{u}_1 \\ u_2 \end{bmatrix} \\
\text{Eigenvalue of} & M_1 & 0 & 0 & 1/2 & -1/2 \\
 & \sqrt{8}M_2 & -1 & 1 & 0 & 0 \\
 & \sqrt{40}M_3 & -1 & -3 & 2 & 2
\end{array} \tag{A9}$$

The J=0 form of the fermions is,

$$\psi^i_{\alpha\ell} = \frac{1}{\sqrt{8\pi r}}\left[\varepsilon_{\alpha\ell}f^i + i(\vec{\tau}\cdot n)_{\alpha\beta}\varepsilon_{\beta\ell}g^i\right] \quad , \tag{A10}$$

where α is the spin index, ℓ is the SU(2) gauge index and $i=1,\ldots,4$ labels the four doublets in Eq. (A9). Substitution of Eq.(A10) yields the following action for the fermions,[18]

$$S = -i\int dr\ dt \sum_{i=1}^{4} \bar{\chi}^i \gamma^\mu (\partial_\mu + \frac{i}{2} \varepsilon_{\mu\nu} A^{\nu,0} - i \sum_{k=1}^{3} Q_i^k A_\mu^k) \chi^i \quad \text{(A11)}$$

where $\chi = \begin{pmatrix} f \\ g \end{pmatrix}$. In this two-dimensional space we define $\gamma^0 = \tau_3$, $\gamma^1 = i\tau_1$, and Q_i^k is the M_k eigenvalue of doublet k.

The bosonized version of the two dimensional fermion Lagrangian defined by Eq. (A11) is,

$$L = \frac{1}{2} \sum_{i=1}^{4} (\partial_\mu \phi_i)^2 + \frac{1}{\sqrt{\pi}} \sum_{i=1}^{4} \partial^\mu \phi_i [\frac{1}{2} \varepsilon_{\mu\nu} A^{\nu,0} - \sum_{k=1}^{3} Q_i^k A_\mu^k]$$

$$+ \frac{1}{2\pi} \sum_{k=1}^{3} (\sum_{i=1}^{4} (Q_i^k)^2 A_\mu^k A^{\mu k})] \quad \text{(A12)}$$

The last term is necessary for gauge invariance.[19] Its effect is to add an extra A-dependent term to the currents. The conserved currents are,

$$J_0^\mu = -\frac{1}{2\sqrt{\pi}} \varepsilon^{\mu\nu} \partial_\nu \sum_{i=1}^{4} \phi_i$$

$$J_k^\mu = -\frac{1}{\sqrt{\pi}} \partial^\mu \sum_{i=1}^{4} (Q_i^k \phi_i)$$

$$+ \frac{1}{\pi} A_k^\mu \sum_{i=1}^{4} (Q_i^k)^2 \quad \text{(A13)}$$

Since the mass term would be rather awkward in this bosonized form, it is convenient to make a canonical transformation,

$$\phi_e = 1/2\ (\phi_1 + \phi_2) + 1/2 \int(\pi_1 - \pi_2)ds$$
$$\phi_d = 1/2\ (\phi_1 + \phi_2) - 1/2 \int(\pi_1 - \pi_2)ds$$
$$\pi_e = 1/2\ (\pi_1 + \pi_2) + 1/2\ (\phi_1' - \phi_2')$$
$$\pi_d = 1/2\ (\pi_1 + \pi_2) - 1/2\ (\phi_1' - \phi_2') \qquad \text{(A14)}$$

and the same with (e,d,1,2) replaced by $(u_1, u_2, 3, 4)$. With these transformations we obtain the following Lagrangian, (including the gauge fields),

$$L = 4\pi r^2[- \frac{1}{4g^2} \sum_{k=0}^{3} F^k_{\mu\nu}F^{\mu\nu,k}] + \frac{1}{2} \sum_{i=1}^{4} (\partial_\mu\phi_i)^2$$
$$+ \frac{1}{\sqrt{\pi}} \sum_{k=0}^{3} A^k_\mu J^{\mu,k} + \frac{1}{5\pi} A^3_\mu A^{\mu,3} \qquad \text{(A15)}$$

with,

$$J^0_\mu = -1/2\ \epsilon_{\mu\nu}\partial^\nu(\phi_e + \phi_d + \phi_{u_1} + \phi_{u_2})$$
$$J^1_\mu = -1/2\ \epsilon_{\mu\nu}\partial^\nu(\phi_{u_2} - \phi_{u_1})$$
$$J^2_\mu = \frac{1}{\sqrt{8}}\ \epsilon_{\mu\nu}\partial^\nu(\phi_e - \phi_d)$$
$$J^3_\mu = -\frac{1}{\sqrt{40}}[\epsilon_{\mu\nu}\partial^\nu(\phi_e - \phi_d) + 2\partial_\mu(\phi_{u_1} + \phi_{u_2} - \phi_e - \phi_d)] \ . \qquad \text{(A16)}$$

The last term in Eq. (A15) is again necessary for gauge invariance. The difference in these gauge terms between Eq. (A12) and Eq. (A15) is due to the transformations from Lagrangian to Hamiltonian and back, which are necessary to make the canonical transformation of Eq. (A14).

The fields A_μ^0, A_μ^1, and A_μ^2 can be eliminated from the Lagrangian since their equations of motion can be solved exactly. (This is most conveniently done in the $A_r = 0$ gauge. Since the action, after a partial integration, depends only quadratically on A_0', this variable can be eliminated as in Ref. 2). Then we obtain the following interaction Lagrangian,

$$L_I = -\frac{g^2}{32\pi^2 r^2}\left[(\phi_e+\phi_d+\phi_{u_1}+\phi_{u_2})^2 + (\phi_{u_1}-\phi_{u_2})^2 + \frac{1}{2}(\phi_e-\phi_d)^2\right] \tag{A17}$$

The first term is the T_3 Coulomb term introduced by Callan.[2]

This expression can be made more recognizable by writing it in the following way,

$$L_I = -\frac{g^2}{8\pi^2 r^2}\left[\frac{3}{8}(\phi_e+\frac{1}{3}\phi_d+\frac{2}{3}\phi_{u_1}+\frac{2}{3}\phi_{u_2})^2 + (\frac{1}{\sqrt{3}}\phi_d + \frac{1}{2\sqrt{3}}(\phi_{u_1}+\phi_{u_2}))^2 + (\frac{1}{2}\phi_{u_1}-\frac{1}{2}\phi_{u_2})^2\right] \tag{A18}$$

This can finally be written as,

$$L_I = -\frac{\alpha}{2r^2}(Q_{em}\frac{\phi}{\sqrt{\pi}})^2 - \frac{\alpha_s}{2r^2}\left[(\frac{1}{2}\lambda_8\frac{\phi}{\sqrt{\pi}})^2 + (\frac{1}{2}\lambda_3\frac{\phi}{\sqrt{\pi}})^2\right] \tag{A19}$$

where Q_{em} is the generator of electric charge, λ_3 and λ_8 are the usual SU(3) matrices and $\phi=(\phi_{u_1},\phi_{u_2},\phi_{d_3},\phi_e)$. Furthermore, we have used the well known SU(5) relation $e = \sqrt{3/8}g$.

The contribution of J_μ^3, (which couples to the Z boson), can not be solved exactly, (this is due to the second term in the expression for J_μ^3 (see Eq. A16), which is present because the corresponding fermion current in four dimensions has an axial vector part). We discuss this problem in Section VI.

Table 1. Possible processes for $N_f=4$. The pairs (a,b) and (c,d) are coupled at r=0. These pairs can represent (e^-, d_3) and ($\overline{u}_1$, $\overline{u}_2$) or any permutation that does not alter the pairing. The mechanisms are: 1) Non-zero mass, 2) Adler-Bell-Jackiw anomaly, 3) Adler-Bell-Jackiw anomaly and non-zero mass, and 4) Boundary conditions at r=0.

	Process	Mechanism
I.	$a_L^+ \to a_R^+$	1
	$\to b_L^- + c_R^+ + d_R^+$	3
II.	$a_L^+ + b_L^+ \to a_R^+ + b_R^+$	1
	$\to c_R^+ + d_R^+$	2
III.	$a_L^+ + c_R^- \to a_R^+ + c_L^-$	1
	$\to b_L^- + d_R^+$	4
IV.	$a_L^+ + c_L^+ \to a_R^+ + c_R^+$	1,2
V.	$a_L^+ + b_R^- \to a_R^+ + b_L^-$	1
	$\to 2a_R^+ + c_L^- + d_L^-$	3
	$\to 2b_L^- + c_R^+ + d_R^+$	3

FOOTNOTES AND REFERENCES

a) Address after July 18, 1983: Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720.

b) Address after September 1, 1983: Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794.

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5. See the discussion in Section IV.B to clarify our use of the word "anomaly."

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15. This assumes that the mass is constant at all r.

16. The correct behavior of the fermion mass near the monopole core may be different as suggested in Ref. 9.

17. Y. Kazama, C.N. Yang, and A.S. Goldhaber, Phys. Rev. D 15, 2287 (1977).

18. We have used left-handed Weyl spinors for the fermions. In Ref. 2, Dirac spinors are used. This leads to a few sign differences in Eqs. (A11) to (A14) when compared with Ref. 2.

19. The action is invariant under the transformations $A_\mu^k \rightarrow A_\mu^k + \partial_\mu \lambda$, $\phi_i \rightarrow \phi_i + (1/\sqrt{\pi}) Q_i^k \lambda$, $(k=1\ldots3)$, and $A_\mu^0 \rightarrow A_\mu^0 + \partial_\mu \lambda$.

FIGURE CAPTIONS

Fig. 1: Solitons corresponding to the possible asymptotic states.

Fig. 2: The helicity flip process $a_L^+ \to a_R^+$. The incoming soliton a_L^+ has a velocity $\beta=.995$ and all of the solitons have mass, $m_i=1$. The units on the vertical axis are $\sqrt{\pi}$. The zero points for a,b,c, and d solitons have been shifted vertically by 1,3,5, and 7 units, respectively. The time of each picture is shown in the upper left-hand corner. This figure shows three half solitons evolving into an a_R^+ soliton. of Eq. 3.4.

Fig. 3: The transition between $a_L^+ \to a_R^+$ and $a_L^+ \to b_L^- + c_R^+ + d_R^+$. The solid line shows the kinematic boundary and the cross hatches the threshold determined by the fermion-monopole interactions. Below the cross hatches $a_L^+ \to a_R^+$ and above them $a_L^+ \to b_L^- + c_R^+ + d_R^+$. $\mu = m_a/m_b$ and $m_b = m_c = m_d$. β_{th} is the minimum velocity for which $a_L^+ \to b_L^- + c_R^+ + d_R^+$.

Fig. 4: The proton decay process $a_L^+ + b_L^+ \to c_R^+ + d_R^+$. The incoming solitons are traveling at $\beta=.9$ and are initially separated by a distance $\delta R = 1/2$. (In the SU(5) model, this process corresponds to $\bar{u}_{1L} + \bar{u}_{2L} \to e_R^- + d_{3R}$.)

Fig. 5: The dependence of β_{th} on the number of lattice points N. β_{th} is determined for two incoming

solitons ($a_L^+ + b_L^+$) traveling together.

Fig. 6: The dependence of β_{th} on δR, the spacial separation of the the incoming solitons. (This figure is derived for incoming $a_L^+ + b_L^+$).

Fig. 7: The dependence of β_{th} on F, the coupling constant. (This figure is derived for incoming $a_L^+ + b_L^+$.)

Fig. 8: $a_L^+ + c_R^- \rightarrow b_L^- + d_R^+$. The incoming solitons are traveling at β=.9 and are initially separated by a distance δR=1/2.

Fig. 9: Final states for the process $u_{1R} + u_{2R} \rightarrow e_L^+ + \bar{d}_{3L}$. The incoming solitons are traveling at β=.95 and are initially separated by a distance δR=1/2. The solid lines include the SU(2) Coulomb interactions, (Eq. 3.4), the dashed lines include also the QCD Coulomb interactions, (Eq. 6.2), and the dashed-dotted lines include both SU(2) and QCD interactions plus the effects of turning on the Z_0 interactions, (Eq. 6.4), at r=.5. The coupling constant F is .2.

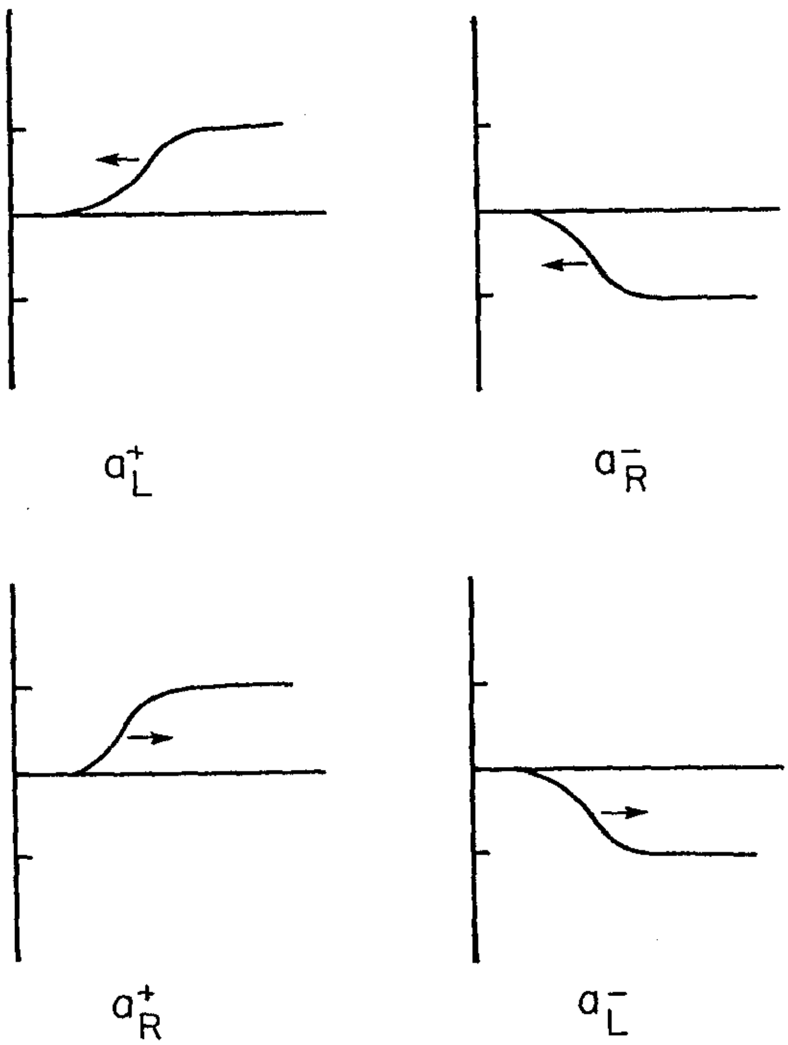

Fig. 1

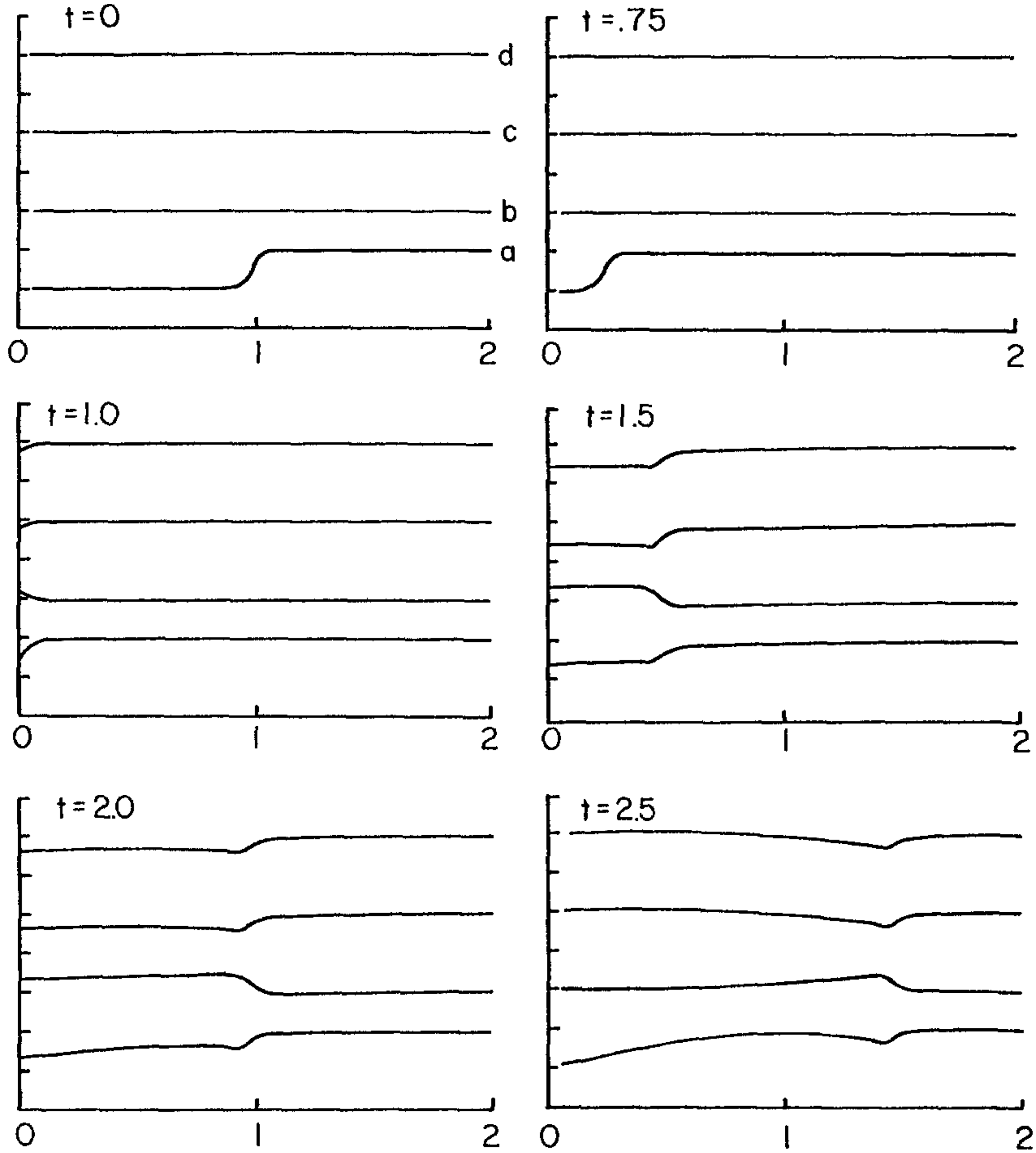

Fig. 2

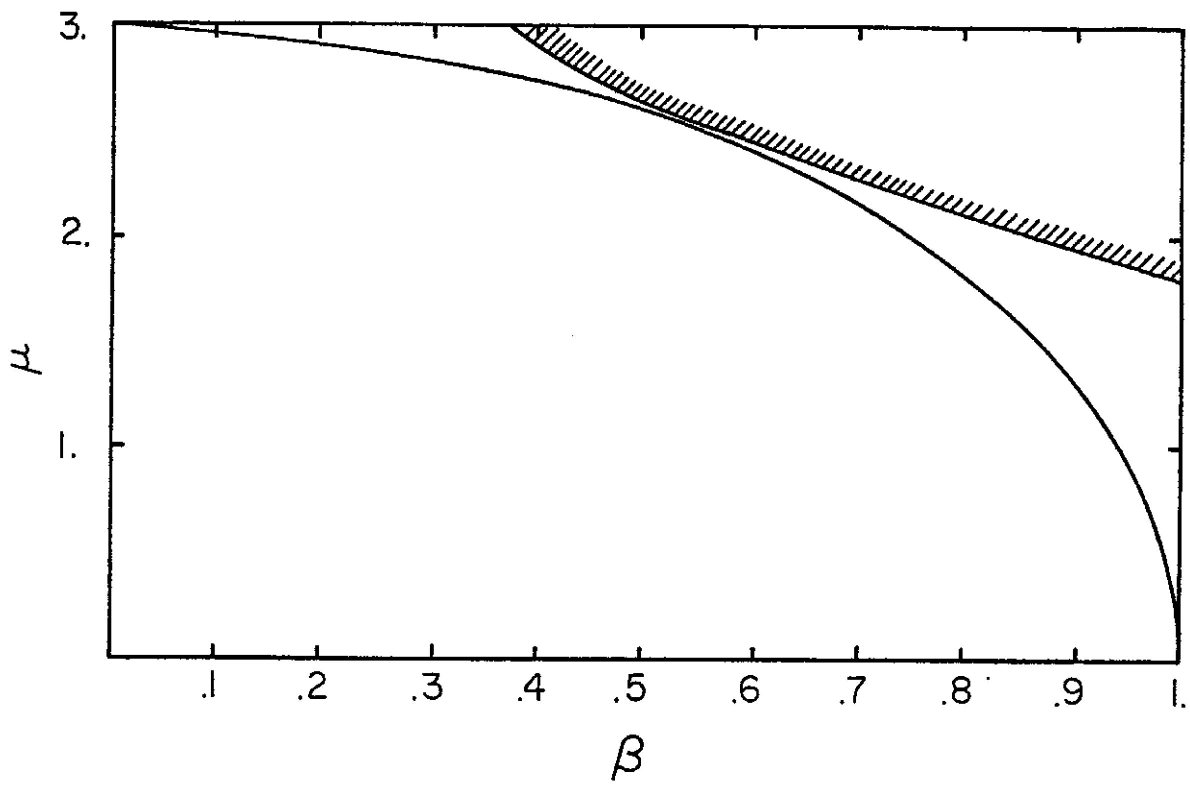

Fig. 3

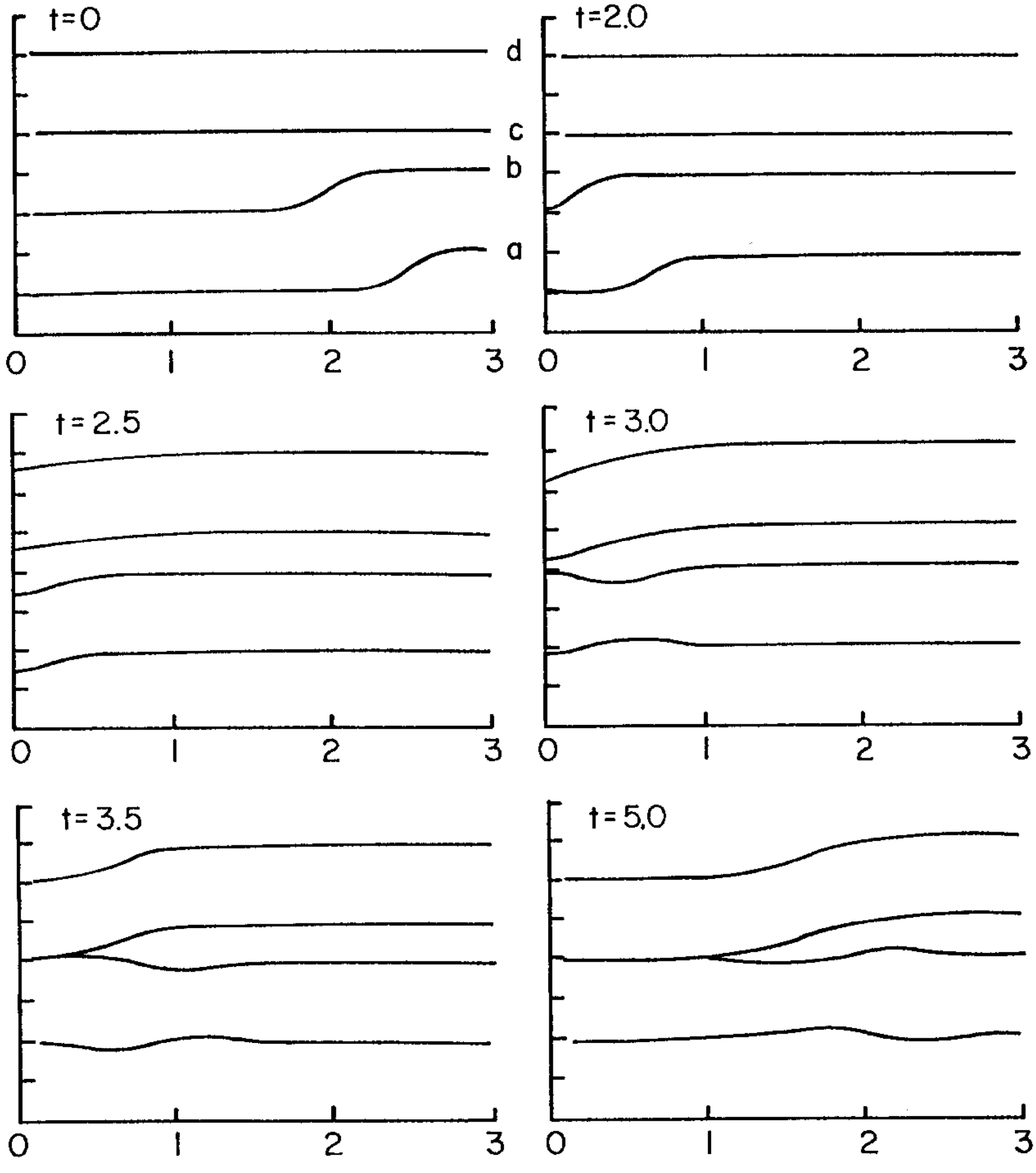

Fig. 4

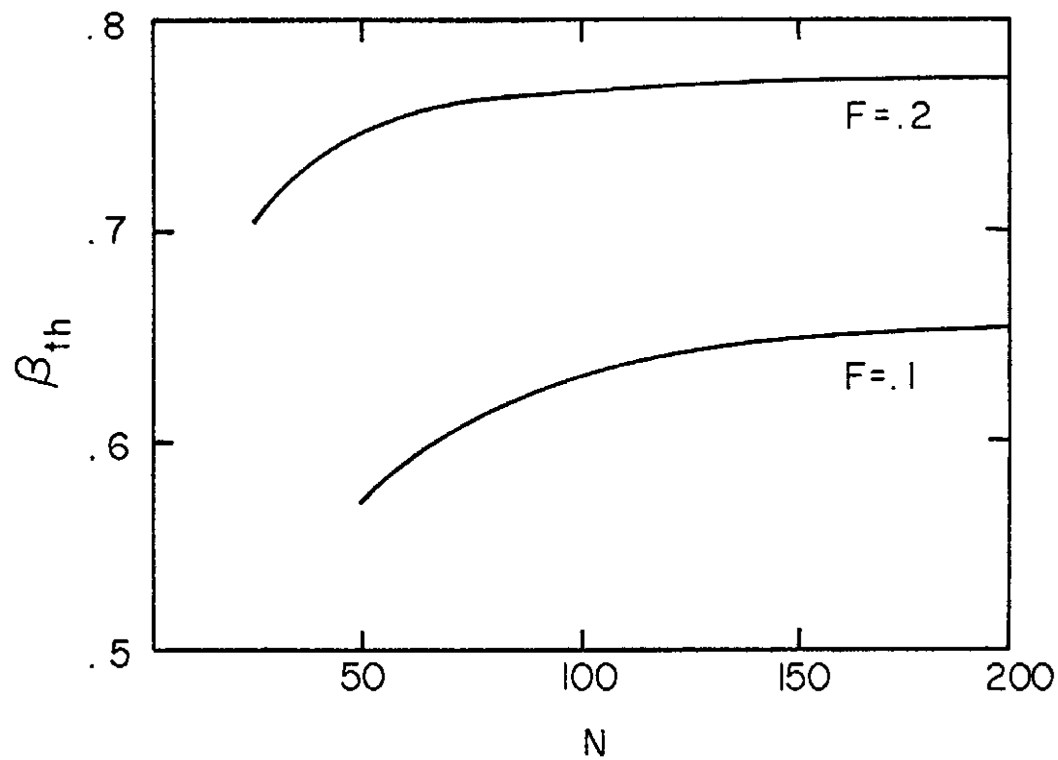

Fig. 5

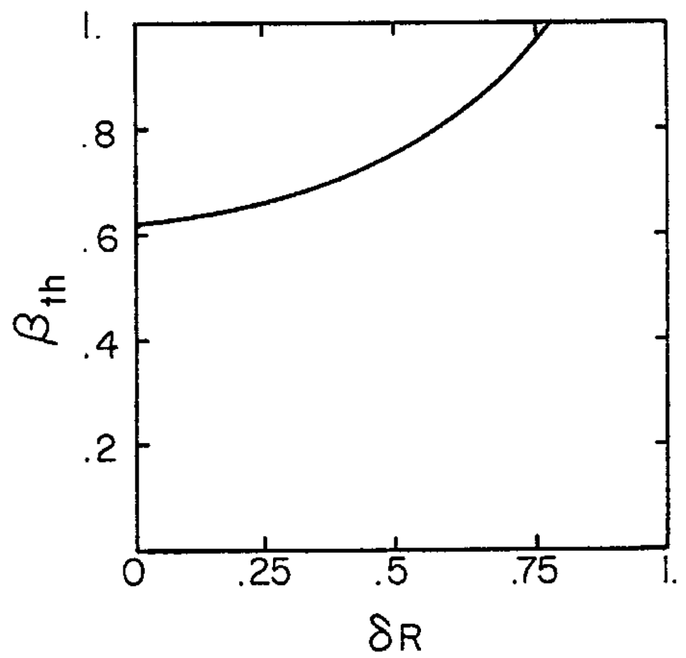

Fig. 6

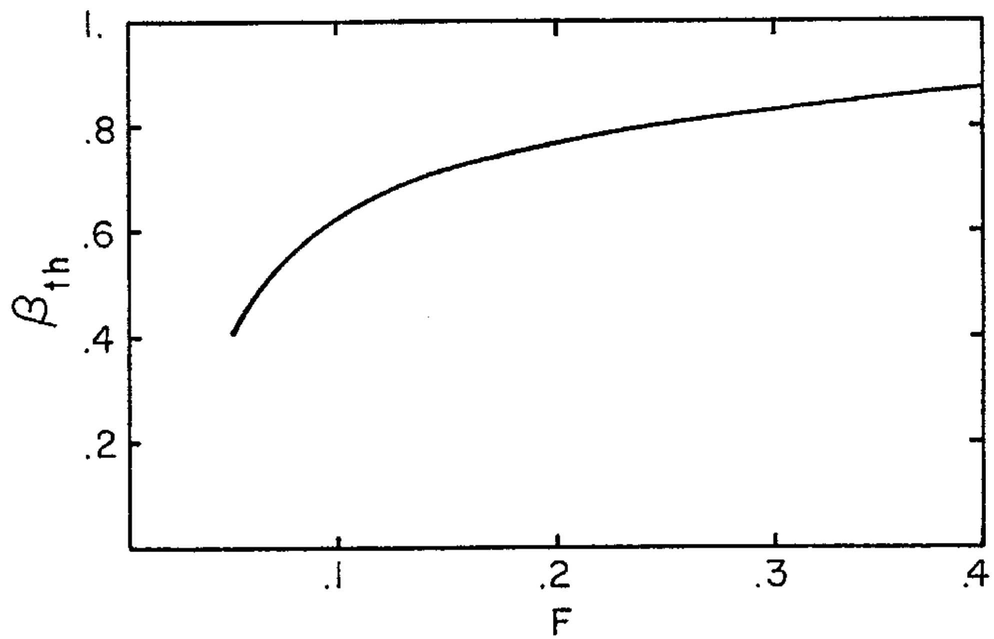

Fig. 7

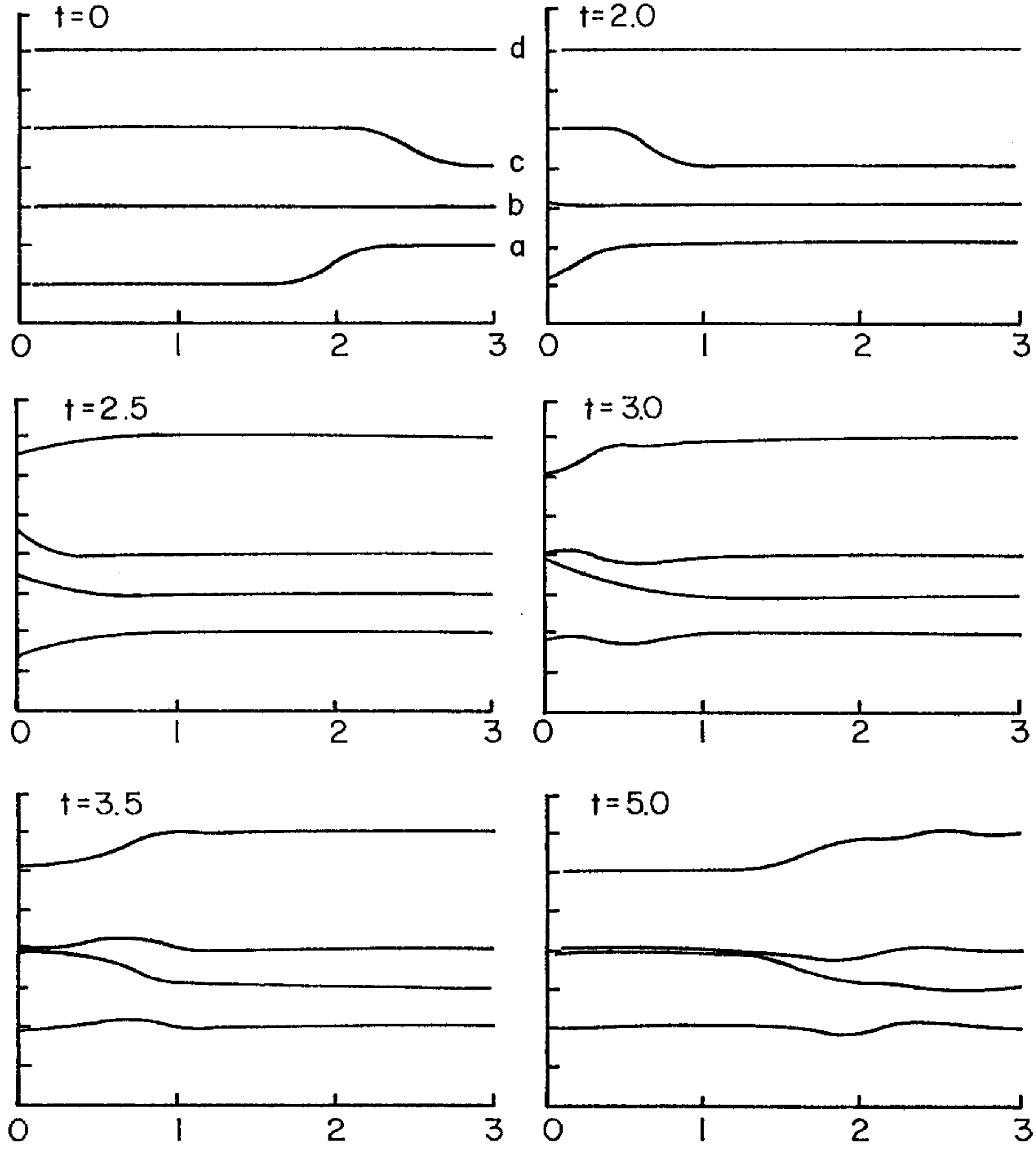

Fig. 8

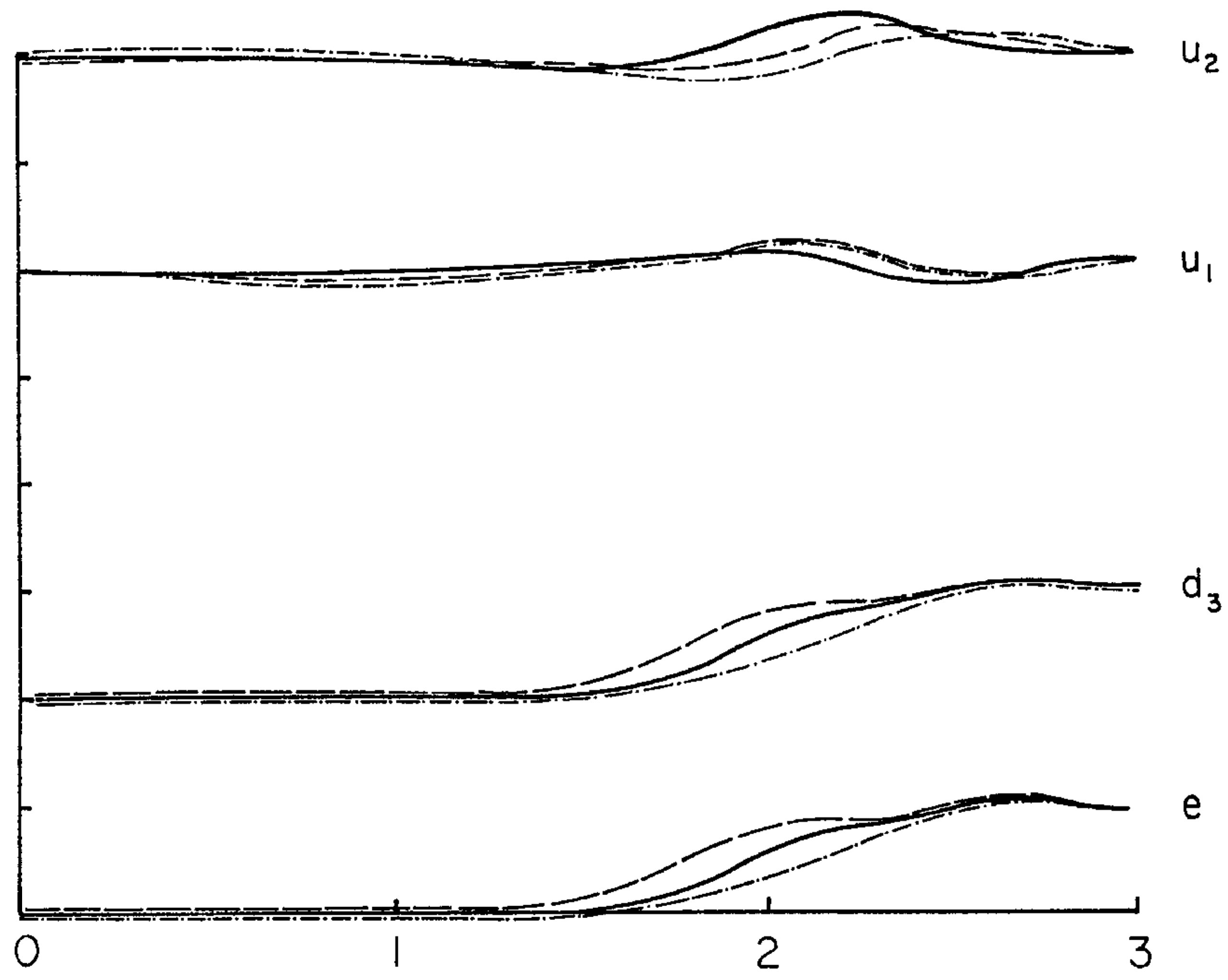

Fig. 9